axiom™

The 30 Year Horizon

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

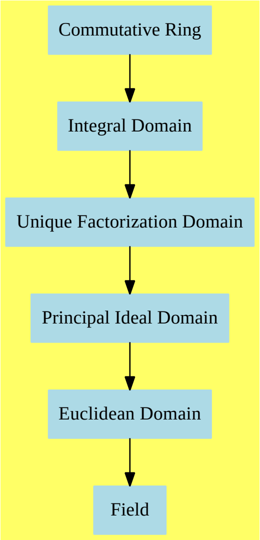
Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Chapter 1

Categories

Axiom has 3 main algebra components, Categories, Domains, and Packages. If we make an analogy to dressmaking, you can consider the Categories to be hierarchies of properties of things, like patterns, colors, or fabrics. Domains are instances of things based on category choices, such as a dress with a particular style, fabric, color, etc. Packages are tools that work with dresses such as irons, sewing machines, etc.

Axiom is based on abstract algebra and uses it as a scaffolding for constructing well-formed algebra. For instance, in abstract algebra there is a strict subset hierarchy, like:


Commutative Ring
Integral Domain
Unique Factorization Domain
Principal Ideal Domain
Euclidean Domain
Field

⟨*algebrahierarchy.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
```

```
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Commutative Ring"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
"Integral Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"Unique Factorization Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"Principal Ideal Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"Euclidean Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"Field"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];

"Commutative Ring" -> "Integral Domain"
"Integral Domain" -> "Unique Factorization Domain"
"Unique Factorization Domain" -> "Principal Ideal Domain"
"Principal Ideal Domain" -> "Euclidean Domain"
"Euclidean Domain" -> "Field"

}
```

Chapter 2

Category Layer 1

In general, we use several colors in the graph images. The “lightblue” color indicates a category that is in the direct inheritance path. The “green” (#00EE00) color indicates a category or domain used in the exports. The “seagreen” (a dark green, indicates a category or domain which is used but does not correspond to the signature of an existing category. The system can infer that this “subsumption node” matches the category. A “yellow” color indicates a domain.

2.0.1 Category (CATEGORY)

This is the root of the category hierarchy and is not represented by code.

See:

⇒ “ArcHyperbolicFunctionCategory” (AHYP) 2.0.2 on page 5
⇒ “ArcTrigonometricFunctionCategory” (ATRIG) 2.0.3 on page 8
⇒ “BasicType” (BASTYPE) 2.0.5 on page 16
⇒ “CoercibleTo” (KOERCE) 2.0.6 on page 19
⇒ “CombinatorialFunctionCategory” (CFCAT) 2.0.7 on page 22
⇒ “ConvertibleTo” (KONVERT) 2.0.8 on page 25
⇒ “ElementaryFunctionCategory” (ELEMFUN) 2.0.9 on page 29
⇒ “Eltable” (ELTAB) 2.0.10 on page 32
⇒ “FullyEvalableOver” (FEVALAB) 4.0.44 on page 178
⇒ “HyperbolicFunctionCategory” (HYPCAT) 2.0.11 on page 35
⇒ “InnerEvalable” (IEVALAB) 2.0.12 on page 39

⇒ “Logic” (LOGIC) 3.0.29 on page 110
⇒ “OpenMath” (OM) 2.0.13 on page 43
⇒ “PartialTranscendentalFunctions” (PTRANFN) 2.0.14 on page 47
⇒ “Patternable” (PATAB) 2.0.15 on page 53
⇒ “PrimitiveFunctionCategory” (PRIMCAT) 2.0.16 on page 56
⇒ “RadicalCategory” (RADCAT) 2.0.17 on page 59
⇒ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇒ “SpecialFunctionCategory” (SPFCAT) 2.0.19 on page 67
⇒ “TrigonometricFunctionCategory” (TRIGCAT) 2.0.20 on page 71
⇒ “Type” (TYPE) 2.0.21 on page 75

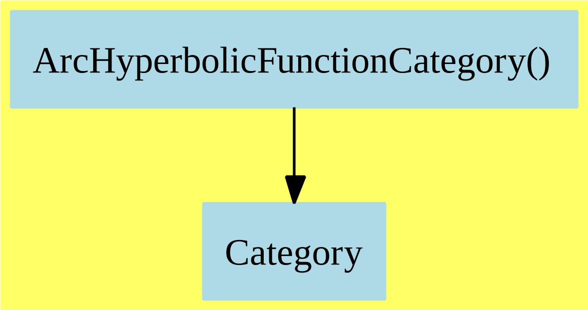
⟨*CATEGORY.dotabb*⟩≡
```
"CATEGORY"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

⟨*CATEGORY.dotfull*⟩≡
```
"Category"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

⟨*CATEGORY.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Category" [color=lightblue];
}
```

2.0.2 ArcHyperbolicFunctionCategory (AHYP)



⟨*ArcHyperbolicFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f ArcHyperbolicFunctionCategory.output
)spool ArcHyperbolicFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArcHyperbolicFunctionCategory
--R ArcHyperbolicFunctionCategory  is a category constructor
--R Abbreviation for ArcHyperbolicFunctionCategory is AHYP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AHYP
--R
--R------------------------------- Operations --------------------------------
--R acosh : % -> %                        acoth : % -> %
--R acsch : % -> %                        asech : % -> %
--R asinh : % -> %                        atanh : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ArcHyperbolicFunctionCategory.help*⟩≡

```
====================================================================
ArcHyperbolicFunctionCategory examples
====================================================================

See Also:
o )show ArcHyperbolicFunctionCategory
```

See:

⇒ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

acosh acoth acsch asech asinh atanh

These are directly exported but not implemented:

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

⟨*category AHYP ArcHyperbolicFunctionCategory*⟩≡

```
)abbrev category AHYP ArcHyperbolicFunctionCategory
++ Category for the inverse hyperbolic trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the inverse hyperbolic trigonometric functions;

ArcHyperbolicFunctionCategory(): Category == with
    acosh: $ -> $ ++ acosh(x) returns the hyperbolic arc-cosine of x.
    acoth: $ -> $ ++ acoth(x) returns the hyperbolic arc-cotangent of x.
    acsch: $ -> $ ++ acsch(x) returns the hyperbolic arc-cosecant of x.
    asech: $ -> $ ++ asech(x) returns the hyperbolic arc-secant of x.
    asinh: $ -> $ ++ asinh(x) returns the hyperbolic arc-sine of x.
    atanh: $ -> $ ++ atanh(x) returns the hyperbolic arc-tangent of x.
```

⟨*AHYP.dotabb*⟩≡

```
"AHYP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"AHYP" -> "CATEGORY"
```

⟨*AHYP.dotfull*⟩≡

```
"ArcHyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"ArcHyperbolicFunctionCategory()" -> "Category"
```

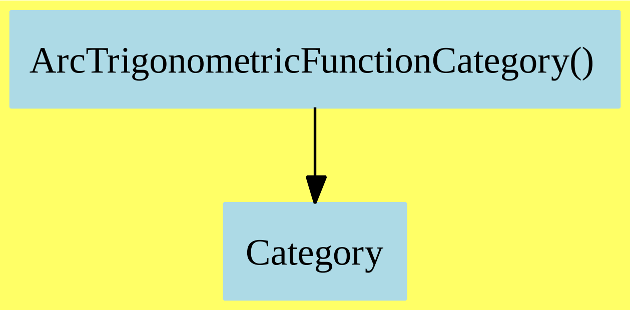
⟨*AHYP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ArcHyperbolicFunctionCategory()" [color=lightblue];
"ArcHyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.3 ArcTrigonometricFunctionCategory (ATRIG)



The `asec` and `acsc` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

⟨*ArcTrigonometricFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f ArcTrigonometricFunctionCategory.output
)spool ArcTrigonometricFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArcTrigonometricFunctionCategory
--R ArcTrigonometricFunctionCategory  is a category constructor
--R Abbreviation for ArcTrigonometricFunctionCategory is ATRIG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATRIG
--R
--R------------------------------- Operations --------------------------------
--R acos : % -> %                         acot : % -> %
--R acsc : % -> %                         asec : % -> %
--R asin : % -> %                         atan : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ArcTrigonometricFunctionCategory.help*⟩≡

```
====================================================================
ArcTrigonometricFunctionCategory examples
====================================================================

See Also:
o )show ArcTrigonometricFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.35 on page 135
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

acos acot acsc asec asin atan

These are directly exported but not implemented:

```
acos : % -> %
acot : % -> %
asin : % -> %
atan : % -> %
```

These are implemented by this category:

```
acsc : % -> %
asec : % -> %
```

⟨*category ATRIG ArcTrigonometricFunctionCategory*⟩≡

```
)abbrev category ATRIG ArcTrigonometricFunctionCategory
++ Category for the inverse trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the inverse trigonometric functions;

ArcTrigonometricFunctionCategory(): Category == with
    acos: $ -> $       ++ acos(x) returns the arc-cosine of x.
    acot: $ -> $       ++ acot(x) returns the arc-cotangent of x.
    acsc: $ -> $       ++ acsc(x) returns the arc-cosecant of x.
    asec: $ -> $       ++ asec(x) returns the arc-secant of x.
    asin: $ -> $       ++ asin(x) returns the arc-sine of x.
    atan: $ -> $       ++ atan(x) returns the arc-tangent of x.
 add
    if $ has Ring then
       asec(x) ==
         (a := recip x) case "failed" => error "asec: no reciprocal"
         acos(a::$)
       acsc(x) ==
         (a := recip x) case "failed" => error "acsc: no reciprocal"
         asin(a::$)
```

⟨*ATRIG.dotabb*⟩≡

```
"ATRIG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
"ATRIG" -> "CATEGORY"
```

⟨*ATRIG.dotfull*⟩≡

```
"ArcTrigonometricFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
"ArcTrigonometricFunctionCategory()" -> "Category"
```

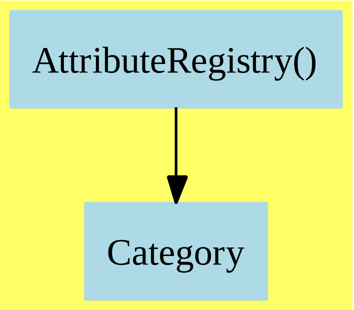
⟨*ATRIG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ArcTrigonometricFunctionCategory()" [color=lightblue];
"ArcTrigonometricFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.4 AttributeRegistry (ATTREG)



See:

⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports: Nothing

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **arbitraryPrecision** means the user can set the precision for subsequent calculations.

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **central** is true if, given an algebra over a ring R, the image of R is the center of the algebra, i.e. the set of members of the algebra which commute with all others is precisely the image of R in the algebra.
- **noetherian** is true if all of its ideals are finitely generated.
- **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **canonical** is true if and only if distinct elements have distinct data structures. For example, a domain of mathematical objects which has the `canonical` attribute means that two objects are mathematically equal if and only if their data structures are equal.
- **approximate** means "is an approximation to the real numbers".
- **complex** means that this domain has $\sqrt{-1}$

⟨*category ATTREG AttributeRegistry*⟩≡

```
)abbrev category ATTREG AttributeRegistry
++ Description:
++ This category exports the attributes in the AXIOM Library

AttributeRegistry(): Category == with
  finiteAggregate
    ++ \spad{finiteAggregate} is true if it is an aggregate with a
    ++ finite number of elements.
  commutative("*")
    ++ \spad{commutative("*")} is true if it has an operation
    ++ \spad{"*": (D,D) -> D} which is commutative.
  shallowlyMutable
    ++ \spad{shallowlyMutable} is true if its values
    ++ have immediate components that are updateable (mutable).
    ++ Note that the properties of any component domain are irrevelant
    ++ to the \spad{shallowlyMutable} proper.
  unitsKnown
    ++ \spad{unitsKnown} is true if a monoid (a multiplicative semigroup
    ++ with a 1) has \spad{unitsKnown} means that
  ++ the operation \spadfun{recip} can only return "failed"
  ++ if its argument is not a unit.
leftUnitary
  ++ \spad{leftUnitary} is true if \spad{1 * x = x} for all x.
rightUnitary
  ++ \spad{rightUnitary} is true if \spad{x * 1 = x} for all x.
noZeroDivisors
  ++ \spad{noZeroDivisors} is true if \spad{x * y \~~= 0} implies
  ++ both x and y are non-zero.
canonicalUnitNormal
  ++ \spad{canonicalUnitNormal} is true if we can choose a canonical
  ++ representative for each class of associate elements, that is
  ++ \spad{associates?(a,b)} returns true if and only if
  ++ \spad{unitCanonical(a) = unitCanonical(b)}.
canonicalsClosed
  ++ \spad{canonicalsClosed} is true if
  ++ \spad{unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)}.
arbitraryPrecision
  ++ \spad{arbitraryPrecision} means the user can set the
  ++ precision for subsequent calculations.
partiallyOrderedSet
  ++ \spad{partiallyOrderedSet} is true if
  ++ a set with \spadop{<} which is transitive,
  ++ but \spad{not(a < b or a = b)}
  ++ does not necessarily imply \spad{b<a}.
central
  ++ \spad{central} is true if, given an algebra over a ring R,
  ++ the image of R is the center
  ++ of the algebra, i.e. the set of members of the algebra which commute
  ++ with all others is precisely the image of R in the algebra.
noetherian
  ++ \spad{noetherian} is true if all of its ideals are finitely generated.
additiveValuation
  ++ \spad{additiveValuation} implies
  ++ \spad{euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)}.
multiplicativeValuation
  ++ \spad{multiplicativeValuation} implies
  ++ \spad{euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)}.
NullSquare
  ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
  ++ See \axiomType{LieAlgebra}.
JacobiIdentity
  ++ \axiom{JacobiIdentity} means that
  ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
  ++ See \axiomType{LieAlgebra}.
canonical
```

```
    ++ \spad{canonical} is true if and only if distinct elements have
    ++ distinct data structures. For example, a domain of mathematical
    ++ objects  which has the \spad{canonical} attribute means that two
    ++ objects are mathematically equal if and only if their data
    ++ structures are equal.
  approximate
    ++ \spad{approximate} means "is an approximation to the real numbers".
```

⟨*ATTREG.dotabb*⟩≡

```
"ATTREG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
"ATTREG" -> "CATEGORY"
```

⟨*ATTREG.dotfull*⟩≡

```
"AttributeRegistry()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
"AttributeRegistry()" -> "Category"
```

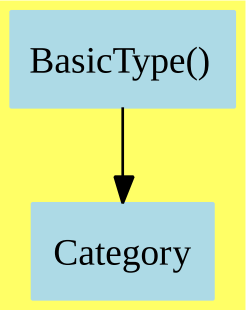
⟨*ATTREG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AttributeRegistry()" [color=lightblue];
"AttributeRegistry()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.5 BasicType (BASTYPE)



⟨*BasicType.input*⟩≡
```
)set break resume
)sys rm -f BasicType.output
)spool BasicType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BasicType
--R BasicType  is a category constructor
--R Abbreviation for BasicType is BASTYPE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BASTYPE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*BasicType.help*⟩≡
```
====================================================================
BasicType examples
====================================================================

See Also:
o )show BasicType
```

See:

⇒ "BlowUpMethodCategory" (BLMETCT) 4.0.37 on page 146
⇒ "SetCategory" (SETCAT) 3.0.34 on page 130
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?=? ?~=?

These are directly exported but not implemented:

```
?=? : (%,%) -> Boolean
```

These are implemented by this category:

```
?~=? : (%,%) -> Boolean
```

⟨*category BASTYPE BasicType*⟩≡

```
)abbrev category BASTYPE BasicType
--% BasicType
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{BasicType} is the basic category for describing a collection
++ of elements with \spadop{=} (equality).

BasicType(): Category == with
      "=": (%,%) -> Boolean    ++ x=y tests if x and y are equal.
      "~=": (%,%) -> Boolean   ++ x~=y tests if x and y are not equal.
   add
      _~_=(x:%,y:%) : Boolean == not(x=y)
```

⟨*BASTYPE.dotabb*⟩≡

```
"BASTYPE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
"BASTYPE" -> "CATEGORY"
```

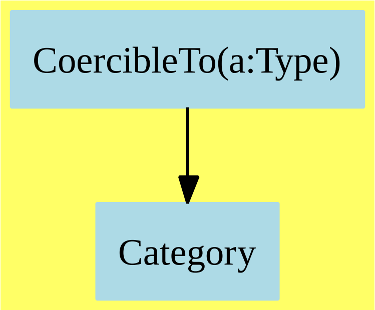
⟨*BASTYPE.dotfull*⟩≡
```
"BasicType()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
"BasicType()" -> "Category"
```

⟨*BASTYPE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.6 CoercibleTo (KOERCE)



⟨*CoercibleTo.input*⟩≡
```
)set break resume
)sys rm -f CoercibleTo.output
)spool CoercibleTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CoercibleTo
--R CoercibleTo S: Type  is a category constructor
--R Abbreviation for CoercibleTo is KOERCE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KOERCE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> S
--R
--E 1

)spool
)lisp (bye)
```

⟨*CoercibleTo.help*⟩≡
```
====================================================================
CoercibleTo examples
====================================================================

See Also:
o )show CoercibleTo
```

See:

⇒ “BlowUpMethodCategory” (BLMETCT) 4.0.37 on page 146
⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “FortranProgramCategory” (FORTCAT) 3.0.26 on page 96
⇒ “PlottablePlaneCurveCategory” (PPCURVE) 3.0.30 on page 114
⇒ “PlottableSpaceCurveCategory” (PSCURVE) 3.0.31 on page 118
⇒ “PolynomialSetCategory” (PSETCAT) 6.0.91 on page 518
⇒ “SetCategory” (SETCAT) 3.0.34 on page 130
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

coerce

This is directly exported but not implemented:

```
coerce : % -> S
```

⟨*category KOERCE CoercibleTo*⟩≡

```
)abbrev category KOERCE CoercibleTo
++ Category for coerce
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is coercible to B means any element of A can automatically be
++ converted into an element of B by the interpreter.

CoercibleTo(S:Type): Category == with
    coerce: % -> S
      ++ coerce(a) transforms a into an element of S.
```

⟨*KOERCE.dotabb*⟩≡

```
"KOERCE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"KOERCE" -> "CATEGORY"
```

⟨*KOERCE.dotfull*⟩≡
```
"CoercibleTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(a:Type)" -> "Category"

"CoercibleTo(OutputForm)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

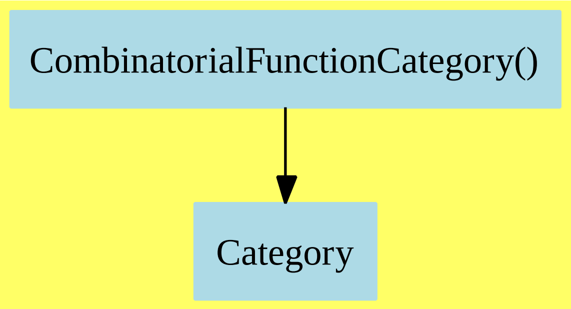
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
  -> "CoercibleTo(a:Type)"
```

⟨*KOERCE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.7 CombinatorialFunctionCategory (CFCAT)



⟨*CombinatorialFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f CombinatorialFunctionCategory.output
)spool CombinatorialFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CombinatorialFunctionCategory
--R CombinatorialFunctionCategory  is a category constructor
--R Abbreviation for CombinatorialFunctionCategory is CFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CFCAT
--R
--R------------------------------- Operations --------------------------------
--R binomial : (%,%) -> %                 factorial : % -> %
--R permutation : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*CombinatorialFunctionCategory.help*⟩≡

```
====================================================================
CombinatorialFunctionCategory examples
====================================================================

See Also:
o )show CombinatorialFunctionCategory
```

See:

⇒ "CombinatorialOpsCategory" (COMBOPC) 3.0.23 on page 82
⇒ "IntegerNumberSystem" (INS) 16.0.186 on page 1355
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

binomial factorial permutation

These are directly exported but not implemented:

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

⟨*category CFCAT CombinatorialFunctionCategory*⟩≡

```
)abbrev category CFCAT CombinatorialFunctionCategory
++ Category for the usual combinatorial functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the usual combinatorial functions;

CombinatorialFunctionCategory(): Category == with
    binomial   : ($, $) -> $
      ++ binomial(n,r) returns the \spad{(n,r)} binomial coefficient
      ++ (often denoted in the literature by \spad{C(n,r)}).
      ++ Note that \spad{C(n,r) = n!/(r!(n-r)!)} where \spad{n >= r >= 0}.
      ++
      ++X [binomial(5,i) for i in 0..5]
    factorial  : $ -> $
      ++ factorial(n) computes the factorial of n
      ++ (denoted in the literature by \spad{n!})
      ++ Note that \spad{n! = n (n-1)! when n > 0}; also, \spad{0! = 1}.
    permutation: ($, $) -> $
      ++ permutation(n, m) returns the number of
      ++ permutations of n objects taken m at a time.
      ++ Note that \spad{permutation(n,m) = n!/(n-m)!}.
```

⟨*CFCAT.dotabb*⟩≡

```
"CFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
"CFCAT" -> "CATEGORY"
```

⟨*CFCAT.dotfull*⟩≡

```
"CombinatorialFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
"CombinatorialFunctionCategory()" -> "Category"
```

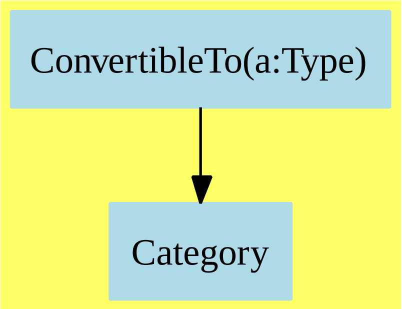
⟨*CFCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CombinatorialFunctionCategory()" [color=lightblue];
"CombinatorialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.8 ConvertibleTo (KONVERT)



⟨*ConvertibleTo.input*⟩≡
```
)set break resume
)sys rm -f ConvertibleTo.output
)spool ConvertibleTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ConvertibleTo
--R ConvertibleTo S: Type  is a category constructor
--R Abbreviation for ConvertibleTo is KONVERT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KONVERT
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> S
--R
--E 1

)spool
)lisp (bye)
```

⟨*ConvertibleTo.help*⟩≡
```
====================================================================
ConvertibleTo examples
====================================================================

See Also:
o )show ConvertibleTo
```

See:

⇒ “Collection” (CLAGG) 5.0.70 on page 322
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “RealConstant” (REAL) 3.0.32 on page 122
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

convert

This is directly exported but not implemented:

```
 convert : % -> S
```

⟨*category KONVERT ConvertibleTo*⟩≡

```
)abbrev category KONVERT ConvertibleTo
++ Category for convert
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is convertible to B means any element of A
++ can be converted into an element of B,
++ but not automatically by the interpreter.

ConvertibleTo(S:Type): Category == with
    convert: % -> S
      ++ convert(a) transforms a into an element of S.
```

⟨*KONVERT.dotabb*⟩≡

```
"KONVERT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
"KONVERT" -> "CATEGORY"
```

⟨*KONVERT.dotfull*⟩≡

```
"ConvertibleTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(a:Type)" -> "Category"

"ConvertibleTo(DoubleFloat)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(InputForm)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Integer)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Pattern(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Integer))" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Pattern(Float))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Float))" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Complex(Float))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Complex(Float))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Complex(DoubleFloat))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Complex(DoubleFloat))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(String)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(String)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Symbol)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(SExpression)"
```

```
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(SExpression)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Pattern(Base))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Base))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(List(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(List(Integer))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(List(Character))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(List(Character))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))" ->
    "ConvertibleTo(a:Type)"
```

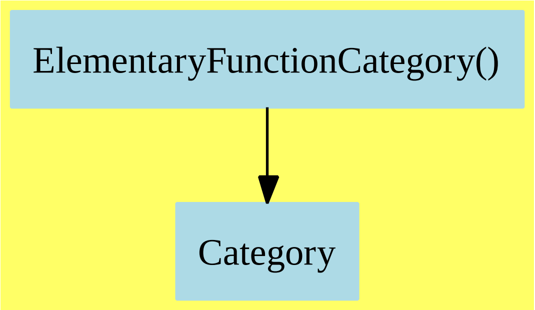
⟨*KONVERT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ConvertibleTo(a:Type)" [color=lightblue];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.9 ElementaryFunctionCategory (ELEMFUN)



⟨*ElementaryFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f ElementaryFunctionCategory.output
)spool ElementaryFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ElementaryFunctionCategory
--R ElementaryFunctionCategory  is a category constructor
--R Abbreviation for ElementaryFunctionCategory is ELEMFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELEMFUN
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                     exp : % -> %
--R log : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ElementaryFunctionCategory.help*⟩≡

```
====================================================================
ElementaryFunctionCategory examples
====================================================================

See Also:
o )show ElementaryFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.35 on page 135
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?**? exp log

These are directly exported but not implemented:

```
exp : % -> %
log : % -> %
```

These are implemented by this category:

```
?**? : (%,%) -> %
```

⟨*category ELEMFUN ElementaryFunctionCategory*⟩≡

```
)abbrev category ELEMFUN ElementaryFunctionCategory
++ Category for the elementary functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the elementary functions;

ElementaryFunctionCategory(): Category == with
    log : $ -> $       ++ log(x) returns the natural logarithm of x.
    exp : $ -> $       ++ exp(x) returns %e to the power x.
    "**": ($, $) -> $  ++ x**y returns x to the power y.
 add
   if $ has Monoid then
     x ** y == exp(y * log x)
```

⟨*ELEMFUN.dotabb*⟩≡

```
"ELEMFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ELEMFUN" -> "CATEGORY"
```

⟨*ELEMFUN.dotfull*⟩≡

```
"ElementaryFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ElementaryFunctionCategory()" -> "Category"
```

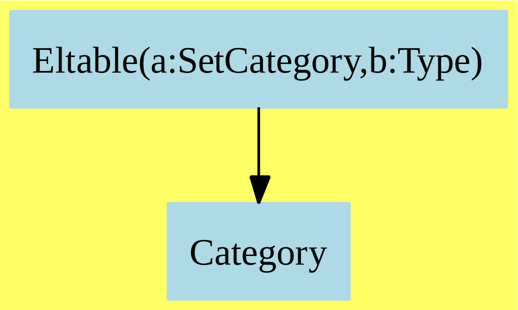
⟨*ELEMFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ElementaryFunctionCategory()" [color=lightblue];
"ElementaryFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.10 Eltable (ELTAB)



⟨*Eltable.input*⟩≡
```
)set break resume
)sys rm -f Eltable.output
)spool Eltable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Eltable
--R Eltable(S: SetCategory,Index: Type)  is a category constructor
--R Abbreviation for Eltable is ELTAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELTAB
--R
--R------------------------------- Operations --------------------------------
--R ?.? : (%,S) -> Index
--R
--E 1

)spool
)lisp (bye)
```

⟨*Eltable.help*⟩≡
```
====================================================================
Eltable examples
====================================================================

See Also:
o )show Eltable
```

See:

⇒ "EltableAggregate" (ELTAGG) 3.0.24 on page 86
⇒ "LinearOrdinaryDifferentialOperatorCategory" (LODOCAT) 11.0.156 on page 1072

⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.204 on page 1620
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?.?

This syntax for elt is supported by the interpreter and compiler.

This is directly exported but not implemented:

```
 ?.? : (%,S) -> Index
```

⟨*category ELTAB Eltable*⟩≡

```
)abbrev category ELTAB Eltable
++ Author: Michael Monagan; revised by Manuel Bronstein and Manuel Bronstein
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An eltable over domains D and I is a structure which can be viewed
++ as a function from D to I.
++ Examples of eltable structures range from data structures, e.g. those
++ of type \spadtype{List}, to algebraic structures like
++ \spadtype{Polynomial}.

Eltable(S:SetCategory, Index:Type): Category == with
  elt : (%, S) -> Index
     ++ elt(u,i) (also written: u . i) returns the element of u indexed by i.
     ++ Error: if i is not an index of u.
```

⟨*ELTAB.dotabb*⟩≡

```
"ELTAB" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
"ELTAB" -> "CATEGORY"
```

⟨*ELTAB.dotfull*⟩≡

```
"Eltable(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:Ring,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:Ring,b:Ring)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:SetCategory)" ->
   "Eltable(a:SetCategory,b:Type)"
```

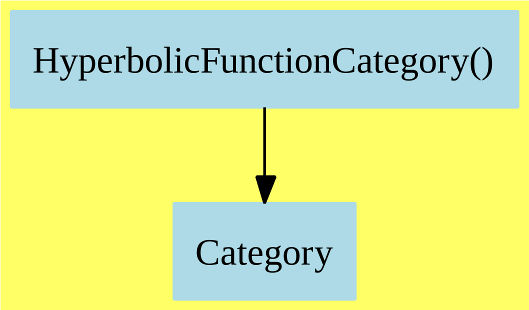
⟨*ELTAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.11 HyperbolicFunctionCategory (HYPCAT)



The `csch` and `sech` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

⟨*HyperbolicFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f HyperbolicFunctionCategory.output
)spool HyperbolicFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HyperbolicFunctionCategory
--R HyperbolicFunctionCategory  is a category constructor
--R Abbreviation for HyperbolicFunctionCategory is HYPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for HYPCAT
--R
--R------------------------------- Operations --------------------------------
--R cosh : % -> %                        coth : % -> %
--R csch : % -> %                        sech : % -> %
--R sinh : % -> %                        tanh : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*HyperbolicFunctionCategory.help*⟩≡

```
====================================================================
HyperbolicFunctionCategory examples
====================================================================

See Also:
o )show HyperbolicFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.35 on page 135
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

cosh	coth	csch	sech	sinh	tanh

These are implemented by this category:

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

⟨*category HYPCAT HyperbolicFunctionCategory*⟩≡

```
)abbrev category HYPCAT HyperbolicFunctionCategory
++ Category for the hyperbolic trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the hyperbolic trigonometric functions;

HyperbolicFunctionCategory(): Category == with
    cosh: $ -> $       ++ cosh(x) returns the hyperbolic cosine of x.
    coth: $ -> $       ++ coth(x) returns the hyperbolic cotangent of x.
    csch: $ -> $       ++ csch(x) returns the hyperbolic cosecant of x.
    sech: $ -> $       ++ sech(x) returns the hyperbolic secant of x.
    sinh: $ -> $       ++ sinh(x) returns the hyperbolic sine of x.
    tanh: $ -> $       ++ tanh(x) returns the hyperbolic tangent of x.
 add
    if $ has Ring then
       csch x ==
         (a := recip(sinh x)) case "failed" => error "csch: no reciprocal"
         a::$
       sech x ==
         (a := recip(cosh x)) case "failed" => error "sech: no reciprocal"
         a::$
       tanh x == sinh x * sech x
       coth x == cosh x * csch x
       if $ has ElementaryFunctionCategory then
         cosh x ==
           e := exp x
           (e + recip(e)::$) * recip(2::$)::$
```

```
          sinh(x):$ ==
            e := exp x
            (e - recip(e)::$) * recip(2::$)::$
```

⟨*HYPCAT.dotabb*⟩≡

```
"HYPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HYPCAT" -> "CATEGORY"
```

⟨*HYPCAT.dotfull*⟩≡

```
"HyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HyperbolicFunctionCategory()" -> "Category"
```

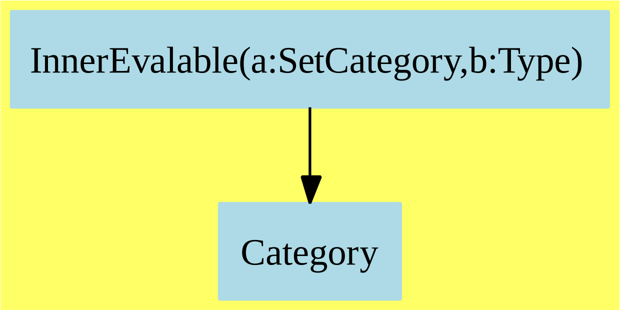
⟨*HYPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"HyperbolicFunctionCategory()" [color=lightblue];
"HyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.12 InnerEvalable (IEVALAB)



⟨*InnerEvalable.input*⟩≡
```
)set break resume
)sys rm -f InnerEvalable.output
)spool InnerEvalable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerEvalable
--R InnerEvalable(A: SetCategory,B: Type)  is a category constructor
--R Abbreviation for InnerEvalable is IEVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IEVALAB
--R
--R------------------------------- Operations --------------------------------
--R eval : (%,A,B) -> %                  eval : (%,List A,List B) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*InnerEvalable.help*⟩≡
```
====================================================================
InnerEvalable examples
====================================================================

See Also:
o )show InnerEvalable
```

See:

⇒ “DesingTreeCategory” (DSTRCAT) 4.0.38 on page 150
⇒ “Evalable” (EVALAB) 3.0.25 on page 91
⇒ “ExpressionSpace” (ES) 5.0.72 on page 337
⇒ “MultivariateTaylorSeriesCategory” (MTSCAT) 15.0.182 on page 1316
⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

eval

These are directly exported but not implemented:

```
eval : (%,A,B) -> %
```

These are implemented by this category:

```
eval : (%,List A,List B) -> %
```

⟨*category IEVALAB InnerEvalable*⟩≡

```
)abbrev category IEVALAB InnerEvalable
-- FOR THE BENEFIT OF LIBAXO GENERATION
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See: Evalable
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This category provides \spadfun{eval} operations.
++ A domain may belong to this category if it is possible to make
++ ``evaluation'' substitutions.  The difference between this
++ and \spadtype{Evalable} is that the operations in this category
++ specify the substitution as a pair of arguments rather than as
++ an equation.

InnerEvalable(A:SetCategory, B:Type): Category == with
    eval: ($, A, B) -> $
       ++ eval(f, x, v) replaces x by v in f.
    eval: ($, List A, List B) -> $
       ++ eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.
```

```
 add
    eval(f:$, x:A, v:B) == eval(f, [x], [v])
```

⟨*IEVALAB.dotabb*⟩≡

```
"IEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"IEVALAB" -> "CATEGORY"
```

⟨*IEVALAB.dotfull*⟩≡

```
"InnerEvalable(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"InnerEvalable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:OrderedSet,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:OrderedSet,b:Ring)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedS
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedS
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
     -> "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
    "InnerEvalable(a:SetCategory,b:Type)"
```

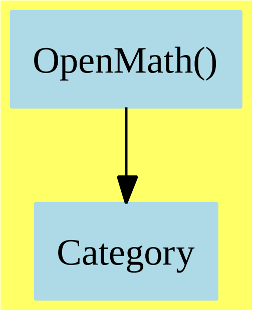
⟨*IEVALAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.13 OpenMath (OM)



⟨*OpenMath.input*⟩≡
```
)set break resume
)sys rm -f OpenMath.output
)spool OpenMath.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMath
--R OpenMath  is a category constructor
--R Abbreviation for OpenMath is OM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OM
--R
--R------------------------------- Operations --------------------------------
--R OMwrite : % -> String                 OMwrite : (%,Boolean) -> String
--R OMwrite : (OpenMathDevice,%) -> Void
--R OMwrite : (OpenMathDevice,%,Boolean) -> Void
--R
--E 1

)spool
)lisp (bye)
```

⟨*OpenMath.help*⟩≡

```
====================================================================
OpenMath examples
====================================================================

See Also:
o )show OpenMath
```

See:

⇒ “StringCategory” (STRICAT) 10.0.149 on page 1014
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

OMwrite

These are directly exported but not implemented:

```
OMwrite : % -> String
OMwrite : (%,Boolean) -> String
OMwrite : (OpenMathDevice,%) -> Void
OMwrite : (OpenMathDevice,%,Boolean) -> Void
```

⟨*category OM OpenMath*⟩≡

```
)abbrev category OM OpenMath
++ Author: Mike Dewar & Vilya Harvey
++ Basic Functions: OMwrite
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMath} provides operations for exporting an object
++ in OpenMath format.

OpenMath(): Category == with
  OMwrite  : % -> String
  ++ OMwrite(u) returns the OpenMath XML encoding of \axiom{u} as a
  ++ complete OpenMath object.
  OMwrite  : (%, Boolean) -> String
  ++ OMwrite(u, true) returns the OpenMath XML encoding of \axiom{u}
  ++ as a complete OpenMath object; OMwrite(u, false) returns the
  ++ OpenMath XML encoding of \axiom{u} as an OpenMath fragment.
  OMwrite  : (OpenMathDevice, %) -> Void
  ++ OMwrite(dev, u) writes the OpenMath form of \axiom{u} to the
  ++ OpenMath device \axiom{dev} as a complete OpenMath object.
  OMwrite  : (OpenMathDevice, %, Boolean) -> Void
  ++ OMwrite(dev, u, true) writes the OpenMath form of \axiom{u} to
  ++ the OpenMath device \axiom{dev} as a complete OpenMath object;
  ++ OMwrite(dev, u, false) writes the object as an OpenMath fragment.
```

⟨*OM.dotabb*⟩≡

```
"OM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
"OM" -> "CATEGORY"
```

⟨*OM.dotfull*⟩≡

```
"OpenMath()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
"OpenMath()" -> "Category"
```

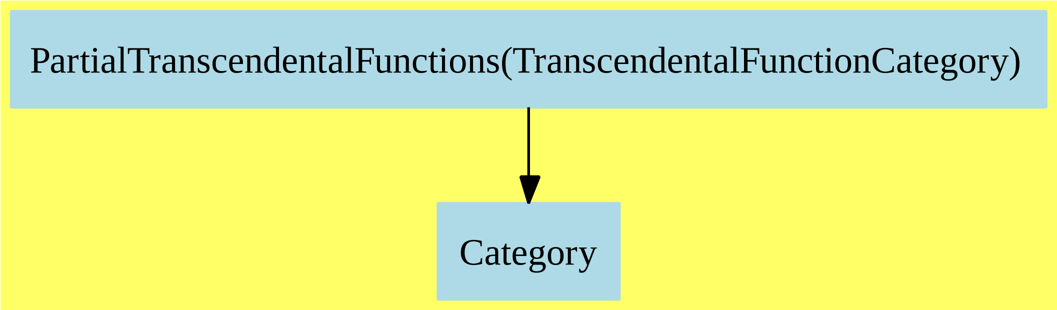
⟨*OM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OpenMath()" [color=lightblue];
"OpenMath()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.14 PartialTranscendentalFunctions (PTRANFN)


PartialTranscendentalFunctions(TranscendentalFunctionCategory)
Category

⟨*PartialTranscendentalFunctions.input*⟩≡

```
)set break resume
)sys rm -f PartialTranscendentalFunctions.output
)spool PartialTranscendentalFunctions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PartialTranscendentalFunctions
--R PartialTranscendentalFunctions K: TranscendentalFunctionCategory  is a category construc
--R Abbreviation for PartialTranscendentalFunctions is PTRANFN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PTRANFN
--R
--R------------------------------- Operations --------------------------------
--R acosIfCan : K -> Union(K,"failed")    acotIfCan : K -> Union(K,"failed")
--R acscIfCan : K -> Union(K,"failed")    asecIfCan : K -> Union(K,"failed")
--R asinIfCan : K -> Union(K,"failed")    atanIfCan : K -> Union(K,"failed")
--R cosIfCan : K -> Union(K,"failed")     coshIfCan : K -> Union(K,"failed")
--R cotIfCan : K -> Union(K,"failed")     cothIfCan : K -> Union(K,"failed")
--R cscIfCan : K -> Union(K,"failed")     cschIfCan : K -> Union(K,"failed")
--R expIfCan : K -> Union(K,"failed")     logIfCan : K -> Union(K,"failed")
--R secIfCan : K -> Union(K,"failed")     sechIfCan : K -> Union(K,"failed")
--R sinIfCan : K -> Union(K,"failed")     sinhIfCan : K -> Union(K,"failed")
--R tanIfCan : K -> Union(K,"failed")     tanhIfCan : K -> Union(K,"failed")
--R acoshIfCan : K -> Union(K,"failed")
--R acothIfCan : K -> Union(K,"failed")
--R acschIfCan : K -> Union(K,"failed")
--R asechIfCan : K -> Union(K,"failed")
--R asinhIfCan : K -> Union(K,"failed")
--R atanhIfCan : K -> Union(K,"failed")
```

```
--R nthRootIfCan : (K,NonNegativeInteger) -> Union(K,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*PartialTranscendentalFunctions.help*⟩≡

```
====================================================================
PartialTranscendentalFunctions examples
====================================================================

See Also:
o )show PartialTranscendentalFunctions
```

See:

⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

acosIfCan
acschIfCan
atanIfCan
cothIfCan
nthRootIfCan
tanIfCan
acoshIfCan
asecIfCan
atanhIfCan
cscIfCan
secIfCan
tanhIfCan
acotIfCan
asechIfCan
cosIfCan
cschIfCan
sechIfCan
acothIfCan
asinIfCan
coshIfCan
expIfCan
sinIfCan
acscIfCan
asinhIfCan
cotIfCan
logIfCan
sinhIfCan

These are directly exported but not implemented:

```
acosIfCan : K -> Union(K,"failed")
acoshIfCan : K -> Union(K,"failed")
acotIfCan : K -> Union(K,"failed")
acothIfCan : K -> Union(K,"failed")
acscIfCan : K -> Union(K,"failed")
acschIfCan : K -> Union(K,"failed")
asecIfCan : K -> Union(K,"failed")
asechIfCan : K -> Union(K,"failed")
asinIfCan : K -> Union(K,"failed")
asinhIfCan : K -> Union(K,"failed")
atanIfCan : K -> Union(K,"failed")
atanhIfCan : K -> Union(K,"failed")
cosIfCan : K -> Union(K,"failed")
coshIfCan : K -> Union(K,"failed")
cotIfCan : K -> Union(K,"failed")
cothIfCan : K -> Union(K,"failed")
cscIfCan : K -> Union(K,"failed")
cschIfCan : K -> Union(K,"failed")
expIfCan : K -> Union(K,"failed")
logIfCan : K -> Union(K,"failed")
nthRootIfCan : (K,NonNegativeInteger) -> Union(K,"failed")
secIfCan : K -> Union(K,"failed")
sechIfCan : K -> Union(K,"failed")
sinIfCan : K -> Union(K,"failed")
sinhIfCan : K -> Union(K,"failed")
tanIfCan : K -> Union(K,"failed")
tanhIfCan : K -> Union(K,"failed")
```

⟨*category PTRANFN PartialTranscendentalFunctions*⟩≡

```
)abbrev category PTRANFN PartialTranscendentalFunctions
++ Description of a package which provides partial transcendental
++ functions, i.e. functions which return an answer or "failed"
++ Author: Clifton J. Williamson
++ Date Created: 12 February 1990
++ Date Last Updated: 14 February 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This is the description of any package which provides partial
++ functions on a domain belonging to TranscendentalFunctionCategory.

PartialTranscendentalFunctions(K): Category == Definition where
  K :     TranscendentalFunctionCategory
  NNI ==> NonNegativeInteger

  Definition ==> with

--% Exponentials and Logarithms

    nthRootIfCan: (K,NNI) -> Union(K,"failed")
      ++ nthRootIfCan(z,n) returns the nth root of z if possible,
      ++ and "failed" otherwise.
    expIfCan: K -> Union(K,"failed")
      ++ expIfCan(z) returns exp(z) if possible, and "failed" otherwise.
    logIfCan: K -> Union(K,"failed")
      ++ logIfCan(z) returns log(z) if possible, and "failed" otherwise.

--% TrigonometricFunctionCategory

    sinIfCan : K -> Union(K,"failed")
      ++ sinIfCan(z) returns sin(z) if possible, and "failed" otherwise.
    cosIfCan: K -> Union(K,"failed")
      ++ cosIfCan(z) returns cos(z) if possible, and "failed" otherwise.
    tanIfCan: K -> Union(K,"failed")
      ++ tanIfCan(z) returns tan(z) if possible, and "failed" otherwise.
    cotIfCan: K -> Union(K,"failed")
      ++ cotIfCan(z) returns cot(z) if possible, and "failed" otherwise.
    secIfCan: K -> Union(K,"failed")
      ++ secIfCan(z) returns sec(z) if possible, and "failed" otherwise.
    cscIfCan: K -> Union(K,"failed")
      ++ cscIfCan(z) returns csc(z) if possible, and "failed" otherwise.

--% ArcTrigonometricFunctionCategory

    asinIfCan: K -> Union(K,"failed")
      ++ asinIfCan(z) returns asin(z) if possible, and "failed" otherwise.
    acosIfCan: K -> Union(K,"failed")
      ++ acosIfCan(z) returns acos(z) if possible, and "failed" otherwise.
    atanIfCan: K -> Union(K,"failed")
      ++ atanIfCan(z) returns atan(z) if possible, and "failed" otherwise.
    acotIfCan: K -> Union(K,"failed")
      ++ acotIfCan(z) returns acot(z) if possible, and "failed" otherwise.
    asecIfCan: K -> Union(K,"failed")
      ++ asecIfCan(z) returns asec(z) if possible, and "failed" otherwise.
    acscIfCan: K -> Union(K,"failed")
      ++ acscIfCan(z) returns acsc(z) if possible, and "failed" otherwise.

--% HyperbolicFunctionCategory

    sinhIfCan: K -> Union(K,"failed")
      ++ sinhIfCan(z) returns sinh(z) if possible, and "failed" otherwise.
    coshIfCan: K -> Union(K,"failed")
      ++ coshIfCan(z) returns cosh(z) if possible, and "failed" otherwise.
    tanhIfCan: K -> Union(K,"failed")
      ++ tanhIfCan(z) returns tanh(z) if possible, and "failed" otherwise.
    cothIfCan: K -> Union(K,"failed")
      ++ cothIfCan(z) returns coth(z) if possible, and "failed" otherwise.
    sechIfCan: K -> Union(K,"failed")
      ++ sechIfCan(z) returns sech(z) if possible, and "failed" otherwise.
    cschIfCan: K -> Union(K,"failed")
      ++ cschIfCan(z) returns csch(z) if possible, and "failed" otherwise.

--% ArcHyperbolicFunctionCategory

    asinhIfCan: K -> Union(K,"failed")
      ++ asinhIfCan(z) returns asinh(z) if possible, and "failed" otherwise.
    acoshIfCan: K -> Union(K,"failed")
      ++ acoshIfCan(z) returns acosh(z) if possible, and "failed" otherwise.
    atanhIfCan: K -> Union(K,"failed")
      ++ atanhIfCan(z) returns atanh(z) if possible, and "failed" otherwise.
    acothIfCan: K -> Union(K,"failed")
      ++ acothIfCan(z) returns acoth(z) if possible, and "failed" otherwise.
    asechIfCan: K -> Union(K,"failed")
      ++ asechIfCan(z) returns asech(z) if possible, and "failed" otherwise.
    acschIfCan: K -> Union(K,"failed")
      ++ acschIfCan(z) returns acsch(z) if possible, and "failed" otherwise.
```

⟨*PTRANFN.dotabb*⟩≡

```
"PTRANFN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PTRANFN" -> "CATEGORY"
```

⟨*PTRANFN.dotfull*⟩≡

```
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category()"
```

⟨*PTRANFN.dotpic*⟩≡

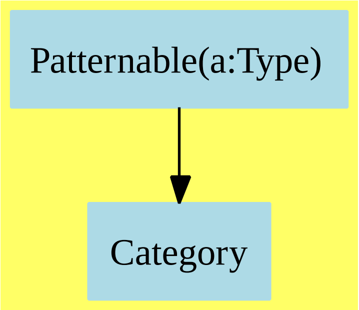
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category"

"Category" [color=lightblue];

}
```

2.0.15 Patternable (PATAB)



⟨*Patternable.input*⟩≡
```
)set break resume
)sys rm -f Patternable.output
)spool Patternable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Patternable
--R Patternable R: Type  is a category constructor
--R Abbreviation for Patternable is PATAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PATAB
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*Patternable.help*⟩≡
```
====================================================================
Patternable examples
====================================================================

See Also:
o )show Patternable
```

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.216 on page 1779
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "IntegerNumberSystem" (INS) 16.0.186 on page 1355
⇒ "QuotientFieldCategory" (QFCAT) 17.0.198 on page 1516
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

convert

These exports come from (p25) ConvertibleTo(Pattern(Integer)):

```
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
```

These exports come from (p25) ConvertibleTo(Pattern(Float)):

```
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
```

⟨*category PATAB Patternable*⟩≡

```
)abbrev category PATAB Patternable
++ Category of sets that can be converted to useful patterns
++ Author: Manuel Bronstein
++ Date Created: 29 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Keywords: pattern, matching.
++ Description:
++ An object S is Patternable over an object R if S can
++ lift the conversions from R into \spadtype{Pattern(Integer)} and
++ \spadtype{Pattern(Float)} to itself;

Patternable(R:Type): Category == with
  if R has ConvertibleTo Pattern Integer then
           ConvertibleTo Pattern Integer
  if R has ConvertibleTo Pattern Float then
           ConvertibleTo Pattern Float
```

⟨*PATAB.dotabb*⟩≡

```
"PATAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"PATAB" -> "CATEGORY"
```

⟨*PATAB.dotfull*⟩≡

```
"Patternable(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(a:Type)" -> "Category"

"Patternable(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(IntegralDomain)" -> "Patternable(a:Type)"

"Patternable(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(OrderedSet)" -> "Patternable(a:Type)"

"Patternable(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(CommutativeRing)" -> "Patternable(a:Type)"
```

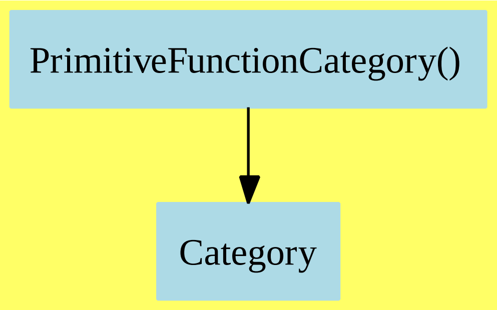
⟨*PATAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Patternable(a:Type)" [color=lightblue];
"Patternable(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.16 PrimitiveFunctionCategory (PRIMCAT)



⟨*PrimitiveFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f PrimitiveFunctionCategory.output
)spool PrimitiveFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimitiveFunctionCategory
--R PrimitiveFunctionCategory  is a category constructor
--R Abbreviation for PrimitiveFunctionCategory is PRIMCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRIMCAT
--R
--R------------------------------- Operations --------------------------------
--R integral : (%,Symbol) -> %
--R integral : (%,SegmentBinding %) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*PrimitiveFunctionCategory.help*⟩≡
```
====================================================================
PrimitiveFunctionCategory examples
====================================================================

See Also:
o )show PrimitiveFunctionCategory
```

See:

⇒ "LiouvillianFunctionCategory" (LFCAT) 4.0.51 on page 215
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

integral

These are directly exported but not implemented:

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

⟨*category PRIMCAT PrimitiveFunctionCategory*⟩≡

```
)abbrev category PRIMCAT PrimitiveFunctionCategory
++ Category for the integral functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the functions defined by integrals;

PrimitiveFunctionCategory(): Category == with
    integral: ($, Symbol) -> $
      ++ integral(f, x) returns the formal integral of f dx.
    integral: ($, SegmentBinding $) -> $
      ++ integral(f, x = a..b) returns the formal definite integral
      ++ of f dx for x between \spad{a} and b.
```

⟨*PRIMCAT.dotabb*⟩≡

```
"PRIMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PRIMCAT" -> "CATEGORY"
```

⟨*PRIMCAT.dotfull*⟩≡

```
"PrimitiveFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PrimitiveFunctionCategory()" -> "Category"
```

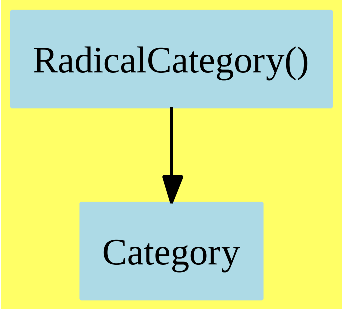
⟨*PRIMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.17 RadicalCategory (RADCAT)



⟨*RadicalCategory.input*⟩≡
```
)set break resume
)sys rm -f RadicalCategory.output
)spool RadicalCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RadicalCategory
--R RadicalCategory  is a category constructor
--R Abbreviation for RadicalCategory is RADCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RADCAT
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,Fraction Integer) -> %      nthRoot : (%,Integer) -> %
--R sqrt : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*RadicalCategory.help*⟩≡
```
====================================================================
RadicalCategory examples
====================================================================

See Also:
o )show RadicalCategory
```

See:

⇒ “AlgebraicallyClosedField” (ACF) 17.0.191 on page 1425
⇒ “IntervalCategory” (INTCAT) 14.0.176 on page 1270
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇒ “UnivariateLaurentSeriesCategory” (ULSCAT) 17.0.202 on page 1595
⇒ “UnivariatePuiseuxSeriesCategory” (UPXSCAT) 17.0.203 on page 1608
⇒ “UnivariateTaylorSeriesCategory” (UTSCAT) 16.0.190 on page 1408
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

nthRoot sqrt ?**?

These are directly exported but not implemented:

```
?**? : (%,Fraction Integer) -> %
```

These are implemented by this category:

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

⟨*category RADCAT RadicalCategory*⟩≡

```
)abbrev category RADCAT RadicalCategory
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: nthRoot, sqrt, **
++ Related Constructors:
++ Keywords: rational numbers
++ Description:
++ The \spad{RadicalCategory} is a model for the rational numbers.

RadicalCategory(): Category == with
  sqrt   : % -> %
      ++ sqrt(x) returns the square root of x.
  nthRoot: (%, Integer) -> %
      ++ nthRoot(x,n) returns the nth root of x.
  _*_*   : (%, Fraction Integer) -> %
      ++ x ** y is the rational exponentiation of x by the power y.
 add
  sqrt x        == x ** inv(2::Fraction(Integer))
  nthRoot(x, n) == x ** inv(n::Fraction(Integer))
```

⟨*RADCAT.dotabb*⟩≡

```
"RADCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
"RADCAT" -> "CATEGORY"
```

⟨*RADCAT.dotfull*⟩≡

```
"RadicalCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
"RadicalCategory()" -> "Category"
```

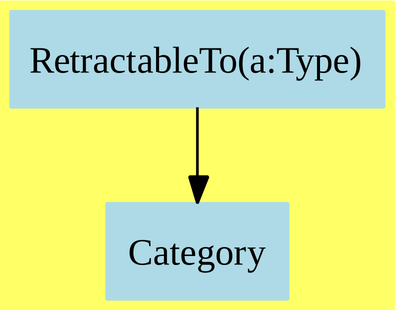
⟨*RADCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.18 RetractableTo (RETRACT)



⟨*RetractableTo.input*⟩≡
```
)set break resume
)sys rm -f RetractableTo.output
)spool RetractableTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RetractableTo
--R RetractableTo S: Type  is a category constructor
--R Abbreviation for RetractableTo is RETRACT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RETRACT
--R
--R------------------------------- Operations --------------------------------
--R coerce : S -> %                       retract : % -> S
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*RetractableTo.help*⟩≡
```
====================================================================
RetractableTo examples
====================================================================

See Also:
o )show RetractableTo
```

See:

⇒ “DifferentialPolynomialCategory” (DPOLCAT) 17.0.192 on page 1439
⇒ “DifferentialVariableCategory” (DVARCAT) 5.0.71 on page 330
⇒ “ExtensionField” (XF) 18.0.206 on page 1662
⇒ “ExpressionSpace” (ES) 5.0.72 on page 337
⇒ “FiniteAlgebraicExtensionField” (FAXF) 19.0.213 on page 1741
⇒ “FortranMachineTypeCategory” (FMTC) 13.0.171 on page 1223
⇒ “FreeAbelianMonoidCategory” (FAMONC) 7.0.103 on page 622
⇒ “FreeModuleCat” (FMCAT) 10.0.140 on page 950
⇒ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “GradedAlgebra” (GRALG) 5.0.73 on page 351
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “IntervalCategory” (INTCAT) 14.0.176 on page 1270
⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇒ “UnivariateLaurentSeriesConstructorCategory” (ULSCCAT) 18.0.211 on page 1709

⇒ “UnivariatePuiseuxSeriesConstructorCategory” (UPXSCCA) 18.0.212 on page 1728

⇒ “XFreeAlgebra” (XFALG) 11.0.159 on page 1094
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

coerce retract retractIfCan

These are directly exported but not implemented:

```
coerce : S -> %
retractIfCan : % -> Union(S,"failed")
```

These are implemented by this category:

```
retract : % -> S
```

⟨*category RETRACT RetractableTo*⟩≡

```
)abbrev category RETRACT RetractableTo
++ Category for retract
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is retractable to B means that some elementsif A can be converted
++ into elements of B and any element of B can be converted into an
++ element of A.

RetractableTo(S: Type): Category == with
    coerce:       S -> %
      ++ coerce(a) transforms a into an element of %.
    retractIfCan: % -> Union(S,"failed")
      ++ retractIfCan(a) transforms a into an element of S if possible.
      ++ Returns "failed" if a cannot be made into an element of S.
    retract:      % -> S
      ++ retract(a) transforms a into an element of S if possible.
      ++ Error: if a cannot be made into an element of S.
  add
    retract(s) ==
      (u:=retractIfCan s) case "failed" => error "not retractable"
      u
```

⟨*RETRACT.dotabb*⟩≡

```
"RETRACT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RETRACT" -> "CATEGORY"
```

⟨RETRACT.dotfull⟩≡

```
"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"RetractableTo(SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Symbol)" -> "RetractableTo(a:Type)"

"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(NonNegativeInteger)" -> "RetractableTo(a:Type)"

"RetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Fraction(Integer))" -> "RetractableTo(a:Type)"

"RetractableTo(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Float)" -> "RetractableTo(a:Type)"

"RetractableTo(Kernel(ExpressionSpace))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
  -> "RetractableTo(a:Type)"
```

```
"RetractableTo(Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(IntegralDomain)" -> "RetractableTo(a:Type)"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"
```

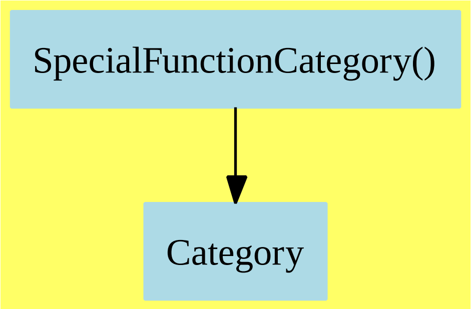
⟨*RETRACT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.0.19 SpecialFunctionCategory (SPFCAT)



⟨*SpecialFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f SpecialFunctionCategory.output
)spool SpecialFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SpecialFunctionCategory
--R SpecialFunctionCategory  is a category constructor
--R Abbreviation for SpecialFunctionCategory is SPFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SPFCAT
--R
--R------------------------------- Operations --------------------------------
--R Beta : (%,%) -> %                     Gamma : % -> %
--R Gamma : (%,%) -> %                    abs : % -> %
--R airyAi : % -> %                       airyBi : % -> %
--R besselI : (%,%) -> %                  besselJ : (%,%) -> %
--R besselK : (%,%) -> %                  besselY : (%,%) -> %
--R digamma : % -> %                      polygamma : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*SpecialFunctionCategory.help*⟩≡

```
====================================================================
SpecialFunctionCategory examples
====================================================================

See Also:
o )show SpecialFunctionCategory
```

See:

⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

abs airyAi airyBi besselI besselJ
besselK besselY Beta digamma Gamma
polygamma

These are directly exported but not implemented:

```
abs : % -> %
airyAi : % -> %
airyBi : % -> %
besselI : (%,%) -> %
besselJ : (%,%) -> %
besselK : (%,%) -> %
besselY : (%,%) -> %
Beta : (%,%) -> %
digamma : % -> %
Gamma : % -> %
Gamma : (%,%) -> %
polygamma : (%,%) -> %
```

⟨*category SPFCAT SpecialFunctionCategory*⟩≡

```
)abbrev category SPFCAT SpecialFunctionCategory
++ Category for the other special functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 11 May 1993
++ Description:
++ Category for the other special functions;

SpecialFunctionCategory(): Category == with
    abs :      $ -> $
        ++ abs(x) returns the absolute value of x.
    Gamma:     $ -> $
        ++ Gamma(x) is the Euler Gamma function.
    Beta:      ($,$)->$
        ++ Beta(x,y) is \spad{Gamma(x) * Gamma(y)/Gamma(x+y)}.
    digamma:   $ -> $
        ++ digamma(x) is the logarithmic derivative of \spad{Gamma(x)}
        ++ (often written \spad{psi(x)} in the literature).
    polygamma: ($, $) -> $
        ++ polygamma(k,x) is the \spad{k-th} derivative of \spad{digamma(x)},
        ++ (often written \spad{psi(k,x)} in the literature).
    Gamma:     ($, $) -> $
```

```
        ++ Gamma(a,x) is the incomplete Gamma function.
    besselJ:   ($,$) -> $
        ++ besselJ(v,z) is the Bessel function of the first kind.
    besselY:   ($,$) -> $
        ++ besselY(v,z) is the Bessel function of the second kind.
    besselI:   ($,$) -> $
        ++ besselI(v,z) is the modified Bessel function of the first kind.
    besselK:   ($,$) -> $
        ++ besselK(v,z) is the modified Bessel function of the second kind.
    airyAi:    $ -> $
        ++ airyAi(x) is the Airy function \spad{Ai(x)}.
    airyBi:    $ -> $
        ++ airyBi(x) is the Airy function \spad{Bi(x)}.
```

⟨*SPFCAT.dotabb*⟩≡

```
"SPFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SPFCAT" -> "CATEGORY"
```

⟨*SPFCAT.dotfull*⟩≡

```
"SpecialFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SpecialFunctionCategory()" -> "Category"
```

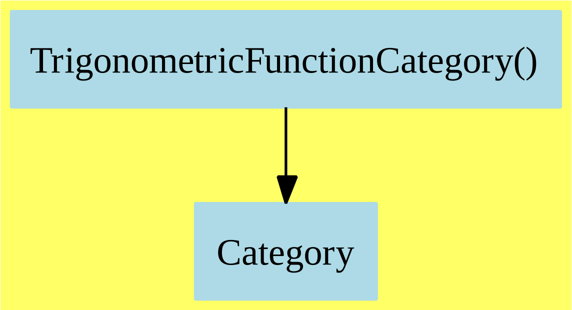
⟨*SPFCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SpecialFunctionCategory()" [color=lightblue];
"SpecialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.20 TrigonometricFunctionCategory (TRIGCAT)



The `csc` and `sec` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

⟨*TrigonometricFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f TrigonometricFunctionCategory.output
)spool TrigonometricFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TrigonometricFunctionCategory
--R TrigonometricFunctionCategory  is a category constructor
--R Abbreviation for TrigonometricFunctionCategory is TRIGCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TRIGCAT
--R
--R------------------------------- Operations --------------------------------
--R cos : % -> %                         cot : % -> %
--R csc : % -> %                         sec : % -> %
--R sin : % -> %                         tan : % -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*TrigonometricFunctionCategory.help*⟩≡

```
====================================================================
TrigonometricFunctionCategory examples
====================================================================

See Also:
o )show TrigonometricFunctionCategory
```

See:

⇒ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Exports:

cos cot csc sec sin tan

These are directly exported but not implemented:

```
cos : % -> %
sin : % -> %
```

These are implemented by this category:

```
cot : % -> %
csc : % -> %
sec : % -> %
tan : % -> %
```

⟨*category TRIGCAT TrigonometricFunctionCategory*⟩≡

```
)abbrev category TRIGCAT TrigonometricFunctionCategory
++ Category for the trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the trigonometric functions;

TrigonometricFunctionCategory(): Category == with
    cos: $ -> $        ++ cos(x) returns the cosine of x.
    cot: $ -> $        ++ cot(x) returns the cotangent of x.
    csc: $ -> $        ++ csc(x) returns the cosecant of x.
    sec: $ -> $        ++ sec(x) returns the secant of x.
    sin: $ -> $        ++ sin(x) returns the sine of x.
    tan: $ -> $        ++ tan(x) returns the tangent of x.
 add
    if $ has Ring then
       csc x ==
         (a := recip(sin x)) case "failed" => error "csc: no reciprocal"
         a::$
       sec x ==
         (a := recip(cos x)) case "failed" => error "sec: no reciprocal"
         a::$
       tan x == sin x * sec x
       cot x == cos x * csc x
```

⟨*TRIGCAT.dotabb*⟩≡
```
"TRIGCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
"TRIGCAT" -> "CATEGORY"
```

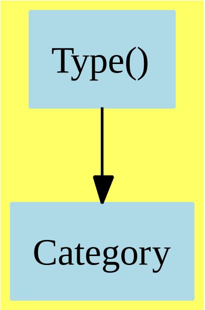
⟨*TRIGCAT.dotfull*⟩≡
```
"TrigonometricFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
"TrigonometricFunctionCategory()" -> "Category"
```

⟨*TRIGCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TrigonometricFunctionCategory()" [color=lightblue];
"TrigonometricFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.21 Type (TYPE)



See:

⇒ “Aggregate” (AGG) 3.0.22 on page 77
⇒ “FortranProgramCategory” (FORTCAT) 3.0.26 on page 96
⇒ “FullyPatternMatchable” (FPATMAB) 3.0.28 on page 105
⇒ “SegmentCategory” (SEGCAT) 3.0.33 on page 125
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Attributes exported:

- **nil**

⟨*category TYPE Type*⟩≡

```
)abbrev category TYPE Type
++ The new fundamental Type (keeping Object for 1.5 as well)
++ Author: Richard Jenks
++ Date Created: 14 May 1992
++ Date Last Updated: 14 May 1992
++ Description:
++ The fundamental Type;

Type(): Category == with nil
```

⟨*TYPE.dotabb*⟩≡

```
"TYPE" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"TYPE" -> "CATEGORY"
```

⟨*TYPE.dotfull*⟩≡

```
"Type()" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"Type()" -> "Category"
```

⟨*TYPE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

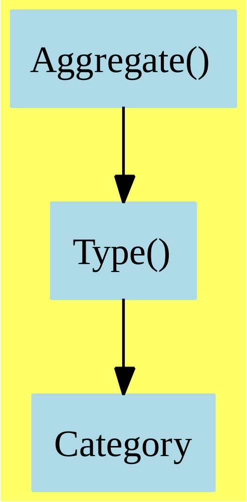
"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 3

Category Layer 2

3.0.22 Aggregate (AGG)


Aggregate()
Type()
Category

⟨*Aggregate.input*⟩≡

```
)set break resume
)sys rm -f Aggregate.output
)spool Aggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Aggregate
```

```
--R Aggregate  is a category constructor
--R Abbreviation for Aggregate is AGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R less? : (%,NonNegativeInteger) -> Boolean
--R more? : (%,NonNegativeInteger) -> Boolean
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*Aggregate.help*⟩≡

```
====================================================================
Aggregate examples
====================================================================

See Also:
o )show Aggregate
```

See:

⇒ “HomogeneousAggregate” (HOAGG) 4.0.49 on page 203
⇐ “Type” (TYPE) 2.0.21 on page 75

Attributes exported:

- **nil**

Exports:

empty? eq? less? more? sample size?

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
empty? : % -> Boolean
eq? : (%,%) -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

⟨*category AGG Aggregate*⟩≡

```
)abbrev category AGG Aggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The notion of aggregate serves to model any data structure aggregate,
++ designating any collection of objects, with heterogenous or homogeneous
++ members, with a finite or infinite number of members, explicitly or
++ implicitly represented. An aggregate can in principle represent
++ everything from a string of characters to abstract sets such
++ as "the set of x satisfying relation r(x)"
++ An attribute \spadatt{finiteAggregate} is used to assert that a domain
++ has a finite number of elements.
```

```
Aggregate: Category == Type with
   eq?: (%,%) -> Boolean
     ++ eq?(u,v) tests if u and v are same objects.
   copy: % -> %
     ++ copy(u) returns a top-level (non-recursive) copy of u.
     ++ Note that for collections, \axiom{copy(u) == [x for x in u]}.
   empty: () -> %
     ++ empty()$D creates an aggregate of type D with 0 elements.
     ++ Note that The $D can be dropped if understood by context,
     ++ e.g. \axiom{u: D := empty()}.
   empty?: % -> Boolean
     ++ empty?(u) tests if u has 0 elements.
   less?: (%,NonNegativeInteger) -> Boolean
     ++ less?(u,n) tests if u has less than n elements.
   more?: (%,NonNegativeInteger) -> Boolean
     ++ more?(u,n) tests if u has greater than n elements.
   size?: (%,NonNegativeInteger) -> Boolean
     ++ size?(u,n) tests if u has exactly n elements.
   sample: constant -> %
     ++ sample yields a value of type %
   if % has finiteAggregate then
     "#": % -> NonNegativeInteger
       ++ # u returns the number of items in u.
 add
  eq?(a,b) == EQ(a,b)$Lisp
  sample() == empty()
  if % has finiteAggregate then
    empty? a   == #a = 0
    less?(a,n) == #a < n
    more?(a,n) == #a > n
    size?(a,n) == #a = n
```

⟨*AGG.dotabb*⟩≡

```
"AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"AGG" -> "TYPE"
```

⟨*AGG.dotfull*⟩≡

```
"Aggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"Aggregate()" -> "Type()"
```

⟨*AGG.dotpic*⟩≡

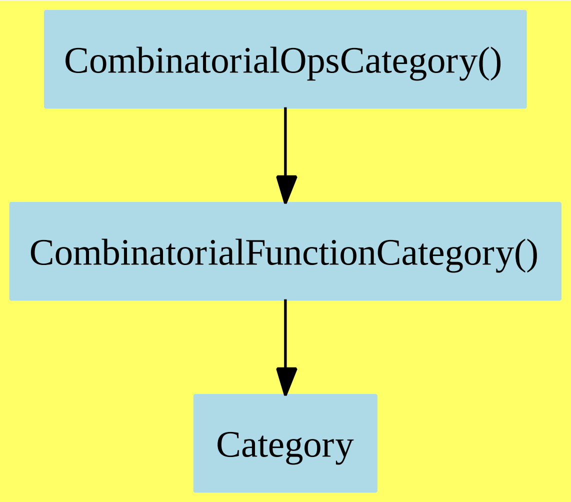
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.23 CombinatorialOpsCategory (COMBOPC)



⟨*CombinatorialOpsCategory.input*⟩≡

```
)set break resume
)sys rm -f CombinatorialOpsCategory.output
)spool CombinatorialOpsCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CombinatorialOpsCategory
--R CombinatorialOpsCategory  is a category constructor
--R Abbreviation for CombinatorialOpsCategory is COMBOPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMBOPC
--R
--R------------------------------- Operations --------------------------------
--R binomial : (%,%) -> %                 factorial : % -> %
--R factorials : (%,Symbol) -> %          factorials : % -> %
--R permutation : (%,%) -> %              product : (%,Symbol) -> %
--R summation : (%,Symbol) -> %
--R product : (%,SegmentBinding %) -> %
--R summation : (%,SegmentBinding %) -> %
--R
--E 1

)spool
```

```
)lisp (bye)
```

⟨*CombinatorialOpsCategory.help*⟩≡

```
====================================================================
CombinatorialOpsCategory examples
====================================================================

See Also:
o )show CombinatorialOpsCategory
```

See:

⇐ "CombinatorialFunctionCategory" (CFCAT) 2.0.7 on page 22

Exports:

binomial factorial factorials permutation product
summation

These are directly exported but not implemented:

```
factorials : % -> %
factorials : (%,Symbol) -> %
product : (%,Symbol) -> %
product : (%,SegmentBinding %) -> %
summation : (%,Symbol) -> %
summation : (%,SegmentBinding %) -> %
```

These exports come from (p22) CombinatorialFunctionCategory():

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

⟨*category COMBOPC CombinatorialOpsCategory*⟩≡

```
)abbrev category COMBOPC CombinatorialOpsCategory
++ Category for summations and products
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 22 February 1993 (JHD/BMT)
++ Description:
++ CombinatorialOpsCategory is the category obtaining by adjoining
++ summations and products to the usual combinatorial operations;

CombinatorialOpsCategory(): Category ==
  CombinatorialFunctionCategory with
    factorials : $ -> $
      ++ factorials(f) rewrites the permutations and binomials in f
      ++ in terms of factorials;
    factorials : ($, Symbol) -> $
      ++ factorials(f, x) rewrites the permutations and binomials in f
      ++ involving x in terms of factorials;
    summation  : ($, Symbol)           -> $
      ++ summation(f(n), n) returns the formal sum S(n) which verifies
      ++ S(n+1) - S(n) = f(n);
    summation  : ($, SegmentBinding $)  -> $
      ++ summation(f(n), n = a..b) returns f(a) + ... + f(b) as a
      ++ formal sum;
    product    : ($, Symbol)            -> $
      ++ product(f(n), n) returns the formal product P(n) which verifies
      ++ P(n+1)/P(n) = f(n);
    product    : ($, SegmentBinding  $) -> $
      ++ product(f(n), n = a..b) returns f(a) * ... * f(b) as a
      ++ formal product;
```

⟨*COMBOPC.dotabb*⟩≡

```
"COMBOPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"COMBOPC" -> "CFCAT"
```

⟨*COMBOPC.dotfull*⟩≡

```
"CombinatorialOpsCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"
```

⟨*COMBOPC.dotpic*⟩≡

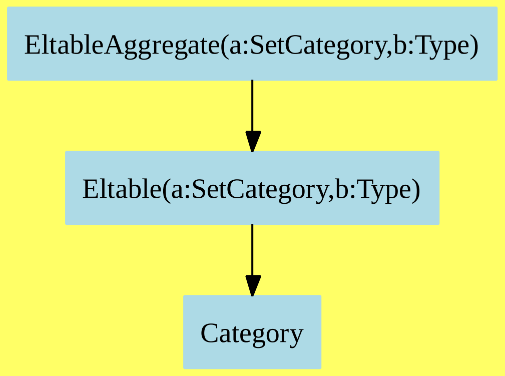
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CombinatorialOpsCategory()" [color=lightblue];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"

"CombinatorialFunctionCategory()" [color=lightblue];
"CombinatorialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.24 EltableAggregate (ELTAGG)


EltableAggregate(a:SetCategory,b:Type)
Eltable(a:SetCategory,b:Type)
Category

⟨*EltableAggregate.input*⟩≡
```
)set break resume
)sys rm -f EltableAggregate.output
)spool EltableAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EltableAggregate
--R EltableAggregate(Dom: SetCategory,Im: Type)  is a category constructor
--R Abbreviation for EltableAggregate is ELTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELTAGG
--R
--R------------------------------- Operations --------------------------------
--R elt : (%,Dom,Im) -> Im                ?.? : (%,Dom) -> Im
--R qelt : (%,Dom) -> Im
--R qsetelt! : (%,Dom,Im) -> Im if $ has shallowlyMutable
--R setelt : (%,Dom,Im) -> Im if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

⟨*EltableAggregate.help*⟩≡

```
====================================================================
EltableAggregate examples
====================================================================

See Also:
o )show EltableAggregate
```

See:

⇒ "IndexedAggregate" (IXAGG) 5.0.74 on page 356
⇐ "Eltable" (ELTAB) 2.0.10 on page 32

Exports:

elt qelt qsetelt! setelt ?.?

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are part of this category's direct exports:

```
elt : (%,Dom,Im) -> Im
setelt : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These are implemented by this category:

```
qelt : (%,Dom) -> Im
qsetelt! : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These exports come from (p32) Eltable():

```
?.? : (%,Dom) -> Im
```

⟨*category ELTAGG EltableAggregate*⟩≡

```
)abbrev category ELTAGG EltableAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An eltable aggregate is one which can be viewed as a function.
++ For example, the list \axiom{[1,7,4]} can applied to 0,1, and 2
++ respectively will return the integers 1,7, and 4; thus this list may
++ be viewed as mapping 0 to 1, 1 to 7 and 2 to 4. In general, an aggregate
++ can map members of a domain Dom to an image domain Im.

EltableAggregate(Dom:SetCategory, Im:Type): Category ==
-- This is separated from Eltable
-- and series won't have to support qelt's and setelt's.
  Eltable(Dom, Im) with
    elt : (%, Dom, Im) -> Im
       ++ elt(u, x, y) applies u to x if x is in the domain of u,
       ++ and returns y otherwise.
       ++ For example, if u is a polynomial in \axiom{x} over the rationals,
       ++ \axiom{elt(u,n,0)} may define the coefficient of \axiom{x}
       ++ to the power n, returning 0 when n is out of range.
    qelt: (%, Dom) -> Im
       ++ qelt(u, x) applies \axiom{u} to \axiom{x} without checking whether
       ++ \axiom{x} is in the domain of \axiom{u}.  If \axiom{x} is not
       ++ in the domain of \axiom{u} a memory-access violation may occur.
       ++ If a check on whether \axiom{x} is in the domain of \axiom{u}
       ++ is required, use the function \axiom{elt}.
    if % has shallowlyMutable then
       setelt : (%, Dom, Im) -> Im
           ++ setelt(u,x,y) sets the image of x to be y under u,
           ++ assuming x is in the domain of u.
           ++ Error: if x is not in the domain of u.
           -- this function will soon be renamed as setelt!.
       qsetelt_!: (%, Dom, Im) -> Im
           ++ qsetelt!(u,x,y) sets the image of \axiom{x} to be \axiom{y}
           ++ under \axiom{u}, without checking that \axiom{x} is in
           ++ the domain of \axiom{u}.
           ++ If such a check is required use the function \axiom{setelt}.
 add
  qelt(a, x) == elt(a, x)
  if % has shallowlyMutable then
    qsetelt_!(a, x, y) == (a.x := y)
```

⟨*ELTAGG.dotabb*⟩≡

```
"ELTAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"ELTAGG" -> "ELTAB"
```

⟨*ELTAGG.dotfull*⟩≡

```
"EltableAggregate(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"
```

⟨*ELTAGG.dotpic*⟩≡

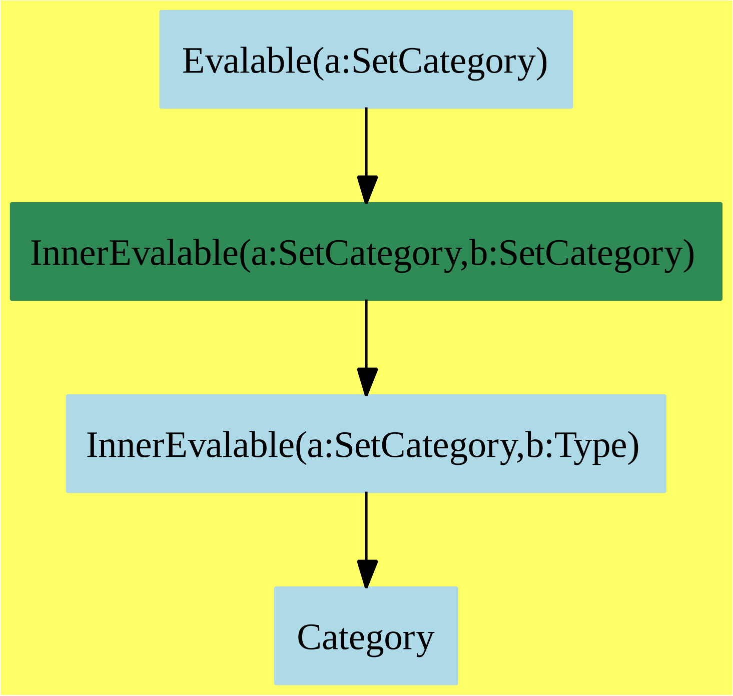
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.25 Evalable (EVALAB)


Evalable(a:SetCategory)
InnerEvalable(a:SetCategory,b:SetCategory)
InnerEvalable(a:SetCategory,b:Type)
Category

⟨*Evalable.input*⟩≡

```
)set break resume
)sys rm -f Evalable.output
)spool Evalable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Evalable
--R Evalable R: SetCategory  is a category constructor
--R Abbreviation for Evalable is EVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for EVALAB
--R
--R------------------------------- Operations --------------------------------
--R eval : (%,List Equation R) -> %       eval : (%,Equation R) -> %
--R eval : (%,R,R) -> %                   eval : (%,List R,List R) -> %
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*Evalable.help*⟩≡

```
====================================================================
Evalable examples
====================================================================

See Also:
o )show Evalable
```

See:

⇒ "ExpressionSpace" (ES) 5.0.72 on page 337
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.0.182 on page 1316
⇒ "PolynomialCategory" (POLYCAT) 16.0.189 on page 1385
⇐ "InnerEvalable" (IEVALAB) 2.0.12 on page 39

Exports:

eval

These are directly exported but not implemented:

```
 eval : (%,List Equation R) -> %
```

These are implemented by this category:

```
eval : (%,Equation R) -> %
eval : (%,List R,List R) -> %
```

These exports come from (p39) InnerEvalable(R:SetCategory,R:SetCategory):

```
 eval : (%,R,R) -> %
```

⟨*category EVALAB Evalable*⟩≡

```
)abbrev category EVALAB Evalable
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See: FullyEvalable
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This category provides \spadfun{eval} operations.
++ A domain may belong to this category if it is possible to make
++ ``evaluation'' substitutions.

Evalable(R:SetCategory): Category == InnerEvalable(R,R) with
    eval: ($, Equation R) -> $
       ++ eval(f,x = v) replaces x by v in f.
    eval: ($, List Equation R) -> $
       ++ eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.
 add
    eval(f:$, eq:Equation R) == eval(f, [eq])
```

```
      eval(f:$, xs:List R,vs:List R) == eval(f,[x=v for x in xs for v in vs])
```

⟨*EVALAB.dotabb*⟩≡

```
  "EVALAB"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "EVALAB" -> "IEVALAB"
```

⟨*EVALAB.dotfull*⟩≡

```
  "Evalable(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

  "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))" ->
      "Evalable(a:SetCategory)"

  "Evalable(ExpressionSpace)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

  "Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
    -> "Evalable(a:SetCategory)"
```

⟨*EVALAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

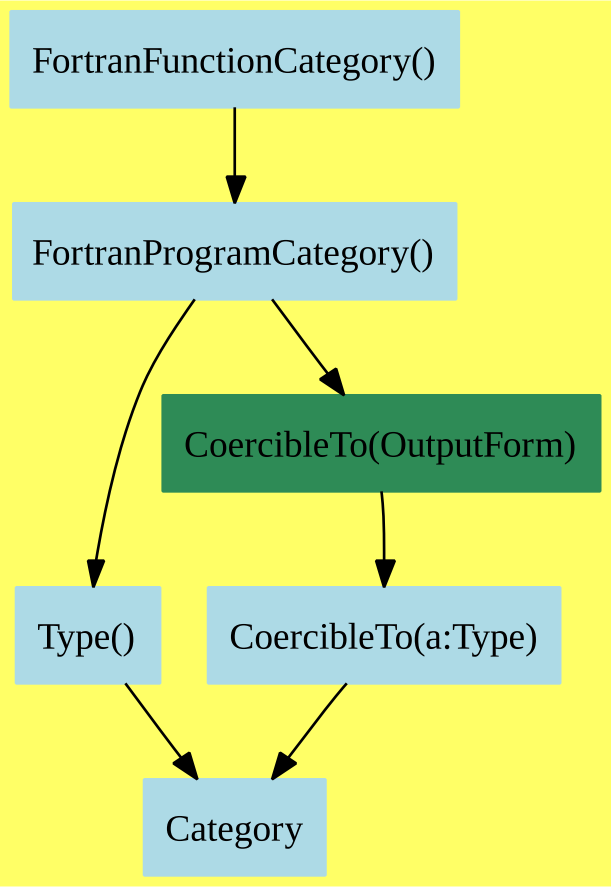
"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.26 FortranProgramCategory (FORTCAT)


FortranFunctionCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranProgramCategory.input*⟩≡
```
)set break resume
)sys rm -f FortranProgramCategory.output
)spool FortranProgramCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranProgramCategory
--R FortranProgramCategory  is a category constructor
--R Abbreviation for FortranProgramCategory is FORTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FORTCAT
```

```
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranProgramCategory.help*⟩≡

```
====================================================================
FortranProgramCategory examples
====================================================================

See Also:
o )show FortranProgramCategory
```

See:

⇒ "FortranFunctionCategory" (FORTFN) 4.0.39 on page 155
⇒ "FortranMatrixCategory" (FMC) 4.0.40 on page 160
⇒ "FortranMatrixFunctionCategory" (FMFUN) 4.0.41 on page 164
⇒ "FortranVectorCategory" (FVC) 4.0.42 on page 169
⇒ "FortranVectorFunctionCategory" (FVFUN) 4.0.43 on page 173
⇐ "CoercibleTo" (KOERCE) 2.0.6 on page 19
⇐ "Type" (TYPE) 2.0.21 on page 75

Exports:

coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
outputAsFortran : % -> Void
```

These exports come from (p19) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category FORTCAT FortranProgramCategory*⟩≡

```
)abbrev category FORTCAT FortranProgramCategory
++ Author: Mike Dewar
++ Date Created: November 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{FortranProgramCategory} provides various models of
++ FORTRAN subprograms.  These can be transformed into actual FORTRAN code.

FortranProgramCategory():Category == Join(Type,CoercibleTo OutputForm) with
    outputAsFortran : $ -> Void
    ++ \axiom{outputAsFortran(u)} translates \axiom{u} into a legal FORTRAN
    ++ subprogram.
```

⟨*FORTCAT.dotabb*⟩≡
```
"FORTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FORTCAT" -> "KOERCE"
"FORTCAT" -> "TYPE"
```

⟨*FORTCAT.dotfull*⟩≡
```
"FortranProgramCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*FORTCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

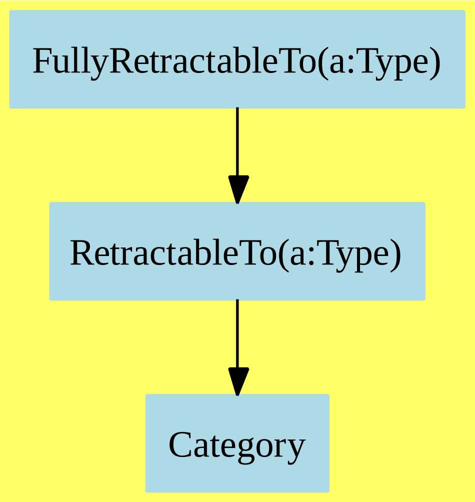
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.27 FullyRetractableTo (FRETRCT)



⟨*FullyRetractableTo.input*⟩≡

```
)set break resume
)sys rm -f FullyRetractableTo.output
)spool FullyRetractableTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyRetractableTo
--R FullyRetractableTo S: Type  is a category constructor
--R Abbreviation for FullyRetractableTo is FRETRCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRETRCT
--R
--R------------------------------- Operations --------------------------------
--R coerce : S -> %                       retract : % -> S
--R coerce : Integer -> % if S has RETRACT INT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT
--R retract : % -> Integer if S has RETRACT INT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC IN
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*FullyRetractableTo.help*⟩≡

```
====================================================================
FullyRetractableTo examples
====================================================================

See Also:
o )show FullyRetractableTo
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.216 on page 1779
⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “FiniteAbelianMonoidRing” (FAMR) 14.0.175 on page 1259
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇒ “OctonionCategory” (OC) 12.0.166 on page 1167
⇒ “QuaternionCategory” (QUATCAT) 12.0.167 on page 1180
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.168 on page 1192
⇒ “UnivariateSkewPolynomialCategory” (OREPCAT) 10.0.150 on page 1024
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62

Exports:

coerce retract retractIfCan

These are implemented by this category:

```
coerce : Integer -> %
    if S has RETRACT INT
coerce : Fraction Integer -> %
    if S has RETRACT FRAC INT
retract : % -> Integer
    if S has RETRACT INT
retract : % -> Fraction Integer
    if S has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if S has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if S has RETRACT FRAC INT
```

These exports come from (p62) RetractableTo(S:Type):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

⟨*category FRETRCT FullyRetractableTo*⟩≡

```
)abbrev category FRETRCT FullyRetractableTo
++ Author: Manuel Bronstein
++ Date Created: March 1990
++ Date Last Updated: 9 April 1991
++ Description:
++ A is fully retractable to B means that A is retractable to B and
++ if B is retractable to the integers or rational numbers then so is A.
++ In particular, what we are asserting is that there are no integers
++ (rationals) in A which don't retract into B.

FullyRetractableTo(S: Type): Category == RetractableTo(S) with
    if (S has RetractableTo Integer) then RetractableTo Integer
    if (S has RetractableTo Fraction Integer) then
              RetractableTo Fraction Integer
  add
    if not(S is Integer) then
      if (S has RetractableTo Integer) then    -- induction
        coerce(n:Integer):%  == n::S::%
        retract(r:%):Integer == retract(retract(r)@S)

        retractIfCan(r:%):Union(Integer, "failed") ==
          (u:= retractIfCan(r)@Union(S,"failed")) case "failed"=> "failed"
          retractIfCan(u::S)

    if not(S is Fraction Integer) then
      if (S has RetractableTo Fraction Integer) then   -- induction
        coerce(n:Fraction Integer):%  == n::S::%
        retract(r:%):Fraction(Integer) == retract(retract(r)@S)

        retractIfCan(r:%):Union(Fraction Integer, "failed") ==
          (u:=retractIfCan(r)@Union(S,"failed")) case "failed"=>"failed"
          retractIfCan(u::S)
```

⟨*FRETRCT.dotabb*⟩≡

```
"FRETRCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FRETRCT" -> "RETRACT"
```

⟨*FRETRCT.dotfull*⟩≡

```
"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:CommutativeRing)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:SetCategory)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(Fraction(Integer))" -> "FullyRetractableTo(a:Type)"
```

⟨*FRETRCT.dotpic*⟩≡

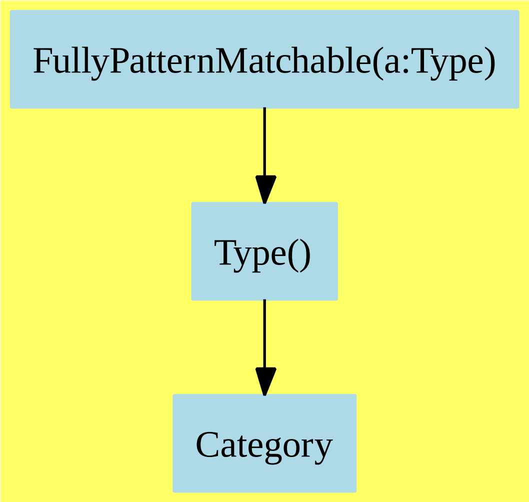
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.28 FullyPatternMatchable (FPATMAB)



⟨*FullyPatternMatchable.input*⟩≡

```
)set break resume
)sys rm -f FullyPatternMatchable.output
)spool FullyPatternMatchable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyPatternMatchable
--R FullyPatternMatchable R: Type  is a category constructor
--R Abbreviation for FullyPatternMatchable is FPATMAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPATMAB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean if R has PATMAB INT or R has PATMAB FLOAT
--R coerce : % -> OutputForm if R has PATMAB INT or R has PATMAB FLOAT
--R hash : % -> SingleInteger if R has PATMAB INT or R has PATMAB FLOAT
--R latex : % -> String if R has PATMAB INT or R has PATMAB FLOAT
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Floa
--R ?~=? : (%,%) -> Boolean if R has PATMAB INT or R has PATMAB FLOAT
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*FullyPatternMatchable.help*⟩≡

```
====================================================================
FullyPatternMatchable examples
====================================================================

See Also:
o )show FullyPatternMatchable
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.216 on page 1779
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇐ “Type” (TYPE) 2.0.21 on page 75

Attributes exported:

- **nil**

Exports:

coerce hash latex patternMatch ?=?
?~=?

These exports come from (p253) PatternMatchable(Integer):

```
coerce : % -> OutputForm
    if R has PATMAB INT
    or R has PATMAB FLOAT
hash : % -> SingleInteger
    if R has PATMAB INT
    or R has PATMAB FLOAT
latex : % -> String
    if R has PATMAB INT
    or R has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%))
     -> PatternMatchResult(Integer,%) if R has PATMAB INT
?=? : (%,%) -> Boolean
    if R has PATMAB INT
    or R has PATMAB FLOAT
?~=? : (%,%) -> Boolean
    if R has PATMAB INT
    or R has PATMAB FLOAT
```

These exports come from (p253) PatternMatchable(Float):

```
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%))
     -> PatternMatchResult(Float,%) if R has PATMAB FLOAT
```

These exports come from (p75) Type():

⟨*category FPATMAB FullyPatternMatchable*⟩≡

```
)abbrev category FPATMAB FullyPatternMatchable
```

```
++ Category of sets that can be pattern-matched on
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Keywords: pattern, matching.
++ Description:
++ A set S is PatternMatchable over R if S can lift the
++ pattern-matching functions of S over the integers and float
++ to itself (necessary for matching in towers).

FullyPatternMatchable(R:Type): Category == Type with
  if R has PatternMatchable Integer then PatternMatchable Integer
  if R has PatternMatchable Float   then PatternMatchable Float
```

⟨*FPATMAB.dotabb*⟩≡

```
"FPATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FPATMAB" -> "TYPE"
```

⟨*FPATMAB.dotfull*⟩≡

```
"FullyPatternMatchable(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(a:Type)" -> "Type()"

"FullyPatternMatchable(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(IntegralDomain)" ->
  "FullyPatternMatchable(a:Type)"

"FullyPatternMatchable(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(OrderedSet)" ->
  "FullyPatternMatchable(a:Type)"

"FullyPatternMatchable(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(CommutativeRing)" ->
  "FullyPatternMatchable(a:Type)"
```

⟨*FPATMAB.dotpic*⟩≡

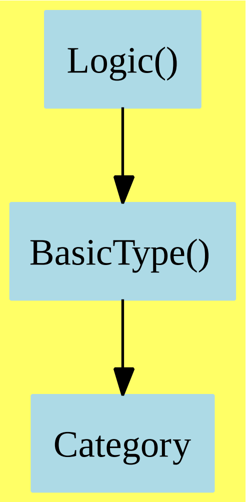
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyPatternMatchable(a:Type)" [color=lightblue];
"FullyPatternMatchable(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.29 Logic (LOGIC)



⟨*Logic.input*⟩≡
```
)set break resume
)sys rm -f Logic.output
)spool Logic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Logic
--R Logic  is a category constructor
--R Abbreviation for Logic is LOGIC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LOGIC
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                      ~? : % -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*Logic.help*⟩≡

```
====================================================================
Logic examples
====================================================================

See Also:
o )show Logic
```

See:

⇒ “BitAggregate” (BTAGG) 9.0.123 on page 806
⇐ “BasicType” (BASTYPE) 2.0.5 on page 16

Exports:

?/\? ?=? ?\/? ~? ?~=?

These are directly exported but not implemented:

```
~? : % -> %
?/\? : (%,%) -> %
```

These are implemented by this category:

```
?\/? : (%,%) -> %
```

These exports come from (p16) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category LOGIC Logic*⟩≡

```
)abbrev category LOGIC Logic
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: ~, /\, \/
++ Related Constructors:
++ Keywords: boolean
++ Description:
++ `Logic' provides the basic operations for lattices, e.g., boolean algebra.

Logic: Category == BasicType with
      _~:        % -> %
       ++ ~(x) returns the logical complement of x.
      _/_\:       (%, %) -> %
       ++ \spadignore { /\ }returns the logical `meet', e.g. `and'.
      _\_/:       (%, %) -> %
       ++ \spadignore{ \/ } returns the logical `join', e.g. `or'.
  add
    _\_/(x: %,y: %) == _~( _/_\(_~(x), _~(y)))
```

⟨*LOGIC.dotabb*⟩≡

```
"LOGIC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
"LOGIC" -> "BASTYPE"
```

⟨*LOGIC.dotfull*⟩≡

```
"Logic()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
"Logic()" -> "BasicType()"
```

⟨*LOGIC.dotpic*⟩≡

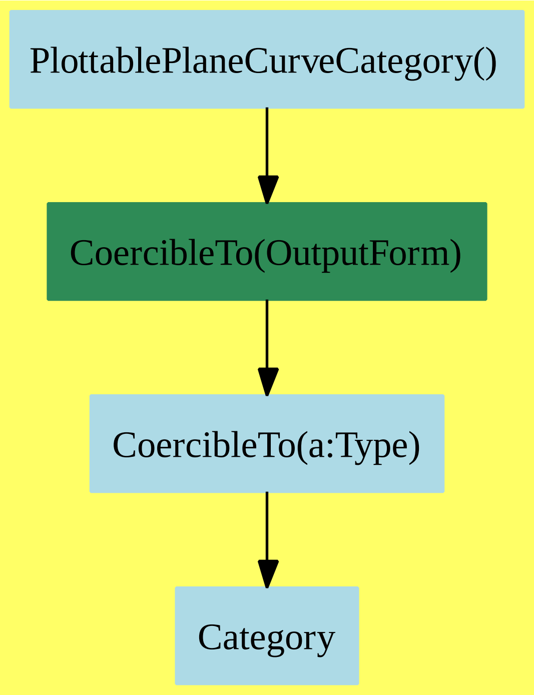
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Logic()" [color=lightblue];
"Logic()" -> "BasicType()"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.30 PlottablePlaneCurveCategory (PPCURVE)


PlottablePlaneCurveCategory()
CoercibleTo(OutputForm)
CoercibleTo(a:Type)
Category

⟨*PlottablePlaneCurveCategory.input*⟩≡
```
)set break resume
)sys rm -f PlottablePlaneCurveCategory.output
)spool PlottablePlaneCurveCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlottablePlaneCurveCategory
--R PlottablePlaneCurveCategory  is a category constructor
--R Abbreviation for PlottablePlaneCurveCategory is PPCURVE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PPCURVE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              xRange : % -> Segment DoubleFloat
--R yRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R
```

```
--E 1

)spool
)lisp (bye)
```

⟨*PlottablePlaneCurveCategory.help*⟩≡

```
====================================================================
PlottablePlaneCurveCategory examples
====================================================================

See Also:
o )show PlottablePlaneCurveCategory
```

See:

⇐ “CoercibleTo” (KOERCE) 2.0.6 on page 19

Exports:

coerce listBranches xRange yRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
```

These exports come from (p19) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category PPCURVE PlottablePlaneCurveCategory*⟩≡

```
)abbrev category PPCURVE PlottablePlaneCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Basic Operations: listBranches, xRange, yRange
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, graphics
++ References:
++ Description:
++ PlottablePlaneCurveCategory is the category of curves in the
++ plane which may be plotted via the graphics facilities.  Functions are
++ provided for obtaining lists of lists of points, representing the
++ branches of the curve, and for determining the ranges of the
++ x-coordinates and y-coordinates of the points on the curve.

PlottablePlaneCurveCategory(): Category == Definition where
  L     ==> List
  SEG   ==> Segment
  SF    ==> DoubleFloat
  POINT ==> Point DoubleFloat

  Definition ==> CoercibleTo OutputForm with

    listBranches: % -> L L POINT
      ++ listBranches(c) returns a list of lists of points, representing the
      ++ branches of the curve c.
    xRange: % -> SEG SF
    ++ xRange(c) returns the range of the x-coordinates of the points
    ++ on the curve c.
  yRange: % -> SEG SF
    ++ yRange(c) returns the range of the y-coordinates of the points
    ++ on the curve c.
```

⟨*PPCURVE.dotabb*⟩≡

```
"PPCURVE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
"PPCURVE" -> "KOERCE"
```

⟨*PPCURVE.dotfull*⟩≡

```
"PlottablePlaneCurveCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
"PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*PPCURVE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

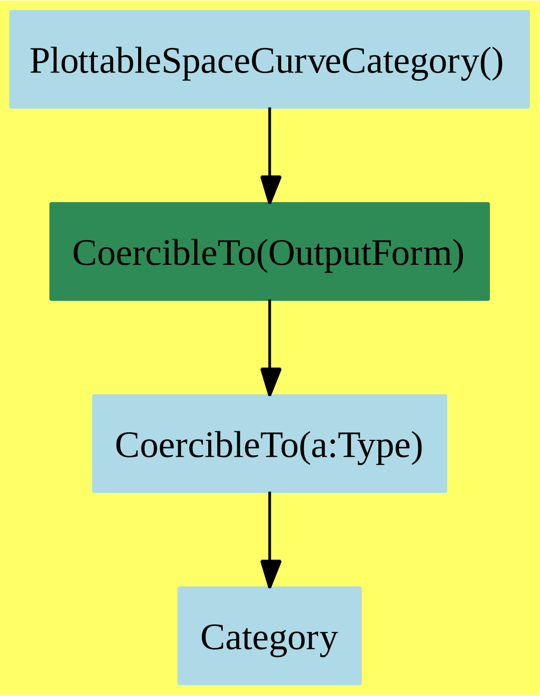
"PlottablePlaneCurveCategory()" [color=lightblue];
"PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.31 PlottableSpaceCurveCategory (PSCURVE)


PlottableSpaceCurveCategory()
CoercibleTo(OutputForm)
CoercibleTo(a:Type)
Category

⟨*PlottableSpaceCurveCategory.input*⟩≡

```
)set break resume
)sys rm -f PlottableSpaceCurveCategory.output
)spool PlottableSpaceCurveCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlottableSpaceCurveCategory
--R PlottableSpaceCurveCategory  is a category constructor
--R Abbreviation for PlottableSpaceCurveCategory is PSCURVE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSCURVE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              xRange : % -> Segment DoubleFloat
--R yRange : % -> Segment DoubleFloat     zRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R
```

```
--E 1

)spool
)lisp (bye)
```

⟨*PlottableSpaceCurveCategory.help*⟩≡

```
====================================================================
PlottableSpaceCurveCategory examples
====================================================================

See Also:
o )show PlottableSpaceCurveCategory
```

See:

⇐ "CoercibleTo" (KOERCE) 2.0.6 on page 19

Exports:

coerce listBranches xRange yRange zRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
zRange : % -> Segment DoubleFloat
```

These exports come from (p19) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category PSCURVE PlottableSpaceCurveCategory*⟩≡

```
)abbrev category PSCURVE PlottableSpaceCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Basic Operations: listBranches, xRange, yRange, zRange
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, graphics
++ References:
++ Description:
++ PlottableSpaceCurveCategory is the category of curves in
++ 3-space which may be plotted via the graphics facilities.  Functions are
++ provided for obtaining lists of lists of points, representing the
++ branches of the curve, and for determining the ranges of the
++ x-, y-, and z-coordinates of the points on the curve.

PlottableSpaceCurveCategory(): Category == Definition where
  L     ==> List
  SEG   ==> Segment
  SF    ==> DoubleFloat
  POINT ==> Point DoubleFloat

  Definition ==> CoercibleTo OutputForm with

    listBranches: % -> L L POINT
      ++ listBranches(c) returns a list of lists of points, representing the
      ++ branches of the curve c.
    xRange: % -> SEG SF
      ++ xRange(c) returns the range of the x-coordinates of the points
      ++ on the curve c.
    yRange: % -> SEG SF
      ++ yRange(c) returns the range of the y-coordinates of the points
      ++ on the curve c.
    zRange: % -> SEG SF
      ++ zRange(c) returns the range of the z-coordinates of the points
      ++ on the curve c.
```

⟨*PSCURVE.dotabb*⟩≡

```
"PSCURVE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PSCURVE" -> "KOERCE"
```

⟨*PSCURVE.dotfull*⟩≡

```
"PlottableSpaceCurveCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*PSCURVE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

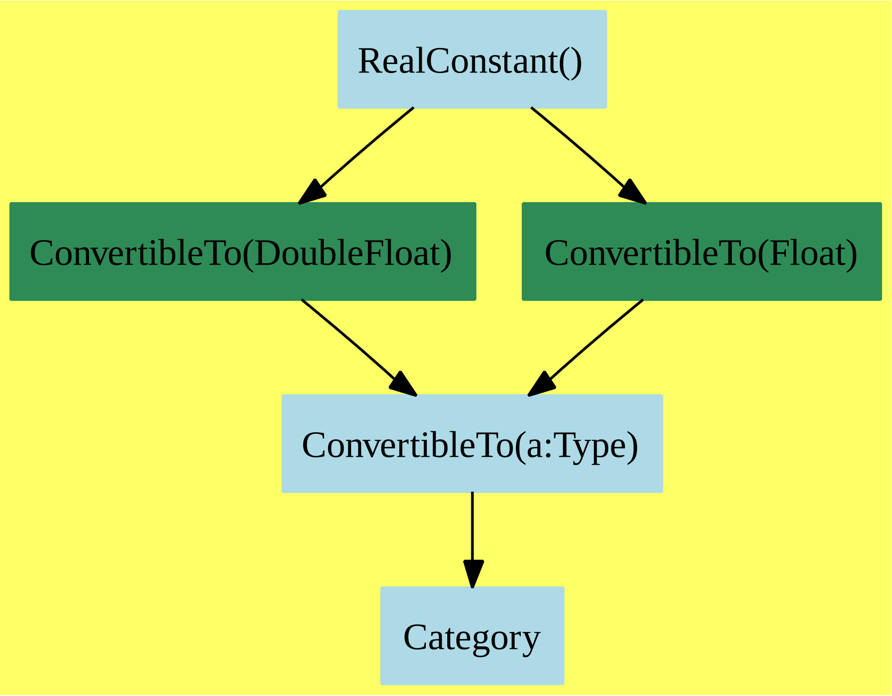
"PlottableSpaceCurveCategory()" [color=lightblue];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.32 RealConstant (REAL)


RealConstant()
ConvertibleTo(DoubleFloat)
ConvertibleTo(Float)
ConvertibleTo(a:Type)
Category

⟨*RealConstant.input*⟩≡

```
)set break resume
)sys rm -f RealConstant.output
)spool RealConstant.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealConstant
--R RealConstant  is a category constructor
--R Abbreviation for RealConstant is REAL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for REAL
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> Float                  convert : % -> DoubleFloat
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*RealConstant.help*⟩≡

```
====================================================================
RealConstant examples
====================================================================

See Also:
o )show RealConstant
```

See:

⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇐ “ConvertibleTo” (KONVERT) 2.0.8 on page 25

Exports:

convert

These exports come from (p25) ConvertibleTo(DoubleFloat):

```
convert : % -> DoubleFloat
```

These exports come from (p25) ConvertibleTo(Float):

```
convert : % -> Float
```

⟨*category REAL RealConstant*⟩≡

```
)abbrev category REAL RealConstant
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of real numeric domains, i.e. convertible to floats.

RealConstant(): Category ==
  Join(ConvertibleTo DoubleFloat, ConvertibleTo Float)
```

⟨REAL.dotabb⟩≡

```
"REAL"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
"REAL" -> "KONVERT"
```

⟨REAL.dotfull⟩≡

```
"RealConstant()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
"RealConstant()" -> "ConvertibleTo(DoubleFloat)"
"RealConstant()" -> "ConvertibleTo(Float)"
```

⟨REAL.dotpic⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealConstant()" [color=lightblue];
"RealConstant()" -> "ConvertibleTo(DoubleFloat)"
"RealConstant()" -> "ConvertibleTo(Float)"

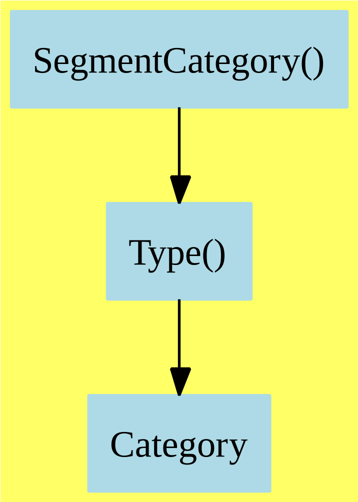
"ConvertibleTo(DoubleFloat)" [color=seagreen];
"ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Float)" [color=seagreen];
"ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color=lightblue];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.33 SegmentCategory (SEGCAT)



⟨*SegmentCategory.input*⟩≡

```
)set break resume
)sys rm -f SegmentCategory.output
)spool SegmentCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SegmentCategory
--R SegmentCategory S: Type  is a category constructor
--R Abbreviation for SegmentCategory is SEGCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEGCAT
--R
--R------------------------------- Operations --------------------------------
--R BY : (%,Integer) -> %                ?..? : (S,S) -> %
--R convert : S -> %                     hi : % -> S
--R high : % -> S                        incr : % -> Integer
--R lo : % -> S                          low : % -> S
--R segment : (S,S) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*SegmentCategory.help*⟩≡
```
====================================================================
SegmentCategory examples
====================================================================

See Also:
o )show SegmentCategory
```

See:

⇒ "SegmentExpansionCategory" (SEGXCAT) 4.0.60 on page 264
⇐ "Type" (TYPE) 2.0.21 on page 75

Exports:

BY	convert	hi	high	incr
lo	low	segment	?..?	

Attributes Exported:

- **nil**

These are directly exported but not implemented:

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
?..? : (S,S) -> %
```

⟨*category SEGCAT SegmentCategory*⟩≡

```
)abbrev category SEGCAT SegmentCategory
++ Author:  Stephen M. Watt
++ Date Created:  December 1986
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: range, segment
++ Examples:
++ References:
++ Description:
++ This category provides operations on ranges, or segments
++ as they are called.

SegmentCategory(S:Type): Category == Type with
    SEGMENT: (S, S) -> %
        ++ \spad{l..h} creates a segment with l and h as the endpoints.
    BY: (%, Integer) -> %
        ++ \spad{s by n} creates a new segment in which only every
        ++ \spad{n}-th element is used.
    lo: % -> S
        ++ lo(s) returns the first endpoint of s.
        ++ Note that \spad{lo(l..h) = l}.
    hi: % -> S
        ++ hi(s) returns the second endpoint of s.
        ++ Note that \spad{hi(l..h) = h}.
    low: % -> S
        ++ low(s) returns the first endpoint of s.
        ++ Note that \spad{low(l..h) = l}.
    high: % -> S
        ++ high(s) returns the second endpoint of s.
        ++ Note that \spad{high(l..h) = h}.
    incr: % -> Integer
        ++ incr(s) returns \spad{n}, where s is a segment in which every
        ++ \spad{n}-th element is used.
        ++ Note that \spad{incr(l..h by n) = n}.
    segment: (S, S) -> %
        ++ segment(i,j) is an alternate way to create the segment
        ++ \spad{i..j}.
    convert: S -> %
        ++ convert(i) creates the segment \spad{i..i}.
```

⟨*SEGCAT.dotabb*⟩≡

```
"SEGCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SEGCAT" -> "TYPE"
```

⟨*SEGCAT.dotfull*⟩≡

```
"SegmentCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(a:Type)" -> "Type()"

"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"
```

⟨*SEGCAT.dotpic*⟩≡

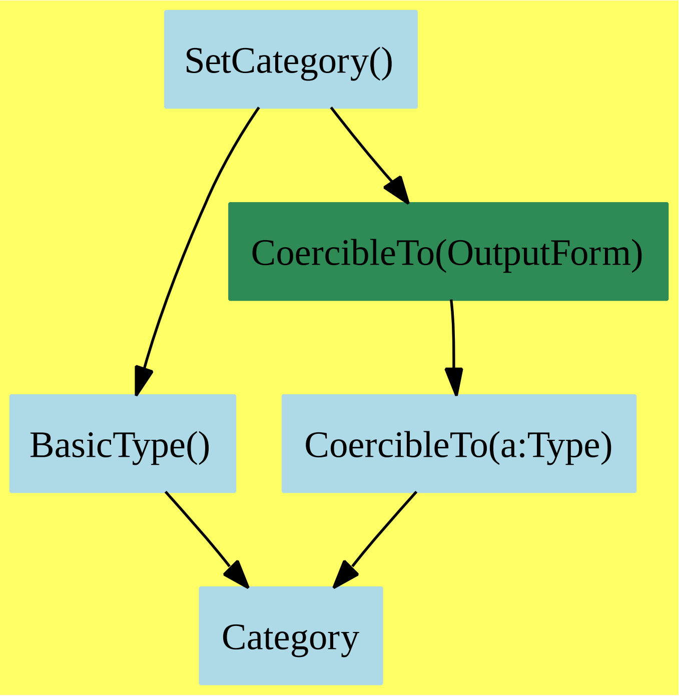
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.34 SetCategory (SETCAT)


SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*SetCategory.input*⟩≡

```
)set break resume
)sys rm -f SetCategory.output
)spool SetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SetCategory
--R SetCategory  is a category constructor
--R Abbreviation for SetCategory is SETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
```

```
--E 1

)spool
)lisp (bye)
```

⟨*SetCategory.help*⟩≡

```
====================================================================
SetCategory examples
====================================================================

See Also:
o )show SetCategory
```

See:

⇒ "AbelianSemiGroup" (ABELSG) 4.0.36 on page 141
⇒ "BlowUpMethodCategory" (BLMETCT) 4.0.37 on page 146
⇒ "FileCategory" (FILECAT) 4.0.45 on page 183
⇒ "FileNameCategory" (FNCAT) 4.0.47 on page 193
⇒ "Finite" (FINITE) 4.0.46 on page 188
⇒ "GradedModule" (GRMOD) 4.0.48 on page 198
⇒ "HomogeneousAggregate" (HOAGG) 4.0.49 on page 203
⇒ "IndexedDirectProductCategory" (IDPC) 4.0.50 on page 210
⇒ "NumericalIntegrationCategory" (NUMINT) 4.0.53 on page 226
⇒ "NumericalOptimizationCategory" (OPTCAT) 4.0.54 on page 232
⇒ "OrderedSet" (ORDSET) 4.0.56 on page 242
⇒ "OrdinaryDifferentialEquationsSolverCategory" (ODECAT) 4.0.55 on page 237

⇒ "PartialDifferentialEquationsSolverCategory" (PDECAT) 4.0.57 on page 247

⇒ "PatternMatchable" (PATMAB) 4.0.58 on page 253
⇒ "PolynomialSetCategory" (PSETCAT) 6.0.91 on page 518
⇒ "RealRootCharacterizationCategory" (RRCC) 4.0.59 on page 258
⇒ "SemiGroup" (SGROUP) 4.0.61 on page 269
⇒ "SetAggregate" (SETAGG) 6.0.94 on page 547
⇒ "SExpressionCategory" (SEXCAT) 4.0.63 on page 277
⇒ "StepThrough" (STEP) 4.0.64 on page 283
⇒ "StringCategory" (STRICAT) 10.0.149 on page 1014
⇒ "ThreeSpaceCategory" (SPACEC) 4.0.65 on page 288
⇐ "BasicType" (BASTYPE) 2.0.5 on page 16
⇐ "CoercibleTo" (KOERCE) 2.0.6 on page 19

Exports:

coerce hash latex ?=? ?~=?

These are implemented by this category:

```
hash : % -> SingleInteger
latex : % -> String
```

These exports come from (p16) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p19) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category SETCAT SetCategory*⟩≡
```
)abbrev category SETCAT SetCategory
++ Author:
++ Date Created:
++ Date Last Updated: November 10, 2009 tpd happy birthday
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{SetCategory} is the basic category for describing a collection
++ of elements with \spadop{=} (equality) and \spadfun{coerce} to
++ output form.
++
++ Conditional Attributes\br
++ \tab{5}canonical\tab{5}data structure equality is the same as \spadop{=}

SetCategory(): Category == Join(BasicType,CoercibleTo OutputForm) with
    hash: % -> SingleInteger
      ++ hash(s) calculates a hash code for s.
    latex: % -> String
      ++ latex(s) returns a LaTeX-printable output
      ++ representation of s.
  add
    hash(s : %):  SingleInteger == SXHASH(s)$Lisp
    latex(s : %): String       == "\mbox{\bf Unimplemented}"
```

⟨*SETCAT.dotabb*⟩≡
```
"SETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
"SETCAT" -> "BASTYPE"
"SETCAT" -> "KOERCE"
```

⟨*SETCAT.dotfull*⟩≡
```
"SetCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*SETCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

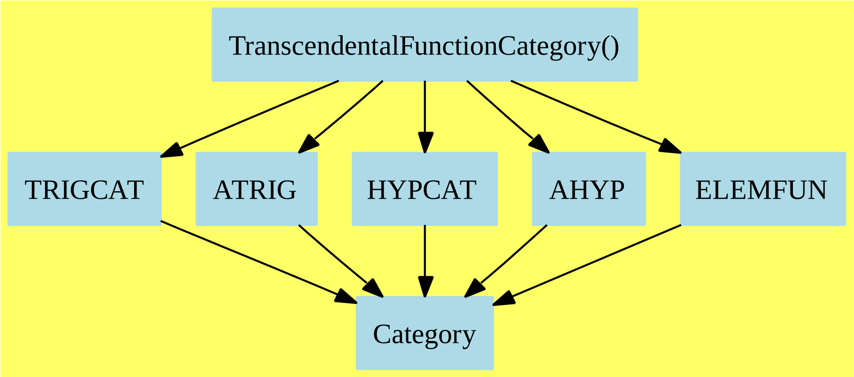
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

3.0.35 TranscendentalFunctionCategory (TRANFUN)


TranscendentalFunctionCategory()
TRIGCAT
ATRIG
HYPCAT
AHYP
ELEMFUN
Category

⟨*TranscendentalFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f TranscendentalFunctionCategory.output
)spool TranscendentalFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TranscendentalFunctionCategory
--R TranscendentalFunctionCategory  is a category constructor
--R Abbreviation for TranscendentalFunctionCategory is TRANFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TRANFUN
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                      acos : % -> %
--R acosh : % -> %                         acot : % -> %
--R acoth : % -> %                         acsc : % -> %
--R acsch : % -> %                         asec : % -> %
--R asech : % -> %                         asin : % -> %
--R asinh : % -> %                         atan : % -> %
--R atanh : % -> %                         cos : % -> %
--R cosh : % -> %                          cot : % -> %
--R coth : % -> %                          csc : % -> %
--R csch : % -> %                          exp : % -> %
--R log : % -> %                           pi : () -> %
--R sec : % -> %                           sech : % -> %
--R sin : % -> %                           sinh : % -> %
--R tan : % -> %                           tanh : % -> %
```

```
--R
--E 1

)spool
)lisp (bye)
```

⟨*TranscendentalFunctionCategory.help*⟩≡

```
====================================================================
TranscendentalFunctionCategory examples
====================================================================

See Also:
o )show TranscendentalFunctionCategory
```

The `acsch`, `asech`, and `acoth` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ "IntervalCategory" (INTCAT) 14.0.176 on page 1270
⇒ "LiouvillianFunctionCategory" (LFCAT) 4.0.51 on page 215
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.0.202 on page 1595
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.0.203 on page 1608
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.0.190 on page 1408
⇐ "ArcHyperbolicFunctionCategory" (AHYP) 2.0.2 on page 5
⇐ "ArcTrigonometricFunctionCategory" (ATRIG) 2.0.3 on page 8
⇐ "ElementaryFunctionCategory" (ELEMFUN) 2.0.9 on page 29
⇐ "HyperbolicFunctionCategory" (HYPCAT) 2.0.11 on page 35
⇐ "TrigonometricFunctionCategory" (TRIGCAT) 2.0.20 on page 71

Exports:

?**?
acsc
asinh
cot
log
sinh
acos
acsch
atan
coth
pi
tan
acosh
asec
atanh
csc
sec
tanh
acot
asech
cos
csch
sech
acoth
asin
cosh
exp
sin

These are implemented by this category:

```
pi : () -> %
```

These exports come from (p71) TrigonometricFunctionCategory():

```
cos : % -> %
cot : % -> %
csc : % -> %
sec : % -> %
sin : % -> %
tan : % -> %
```

These exports come from (p8) ArcTrigonometricFunctionCategory():

```
acos : % -> %
acot : % -> %
acsc : % -> %
asec : % -> %
asin : % -> %
atan : % -> %
```

These exports come from (p35) HyperbolicFunctionCategory():

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

These exports come from (p5) ArcHyperbolicFunctionCategory():

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

These exports come from (p29) ElementaryFunctionCategory():

```
?**? : (%,%) -> %
exp : % -> %
log : % -> %
```

⟨*category TRANFUN TranscendentalFunctionCategory*⟩≡

```
)abbrev category TRANFUN TranscendentalFunctionCategory
++ Category for the transcendental elementary functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the transcendental elementary functions;

TranscendentalFunctionCategory(): Category ==
    Join(TrigonometricFunctionCategory,ArcTrigonometricFunctionCategory,
         HyperbolicFunctionCategory,ArcHyperbolicFunctionCategory,
         ElementaryFunctionCategory) with
           pi : () -> $        ++ pi() returns the constant pi.
   add
     if $ has Ring then
       pi()   == 2*asin(1)
       acsch x ==
         (a := recip x) case "failed" => error "acsch: no reciprocal"
         asinh(a::$)
       asech x ==
         (a := recip x) case "failed" => error "asech: no reciprocal"
         acosh(a::$)
       acoth x ==
         (a := recip x) case "failed" => error "acoth: no reciprocal"
         atanh(a::$)
```

```
      if $ has Field and $ has sqrt: $ -> $ then
        asin x == atan(x/sqrt(1-x**2))
        acos x == pi()/2::$ - asin x
        acot x == pi()/2::$ - atan x
        asinh x == log(x + sqrt(x**2 + 1))
        acosh x == 2*log(sqrt((x+1)/2::$) + sqrt((x-1)/2::$))
        atanh x == (log(1+x)-log(1-x))/2::$
```

⟨*TRANFUN.dotabb*⟩≡

```
"TRANFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TRANFUN" -> "TRIGCAT"
"TRANFUN" -> "ATRIG"
"TRANFUN" -> "HYPCAT"
"TRANFUN" -> "AHYP"
"TRANFUN" -> "ELEMFUN"
```

⟨*TRANFUN.dotfull*⟩≡

```
"TranscendentalFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TranscendentalFunctionCategory()" ->
    "TrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcTrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "HyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcHyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ElementaryFunctionCategory()"
```

⟨*TRANFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"

"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

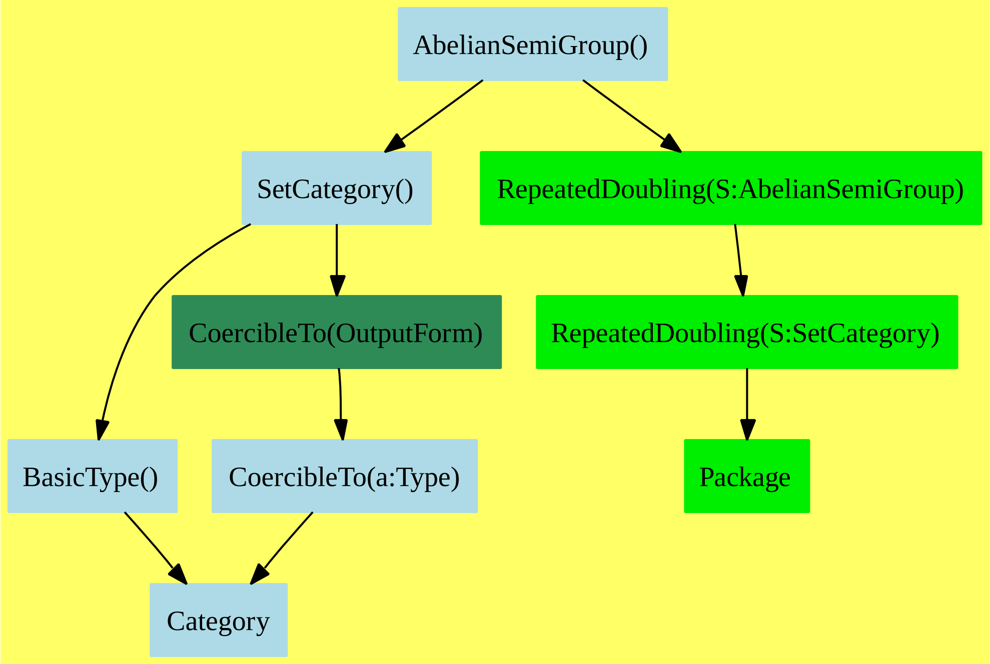
"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 4

Category Layer 3

4.0.36 AbelianSemiGroup (ABELSG)


AbelianSemiGroup()
SetCategory()
RepeatedDoubling(S:AbelianSemiGroup)
CoercibleTo(OutputForm)
RepeatedDoubling(S:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*AbelianSemiGroup.input*⟩≡

```
)set break resume
)sys rm -f AbelianSemiGroup.output
)spool AbelianSemiGroup.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show AbelianSemiGroup
--R AbelianSemiGroup  is a category constructor
--R Abbreviation for AbelianSemiGroup is ABELSG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELSG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*AbelianSemiGroup.help*⟩≡

```
====================================================================
AbelianSemiGroup examples
====================================================================

See Also:
o )show AbelianSemiGroup
```

See:

⇒ "AbelianMonoid" (ABELMON) 5.0.66 on page 301
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	?*?	?+?
?=?	?~=?			

These are directly exported but not implemented:

```
?+? : (%,%) -> %
```

These are implemented by this category:

```
?*? : (PositiveInteger,%) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ABELSG AbelianSemiGroup*⟩≡

```
)abbrev category ABELSG AbelianSemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of all additive (commutative) semigroups, i.e.
++ a set with a commutative and associative operation \spadop{+}.
++
++ Axioms\br
++ \tab{5}\spad{associative("+":(%,%)->%)}\tab{5}\spad{ (x+y)+z = x+(y+z) }\br
++ \tab{6}\spad{commutative("+":(%,%)->%)}\tab{5}\spad{ x+y = y+x }

AbelianSemiGroup(): Category == SetCategory with
      "+": (%,%) -> %                  ++ x+y computes the sum of x and y.
```

```
        "*": (PositiveInteger,%) -> %
          ++ n*x computes the left-multiplication of x by the positive
          ++ integer n. This is equivalent to adding x to itself n times.
      add
        import RepeatedDoubling(%)
        if not (% has Ring) then
          n:PositiveInteger * x:% == double(n,x)
```

⟨ABELSG.dotabb⟩≡

```
  "ABELSG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
  "ABELSG" -> "SETCAT"
  "ABELSG" -> "REPDB"
```

⟨ABELSG.dotfull⟩≡

```
  "AbelianSemiGroup()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
  "AbelianSemiGroup()" -> "SetCategory()"
  "AbelianSemiGroup()" -> "RepeatedDoubling(a:SetCategory)"
```

⟨*ABELSG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

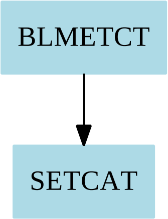
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

4.0.37 BlowUpMethodCategory (BLMETCT)



⟨*BlowUpMethodCategory.input*⟩≡

```
)set break resume
)sys rm -f BlowUpMethodCategory.output
)spool BlowUpMethodCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpMethodCategory
--R BlowUpMethodCategory  is a category constructor
--R Abbreviation for BlowUpMethodCategory is BLMETCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BLMETCT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                 chartCoord : % -> Integer
--R coerce : List Integer -> %             coerce : % -> OutputForm
--R excepCoord : % -> Integer              hash : % -> SingleInteger
--R infClsPt? : % -> Boolean               latex : % -> String
--R quotValuation : % -> Integer           ramifMult : % -> Integer
--R transCoord : % -> Integer              ?~=? : (%,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center
--R type : % -> Union(left,center,right,vertical,horizontal)
--R
--E 1

)spool
)lisp (bye)
```

⟨*BlowUpMethodCategory.help*⟩≡

```
====================================================================
BlowUpMethodCategory examples
====================================================================

See Also:
o )show BlowUpMethodCategory
```

See:

⇐ “BasicType” (BASTYPE) 2.0.5 on page 16
⇐ “CoercibleTo” (KOERCE) 2.0.6 on page 19
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

?=?	?~=?	chartCoord	coerce	createHN
excepCoord	hash	infClsPt?	latex	quotValuation
ramifMult	transCoord	type		

These are directly exported but not implemented:

```
coerce: List Integer -> %
excepCoord: % -> Integer
chartCoord: % -> Integer
transCoord: % -> Integer
createHN:  ( Integer, Integer, Integer , Integer, Integer, Boolean,_
             Union("left","center","right","vertical","horizontal")) -> %
ramifMult: % -> Integer
infClsPt_? : % -> Boolean
quotValuation : % -> Integer
type: % -> Union("left","center","right","vertical","horizontal")
```

These exports come from (p130) SetCategory():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
```

⟨*category BLMETCT BlowUpMethodCategory*⟩≡

```
)abbrev category BLMETCT BlowUpMethodCategory
BlowUpMethodCategory:Category ==  SetCategory with

  coerce: List Integer -> %

  excepCoord: % -> Integer

  chartCoord: % -> Integer

  transCoord: % -> Integer

  createHN:  ( Integer, Integer, Integer , Integer, Integer, Boolean,_
               Union("left","center","right","vertical","horizontal")) -> %

  ramifMult: % -> Integer
  infClsPt_? : % -> Boolean

  quotValuation : % -> Integer

  type: % -> Union("left","center","right","vertical","horizontal")
```

⟨*BLMETCT.dotabb*⟩≡

```
"BLMETCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BLMETCT"];
"BLMETCT" -> "SETCAT"
```

⟨*BLMETCT.dotfull*⟩≡

```
"BlowUpMethodCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BLMETCT"];
"BlowUpMethodCategory()" -> "SetCategory()"
```

⟨*BLMETCT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BlowUpMethodCategory()" [color=lightblue];
"BlowUpMethodCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

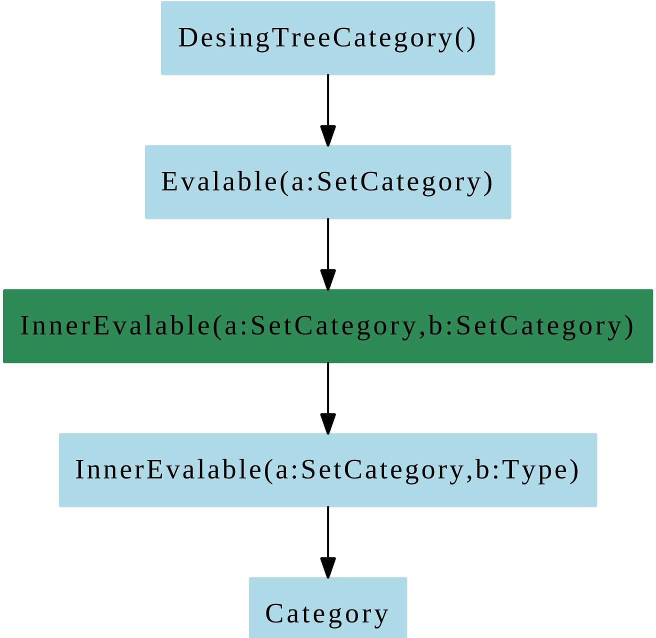
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.38 DesingTreeCategory (DSTRCAT)


DesingTreeCategory()
Evalable(a:SetCategory)
InnerEvalable(a:SetCategory,b:SetCategory)
InnerEvalable(a:SetCategory,b:Type)
Category

⟨*DesingTreeCategory.input*⟩≡

```
)set break resume
)sys rm -f DesingTreeCategory.output
)spool DesingTreeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DesingTreeCategory
--R DesingTreeCategory S: SetCategory  is a category constructor
--R Abbreviation for DesingTreeCategory is DSTRCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DSTRCAT
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %                copy : % -> %
--R cyclic? : % -> Boolean                distance : (%,%) -> Integer
--R ?.value : (%,value) -> S              empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                  leaves : % -> List S
```

```
--R map : ((S -> S),%) -> %              nodes : % -> List %
--R sample : () -> %                     tree : List S -> %
--R tree : S -> %                        tree : (S,List %) -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*DesingTreeCategory.help*⟩≡

```
====================================================================
DesingTreeCategory examples
====================================================================

See Also:
o )show DesingTreeCategory
```

See:

⇒ “Evalable” (EVALAB) 3.0.25 on page 91

Exports:

#?
?.value
?=?
?~=?
any?
child?
children
coerce
copy
count
count
cyclic?
distance
empty
empty?
eq?
eval
every?
hash
latex
leaf?
leaves
less?
map
map!
member?
members
more?
node?
nodes
parts
sample
setchildren!
setelt
setvalue!
size?
tree
value

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
tree : (S,List %) -> %
tree : List S -> %
tree : S -> %
```

These exports come from (p392) RecursiveAggregate(Type)

```
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.value : (%,value) -> S
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
node? : (%,%) -> Boolean if S has SETCAT
nodes : % -> List %
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
```

⟨*category DSTRCAT DesingTreeCategory*⟩≡

```
)abbrev category DSTRCAT DesingTreeCategory
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This category is part of the PAFF package
DesingTreeCategory(S: SetCategory):Category == RecursiveAggregate(S) with

  shallowlyMutable
  finiteAggregate

  tree: (S,List %) -> %
    ++ tree(nd,ls) creates a tree with value nd, and children ls.

  tree: S -> %
    ++ tree(nd) creates a tree with value nd, and no children

  tree: List(S) -> %
    ++ tree(l) creates a chain tree from the list l
```

⟨*DSTRCAT.dotabb*⟩≡
```
"DSTRCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DSTRCAT"];
"EVALAB" [color="#4488FF",href="bookvol10.2.pdf#nameddest=EVALAB"]
"DSTRCAT" -> "EVALAB"
```

⟨*DSTRCAT.dotfull*⟩≡
```
"DesingTreeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DSTRCAT"];
"DesingTreeCategory()" -> "Evalable()"
```

⟨*DSTRCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DesingTreeCategory()" [color=lightblue];
"DesingTreeCategory()" -> "Evalable(a:SetCategory)"

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

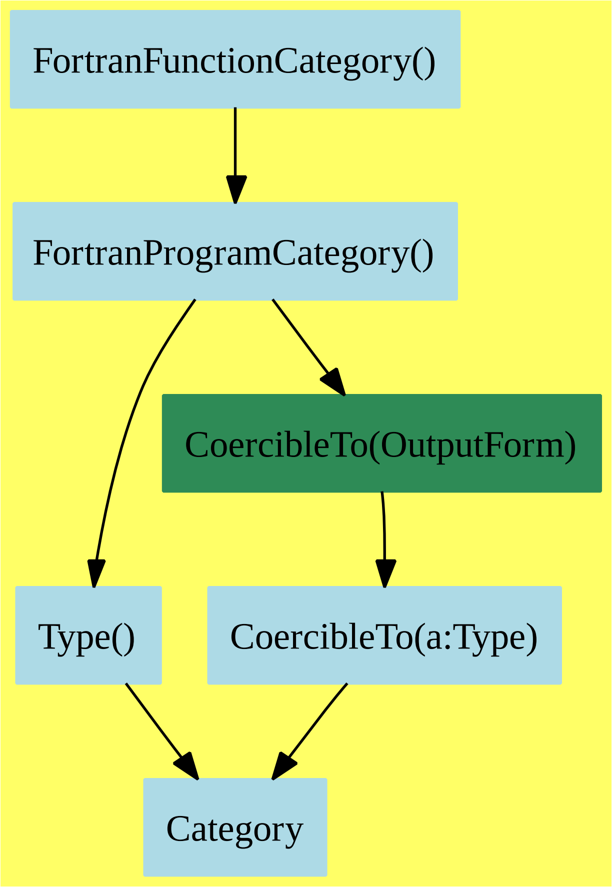
"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.39 FortranFunctionCategory (FORTFN)


FortranFunctionCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f FortranFunctionCategory.output
)spool FortranFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranFunctionCategory
--R FortranFunctionCategory  is a category constructor
--R Abbreviation for FortranFunctionCategory is FORTFN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FORTFN
```

```
--R
--R------------------------------- Operations -------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %     retract : Polynomial Float -> %
--R retract : Expression Integer -> %     retract : Expression Float -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranFunctionCategory.help*⟩≡

```
====================================================================
FortranFunctionCategory examples
====================================================================

See Also:
o )show FortranFunctionCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.26 on page 96

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Expression Float -> %
retract : Expression Integer -> %
retract : Polynomial Float -> %
retract : Polynomial Integer -> %
retract : Fraction Polynomial Integer -> %
retract : Fraction Polynomial Float -> %
retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Polynomial Integer -> Union(%,"failed")
retractIfCan : Polynomial Float -> Union(%,"failed")
retractIfCan : Expression Integer -> Union(%,"failed")
retractIfCan : Expression Float -> Union(%,"failed")
```

These exports come from (p96) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FORTFN FortranFunctionCategory*⟩≡

```
)abbrev category FORTFN FortranFunctionCategory
++ Author: Mike Dewar
++ Date Created: 13 January 1994
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranFunctionCategory} is the category of arguments to
++ NAG Library routines which return (sets of) function values.

FortranFunctionCategory():Category == FortranProgramCategory with
  coerce : List FortranCode -> $
    ++ coerce(e) takes an object from \spadtype{List FortranCode} and
    ++  uses it as the body of an ASP.
```

```
coerce : FortranCode -> $
  ++ coerce(e) takes an object from \spadtype{FortranCode} and
  ++  uses it as the body of an ASP.
coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
  ++ coerce(e) takes the component of \spad{e} from
  ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
  ++ making the declarations in the \spadtype{SymbolTable} component.
retract : Expression Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Expression Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Expression Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Expression Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Fraction Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Fraction Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Fraction Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Fraction Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

-- NB: These ASPs also have a coerce from an appropriate instantiation
--     of FortranExpression.
```

⟨*FORTFN.dotabb*⟩≡

```
"FORTFN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
"FORTFN" -> "FORTCAT"
```

⟨*FORTFN.dotfull*⟩≡

```
"FortranFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
"FortranFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FORTFN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranFunctionCategory()" [color=lightblue];
"FortranFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

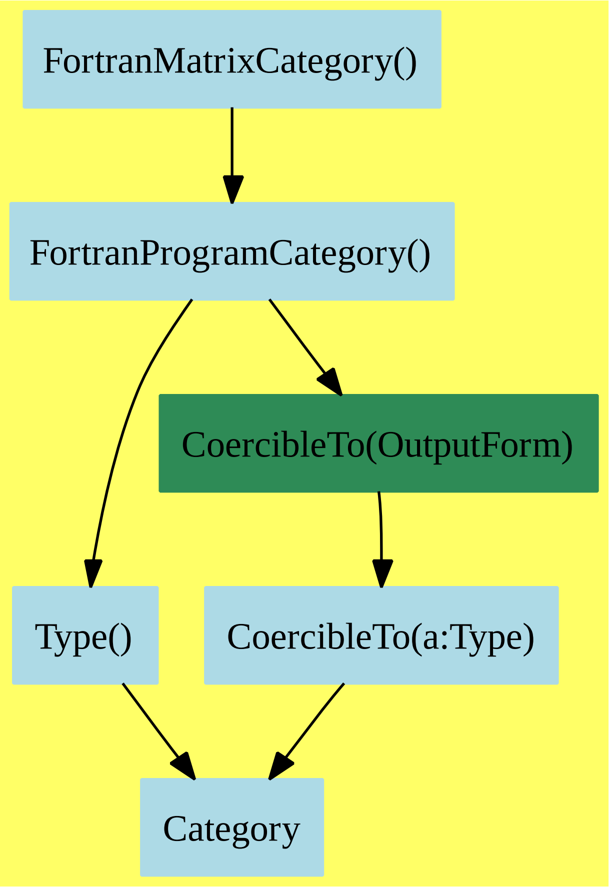
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.40 FortranMatrixCategory (FMC)


FortranMatrixCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranMatrixCategory.input*⟩≡

```
)set break resume
)sys rm -f FortranMatrixCategory.output
)spool FortranMatrixCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranMatrixCategory
--R FortranMatrixCategory  is a category constructor
--R Abbreviation for FortranMatrixCategory is FMC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMC
```

```
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranMatrixCategory.help*⟩≡

```
====================================================================
FortranMatrixCategory examples
====================================================================

See Also:
o )show FortranMatrixCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.26 on page 96

Exports:

coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : Matrix MachineFloat -> %
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p96) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FMC FortranMatrixCategory*⟩≡

```
)abbrev category FMC FortranMatrixCategory
++ Author: Mike Dewar
++ Date Created: 21 March 1994
++ Date Last Updated:
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranMatrixCategory} provides support for
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Matrix} or in domains
++ involving \axiomType{FortranCode}.

FortranMatrixCategory():Category == FortranProgramCategory with
    coerce : Matrix MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
```

⟨*FMC.dotabb*⟩≡

```
"FMC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FMC" -> "FORTCAT"
```

⟨*FMC.dotfull*⟩≡

```
"FortranMatrixCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FortranMatrixCategory()" -> "FortranProgramCategory()"
```

⟨*FMC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMatrixCategory()" [color=lightblue];
"FortranMatrixCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

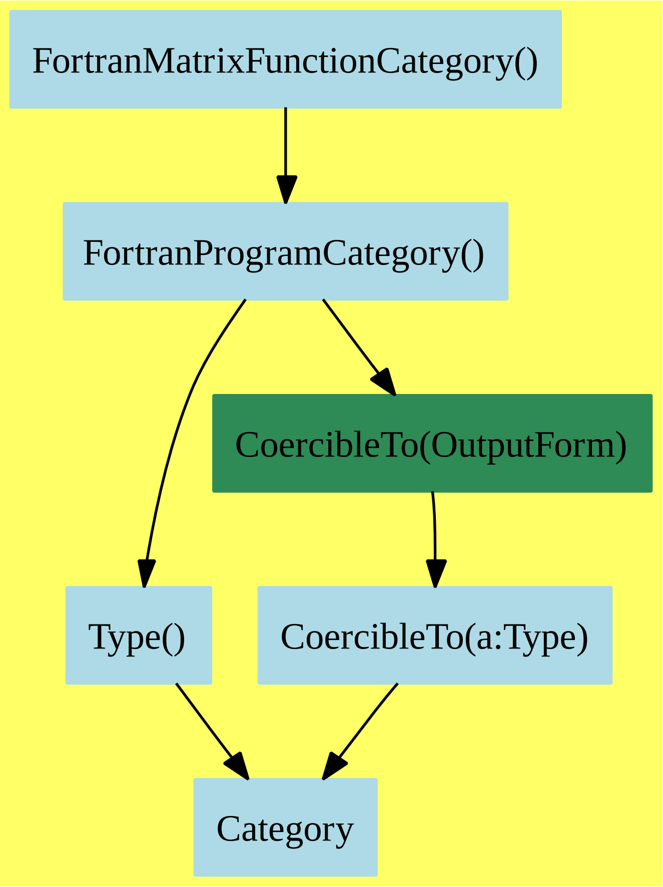
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.41 FortranMatrixFunctionCategory (FMFUN)


FortranMatrixFunctionCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranMatrixFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f FortranMatrixFunctionCategory.output
)spool FortranMatrixFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranMatrixFunctionCategory
--R FortranMatrixFunctionCategory  is a category constructor
--R Abbreviation for FortranMatrixFunctionCategory is FMFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMFUN
```

```
--R
--R------------------------------ Operations -------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranMatrixFunctionCategory.help*⟩≡

```
====================================================================
FortranMatrixFunctionCategory examples
====================================================================

See Also:
o )show FortranMatrixFunctionCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.26 on page 96

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Matrix Expression Float -> %
retract : Matrix Expression Integer -> %
retract : Matrix Polynomial Float -> %
retract : Matrix Polynomial Integer -> %
retract : Matrix Fraction Polynomial Float -> %
retract : Matrix Fraction Polynomial Integer -> %
retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Expression Integer -> Union(%,"failed")
retractIfCan : Matrix Expression Float -> Union(%,"failed")
```

These exports come from (p96) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FMFUN FortranMatrixFunctionCategory*⟩≡

```
)abbrev category FMFUN FortranMatrixFunctionCategory
++ Author: Mike Dewar
++ Date Created: March 18 1994
++ Date Last Updated:
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranMatrixFunctionCategory} provides support for
++ producing Functions and Subroutines representing matrices of
++ expressions.

FortranMatrixFunctionCategory():Category == FortranProgramCategory with
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
```

```
  ++  uses it as the body of an ASP.
coerce : FortranCode -> $
  ++ coerce(e) takes an object from \spadtype{FortranCode} and
  ++  uses it as the body of an ASP.
coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
  ++ coerce(e) takes the component of \spad{e} from
  ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
  ++ making the declarations in the \spadtype{SymbolTable} component.
retract : Matrix Expression Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Expression Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Matrix Expression Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Expression Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Matrix Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Matrix Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Matrix Fraction Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Fraction Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Matrix Fraction Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Matrix Fraction Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

-- NB: These ASPs also have a coerce from an appropriate instantiation
```

```
    --     of Matrix FortranExpression.
```

⟨*FMFUN.dotabb*⟩≡

```
"FMFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
"FMFUN" -> "FORTCAT"
```

⟨*FMFUN.dotfull*⟩≡

```
"FortranMatrixFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
"FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FMFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMatrixFunctionCategory()" [color=lightblue];
"FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

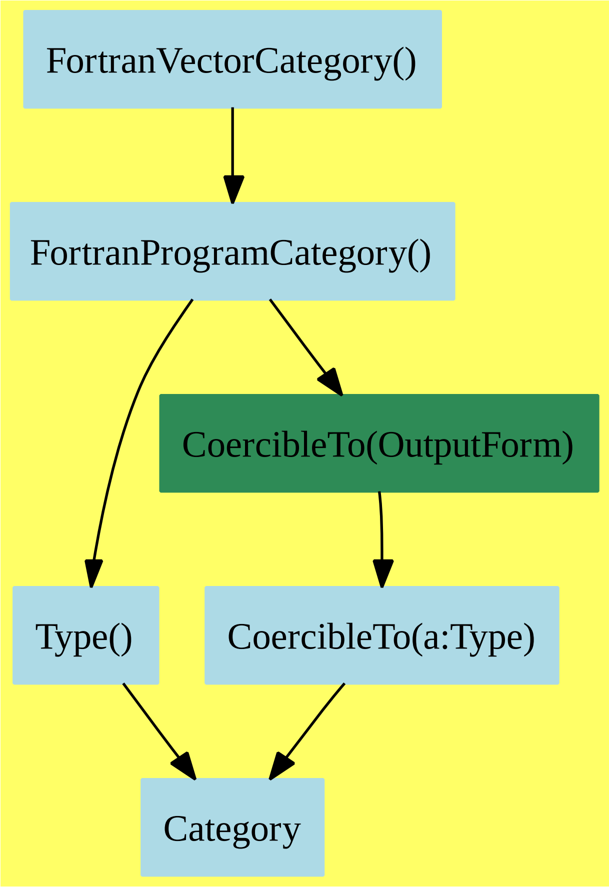
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.42 FortranVectorCategory (FVC)


FortranVectorCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranVectorCategory.input*⟩≡

```
)set break resume
)sys rm -f FortranVectorCategory.output
)spool FortranVectorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranVectorCategory
--R FortranVectorCategory  is a category constructor
--R Abbreviation for FortranVectorCategory is FVC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FVC
```

```
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Vector MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranVectorCategory.help*⟩≡

```
====================================================================
FortranVectorCategory examples
====================================================================

See Also:
o )show FortranVectorCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.26 on page 96

Exports:

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Vector MachineFloat -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p96) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FVC FortranVectorCategory*⟩≡

```
)abbrev category FVC FortranVectorCategory
++ Author: Mike Dewar
++ Date Created: October 1993
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranVectorCategory} provides support for
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Vector} or in domains
++ involving \axiomType{FortranCode}.

FortranVectorCategory():Category == FortranProgramCategory with
    coerce : Vector MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
        ++ making the declarations in the \spadtype{SymbolTable} component.
```

⟨*FVC.dotabb*⟩≡

```
"FVC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FVC" -> "FORTCAT"
```

⟨*FVC.dotfull*⟩≡

```
"FortranVectorCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FortranVectorCategory()" -> "FortranProgramCategory()"
```

⟨*FVC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranVectorCategory()" [color=lightblue];
"FortranVectorCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

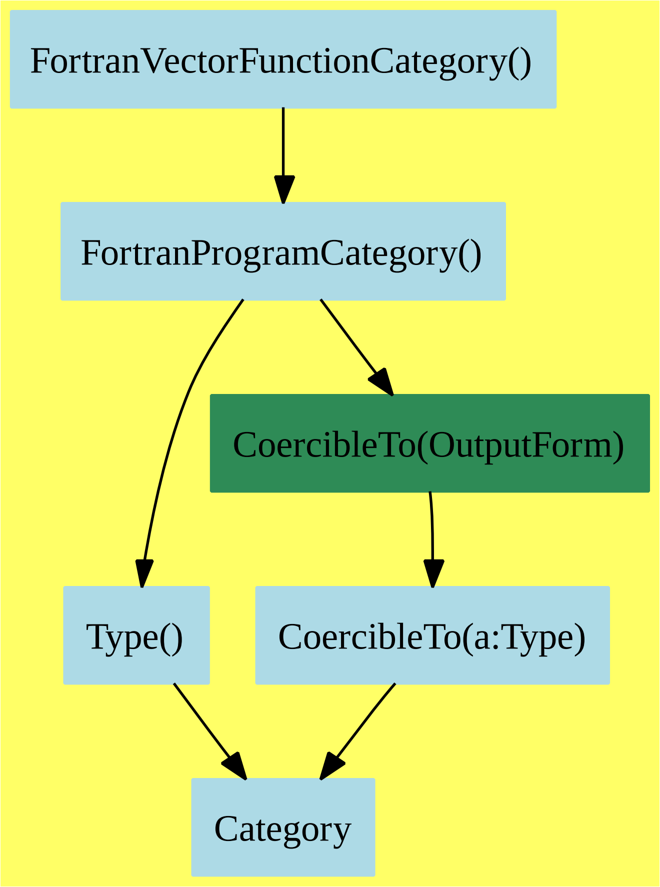
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.43 FortranVectorFunctionCategory (FVFUN)


FortranVectorFunctionCategory()
FortranProgramCategory()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*FortranVectorFunctionCategory.input*⟩≡

```
)set break resume
)sys rm -f FortranVectorFunctionCategory.output
)spool FortranVectorFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranVectorFunctionCategory
--R FortranVectorFunctionCategory  is a category constructor
--R Abbreviation for FortranVectorFunctionCategory is FVFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FVFUN
```

```
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranVectorFunctionCategory.help*⟩≡

```
====================================================================
FortranVectorFunctionCategory examples
====================================================================

See Also:
o )show FortranVectorFunctionCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.26 on page 96

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Vector Fraction Polynomial Integer -> %
retract : Vector Fraction Polynomial Float -> %
retract : Vector Polynomial Integer -> %
retract : Vector Polynomial Float -> %
retract : Vector Expression Integer -> %
retract : Vector Expression Float -> %
retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Expression Integer -> Union(%,"failed")
retractIfCan : Vector Expression Float -> Union(%,"failed")
```

These exports come from (p96) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FVFUN FortranVectorFunctionCategory*⟩≡

```
)abbrev category FVFUN FortranVectorFunctionCategory
++ Author: Mike Dewar
++ Date Created: 11 March 1994
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranVectorFunctionCategory} is the catagory of arguments
++ to NAG Library routines which return the values of vectors of functions.

FortranVectorFunctionCategory():Category == FortranProgramCategory with
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
```

```
coerce : FortranCode -> $
  ++ coerce(e) takes an object from \spadtype{FortranCode} and
  ++  uses it as the body of an ASP.
coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
  ++ coerce(e) takes the component of \spad{e} from
  ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
  ++ making the declarations in the \spadtype{SymbolTable} component.
retract : Vector Expression Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Expression Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Vector Expression Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Expression Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Vector Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Vector Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Vector Fraction Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Fraction Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retract : Vector Fraction Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.
retractIfCan : Vector Fraction Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

-- NB: These ASPs also have a coerce from an appropriate instantiation
--     of Vector FortranExpression.
```

⟨*FVFUN.dotabb*⟩≡
```
"FVFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FVFUN" -> "FORTCAT"
```

⟨*FVFUN.dotfull*⟩≡
```
"FortranVectorFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FVFUN.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranVectorFunctionCategory()" [color=lightblue];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

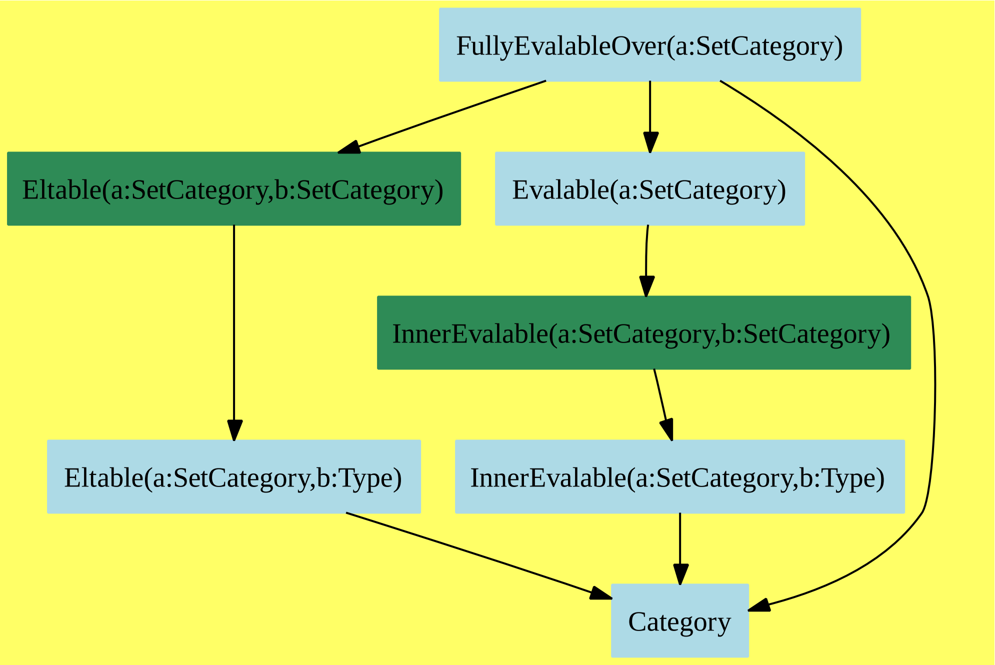
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.44 FullyEvalableOver (FEVALAB)


FullyEvalableOver(a:SetCategory)
Eltable(a:SetCategory,b:SetCategory)
Evalable(a:SetCategory)
InnerEvalable(a:SetCategory,b:SetCategory)
Eltable(a:SetCategory,b:Type)
InnerEvalable(a:SetCategory,b:Type)
Category

⟨*FullyEvalableOver.input*⟩≡

```
)set break resume
)sys rm -f FullyEvalableOver.output
)spool FullyEvalableOver.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyEvalableOver
--R FullyEvalableOver R: SetCategory  is a category constructor
--R Abbreviation for FullyEvalableOver is FEVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FEVALAB
--R
--R------------------------------- Operations --------------------------------
--R map : ((R -> R),%) -> %
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R eval : (%,List R,List R) -> % if R has EVALAB R
--R eval : (%,R,R) -> % if R has EVALAB R
--R eval : (%,Equation R) -> % if R has EVALAB R
```

```
--R eval : (%,List Equation R) -> % if R has EVALAB R
--R eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FullyEvalableOver.help*⟩≡

```
====================================================================
FullyEvalableOver examples
====================================================================

See Also:
o )show FullyEvalableOver
```

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.216 on page 1779
⇒ "OctonionCategory" (OC) 12.0.166 on page 1167
⇒ "QuaternionCategory" (QUATCAT) 12.0.167 on page 1180
⇒ "QuotientFieldCategory" (QFCAT) 17.0.198 on page 1516
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

eval map ?.?

These are directly exported but not implemented:

```
map : ((R -> R),%) -> %
```

These are implemented by this category:

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p91) Evalable(a:Type):

```
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
```

These exports come from (p39) InnerEvalable(a:Symbol,b:SetCategory):

```
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

⟨*category FEVALAB FullyEvalableOver*⟩≡

```
)abbrev category FEVALAB FullyEvalableOver
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains: Equation
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This category provides a selection of evaluation operations
++ depending on what the argument type R provides.
```

```
FullyEvalableOver(R:SetCategory): Category == with
    map: (R -> R, $) -> $
        ++ map(f, ex) evaluates ex, applying f to values of type R in ex.
    if R has Eltable(R, R) then Eltable(R, $)
    if R has Evalable(R) then Evalable(R)
    if R has InnerEvalable(Symbol, R) then InnerEvalable(Symbol, R)
 add
    if R has Eltable(R, R) then
      elt(x:$, r:R) == map(y +-> y(r), x)

    if R has Evalable(R) then
      eval(x:$, l:List Equation R) == map(y +-> eval(y, l), x)

    if R has InnerEvalable(Symbol, R) then
      eval(x:$, ls:List Symbol, lv:List R) == map(y +-> eval(y, ls, lv), x)
```

⟨*FEVALAB.dotabb*⟩≡

```
"FEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FEVALAB" -> "ELTAB"
"FEVALAB" -> "EVALAB"
"FEVALAB" -> "IEVALAB"
"FEVALAB" -> "CATEGORY"
```

⟨*FEVALAB.dotfull*⟩≡

```
"FullyEvalableOver(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:Type)"
"FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Category"

"FullyEvalableOver(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(IntegralDomain)" ->
  "FullyEvalableOver(a:SetCategory)"

"FullyEvalableOver(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(CommutativeRing)" ->
  "FullyEvalableOver(a:SetCategory)"
```

⟨*FEVALAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyEvalableOver(a:SetCategory)" [color=lightblue];
"FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Category"

"Eltable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:SetCategory)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

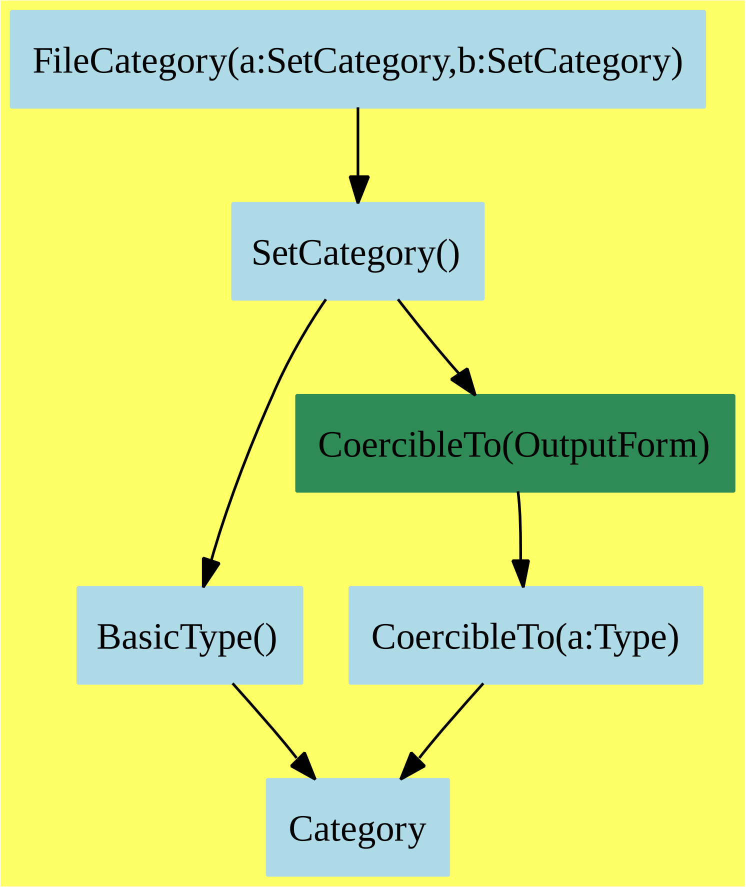
"Evalable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.45 FileCategory (FILECAT)


FileCategory(a:SetCategory,b:SetCategory)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*FileCategory.input*⟩≡

```
)set break resume
)sys rm -f FileCategory.output
)spool FileCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FileCategory
--R FileCategory(Name: SetCategory,S: SetCategory)  is a category constructor
--R Abbreviation for FileCategory is FILECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FILECAT
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                close! : % -> %
--R coerce : % -> OutputForm              flush : % -> Void
--R hash : % -> SingleInteger             iomode : % -> String
--R latex : % -> String                   name : % -> Name
--R open : (Name,String) -> %             open : Name -> %
--R read! : % -> S                        reopen! : (%,String) -> %
--R write! : (%,S) -> S                   ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*FileCategory.help*⟩≡

```
====================================================================
FileCategory examples
====================================================================

See Also:
o )show FileCategory
```

See:

⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

close! coerce hash iomode latex
name open read! reopen! write!
?=? ?~=?

These are directly exported but not implemented:

```
close! : % -> %
iomode : % -> String
name : % -> Name
open : Name -> %
open : (Name,String) -> %
read! : % -> S
reopen! : (%,String) -> %
write! : (%,S) -> S
```

These exports come from SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FILECAT FileCategory*⟩≡

```
)abbrev category FILECAT FileCategory
++ Author: Stephen M. Watt, Victor Miller
++ Date Created:
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: File
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This category provides an interface to operate on files in the
++ computer's file system.  The precise method of naming files
++ is determined by the Name parameter.  The type of the contents
++ of the file is determined by S.

FileCategory(Name, S): Category == FCdefinition where
    Name:      SetCategory
    S:         SetCategory
    IOMode ==> String  -- Union("input", "output", "closed")

    FCdefinition == SetCategory with
        open: Name -> %
          ++ open(s) returns the file s open for input.

        open: (Name, IOMode) -> %
          ++ open(s,mode) returns a file s open for operation in the
          ++ indicated mode: "input" or "output".

        reopen_!: (%, IOMode) -> %
          ++ reopen!(f,mode) returns a file f reopened for operation in the
          ++ indicated mode: "input" or "output".
          ++ \spad{reopen!(f,"input")} will reopen the file f for input.

        close_!: % -> %
          ++ close!(f) returns the file f closed to input and output.

        name: % -> Name
          ++ name(f) returns the external name of the file f.

        iomode: % -> IOMode
          ++ iomode(f) returns the status of the file f. The input/output
          ++ status of f may be "input", "output" or "closed" mode.

        read_!: % -> S
          ++ read!(f) extracts a value from file f.  The state of f is
          ++ modified so a subsequent call to \spadfun{read!} will return
          ++ the next element.

        write_!: (%,S) -> S
          ++ write!(f,s) puts the value s into the file f.
          ++ The state of f is modified so subsequents call to \spad{write!}
          ++ will append one after another.

        flush: % -> Void
          ++ flush(f) makes sure that buffered data is written out
```

⟨*FILECAT.dotabb*⟩≡

```
  "FILECAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
  "FILECAT" -> "SETCAT"
```

⟨*FILECAT.dotfull*⟩≡

```
"FileCategory(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"
```

⟨*FILECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FileCategory(a:SetCategory,b:SetCategory)" [color=lightblue];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

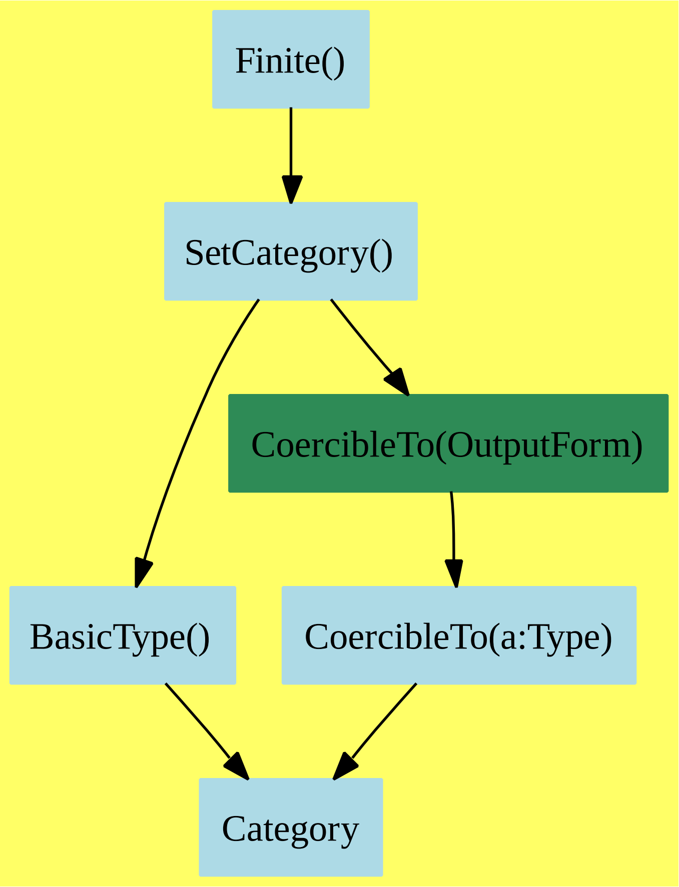
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.46 Finite (FINITE)


Finite()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*Finite.input*⟩≡
```
)set break resume
)sys rm -f Finite.output
)spool Finite.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Finite
--R Finite  is a category constructor
--R Abbreviation for Finite is FINITE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINITE
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             index : PositiveInteger -> %
--R latex : % -> String                   lookup : % -> PositiveInteger
--R random : () -> %                      size : () -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*Finite.help*⟩≡

```
====================================================================
Finite examples
====================================================================

See Also:
o )show Finite
```

See:

⇒ "DirectProductCategory" (DIRPCAT) 12.0.160 on page 1103
⇒ "FiniteFieldCategory" (FFIELDC) 18.0.207 on page 1671
⇒ "OrderedFinite" (ORDFIN) 5.0.77 on page 376
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce hash	index	latex	lookup	random
size	?=?	?~=?		

These are directly exported but not implemented:

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FINITE Finite*⟩≡

```
)abbrev category FINITE Finite
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of domains composed of a finite set of elements.
++ We include the functions \spadfun{lookup} and \spadfun{index}
++ to give a bijection between the finite set and an initial
++ segment of positive integers.
++
++ Axioms:\br
++ \tab{5}\spad{lookup(index(n)) = n}\br
++ \tab{5}\spad{index(lookup(s)) = s}
```

```
Finite(): Category == SetCategory with
      size: () ->  NonNegativeInteger
        ++ size() returns the number of elements in the set.
      index: PositiveInteger -> %
        ++ index(i) takes a positive integer i less than or equal
        ++ to \spad{size()} and
        ++ returns the \spad{i}-th element of the set.
        ++ This operation establishs a bijection
        ++ between the elements of the finite set and \spad{1..size()}.
      lookup: % -> PositiveInteger
        ++ lookup(x) returns a positive integer such that
        ++ \spad{x = index lookup x}.
      random: () -> %
        ++ random() returns a random element from the set.
```

⟨*FINITE.dotabb*⟩≡

```
"FINITE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"FINITE" -> "SETCAT"
```

⟨*FINITE.dotfull*⟩≡

```
"Finite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"Finite()" -> "SetCategory()"
```

⟨*FINITE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

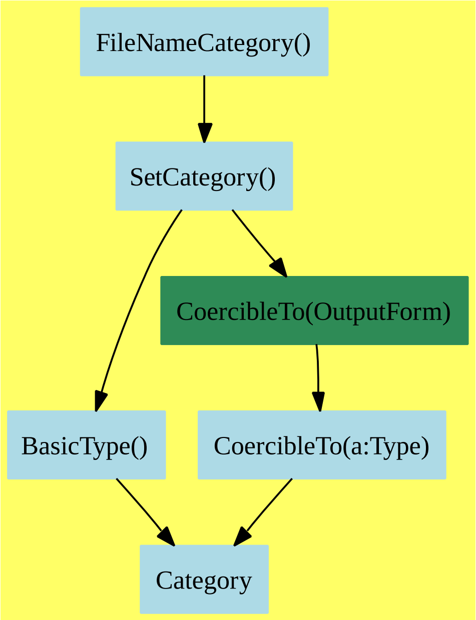
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.47 FileNameCategory (FNCAT)


FileNameCategory()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*FileNameCategory.input*⟩≡

```
)set break resume
)sys rm -f FileNameCategory.output
)spool FileNameCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FileNameCategory
--R FileNameCategory  is a category constructor
--R Abbreviation for FileNameCategory is FNCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FNCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> String
--R coerce : String -> %                  coerce : % -> OutputForm
--R directory : % -> String               exists? : % -> Boolean
--R extension : % -> String               hash : % -> SingleInteger
--R latex : % -> String                   name : % -> String
--R new : (String,String,String) -> %     readable? : % -> Boolean
--R writable? : % -> Boolean              ?~=? : (%,%) -> Boolean
```

```
--R filename : (String,String,String) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*FileNameCategory.help*⟩≡

```
====================================================================
FileNameCategory examples
====================================================================

See Also:
o )show FileNameCategory
```

See:

⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

coerce directory exists? extension filename
hash latex name new readable?
writable? ?=? ?~=?

These are directly exported but not implemented:

```
coerce : String -> %
coerce : % -> String
directory : % -> String
exists? : % -> Boolean
extension : % -> String
filename : (String,String,String) -> %
name : % -> String
new : (String,String,String) -> %
readable? : % -> Boolean
writable? : % -> Boolean
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FNCAT FileNameCategory*⟩≡

```
)abbrev category FNCAT FileNameCategory
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 20, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This category provides an interface to names in the file system.

FileNameCategory(): Category == SetCategory with
   coerce: String -> %
     ++ coerce(s) converts a string to a file name
```

```
      ++ according to operating system-dependent conventions.
    coerce: % -> String
      ++ coerce(fn) produces a string for a file name
      ++ according to operating system-dependent conventions.
    filename: (String, String, String) -> %
      ++ filename(d,n,e) creates a file name with
      ++ d as its directory, n as its name and e as its extension.
      ++ This is a portable way to create file names.
      ++ When d or t is the empty string, a default is used.
    directory: % -> String
      ++ directory(f) returns the directory part of the file name.
    name: % -> String
      ++ name(f) returns the name part of the file name.
    extension: % -> String
      ++ extension(f) returns the type part of the file name.
    exists?: % -> Boolean
      ++ exists?(f) tests if the file exists in the file system.
    readable?: % -> Boolean
      ++ readable?(f) tests if the named file exist and can it be opened
      ++ for reading.
    writable?: % -> Boolean
      ++ writable?(f) tests if the named file be opened for writing.
      ++ The named file need not already exist.
    new: (String, String, String) -> %
      ++ new(d,pref,e) constructs the name of a new writable file with
      ++ d as its directory, pref as a prefix of its name and
      ++ e as its extension.
      ++ When d or t is the empty string, a default is used.
      ++ An error occurs if a new file cannot be written in the given
      ++ directory.
```

⟨*FNCAT.dotabb*⟩≡

```
"FNCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FNCAT" -> "SETCAT"
```

⟨*FNCAT.dotfull*⟩≡

```
"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FileNameCategory()" -> "SetCategory()"
```

⟨*FNCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FileNameCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

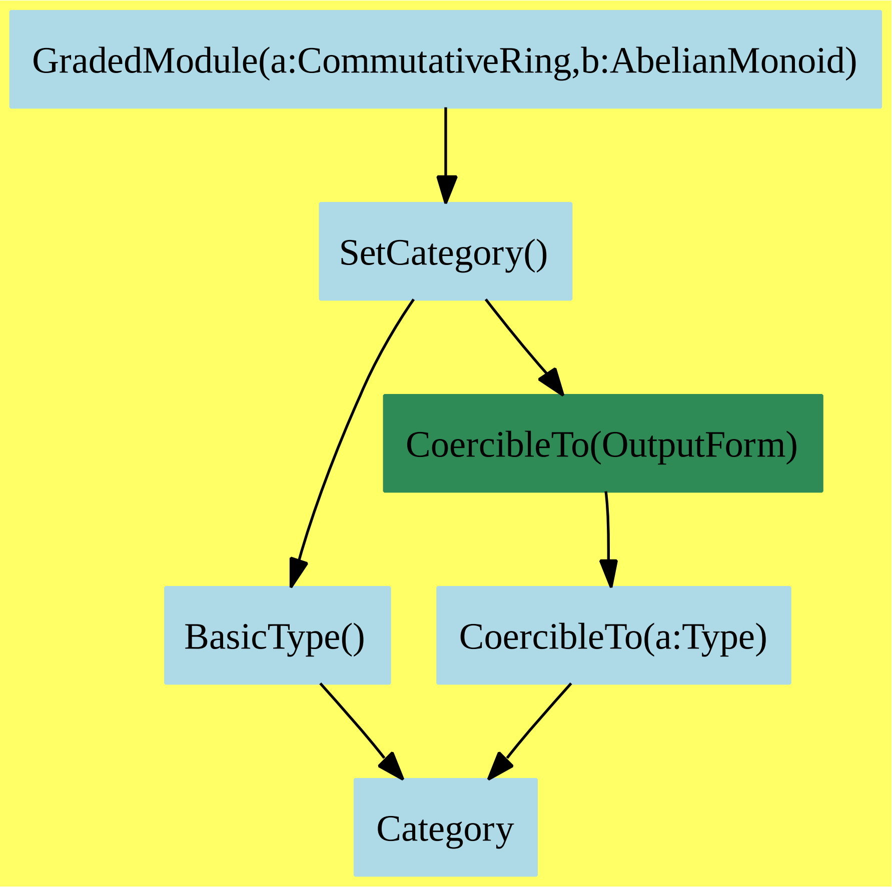
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.48 GradedModule (GRMOD)


GradedModule(a:CommutativeRing,b:AbelianMonoid)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*GradedModule.input*⟩≡

```
)set break resume
)sys rm -f GradedModule.output
)spool GradedModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GradedModule
--R GradedModule(R: CommutativeRing,E: AbelianMonoid)  is a category constructor
--R Abbreviation for GradedModule is GRMOD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GRMOD
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R degree : % -> E                       hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*GradedModule.help*⟩≡

```
====================================================================
GradedModule examples
====================================================================

See Also:
o )show GradedModule
```

See:

⇒ “GradedAlgebra” (GRALG) 5.0.73 on page 351
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

0 coerce degree hash latex
?~=? ?*? ?+? ?-? -?
?=?

These are directly exported but not implemented:

```
0 : () -> %
degree : % -> E
?*? : (%,R) -> %
?*? : (R,%) -> %
-? : % -> %
?+? : (%,%) -> %
```

These are implemented by this category:

```
?-? : (%,%) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category GRMOD GradedModule*⟩≡

```
)abbrev category GRMOD GradedModule
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ Basic Operations: +, *, degree
++ Related Domains: CartesianTensor(n,dim,R)
++ Also See:
++ AMS Classifications:
++ Keywords: graded module, tensor, multi-linear algebra
++ Examples:
++ References: Algebra 2d Edition, MacLane and Birkhoff, MacMillan 1979
++ Description:
++ GradedModule(R,E) denotes ``E-graded R-module'', i.e. collection of
++ R-modules indexed by an abelian monoid E.
++ An element \spad{g} of \spad{G[s]} for some specific \spad{s} in \spad{E}
++ is said to be an element of \spad{G} with degree \spad{s}.
++ Sums are defined in each module \spad{G[s]} so two elements of \spad{G}
++ have a sum if they have the same degree.
++
++ Morphisms can be defined and composed by degree to give the
++ mathematical category of graded modules.

GradedModule(R: CommutativeRing, E: AbelianMonoid): Category ==
    SetCategory with
        degree: % -> E
            ++ degree(g) names the degree of g.  The set of all elements
            ++ of a given degree form an R-module.
        0: constant -> %
            ++ 0 denotes the zero of degree 0.
        _*: (R, %) -> %
            ++ r*g is left module multiplication.
        _*: (%, R) -> %
            ++ g*r is right module multiplication.

        _-: % -> %
            ++ -g is the additive inverse of g in the module of elements
            ++ of the same grade as g.
        _+: (%, %) -> %
            ++ g+h is the sum of g and h in the module of elements of
            ++ the same degree as g and h.  Error: if g and h
            ++ have different degrees.
        _-: (%, %) -> %
            ++ g-h is the difference of g and h in the module of elements of
            ++ the same degree as g and h.  Error: if g and h
            ++ have different degrees.
  add
        (x: %) - (y: %) == x+(-y)
```

⟨*GRMOD.dotabb*⟩≡

```
"GRMOD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
"GRMOD" -> "SETCAT"
```

⟨*GRMOD.dotfull*⟩≡

```
"GradedModule(a:CommutativeRing,b:AbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"
```

⟨*GRMOD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

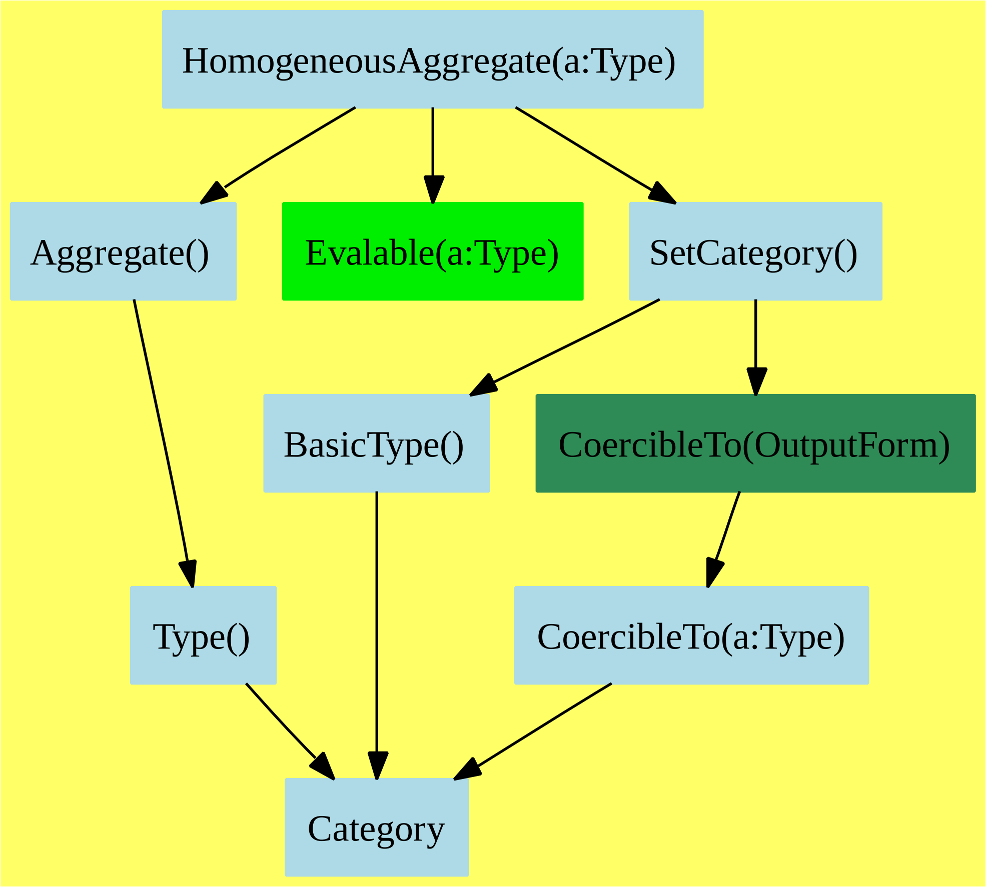
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.49 HomogeneousAggregate (HOAGG)


HomogeneousAggregate(a:Type)
Aggregate()
Evalable(a:Type)
SetCategory()
BasicType()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*HomogeneousAggregate.input*⟩≡

```
)set break resume
)sys rm -f HomogeneousAggregate.output
)spool HomogeneousAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HomogeneousAggregate
--R HomogeneousAggregate S: Type  is a category constructor
--R Abbreviation for HomogeneousAggregate is HOAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for HOAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                        empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R map : ((S -> S),%) -> %              sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*HomogeneousAggregate.help*⟩≡

```
====================================================================
HomogeneousAggregate examples
====================================================================

See Also:
o )show HomogeneousAggregate
```

See:

⇒ “BagAggregate” (BGAGG) 5.0.68 on page 312
⇒ “Collection” (CLAGG) 5.0.70 on page 322
⇒ “IndexedAggregate” (IXAGG) 5.0.74 on page 356
⇒ “RectangularMatrixCategory” (RMATCAT) 10.0.147 on page 994
⇒ “RecursiveAggregate” (RCAGG) 5.0.80 on page 392
⇒ “TwoDimensionalArrayCategory” (ARR2CAT) 5.0.81 on page 399
⇐ “Aggregate” (AGG) 3.0.22 on page 77
⇐ “Evalable” (EVALAB) 3.0.25 on page 91
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Attributes exported:

- **nil**

Exports:

any?	coerce	copy	count	empty
empty?	eq?	eval	every?	hash
latex	less?	map	map!	members
member?	more?	parts	sample	size?
#?	?=?	?~=?		

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
parts : % -> List S if $ has finiteAggregate
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
```

```
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p77) Aggregate:

```
copy : % -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

These exports come from (p91) Evalable(a:Type):

```
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

These exports come from (p130) SetCategory():

```
hash : % -> SingleInteger if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
latex : % -> String if S has SETCAT
```

⟨*category HOAGG HomogeneousAggregate*⟩≡

```
)abbrev category HOAGG HomogeneousAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991, May 1995
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A homogeneous aggregate is an aggregate of elements all of the
++ same type.
++ In the current system, all aggregates are homogeneous.
++ Two attributes characterize classes of aggregates.
++ Aggregates from domains with attribute \spadatt{finiteAggregate}
++ have a finite number of members.
++ Those with attribute \spadatt{shallowlyMutable} allow an element
++ to be modified or updated without changing its overall value.

HomogeneousAggregate(S:Type): Category == Aggregate with
   if S has SetCategory then SetCategory
   if S has SetCategory then
      if S has Evalable S then Evalable S
   map           : (S->S,%) -> %
     ++ map(f,u) returns a copy of u with each element x replaced by f(x).
     ++ For collections, \axiom{map(f,u) = [f(x) for x in u]}.
   if % has shallowlyMutable then
     map_!: (S->S,%) -> %
        ++ map!(f,u) destructively replaces each element x of u
        ++ by \axiom{f(x)}.
   if % has finiteAggregate then
      any?: (S->Boolean,%) -> Boolean
        ++ any?(p,u) tests if \axiom{p(x)} is true for any element x of u.
        ++ Note that for collections,
        ++ \axiom{any?(p,u) = reduce(or,map(f,u),false,true)}.
      every?: (S->Boolean,%) -> Boolean
        ++ every?(f,u) tests if p(x) is true for all elements x of u.
        ++ Note that for collections,
        ++ \axiom{every?(p,u) = reduce(and,map(f,u),true,false)}.
      count: (S->Boolean,%) -> NonNegativeInteger
        ++ count(p,u) returns the number of elements x in u
        ++ such that \axiom{p(x)} is true. For collections,
        ++ \axiom{count(p,u) = reduce(+,[1 for x in u | p(x)],0)}.
      parts: % -> List S
        ++ parts(u) returns a list of the consecutive elements of u.
        ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.
      members: % -> List S
        ++ members(u) returns a list of the consecutive elements of u.
        ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.
      if S has SetCategory then
        count: (S,%) -> NonNegativeInteger
          ++ count(x,u) returns the number of occurrences of x in u. For
          ++ collections, \axiom{count(x,u) = reduce(+,[x=y for y in u],0)}.
        member?: (S,%) -> Boolean
          ++ member?(x,u) tests if x is a member of u.
          ++ For collections,
          ++ \axiom{member?(x,u) = reduce(or,[x=y for y in u],false)}.
  add
```

```
if S has Evalable S then
  eval(u:%,l:List Equation S):% == map(x +-> eval(x,l),u)
if % has finiteAggregate then
  #c          == # parts c
  any?(f, c)  == _or/[f x for x in parts c]
  every?(f, c) == _and/[f x for x in parts c]
  count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]
  members x                  == parts x
  if S has SetCategory then
    count(s:S, x:%) == count(y +-> s = y, x)
    member?(e, c)   == any?(x +-> e = x,c)
    x = y ==
       size?(x, #y) and _and/[a = b for a in parts x for b in parts y]
    coerce(x:%):OutputForm ==
      bracket
         commaSeparate [a::OutputForm for a in parts x]$List(OutputForm)
```

⟨*HOAGG.dotabb*⟩≡

```
"HOAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HOAGG" -> "AGG"
```

⟨*HOAGG.dotfull*⟩≡

```
"HomogeneousAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"HomogeneousAggregate(Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(Ring)"
  -> "HomogeneousAggregate(a:Type)"
```

⟨*HOAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

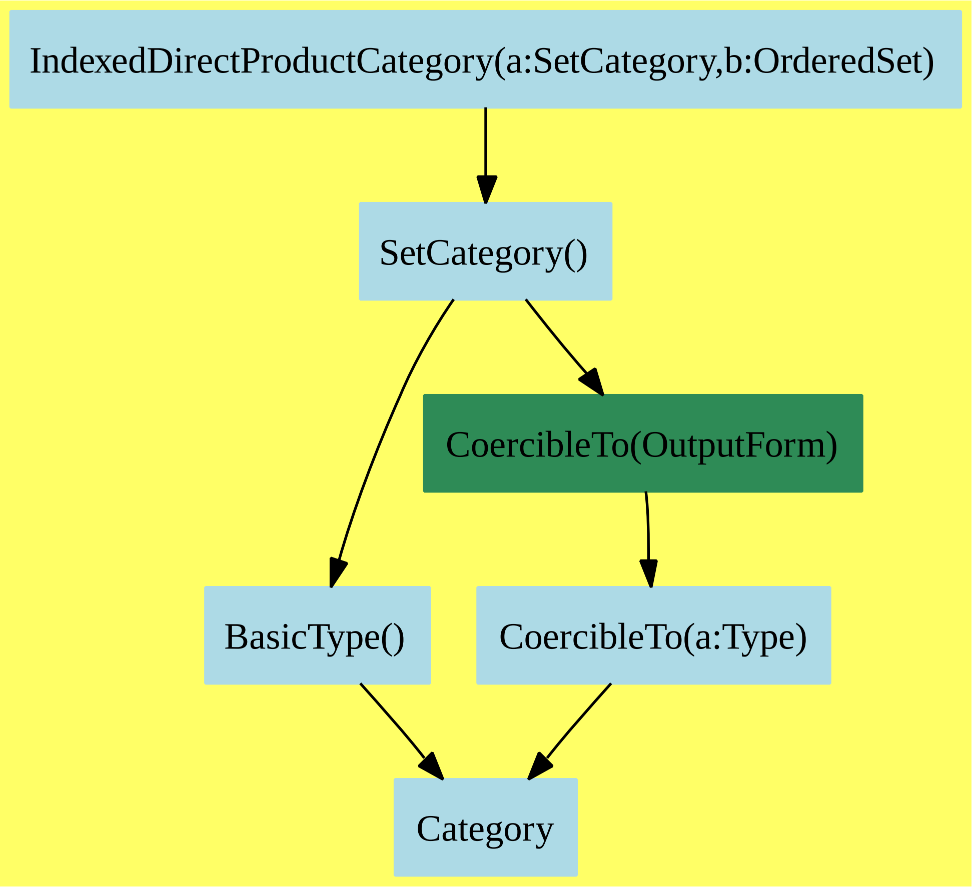
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

4.0.50 IndexedDirectProductCategory (IDPC)


IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*IndexedDirectProductCategory.input*⟩≡

```
)set break resume
)sys rm -f IndexedDirectProductCategory.output
)spool IndexedDirectProductCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductCategory
--R IndexedDirectProductCategory(A: SetCategory,S: OrderedSet)  is a category con
--R Abbreviation for IndexedDirectProductCategory is IDPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IDPC
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> A           leadingSupport : % -> S
--R map : ((A -> A),%) -> %               monomial : (A,S) -> %
--R reductum : % -> %                     ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*IndexedDirectProductCategory.help*⟩≡

```
====================================================================
IndexedDirectProductCategory examples
====================================================================

See Also:
o )show IndexedDirectProductCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	leadingCoefficient	leadingSupport
map	monomial	reductum	?=?	?~=?

These are directly exported but not implemented:

```
leadingCoefficient : % -> A
leadingSupport : % -> S
map : ((A -> A),%) -> %
monomial : (A,S) -> %
reductum : % -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category IDPC IndexedDirectProductCategory*⟩≡

```
)abbrev category IDPC IndexedDirectProductCategory
++ Author: James Davenport
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category represents the direct product of some set with
++ respect to an ordered indexing set.

IndexedDirectProductCategory(A:SetCategory,S:OrderedSet): Category ==
  SetCategory with
    map:           (A -> A, %) -> %
       ++ map(f,z) returns the new element created by applying the
       ++ function f to each component of the direct product element z.
    monomial:         (A, S) -> %
       ++ monomial(a,s) constructs a direct product element with the s
       ++ component set to \spad{a}
```

```
      leadingCoefficient:   % -> A
         ++ leadingCoefficient(z) returns the coefficient of the leading
         ++ (with respect to the ordering on the indexing set)
         ++ monomial of z.
         ++ Error: if z has no support.
      leadingSupport:   % -> S
         ++ leadingSupport(z) returns the index of leading
         ++ (with respect to the ordering on the indexing set) monomial of z.
         ++ Error: if z has no support.
      reductum:      % -> %
         ++ reductum(z) returns a new element created by removing the
         ++ leading coefficient/support pair from the element z.
         ++ Error: if z has no support.
```

⟨*IDPC.dotabb*⟩≡

```
  "IDPC"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
  "IDPC" -> "SETCAT"
```

⟨*IDPC.dotfull*⟩≡

```
  "IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
  "IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
     "SetCategory()"
```

⟨*IDPC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" [color=lightblue];
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

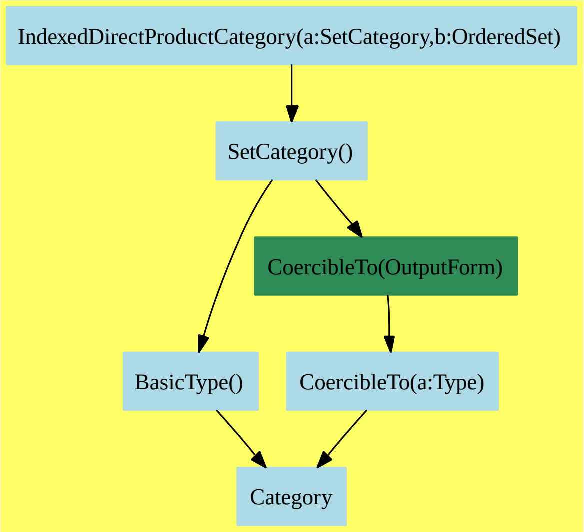
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.51 LiouvillianFunctionCategory (LFCAT)


IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*LiouvillianFunctionCategory.input*⟩≡
```
)set break resume
)sys rm -f LiouvillianFunctionCategory.output
)spool LiouvillianFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LiouvillianFunctionCategory
--R LiouvillianFunctionCategory  is a category constructor
--R Abbreviation for LiouvillianFunctionCategory is LFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                     Ci : % -> %
--R Ei : % -> %                           Si : % -> %
--R acos : % -> %                         acosh : % -> %
--R acot : % -> %                         acoth : % -> %
--R acsc : % -> %                         acsch : % -> %
--R asec : % -> %                         asech : % -> %
--R asin : % -> %                         asinh : % -> %
--R atan : % -> %                         atanh : % -> %
--R cos : % -> %                          cosh : % -> %
```

```
--R cot : % -> %                          coth : % -> %
--R csc : % -> %                          csch : % -> %
--R dilog : % -> %                        erf : % -> %
--R exp : % -> %                          fresnelC : % -> %
--R fresnelS : % -> %                     integral : (%,Symbol) -> %
--R li : % -> %                            log : % -> %
--R pi : () -> %                          sec : % -> %
--R sech : % -> %                         sin : % -> %
--R sinh : % -> %                          tan : % -> %
--R tanh : % -> %
--R integral : (%,SegmentBinding %) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*LiouvillianFunctionCategory.help*⟩≡

```
====================================================================
LiouvillianFunctionCategory examples
====================================================================

See Also:
o )show LiouvillianFunctionCategory
```

See:

⇐ “PrimitiveFunctionCategory” (PRIMCAT) 2.0.16 on page 56
⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135

Exports:

Ci	Ei	Si	acos	acosh
acot	acoth	acsc	acsch	asec
asech	asin	asinh	atan	atanh
cos	cosh	cot	coth	csc
csch	dilog	erf	exp	frsenelC
frsenelS	integral	li	log	pi
sec	sech	sin	sinh	tan
tanh	?**?			

These are directly exported but not implemented:

```
Ci : % -> %
dilog : % -> %
Ei : % -> %
erf : % -> %
li : % -> %
Si : % -> %
fresnelC : % -> %
fresnelS : % -> %
```

These exports come from (p56) PrimitiveFunctionCategory()

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

These exports come from (p135) TranscendentalFunctionCategory():

```
?**? : (%,%) -> %
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
```

```
cot : % -> %
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
```

⟨*category LFCAT LiouvillianFunctionCategory*⟩≡

```
)abbrev category LFCAT LiouvillianFunctionCategory
++ Category for the transcendental Liouvillian functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the transcendental Liouvillian functions;

LiouvillianFunctionCategory(): Category ==
  Join(PrimitiveFunctionCategory, TranscendentalFunctionCategory) with
    Ei      : $  -> $
      ++ Ei(x) returns the exponential integral of x, i.e.
      ++ the integral of \spad{exp(x)/x dx}.
    Si      : $  -> $
      ++ Si(x) returns the sine integral of x, i.e.
      ++ the integral of \spad{sin(x) / x dx}.
    Ci      : $  -> $
      ++ Ci(x) returns the cosine integral of x, i.e.
      ++ the integral of \spad{cos(x) / x dx}.
    li      : $  -> $
      ++ li(x) returns the logarithmic integral of x, i.e.
      ++ the integral of \spad{dx / log(x)}.
    dilog   : $  -> $
      ++ dilog(x) returns the dilogarithm of x, i.e.
      ++ the integral of \spad{log(x) / (1 - x) dx}.
    erf     : $  -> $
      ++ erf(x) returns the error function of x, i.e.
      ++ \spad{2 / sqrt(%pi)} times the integral of \spad{exp(-x**2) dx}.
    fresnelS : $ -> $
      ++ fresnelS is the Fresnel integral S, defined by
      ++ S(x) = integrate(sin(t^2),t=0..x)
    fresnelC : $ -> $
```

```
++ fresnelC is the Fresnel integral C, defined by
++ C(x) = integrate(cos(t^2),t=0..x)
```

⟨*LFCAT.dotabb*⟩≡

```
"LFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LFCAT" -> "PRIMCAT"
"LFCAT" -> "TRANFUN"
```

⟨*LFCAT.dotfull*⟩≡

```
"LiouvillianFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"
```

⟨*LFCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LiouvillianFunctionCategory()" [color=lightblue];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"

"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

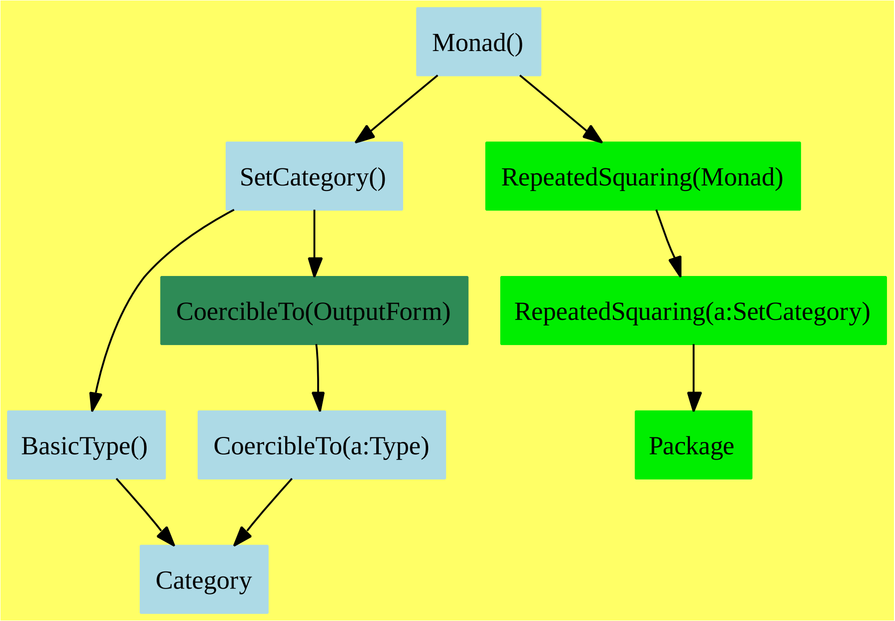
"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

4.0.52 Monad (MONAD)


Monad()
SetCategory()
RepeatedSquaring(Monad)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*Monad.input*⟩≡

```
)set break resume
)sys rm -f Monad.output
)spool Monad.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Monad
--R Monad  is a category constructor
--R Abbreviation for Monad is MONAD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONAD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R
```

```
--E 1

)spool
)lisp (bye)
```

⟨*Monad.help*⟩≡

```
====================================================================
Monad examples
====================================================================

See Also:
o )show Monad
```

See:

⇒ "MonadWithUnit" (MONADWU) 5.0.75 on page 365
⇒ "NonAssociativeRng" (NARNG) 8.0.116 on page 749

Exports:

coerce	hash	latex	leftPower	rightPower
?**?	?*?	?=?	?~=?	

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category MONAD Monad*⟩≡

```
)abbrev category MONAD Monad
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: *, **
++ Related Constructors: SemiGroup, Monoid, MonadWithUnit
++ Also See:
++ AMS Classifications:
++ Keywords: Monad,  binary operation
++ Reference:
++  N. Jacobson: Structure and Representations of Jordan Algebras
++  AMS, Providence, 1968
++ Description:
++ Monad is the class of all multiplicative monads, i.e. sets
++ with a binary operation.

Monad(): Category == SetCategory with
      "*": (%,%) -> %
        ++ a*b is the product of \spad{a} and b in a set with
```

```
          ++ a binary operation.
        rightPower: (%,PositiveInteger) -> %
          ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
          ++ i.e. \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
          ++ \spad{rightPower(a,1) := a}.
        leftPower: (%,PositiveInteger) -> %
          ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
          ++ i.e. \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
          ++ \spad{leftPower(a,1) := a}.
        "**": (%,PositiveInteger) -> %
          ++ a**n returns the \spad{n}-th power of \spad{a},
          ++ defined by repeated squaring.
      add
        import RepeatedSquaring(%)
        x:% ** n:PositiveInteger == expt(x,n)
        rightPower(a,n) ==
--         one? n => a
          (n = 1) => a
          res := a
          for i in 1..(n-1) repeat res := res * a
          res
        leftPower(a,n) ==
--         one? n => a
          (n = 1) => a
          res := a
          for i in 1..(n-1) repeat res := a * res
          res
```

⟨*MONAD.dotabb*⟩≡

```
  "MONAD"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
  "MONAD" -> "SETCAT"
```

⟨*MONAD.dotfull*⟩≡

```
  "Monad()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
  "Monad()" -> "SetCategory()"
  "Monad()" -> "RepeatedSquaring(Monad)"
```

⟨*MONAD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

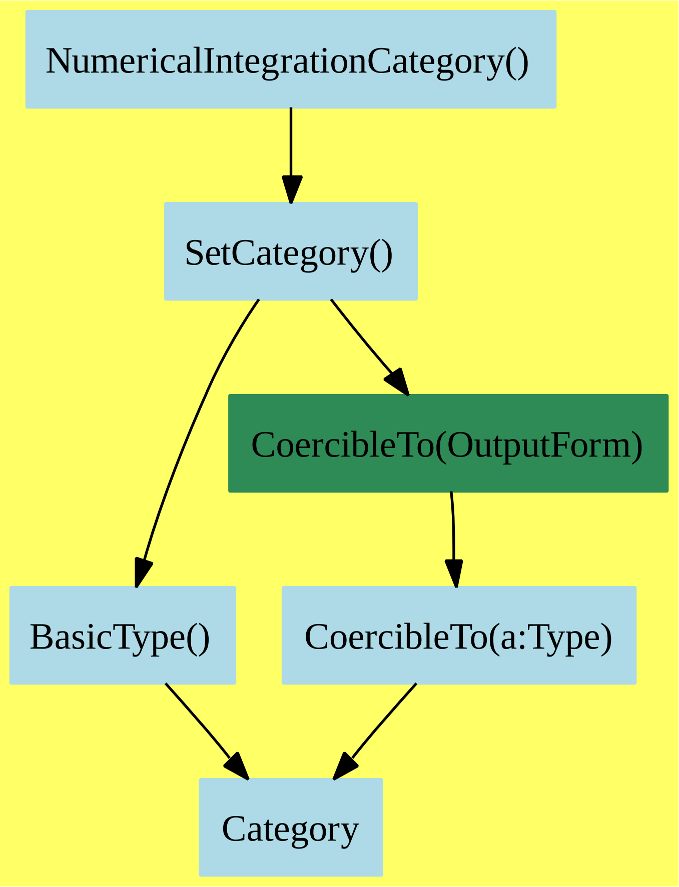
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.53 NumericalIntegrationCategory (NUMINT)


NumericalIntegrationCategory()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*NumericalIntegrationCategory.input*⟩≡

```
)set break resume
)sys rm -f NumericalIntegrationCategory.output
)spool NumericalIntegrationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalIntegrationCategory
--R NumericalIntegrationCategory  is a category constructor
--R Abbreviation for NumericalIntegrationCategory is NUMINT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NUMINT
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCo
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCom
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1

)spool
)lisp (bye)
```

⟨*NumericalIntegrationCategory.help*⟩≡

```
====================================================================
NumericalIntegrationCategory examples
====================================================================

See Also:
o )show NumericalIntegrationCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	measure
numericalIntegration	?=?	?~=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(fn: Expression DoubleFloat,
         range: List Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
    Record(measure: Float,explanations: String,extra: Result)
measure :
 (RoutinesTable,
  Record(var: Symbol,
         fn: Expression DoubleFloat,
         range: Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
   Record(measure: Float,explanations: String,extra: Result)
numericalIntegration :
 (Record(fn: Expression DoubleFloat,
         range: List Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,relerr: DoubleFloat),
  Result) ->
 Result
numericalIntegration :
 (Record(var: Symbol,
         fn: Expression DoubleFloat,
         range: Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat),
  Result) ->
 Result
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category NUMINT NumericalIntegrationCategory*⟩≡

```
)abbrev category NUMINT NumericalIntegrationCategory
++ Author: Brian Dupee
++ Date Created: February 1994
++ Date Last Updated: March 1996
++ Description:
++ \axiomType{NumericalIntegrationCategory} is the \axiom{category} for
++ describing the set of Numerical Integration \axiom{domains} with
++ \axiomFun{measure} and \axiomFun{numericalIntegration}.

NumericalIntegrationCategory(): Category == Exports where

 EDFE   ==> Expression DoubleFloat
 SOCDFE ==> Segment OrderedCompletion DoubleFloat
 DFE    ==> DoubleFloat
 NIAE   ==> Record(var:Symbol,fn:EDFE,range:SOCDFE,abserr:DFE,relerr:DFE)
 MDNIAE ==> Record(fn:EDFE,range:List SOCDFE,abserr:DFE,relerr:DFE)

 Exports ==> SetCategory with

  measure:(RoutinesTable,NIAE) -> _
   Record(measure:Float,explanations:String,extra:Result)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve a problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  numericalIntegration: (NIAE, Result) -> Result
    ++ numericalIntegration(args,hints) performs the integration of the
    ++ function given the strategy or method returned by \axiomFun{measure}.

  measure:(RoutinesTable,MDNIAE) -> _
   Record(measure:Float,explanations:String,extra:Result)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve a problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalIntegration: (MDNIAE, Result) -> Result
      ++ numericalIntegration(args,hints) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*NUMINT.dotabb*⟩≡

```
  "NUMINT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
  "NUMINT" -> "SETCAT"
```

⟨*NUMINT.dotfull*⟩≡

```
  "NumericalIntegrationCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
  "NumericalIntegrationCategory()" -> "SetCategory()"
```

⟨*NUMINT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NumericalIntegrationCategory()" [color=lightblue];
"NumericalIntegrationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

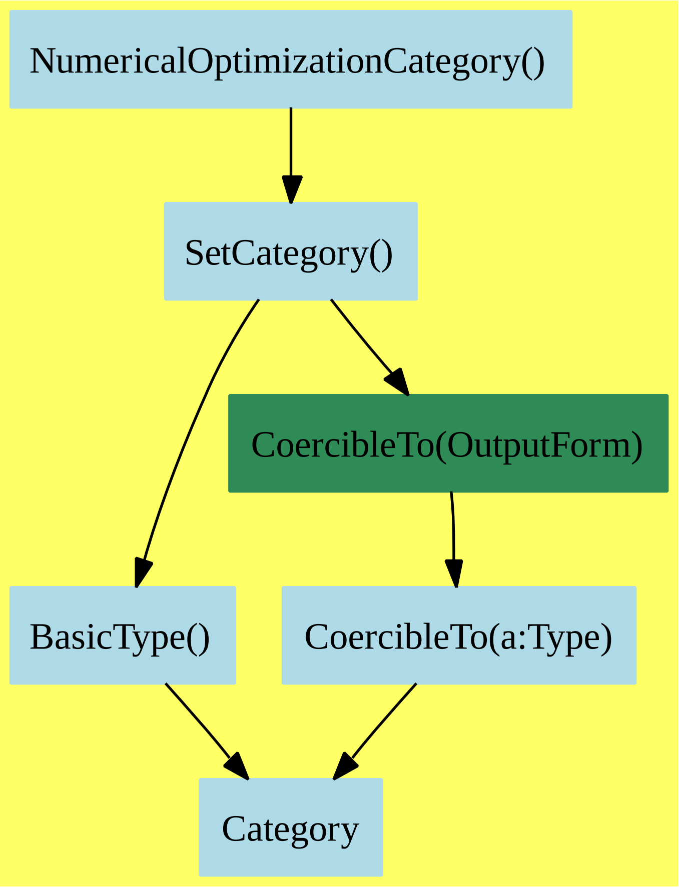
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.54 NumericalOptimizationCategory (OPTCAT)


NumericalOptimizationCategory()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*NumericalOptimizationCategory.input*⟩≡
```
)set break resume
)sys rm -f NumericalOptimizationCategory.output
)spool NumericalOptimizationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalOptimizationCategory
--R NumericalOptimizationCategory  is a category constructor
--R Abbreviation for NumericalOptimizationCategory is OPTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OPTCAT
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Li
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Lis
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R
--E 1

)spool
)lisp (bye)
```

⟨*NumericalOptimizationCategory.help*⟩≡

```
====================================================================
NumericalOptimizationCategory examples
====================================================================

See Also:
o )show NumericalOptimizationCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	measure
numericalOptimization	?=?	?˜=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(lfn: List Expression DoubleFloat,
         init: List DoubleFloat)) ->
   Record(measure: Float,explanations: String)
measure :
 (RoutinesTable,
  Record(fn: Expression DoubleFloat,
         init: List DoubleFloat,
         lb: List OrderedCompletion DoubleFloat,
         cf: List Expression DoubleFloat,
         ub: List OrderedCompletion DoubleFloat)) ->
   Record(measure: Float,explanations: String)
numericalOptimization :
 Record(fn: Expression DoubleFloat,
        init: List DoubleFloat,
        lb: List OrderedCompletion DoubleFloat,
        cf: List Expression DoubleFloat,
        ub: List OrderedCompletion DoubleFloat) ->
  Result
numericalOptimization :
 Record(lfn: List Expression DoubleFloat,
        init: List DoubleFloat) ->
  Result
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category OPTCAT NumericalOptimizationCategory*⟩≡

```
)abbrev category OPTCAT NumericalOptimizationCategory
++ Author: Brian Dupee
++ Date Created: January 1996
++ Date Last Updated: March 1996
++ Description:
```

```
++ \axiomType{NumericalOptimizationCategory} is the \axiom{category} for
++ describing the set of Numerical Optimization \axiom{domains} with
++ \axiomFun{measure} and \axiomFun{optimize}.

NumericalOptimizationCategory(): Category == Exports where

 LDFH   ==> List DoubleFloat
 LEDFH  ==> List Expression DoubleFloat
 LSAH   ==> Record(lfn:LEDFH, init:LDFH)
 EDFH   ==> Expression DoubleFloat
 LOCDFH ==> List OrderedCompletion DoubleFloat
 NOAH   ==> Record(fn:EDFH, init:LDFH, lb:LOCDFH, cf:LEDFH, ub:LOCDFH)

 Exports ==> SetCategory with

  measure:(RoutinesTable,NOAH)->Record(measure:Float,explanations:String)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve an optimization problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  measure:(RoutinesTable,LSAH)->Record(measure:Float,explanations:String)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve an optimization problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  numericalOptimization:LSAH -> Result
    ++ numericalOptimization(args) performs the optimization of the
    ++ function given the strategy or method returned by \axiomFun{measure}.

  numericalOptimization:NOAH -> Result
```

```
    ++ numericalOptimization(args) performs the optimization of the
    ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨OPTCAT.dotabb⟩≡

```
"OPTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
"OPTCAT" -> "SETCAT"
```

⟨OPTCAT.dotfull⟩≡

```
"NumericalOptimizationCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
"NumericalOptimizationCategory()" -> "SetCategory()"
```

⟨OPTCAT.dotpic⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NumericalOptimizationCategory()" [color=lightblue];
"NumericalOptimizationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

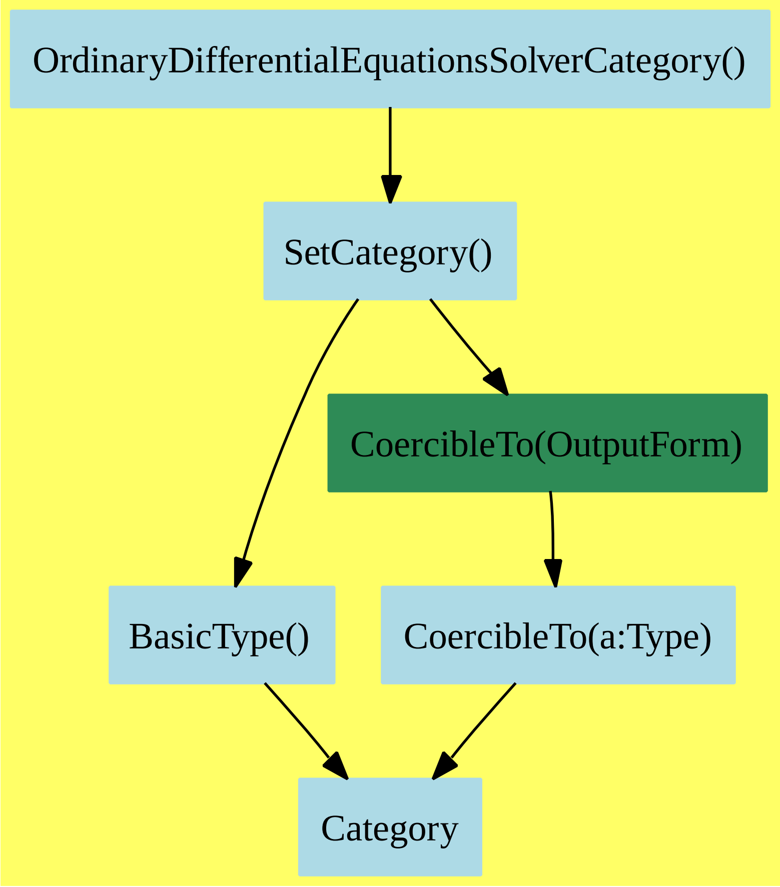
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.55 OrdinaryDifferentialEquationsSolverCategory (ODECAT)


OrdinaryDifferentialEquationsSolverCategory()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*OrdinaryDifferentialEquationsSolverCategory.input*⟩≡

```
)set break resume
)sys rm -f OrdinaryDifferentialEquationsSolverCategory.output
)spool OrdinaryDifferentialEquationsSolverCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryDifferentialEquationsSolverCategory
--R OrdinaryDifferentialEquationsSolverCategory  is a category constructor
--R Abbreviation for OrdinaryDifferentialEquationsSolverCategory is ODECAT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ODECAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression 
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vect
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrdinaryDifferentialEquationsSolverCategory.help*⟩≡

```
====================================================================
OrdinaryDifferentialEquationsSolverCategory examples
====================================================================

See Also:
o )show OrdinaryDifferentialEquationsSolverCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce hash latex measure ODESolve
?=? ?~=?

These are directly exported but not implemented:

```
ODESolve :
 Record(xinit: DoubleFloat,
        xend: DoubleFloat,
        fn: Vector Expression DoubleFloat,
        yinit: List DoubleFloat,
        intvals: List DoubleFloat,
        g: Expression DoubleFloat,
        abserr: DoubleFloat,
        relerr: DoubleFloat) ->
  Result
measure :
 (RoutinesTable,
  Record(xinit: DoubleFloat,
         xend: DoubleFloat,
         fn: Vector Expression DoubleFloat,
         yinit: List DoubleFloat,
         intvals: List DoubleFloat,
         g: Expression DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
   Record(measure: Float,explanations: String)
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ODECAT OrdinaryDifferentialEquationsSolverCategory*⟩≡

```
)abbrev category ODECAT OrdinaryDifferentialEquationsSolverCategory
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: June 1995
++ Basic Operations:
++ Description:
++ \axiomType{OrdinaryDifferentialEquationsSolverCategory} is the
++ \axiom{category} for describing the set of ODE solver \axiom{domains}
++ with \axiomFun{measure} and \axiomFun{ODEsolve}.

OrdinaryDifferentialEquationsSolverCategory(): Category == Exports where

 DFF   ==> DoubleFloat
 VEDFF ==> Vector Expression DoubleFloat
 LDFF  ==> List DoubleFloat
 EDFF  ==> Expression DoubleFloat
 ODEAF ==> Record(xinit:DFF,xend:DFF,fn:VEDFF,yinit:LDFF,intvals:LDFF,g:EDFF,abse

 Exports ==> SetCategory with

  measure:(RoutinesTable,ODEAF) -> Record(measure:Float,explanations:String)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve a problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  ODESolve: ODEAF -> Result
    ++ ODESolve(args) performs the integration of the
    ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*ODECAT.dotabb*⟩≡

```
"ODECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"ODECAT" -> "SETCAT"
```

⟨*ODECAT.dotfull*⟩≡

```
"OrdinaryDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

⟨*ODECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrdinaryDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

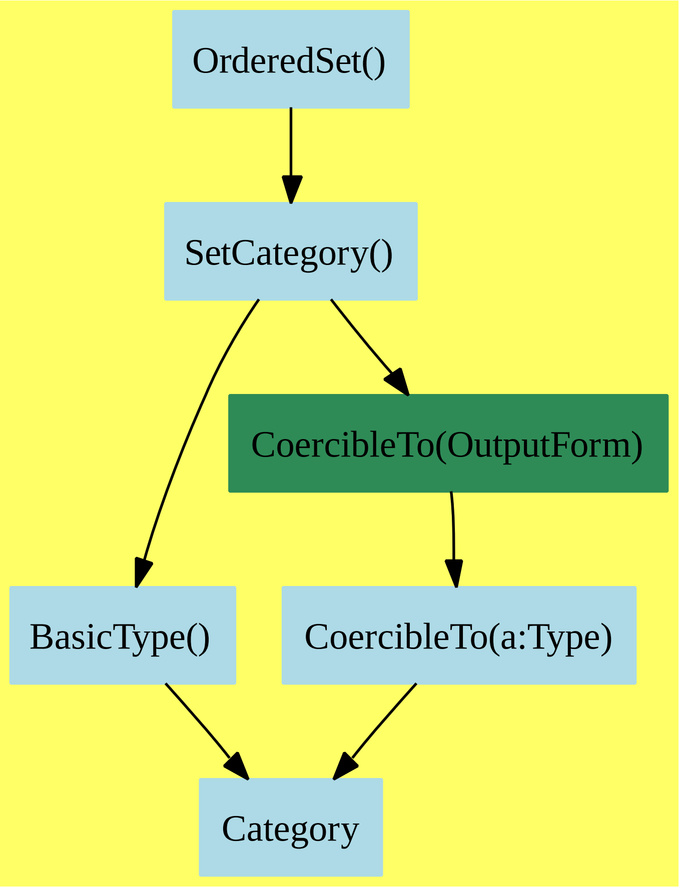
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.56 OrderedSet (ORDSET)


OrderedSet()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*OrderedSet.input*⟩≡
```
)set break resume
)sys rm -f OrderedSet.output
)spool OrderedSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedSet
--R OrderedSet  is a category constructor
--R Abbreviation for OrderedSet is ORDSET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDSET
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedSet.help*⟩≡

```
====================================================================
OrderedSet examples
====================================================================

See Also:
o )show OrderedSet
```

See:

⇒ "BitAggregate" (BTAGG) 9.0.123 on page 806
⇒ "CachableSet" (CACHSET) 5.0.69 on page 318
⇒ "DifferentialVariableCategory" (DVARCAT) 5.0.71 on page 330
⇒ "ExpressionSpace" (ES) 5.0.72 on page 337
⇒ "FortranMachineTypeCategory" (FMTC) 13.0.171 on page 1223
⇒ "IntervalCategory" (INTCAT) 14.0.176 on page 1270
⇒ "OrderedAbelianSemiGroup" (OASGP) 6.0.89 on page 507
⇒ "OrderedFinite" (ORDFIN) 5.0.77 on page 376
⇒ "OrderedMonoid" (ORDMON) 6.0.90 on page 512
⇒ "PolynomialCategory" (POLYCAT) 16.0.189 on page 1385
⇒ "QuotientFieldCategory" (QFCAT) 17.0.198 on page 1516
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	max	min
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These are directly exported but not implemented:

```
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
max : (%,%) -> %
min : (%,%) -> %
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ORDSET OrderedSet*⟩≡

```
)abbrev category ORDSET OrderedSet
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
```

```
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of totally ordered sets, that is, sets such that for each
++ pair of elements \spad{(a,b)}
++ exactly one of the following relations holds \spad{a<b or a=b or b<a}
++ and the relation is transitive, i.e.  \spad{a<b and b<c => a<c}.

OrderedSet(): Category == SetCategory with
    "<": (%,%) -> Boolean
      ++ x < y is a strict total ordering on the elements of the set.
    ">":        (%, %) -> Boolean
      ++ x > y is a greater than test.
    ">=":       (%, %) -> Boolean
      ++ x >= y is a greater than or equal test.
    "<=":       (%, %) -> Boolean
      ++ x <= y is a less than or equal test.
    max: (%,%) -> %
      ++ max(x,y) returns the maximum of x and y relative to "<".
    min: (%,%) -> %
      ++ min(x,y) returns the minimum of x and y relative to "<".
  add
    x,y: %
  -- These really ought to become some sort of macro
    max(x,y) ==
      x > y => x
      y
    min(x,y) ==
      x > y => y
      x
    ((x: %) >  (y: %)) : Boolean == y < x
    ((x: %) >= (y: %)) : Boolean == not (x < y)
    ((x: %) <= (y: %)) : Boolean == not (y < x)
```

⟨*ORDSET.dotabb*⟩≡

```
"ORDSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"ORDSET" -> "SETCAT"
```

⟨*ORDSET.dotfull*⟩≡
```
"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"
```

⟨*ORDSET.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

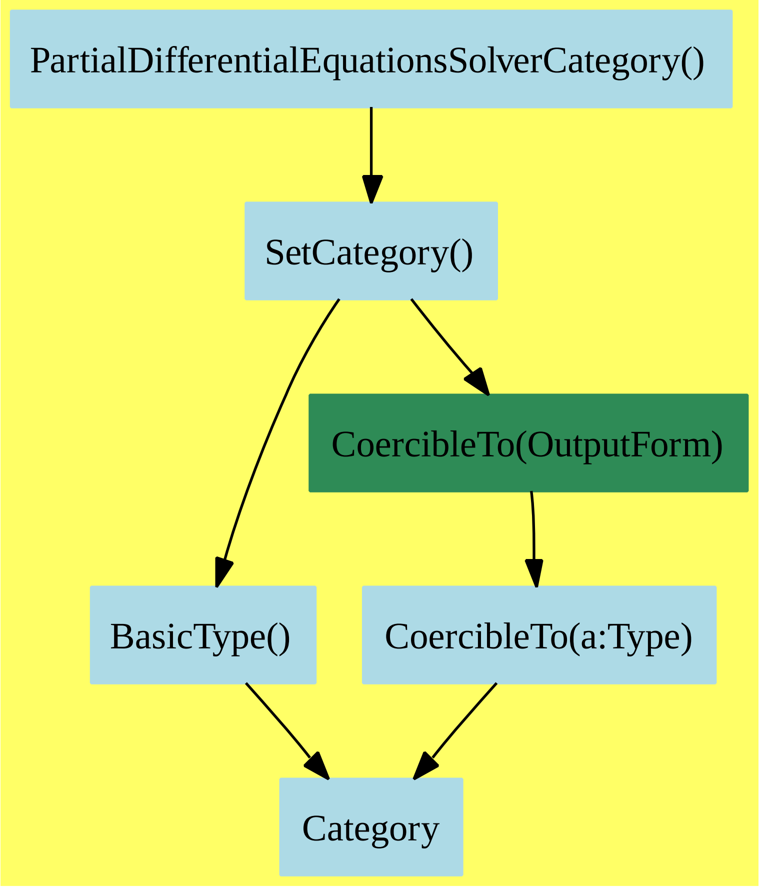
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.57 PartialDifferentialEquationsSolverCategory (PDECAT)


PartialDifferentialEquationsSolverCategory()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*PartialDifferentialEquationsSolverCategory.input*⟩≡

```
)set break resume
)sys rm -f PartialDifferentialEquationsSolverCategory.output
)spool PartialDifferentialEquationsSolverCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PartialDifferentialEquationsSolverCategory
--R PartialDifferentialEquationsSolverCategory  is a category constructor
--R Abbreviation for PartialDifferentialEquationsSolverCategory is PDECAT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PDECAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R PDESolve : Record(pde: List Expression DoubleFloat,constraints: List Record(s
--R measure : (RoutinesTable,Record(pde: List Expression DoubleFloat,constraints:
--R
--E 1

)spool
)lisp (bye)
```

⟨*PartialDifferentialEquationsSolverCategory.help*⟩≡

```
====================================================================
PartialDifferentialEquationsSolverCategory examples
====================================================================

See Also:
o )show PartialDifferentialEquationsSolverCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	measure	PDESolve
?=?	?~=?			

These are directly exported but not implemented:

```
measure :
(RoutinesTable,
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
                     finish: DoubleFloat,
                     grid: NonNegativeInteger,
                     boundaryType: Integer,
                     dStart: Matrix DoubleFloat,
                     dFinish: Matrix DoubleFloat),
        f: List List Expression DoubleFloat,
        st: String,
        tol: DoubleFloat)) ->
   Record(measure: Float,explanations: String)
PDESolve :
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
                     finish: DoubleFloat,
                     grid: NonNegativeInteger,
                     boundaryType: Integer,
                     dStart: Matrix DoubleFloat,
                     dFinish: Matrix DoubleFloat),
        f: List List Expression DoubleFloat,
        st: String,
        tol: DoubleFloat) ->
   Result
```

These exports come from (p589) Dictionary(S:SetCategory):

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category PDECAT PartialDifferentialEquationsSolverCategory*⟩≡

```
)abbrev category PDECAT PartialDifferentialEquationsSolverCategory
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: June 1995
++ Basic Operations:
++ Description:
++ \axiomType{PartialDifferentialEquationsSolverCategory} is the
++ \axiom{category} for describing the set of PDE solver \axiom{domains}
++ with \axiomFun{measure} and \axiomFun{PDEsolve}.

-- PDEA        ==> Record(xmin:F,xmax:F,ymin:F,ymax:F,ngx:NNI,ngy:NNI,_
--              pde:List Expression Float, bounds:List List Expression Float,_
--              st:String, tol:DF)

--  measure:(RoutinesTable,PDEA) -> Record(measure:F,explanations:String)
--    ++ measure(R,args) calculates an estimate of the ability of a particular
--    ++ method to solve a problem.
--    ++
--    ++ This method may be either a specific NAG routine or a strategy (such
--    ++ as transforming the function from one which is difficult to one which
--    ++ is easier to solve).
--    ++
--    ++ It will call whichever agents are needed to perform analysis on the
--    ++ problem in order to calculate the measure. There is a parameter,
--    ++ labelled \axiom{sofar}, which would contain the best compatibility
--    ++ found so far.

--  PDESolve: PDEA -> Result
--    ++ PDESolve(args) performs the integration of the
--    ++ function given the strategy or method returned by \axiomFun{measure}.

PartialDifferentialEquationsSolverCategory(): Category == Exports where

 DFG   ==> DoubleFloat
 NNIG  ==> NonNegativeInteger
 INTG  ==> Integer
 MDFG  ==> Matrix DoubleFloat
 PDECG ==> Record(start:DFG, finish:DFG, grid:NNIG, boundaryType:INTG,
                  dStart:MDFG, dFinish:MDFG)
 LEDFG ==> List Expression DoubleFloat
 PDEBG ==> Record(pde:LEDFG, constraints:List PDECG, f:List LEDFG,
                    st:String, tol:DFG)
 Exports ==> SetCategory with

  measure:(RoutinesTable,PDEBG) -> Record(measure:Float,explanations:String)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve a problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  PDESolve: PDEBG -> Result
    ++ PDESolve(args) performs the integration of the
    ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*PDECAT.dotabb*⟩≡

```
"PDECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PDECAT" -> "SETCAT"
```

⟨*PDECAT.dotfull*⟩≡

```
"PartialDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

⟨*PDECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialDifferentialEquationsSolverCategory()" [color=lightblue];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

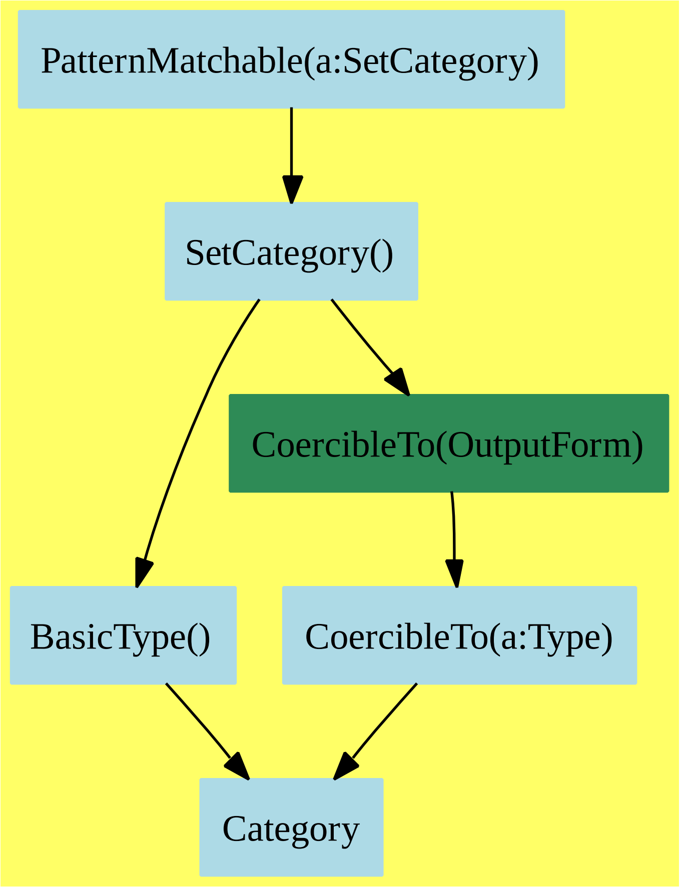
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.58 PatternMatchable (PATMAB)


PatternMatchable(a:SetCategory)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*PatternMatchable.input*⟩≡

```
)set break resume
)sys rm -f PatternMatchable.output
)spool PatternMatchable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchable
--R PatternMatchable S: SetCategory  is a category constructor
--R Abbreviation for PatternMatchable is PATMAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PATMAB
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R patternMatch : (%,Pattern S,PatternMatchResult(S,%)) -> PatternMatchResult(S,
--R
--E 1

)spool
)lisp (bye)
```

⟨*PatternMatchable.help*⟩≡

```
====================================================================
PatternMatchable examples
====================================================================

See Also:
o )show PatternMatchable
```

See:

⇒ "RealNumberSystem" (RNS) 17.0.200 on page 1543
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce hash latex patternMatch ?=?
?~=?

These are directly exported but not implemented:

```
patternMatch :
  (%,Pattern S,PatternMatchResult(S,%))
      -> PatternMatchResult(S,%)
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category PATMAB PatternMatchable*⟩≡

```
)abbrev category PATMAB PatternMatchable
++ Category of sets that can be pattern-matched on
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 15 Mar 1990
++ Keywords: pattern, matching.
++ Description:
++ A set R is PatternMatchable over S if elements of R can
++ be matched to patterns over S.

PatternMatchable(S:SetCategory): Category == SetCategory with
  patternMatch: (%, Pattern S, PatternMatchResult(S, %)) ->
                                                 PatternMatchResult(S, %)
    ++ patternMatch(expr, pat, res) matches the pattern pat to the
    ++ expression expr. res contains the variables of pat which
    ++ are already matched and their matches (necessary for recursion).
    ++ Initially, res is just the result of \spadfun{new}
    ++ which is an empty list of matches.
```

⟨*PATMAB.dotabb*⟩≡
```
"PATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PATMAB" -> "SETCAT"
```

⟨*PATMAB.dotfull*⟩≡
```
"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"PatternMatchable(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Integer)" -> "PatternMatchable(a:SetCategory)"

"PatternMatchable(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Float)" -> "PatternMatchable(a:SetCategory)"
```

⟨*PATMAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

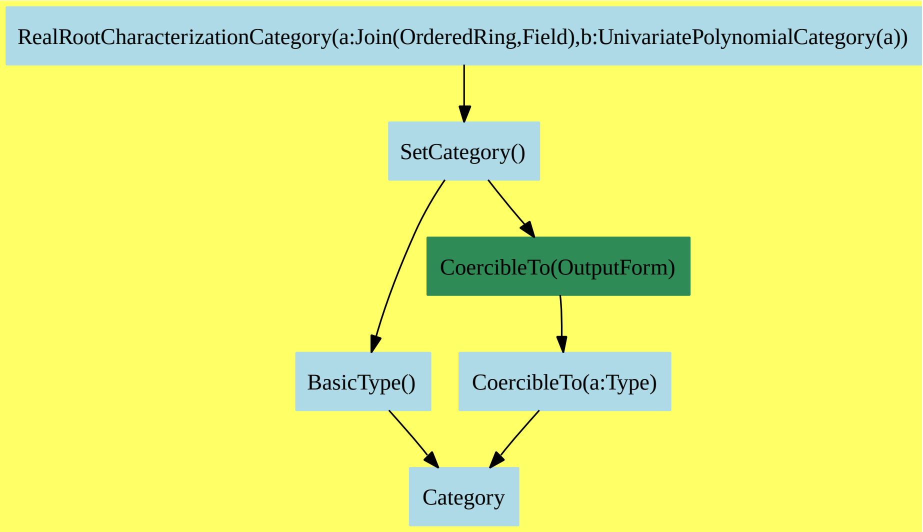
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.59 RealRootCharacterizationCategory (RRCC)


RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a))
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*RealRootCharacterizationCategory.input*⟩≡

```
  )set break resume
  )sys rm -f RealRootCharacterizationCategory.output
  )spool RealRootCharacterizationCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show RealRootCharacterizationCategory
  --R RealRootCharacterizationCategory(TheField: Join(OrderedRing,Field),ThePols: U
  --R Abbreviation for RealRootCharacterizationCategory is RRCC
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for RRCC
  --R
  --R------------------------------- Operations --------------------------------
  --R ?=? : (%,%) -> Boolean                allRootsOf : ThePols -> List %
  --R coerce : % -> OutputForm              definingPolynomial : % -> ThePols
  --R hash : % -> SingleInteger             latex : % -> String
  --R sign : (ThePols,%) -> Integer         zero? : (ThePols,%) -> Boolean
  --R ?~=? : (%,%) -> Boolean
  --R approximate : (ThePols,%,TheField) -> TheField
  --R negative? : (ThePols,%) -> Boolean
  --R positive? : (ThePols,%) -> Boolean
  --R recip : (ThePols,%) -> Union(ThePols,"failed")
```

```
--R relativeApprox : (ThePols,%,TheField) -> TheField
--R rootOf : (ThePols,PositiveInteger) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*RealRootCharacterizationCategory.help*⟩≡

```
====================================================================
RealRootCharacterizationCategory examples
====================================================================

See Also:
o )show RealRootCharacterizationCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

allRootsOf	approximate	coerce	definingPolynomial
hash	latex	negative?	positive?
recip	relativeApprox	rootOf	sign
zero?	?=?	? =?	

These are directly exported but not implemented:

```
approximate : (ThePols,%,TheField) -> TheField
allRootsOf : ThePols -> List %
definingPolynomial : % -> ThePols
relativeApprox : (ThePols,%,TheField) -> TheField
sign : (ThePols,%) -> Integer
```

These are implemented by this category:

```
negative? : (ThePols,%) -> Boolean
positive? : (ThePols,%) -> Boolean
recip : (ThePols,%) -> Union(ThePols,"failed")
rootOf : (ThePols,PositiveInteger) -> Union(%,"failed")
zero? : (ThePols,%) -> Boolean
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category RRCC RealRootCharacterizationCategory*⟩≡

```
)abbrev category RRCC RealRootCharacterizationCategory
++ Author: Renaud Rioboo
++ Date Created: summer 1992
++ Date Last Updated: January 2004
++ Basic Functions: provides operations with generic real roots of
++                  polynomials
++ Related Constructors: RealClosure, RightOpenIntervalRootCharacterization
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ \axiomType{RealRootCharacterizationCategory} provides common acces
++ functions for all real root codings.

RealRootCharacterizationCategory(TheField, ThePols ) : Category == PUB where

   TheField : Join(OrderedRing, Field)
   ThePols : UnivariatePolynomialCategory(TheField)

   Z ==> Integer
   N ==> PositiveInteger

   PUB ==>
     SetCategory with

        sign:               ( ThePols, $ )   ->            Z
              ++ \axiom{sign(pol,aRoot)} gives the sign of \axiom{pol}
              ++ interpreted as \axiom{aRoot}
        zero? :             ( ThePols, $ )   ->         Boolean
              ++ \axiom{zero?(pol,aRoot)} answers if \axiom{pol}
              ++ interpreted as \axiom{aRoot} is \axiom{0}
        negative?:          ( ThePols, $ )   ->         Boolean
              ++ \axiom{negative?(pol,aRoot)} answers if \axiom{pol}
              ++ interpreted as \axiom{aRoot} is negative
        positive?:          ( ThePols, $ )   ->         Boolean
              ++ \axiom{positive?(pol,aRoot)} answers if \axiom{pol}
              ++ interpreted as \axiom{aRoot} is positive
        recip:              ( ThePols, $ )   ->   Union(ThePols,"failed")
              ++ \axiom{recip(pol,aRoot)} tries to inverse \axiom{pol}
              ++ interpreted as \axiom{aRoot}
        definingPolynomial:        $         ->         ThePols
              ++ \axiom{definingPolynomial(aRoot)} gives a polynomial
              ++ such that \axiom{definingPolynomial(aRoot).aRoot = 0}
        allRootsOf:              ThePols     ->          List $
              ++ \axiom{allRootsOf(pol)} creates all the roots of \axiom{pol}
              ++ in the Real Closure, assumed in order.
        rootOf:             ( ThePols, N )   ->     Union($,"failed")
              ++ \axiom{rootOf(pol,n)} gives the nth root for the order of the
              ++ Real Closure
        approximate :  (ThePols,$,TheField)   ->    TheField
              ++ \axiom{approximate(term,root,prec)} gives an approximation
              ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

        relativeApprox :  (ThePols,$,TheField)   ->    TheField
              ++ \axiom{approximate(term,root,prec)} gives an approximation
              ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

      add
```

```
        zero?(toTest, rootChar) ==
          sign(toTest, rootChar) = 0

        negative?(toTest, rootChar) ==
          sign(toTest, rootChar) < 0

        positive?(toTest, rootChar) ==
          sign(toTest, rootChar) > 0

        rootOf(pol,n) ==
          liste:List($):= allRootsOf(pol)
          # liste > n => "failed"
          liste.n

        recip(toInv,rootChar) ==
          degree(toInv) = 0 =>
            res := recip(leadingCoefficient(toInv))
            if (res case "failed") then "failed" else (res::TheField::ThePols)
          defPol := definingPolynomial(rootChar)
          d := principalIdeal([defPol,toInv])
          zero?(d.generator,rootChar) => "failed"
          if (degree(d.generator) ^= 0 )
          then
            defPol := (defPol exquo (d.generator))::ThePols
            d := principalIdeal([defPol,toInv])
          d.coef.2
```

⟨*RRCC.dotabb*⟩≡

```
  "RRCC"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
  "RRCC" -> "SETCAT"
```

⟨*RRCC.dotfull*⟩≡

```
  "RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
  "RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
    -> "SetCategory()"
```

⟨*RRCC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a
 [color=lightblue];
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a
  -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

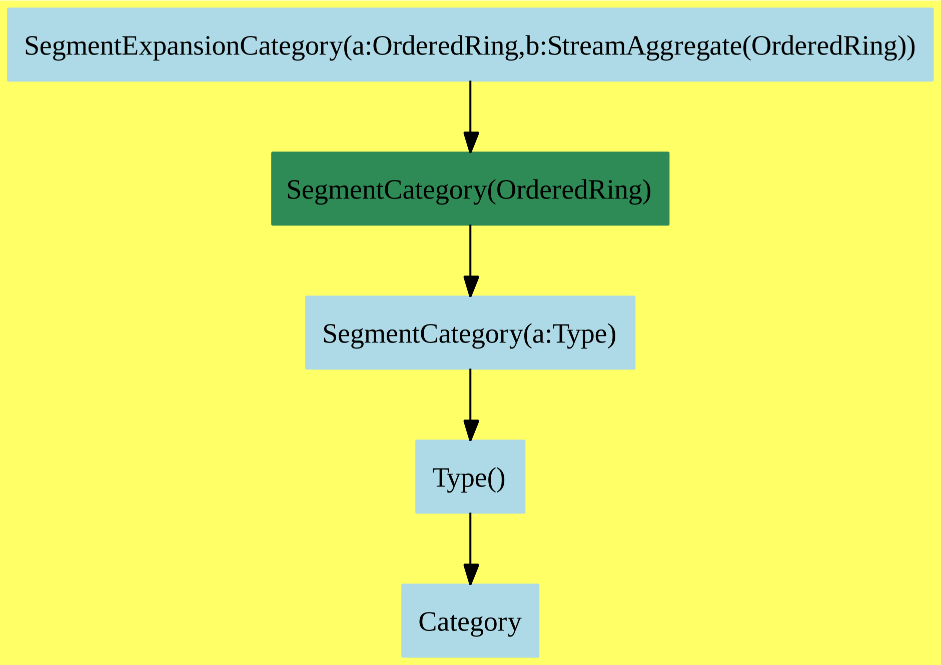
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.60 SegmentExpansionCategory (SEGXCAT)


SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))
SegmentCategory(OrderedRing)
SegmentCategory(a:Type)
Type()
Category

⟨*SegmentExpansionCategory.input*⟩≡

```
)set break resume
)sys rm -f SegmentExpansionCategory.output
)spool SegmentExpansionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SegmentExpansionCategory
--R SegmentExpansionCategory(S: OrderedRing,L: StreamAggregate t#1)  is a categor
--R Abbreviation for SegmentExpansionCategory is SEGXCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEGXCAT
--R
--R------------------------------- Operations --------------------------------
--R BY : (%,Integer) -> %                 ?..? : (S,S) -> %
--R convert : S -> %                      expand : % -> L
--R expand : List % -> L                  hi : % -> S
--R high : % -> S                         incr : % -> Integer
--R lo : % -> S                           low : % -> S
```

```
--R map : ((S -> S),%) -> L               segment : (S,S) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*SegmentExpansionCategory.help*⟩≡

```
====================================================================
SegmentExpansionCategory examples
====================================================================

See Also:
o )show SegmentExpansionCategory
```

See:

⇐ "SegmentCategory" (SEGCAT) 3.0.33 on page 125

Exports:

BY convert expand hi high
incr lo low map segment
?..?

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
expand : % -> L
expand : List % -> L
map : ((S -> S),%) -> L
```

These exports come from (p125) SegmentCategory(OrderedRing):

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
?..? : (S,S) -> %
```

⟨*category SEGXCAT SegmentExpansionCategory*⟩≡

```
)abbrev category SEGXCAT SegmentExpansionCategory
++ Author:  Stephen M. Watt
++ Date Created: June 5, 1991
++ Date Last Updated:
++ Basic Operations:
++ Related Domains: Segment, UniversalSegment
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This category provides an interface for expanding segments to
++ a stream of elements.
```

```
SegmentExpansionCategory(S: OrderedRing, L: StreamAggregate(S)): Category ==
    SegmentCategory(S) with
      expand: List % -> L
        ++ expand(l) creates a new value of type L in which each segment
        ++ \spad{l..h by k} is replaced with \spad{l, l+k, ... lN},
        ++ where \spad{lN <= h < lN+k}.
        ++ For example, \spad{expand [1..4, 7..9] = [1,2,3,4,7,8,9]}.
      expand: % -> L
        ++ expand(l..h by k) creates value of type L with elements
        ++ \spad{l, l+k, ... lN} where \spad{lN <= h < lN+k}.
        ++ For example, \spad{expand(1..5 by 2) = [1,3,5]}.
      map: (S -> S, %) -> L
        ++ map(f,l..h by k) produces a value of type L by applying f
        ++ to each of the succesive elements of the segment, that is,
        ++ \spad{[f(l), f(l+k), ..., f(lN)]}, where \spad{lN <= h < lN+k}.
```

⟨*SEGXCAT.dotabb*⟩≡

```
"SEGXCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SEGXCAT" -> "SEGCAT"
```

⟨*SEGXCAT.dotfull*⟩≡

```
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"
```

⟨*SEGXCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"

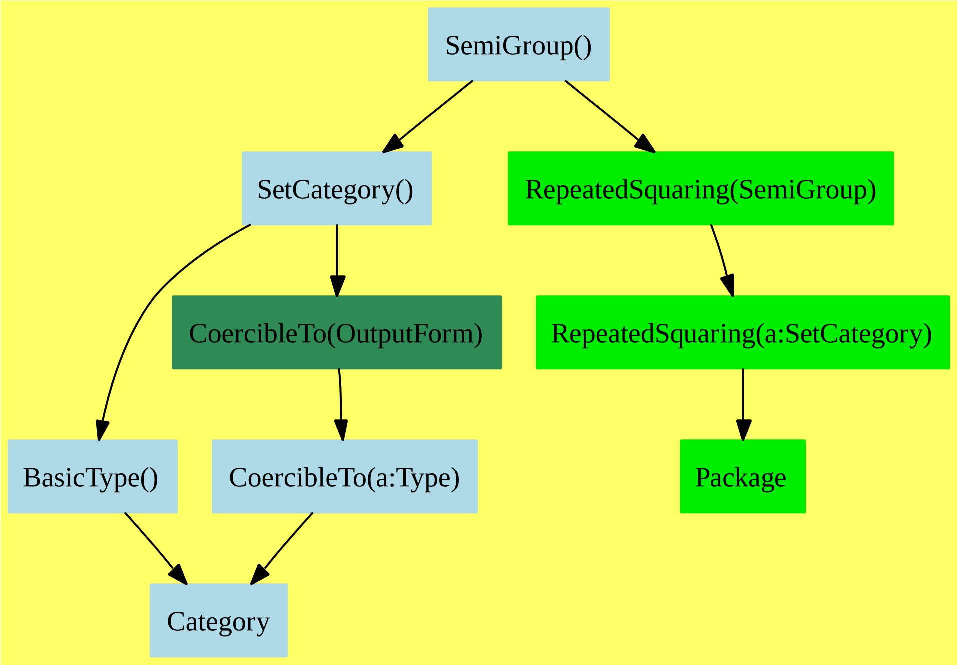
"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

4.0.61 SemiGroup (SGROUP)


SemiGroup()
SetCategory()
RepeatedSquaring(SemiGroup)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

A Semigroup is defined as a set S with a binary multiplicative operator "*". A Semigroup $G(S,*)$ is:

- a set S which can be null
- a binary multiplicative operator "*"
- associative. $\forall a,b,c \in S, a*(b*c) = (a*b)*c$

⟨*SemiGroup.input*⟩≡

```
)set break resume
)sys rm -f SemiGroup.output
)spool SemiGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SemiGroup
--R SemiGroup  is a category constructor
--R Abbreviation for SemiGroup is SGROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SGROUP
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean                 ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm               hash : % -> SingleInteger
--R latex : % -> String                    ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*SemiGroup.help*⟩≡

```
====================================================================
SemiGroup examples
====================================================================

See Also:
o )show SemiGroup
```

See:

⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “Monoid” (MONOID) 5.0.76 on page 371
⇒ “Rng” (RNG) 8.0.121 on page 793
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

coerce	hash	latex	?*?	?**?
?=?	?^?	?~=?		

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SGROUP SemiGroup*⟩≡
```
)abbrev category SGROUP SemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ the class of all multiplicative semigroups, i.e. a set
++ with an associative operation \spadop{*}.
++
++ Axioms\br
++ \tab{5}\spad{associative("*":(%,%)->%)}\tab{5}\spad{ (x*y)*z = x*(y*z)}
++
++ Conditional attributes\br
++ \tab{5}\spad{commutative("*":(%,%)->%)}\tab{5}\spad{ x*y = y*x }

SemiGroup(): Category == SetCategory with
      "*": (%,%) -> %                 ++ x*y returns the product of x and y.
      "**": (%,PositiveInteger) -> %   ++ x**n returns the repeated product
                                       ++ of x n times, i.e. exponentiation.
      "^": (%,PositiveInteger) -> %    ++ x^n returns the repeated product
                                       ++ of x n times, i.e. exponentiation.
    add
      import RepeatedSquaring(%)
      x:% ** n:PositiveInteger == expt(x,n)
      _^(x:%, n:PositiveInteger):% == x ** n
```

⟨*SGROUP.dotabb*⟩≡

```
"SGROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SGROUP" -> "SETCAT"
```

⟨*SGROUP.dotfull*⟩≡

```
"SemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"
```

⟨*SGROUP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

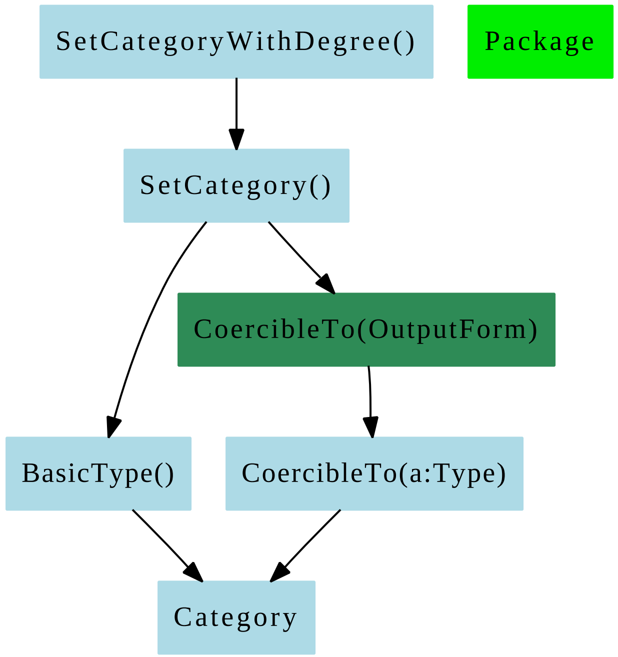
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

4.0.62 SetCategoryWithDegree (SETCATD)


SetCategoryWithDegree()
Package
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*SetCategoryWithDegree.input*⟩≡

```
)set break resume
)sys rm -f SetCategoryWithDegree.output
)spool SetCategoryWithDegree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SetCategoryWithDegree
--R SetCategoryWithDegree  is a category constructor
--R Abbreviation for SetCategoryWithDegree is SETCATD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETCATD
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R degree : % -> PositiveInteger         hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*SetCategoryWithDegree.help*⟩≡

```
====================================================================
SetCategoryWithDegree examples
====================================================================

See Also:
o )show SetCategoryWithDegree
```

See:

⇒ "ProjectiveSpaceCategory" (PRSPCAT) 5.0.79 on page 386
⇐ "BasicType" (BASTYPE) 2.0.5 on page 16
⇐ "CoercibleTo" (KOERCE) 2.0.6 on page 19
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce degree hash latex ?=? ?~=?

These are directly exported but not implemented:

```
degree : % -> PositiveInteger
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SETCATD SetCategoryWithDegree*⟩≡

```
)abbrev category SETCATD SetCategoryWithDegree
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Keywords:
++ Description:
++ This is part of the PAFF package, related to projective space.
SetCategoryWithDegree:Category == SetCategory with
    degree: % -> PositiveInteger
```

⟨*SETCATD.dotabb*⟩≡
```
"SETCATD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCATD"];
"SETCATD" -> "SETCAT"
```

⟨*SETCATD.dotfull*⟩≡
```
"SetCategoryWithDegree()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCATD"];
"SetCategoryWithDegree()" -> "SetCategory()"
```

⟨*SETCATD.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

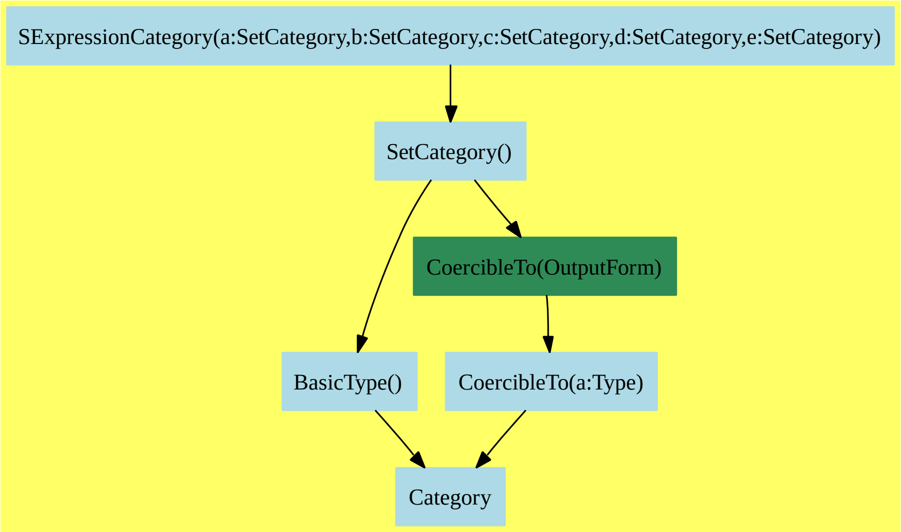
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

4.0.63 SExpressionCategory (SEXCAT)


SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*SExpressionCategory.input*⟩≡

```
)set break resume
)sys rm -f SExpressionCategory.output
)spool SExpressionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpressionCategory
--R SExpressionCategory(Str: SetCategory,Sym: SetCategory,Int: SetCategory,Flt: SetCategory
--R Abbreviation for SExpressionCategory is SEXCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEXCAT
--R
--R------------------------------- Operations --------------------------------
--R #? : % -> Integer                     ?=? : (%,%) -> Boolean
--R atom? : % -> Boolean                  car : % -> %
--R cdr : % -> %                          coerce : % -> OutputForm
--R convert : Expr -> %                   convert : Flt -> %
--R convert : Int -> %                    convert : Sym -> %
--R convert : Str -> %                    convert : List % -> %
--R destruct : % -> List %                ?.? : (%,List Integer) -> %
--R ?.? : (%,Integer) -> %                eq : (%,%) -> Boolean
--R expr : % -> Expr                      float : % -> Flt
```

```
--R float? : % -> Boolean                hash : % -> SingleInteger
--R integer : % -> Int                   integer? : % -> Boolean
--R latex : % -> String                  list? : % -> Boolean
--R null? : % -> Boolean                 pair? : % -> Boolean
--R string : % -> Str                    string? : % -> Boolean
--R symbol : % -> Sym                    symbol? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*SExpressionCategory.help*⟩≡

```
====================================================================
SExpressionCategory examples
====================================================================

See Also:
o )show SExpressionCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

atom?	car	cdr	coerce	convert
destruct	eq	expr	float	float?
hash	integer	integer?	latex	list?
null?	pair?	string	string?	symbol
symbol?	#?	?=?	?~=?	?.?

These are directly exported but not implemented:

```
atom? : % -> Boolean
car : % -> %
cdr : % -> %
convert : Expr -> %
convert : Flt -> %
convert : Int -> %
convert : Sym -> %
convert : Str -> %
convert : List % -> %
destruct : % -> List %
eq : (%,%) -> Boolean
expr : % -> Expr
float : % -> Flt
float? : % -> Boolean
integer : % -> Int
integer? : % -> Boolean
list? : % -> Boolean
null? : % -> Boolean
pair? : % -> Boolean
string : % -> Str
string? : % -> Boolean
symbol : % -> Sym
symbol? : % -> Boolean
#? : % -> Integer
?.? : (%,List Integer) -> %
?.? : (%,Integer) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SEXCAT SExpressionCategory*⟩≡

```
)abbrev category SEXCAT SExpressionCategory
++ Category for Lisp values
++ Author: S.M.Watt
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This category allows the manipulation of Lisp values while keeping
++ the grunge fairly localized.
--  The coerce to expression lets the
--  values be displayed in the usual parenthesized way (displaying
--  them as type Expression can cause the formatter to die, since
--  certain magic cookies are in unexpected places).
--  SMW July 87

SExpressionCategory(Str, Sym, Int, Flt, Expr): Category == Decl where
    Str, Sym, Int, Flt, Expr: SetCategory

    Decl ==> SetCategory with
        eq:        (%,%) -> Boolean
          ++ eq(s, t) is true if EQ(s,t) is true in Lisp.
        null?:     % -> Boolean
          ++ null?(s) is true if s is the S-expression ().
        atom?:     % -> Boolean
          ++ atom?(s) is true if s is a Lisp atom.
        pair?:     % -> Boolean
          ++ pair?(s) is true if s has is a non-null Lisp list.
        list?:     % -> Boolean
          ++ list?(s) is true if s is a Lisp list, possibly ().
        string?:   % -> Boolean
          ++ string?(s) is true if s is an atom and belong to Str.
        symbol?:   % -> Boolean
          ++ symbol?(s) is true if s is an atom and belong to Sym.
        integer?:  % -> Boolean
          ++ integer?(s) is true if s is an atom and belong to Int.
        float?:    % -> Boolean
          ++ float?(s) is true if s is an atom and belong to Flt.
        destruct:  % -> List %
          ++ destruct((a1,...,an)) returns the list [a1,...,an].
        string:    % -> Str
          ++ string(s) returns s as an element of Str.
          ++ Error: if s is not an atom that also belongs to Str.
        symbol:    % -> Sym
          ++ symbol(s) returns s as an element of Sym.
          ++ Error: if s is not an atom that also belongs to Sym.
        integer:   % -> Int
          ++ integer(s) returns s as an element of Int.
```

```
      ++ Error: if s is not an atom that also belongs to Int.
    float:     % -> Flt
      ++ float(s) returns s as an element of Flt;
      ++ Error: if s is not an atom that also belongs to Flt.
    expr:      % -> Expr
      ++ expr(s) returns s as an element of Expr;
      ++ Error: if s is not an atom that also belongs to Expr.
    convert:   List % -> %
      ++ convert([a1,...,an]) returns an S-expression \spad{(a1,...,an)}.
    convert:   Str    -> %
      ++ convert(x) returns the Lisp atom x;
    convert:   Sym    -> %
      ++ convert(x) returns the Lisp atom x.
    convert:   Int    -> %
      ++ convert(x) returns the Lisp atom x.
    convert:   Flt    -> %
      ++ convert(x) returns the Lisp atom x.
    convert:   Expr   -> %
      ++ convert(x) returns the Lisp atom x.
    car:       % -> %
      ++ car((a1,...,an)) returns a1.
    cdr:       % -> %
      ++ cdr((a1,...,an)) returns \spad{(a2,...,an)}.
    "#":       % -> Integer
      ++ #((a1,...,an)) returns n.
    elt:       (%, Integer)      -> %
      ++ elt((a1,...,an), i) returns \spad{ai}.
    elt:       (%, List Integer) -> %
      ++ elt((a1,...,an), [i1,...,im]) returns \spad{(a_i1,...,a_im)}.
```

⟨*SEXCAT.dotabb*⟩≡

```
  "SEXCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
  "SEXCAT" -> "SETCAT"
```

⟨*SEXCAT.dotfull*⟩≡

```
  "SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)'
   [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
  "SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)'
     "SetCategory()"
```

⟨*SEXCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
 [color=lightblue];
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

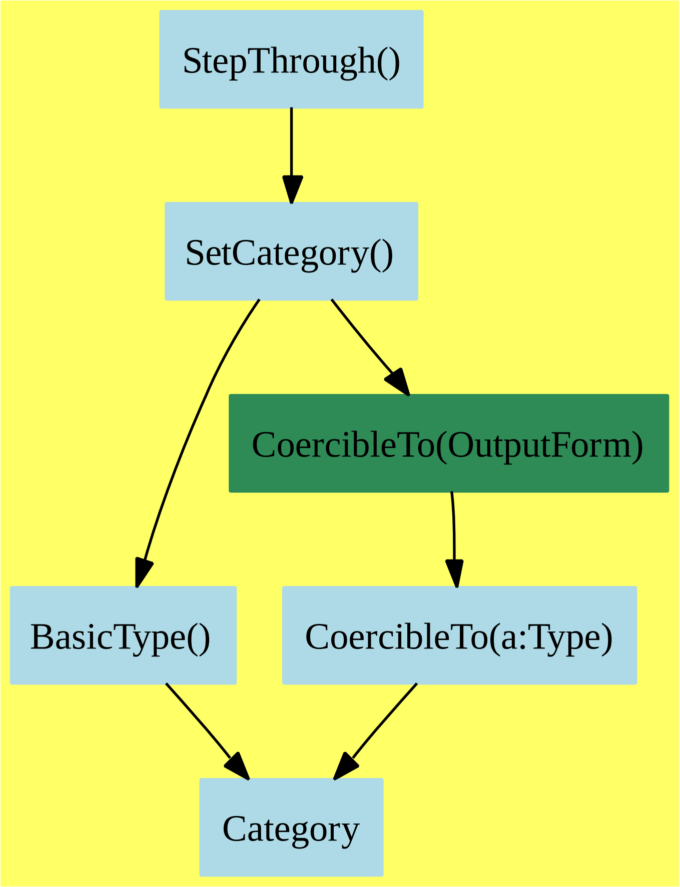
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.64 StepThrough (STEP)


StepThrough()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*StepThrough.input*⟩≡
```
)set break resume
)sys rm -f StepThrough.output
)spool StepThrough.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StepThrough
--R StepThrough  is a category constructor
--R Abbreviation for StepThrough is STEP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STEP
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             init : () -> %
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R nextItem : % -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*StepThrough.help*⟩≡

```
====================================================================
StepThrough examples
====================================================================

See Also:
o )show StepThrough
```

See:

⇒ "FiniteFieldCategory" (FFIELDC) 18.0.207 on page 1671
⇒ "IntegerNumberSystem" (INS) 16.0.186 on page 1355
⇒ "QuotientFieldCategory" (QFCAT) 17.0.198 on page 1516
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.204 on page 1620
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

coerce hash init latex ?=?
?~=? nextItem

These are directly exported but not implemented:

```
init : () -> %
nextItem : % -> Union(%,"failed")
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category STEP StepThrough*⟩≡

```
)abbrev category STEP StepThrough
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A class of objects which can be 'stepped through'.
++ Repeated applications of \spadfun{nextItem} is guaranteed never to
++ return duplicate items and only return "failed" after exhausting
++ all elements of the domain.
++ This assumes that the sequence starts with \spad{init()}.
++ For infinite domains, repeated application
++ of \spadfun{nextItem} is not required to reach all possible domain elements
++ starting from any initial element.
++
++ Conditional attributes\br
++ \tab{5}infinite\tab{5}repeated nextItem's are never "failed".

StepThrough(): Category == SetCategory with
      init: constant -> %
        ++ init() chooses an initial object for stepping.
      nextItem: % -> Union(%,"failed")
        ++ nextItem(x) returns the next item, or "failed"
        ++ if domain is exhausted.
```

⟨*STEP.dotabb*⟩≡

```
"STEP" [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"STEP" -> "SETCAT"
```

⟨*STEP.dotfull*⟩≡

```
"StepThrough()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"StepThrough()" -> "SetCategory()"
```

⟨*STEP.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StepThrough()" [color=lightblue];
"StepThrough()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

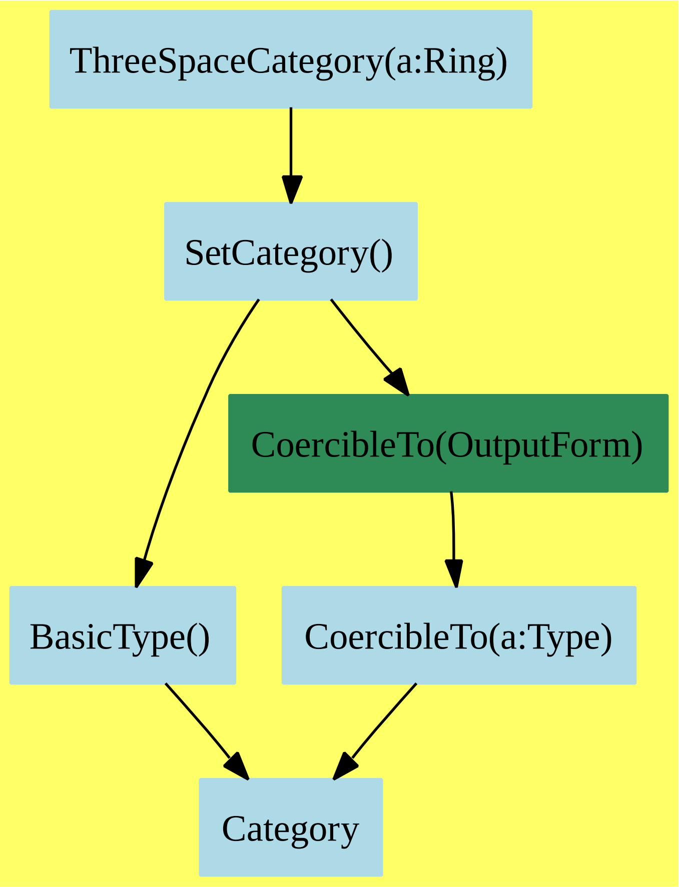
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.65 ThreeSpaceCategory (SPACEC)


ThreeSpaceCategory(a:Ring)
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*ThreeSpaceCategory.input*⟩≡

```
)set break resume
)sys rm -f ThreeSpaceCategory.output
)spool ThreeSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeSpaceCategory
--R ThreeSpaceCategory R: Ring  is a category constructor
--R Abbreviation for ThreeSpaceCategory is SPACEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SPACEC
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                check : % -> %
--R closedCurve : % -> List Point R       closedCurve : List Point R -> %
--R closedCurve? : % -> Boolean           coerce : % -> OutputForm
--R components : % -> List %              composite : List % -> %
--R composites : % -> List %              copy : % -> %
--R create3Space : SubSpace(3,R) -> %     create3Space : () -> %
--R curve : % -> List Point R             curve : List Point R -> %
--R curve : (%,List List R) -> %         curve : (%,List Point R) -> %
--R curve? : % -> Boolean                 hash : % -> SingleInteger
--R latex : % -> String                   lp : % -> List Point R
--R merge : (%,%) -> %                    merge : List % -> %
--R mesh : % -> List List Point R         mesh : List List Point R -> %
--R mesh? : % -> Boolean                  point : % -> Point R
--R point : Point R -> %                  point : (%,List R) -> %
--R point : (%,Point R) -> %              point? : % -> Boolean
--R polygon : % -> List Point R           polygon : List Point R -> %
--R polygon : (%,List List R) -> %       polygon : (%,List Point R) -> %
--R polygon? : % -> Boolean               subspace : % -> SubSpace(3,R)
--R ?~=? : (%,%) -> Boolean
--R closedCurve : (%,List List R) -> %
--R closedCurve : (%,List Point R) -> %
--R enterPointData : (%,List Point R) -> NonNegativeInteger
--R lllip : % -> List List List NonNegativeInteger
--R lllp : % -> List List List Point R
--R llprop : % -> List List SubSpaceComponentProperty
--R lprop : % -> List SubSpaceComponentProperty
--R mesh : (List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,Boolean,Boolean) -> %
--R mesh : (%,List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,List SubSpaceComponentProperty,SubSpaceComponentProperty) ->
--R mesh : (%,List List Point R,List SubSpaceComponentProperty,SubSpaceComponentProperty) -
--R modifyPointData : (%,NonNegativeInteger,Point R) -> %
--R numberOfComponents : % -> NonNegativeInteger
--R numberOfComposites : % -> NonNegativeInteger
--R objects : % -> Record(points: NonNegativeInteger,curves: NonNegativeInteger,polygons: N
--R point : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ThreeSpaceCategory.help*⟩≡

```
====================================================================
ThreeSpaceCategory examples
====================================================================

See Also:
o )show ThreeSpaceCategory
```

See:

⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

check	closedCurve	closedCurve?	coerce
components	composite	composites	copy
create3Space	curve	curve?	enterPointData
hash	latex	lllip	lllp
llprop	lp	lprop	merge
mesh	mesh?	modifyPointData	numberOfComponents
numberOfComposites	objects	point	point?
polygon	polygon?	subspace	?=?
?~=?			

These are directly exported but not implemented:

```
check : % -> %
closedCurve : (%,List List R) -> %
closedCurve : (%,List Point R) -> %
closedCurve : List Point R -> %
closedCurve : % -> List Point R
closedCurve? : % -> Boolean
coerce : % -> OutputForm
components : % -> List %
composite : List % -> %
composites : % -> List %
copy : % -> %
create3Space : () -> %
create3Space : SubSpace(3,R) -> %
curve : (%,List List R) -> %
curve : (%,List Point R) -> %
curve : List Point R -> %
curve : % -> List Point R
curve? : % -> Boolean
enterPointData : (%,List Point R) -> NonNegativeInteger
lllip : % -> List List List NonNegativeInteger
lllp : % -> List List List Point R
llprop : % -> List List SubSpaceComponentProperty
lp : % -> List Point R
lprop : % -> List SubSpaceComponentProperty
merge : List % -> %
merge : (%,%) -> %
mesh : % -> List List Point R
mesh : List List Point R -> %
mesh : (List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,Boolean,Boolean) -> %
mesh : (%,List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh : (%,List List Point R,
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh? : % -> Boolean
modifyPointData : (%,NonNegativeInteger,Point R) -> %
numberOfComponents : % -> NonNegativeInteger
numberOfComposites : % -> NonNegativeInteger
objects : % ->
  Record(points: NonNegativeInteger,
         curves: NonNegativeInteger,
         polygons: NonNegativeInteger,
         constructs: NonNegativeInteger)
point : (%,Point R) -> %
point : (%,List R) -> %
point : (%,NonNegativeInteger) -> %
point : Point R -> %
point : % -> Point R
point? : % -> Boolean
polygon : (%,List Point R) -> %
polygon : (%,List List R) -> %
polygon : List Point R -> %
polygon : % -> List Point R
polygon? : % -> Boolean
subspace : % -> SubSpace(3,R)
```

These exports come from (p130) SetCategory():

```
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SPACEC ThreeSpaceCategory*⟩≡

```
)abbrev category SPACEC ThreeSpaceCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Operations: create3Space, numberOfComponents, numberOfComposites,
++ merge, composite, components, copy, enterPointData, modifyPointData,
++ point, point?, curve, curve?, closedCurve, closedCurve?, polygon,
++ polygon? mesh, mesh?, lp, lllip, lllp, llprop, lprop, objects,
++ check, subspace, coerce
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
```

```
++ References:
++ Description:
++ The category ThreeSpaceCategory is used for creating
++ three dimensional objects using functions for defining points, curves,
++ polygons, constructs and the subspaces containing them.

ThreeSpaceCategory(R:Ring): Exports == Implementation where
 I    ==> Integer
 PI   ==> PositiveInteger
 NNI  ==> NonNegativeInteger
 L    ==> List
 B    ==> Boolean
 O    ==> OutputForm
 SUBSPACE ==> SubSpace(3,R)
 POINT   ==> Point(R)
 PROP    ==> SubSpaceComponentProperty()
 REP3D ==> Record(lp:L POINT,llliPt:L L L NNI, llProp:L L PROP, lProp:L PROP)
 OBJ3D   ==> Record(points:NNI, curves:NNI, polygons:NNI, constructs:NNI)

 Exports ==> Category
 Implementation ==>
  SetCategory with
    create3Space : () -> %
      ++ create3Space() creates a \spadtype{ThreeSpace} object capable of
      ++ holding point, curve, mesh components and any combination.
    create3Space : SUBSPACE -> %
      ++ create3Space(s) creates a \spadtype{ThreeSpace} object containing
      ++ objects pre-defined within some \spadtype{SubSpace} s.
    numberOfComponents : % -> NNI
      ++ numberOfComponents(s) returns the number of distinct
      ++ object components in the indicated \spadtype{ThreeSpace}, s, such
      ++ as points, curves, polygons, and constructs.
    numberOfComposites : % -> NNI
      ++ numberOfComposites(s) returns the number of supercomponents,
      ++ or composites, in the \spadtype{ThreeSpace}, s; Composites are
      ++ arbitrary groupings of otherwise distinct and unrelated components;
      ++ A \spadtype{ThreeSpace} need not have any composites defined at all
      ++ and, outside of the requirement that no component can belong
      ++ to more than one composite at a time, the definition and
      ++ interpretation of composites are unrestricted.
    merge : L % -> %
      ++ merge([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace} that
      ++ has the components of all the ones in the list; Groupings of
      ++ components into composites are maintained.
    merge : (%,%) -> %
      ++ merge(s1,s2) will create a new \spadtype{ThreeSpace} that has the
  ++ components of \spad{s1} and \spad{s2}; Groupings of components
  ++ into composites are maintained.
composite : L % -> %
  ++ composite([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace}
  ++ that is a union of all the components from each
  ++ \spadtype{ThreeSpace} in the parameter list, grouped as a composite.
components : % -> L %
  ++ components(s) takes the \spadtype{ThreeSpace} s, and creates a list
  ++ containing a unique \spadtype{ThreeSpace} for each single component
  ++ of s. If s has no components defined, the list returned is empty.
composites : % -> L %
  ++ composites(s) takes the \spadtype{ThreeSpace} s, and creates a list
  ++ containing a unique \spadtype{ThreeSpace} for each single composite
  ++ of s. If s has no composites defined (composites need to be
  ++ explicitly created), the list returned is empty. Note that not all
  ++ the components need to be part of a composite.
copy : % -> %
  ++ copy(s) returns a new \spadtype{ThreeSpace} that is an exact copy
  ++ of s.
enterPointData  : (%,L POINT) -> NNI
  ++ enterPointData(s,[p0,p1,...,pn]) adds a list of points from p0
  ++ through pn to the \spadtype{ThreeSpace}, s, and returns the index,
  ++ to the starting point of the list;
modifyPointData : (%,NNI,POINT) -> %
  ++ modifyPointData(s,i,p) changes the point at the indexed
  ++ location i in the \spadtype{ThreeSpace}, s, to that of point p.
  ++ This is useful for making changes to a point which has been
  ++ transformed.

-- 3D primitives
point       : (%,POINT) -> %
  ++ point(s,p) adds a point component defined by the point, p,
  ++ specified as a list from \spad{List(R)}, to the
  ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
  ++ which the point is defined.
point       : (%,L R) -> %
  ++ point(s,[x,y,z]) adds a point component defined by a list of
  ++ elements which are from the \spad{PointDomain(R)} to the
  ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
  ++ which the point elements are defined.
point       : (%,NNI) -> %
  ++ point(s,i) adds a point component which is placed into a component
  ++ list of the \spadtype{ThreeSpace}, s, at the index given by i.
point       : POINT -> %
  ++ point(p) returns a \spadtype{ThreeSpace} object which is composed
  ++ of one component, the point p.
```

```
point        : % -> POINT
  ++ point(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of only a single point and if so, returns the point.
  ++ An error is signaled otherwise.
point?       : % -> B
  ++ point?(s) queries whether the \spadtype{ThreeSpace}, s, is
  ++ composed of a single component which is a point and returns the
  ++ boolean result.
curve        : (%,L POINT) -> %
  ++ curve(s,[p0,p1,...,pn]) adds a space curve component defined by a
  ++ list of points \spad{p0} through \spad{pn}, to the
  ++ \spadtype{ThreeSpace} s.
curve        : (%,L L R) -> %
  ++ curve(s,[[p0],[p1],...,[pn]]) adds a space curve which is a list of
  ++ points p0 through pn defined by lists of elements from the domain
  ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
  ++ the point elements are defined and m is the dimension of the
  ++ points, to the \spadtype{ThreeSpace} s.
curve         : L POINT -> %
  ++ curve([p0,p1,p2,...,pn]) creates a space curve defined
  ++ by the list of points \spad{p0} through \spad{pn}, and returns the
  ++ \spadtype{ThreeSpace} whose component is the curve.
curve         : % -> L POINT
  ++ curve(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single curve defined by a list of points and if so,
  ++ returns the curve, i.e., list of points. An error is signaled
  ++ otherwise.
curve?        : % -> B
  ++ curve?(s) queries whether the \spadtype{ThreeSpace}, s, is a curve,
  ++ i.e., has one component, a list of list of points, and returns
  ++ true if it is, or false otherwise.
closedCurve  : (%,L POINT) -> %
  ++ closedCurve(s,[p0,p1,...,pn,p0]) adds a closed curve component
  ++ which is a list of points defined by the first element p0 through
  ++ the last element pn and back to the first element p0 again, to the
  ++ \spadtype{ThreeSpace} s.
closedCurve  : (%,L L R) -> %
  ++ closedCurve(s,[[lr0],[lr1],...,[lrn],[lr0]]) adds a closed curve
  ++ component defined by a list of points \spad{lr0} through
  ++ \spad{lrn}, which are lists of elements from the domain
  ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
  ++ the point elements are defined and m is the dimension of the
  ++ points, in which the last element of the list of points contains
  ++ a copy of the first element list, lr0.
  ++ The closed curve is added to the \spadtype{ThreeSpace}, s.
closedCurve         : L POINT -> %
```

```
  ++ closedCurve(lp) sets a list of points defined by the first element
  ++ of lp through the last element of lp and back to the first elelment
  ++ again and returns a \spadtype{ThreeSpace} whose component is the
  ++ closed curve defined by lp.
closedCurve         : % -> L POINT
  ++ closedCurve(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single closed curve component defined by a list of
  ++ points in which the first point is also the last point, all of
  ++ which are from the domain \spad{PointDomain(m,R)} and if so,
  ++ returns the list of points.  An error is signaled otherwise.
closedCurve?        : % -> B
  ++ closedCurve?(s) returns true if the \spadtype{ThreeSpace} s
  ++ contains a single closed curve component, i.e., the first element
  ++ of the curve is also the last element, or false otherwise.
polygon       : (%,L POINT) -> %
  ++ polygon(s,[p0,p1,...,pn]) adds a polygon component defined by a
  ++ list of points, p0 throught pn, to the \spadtype{ThreeSpace} s.
polygon       : (%,L L R) -> %
  ++ polygon(s,[[r0],[r1],...,[rn]]) adds a polygon component defined
  ++ by a list of points \spad{r0} through \spad{rn}, which are lists of
  ++ elements from the domain \spad{PointDomain(m,R)} to the
  ++ \spadtype{ThreeSpace} s, where m is the dimension of the points
  ++ and R is the \spadtype{Ring} over which the points are defined.
polygon         : L POINT -> %
  ++ polygon([p0,p1,...,pn]) creates a polygon defined by a list of
  ++ points, p0 through pn, and returns a \spadtype{ThreeSpace} whose
  ++ component is the polygon.
polygon         : % -> L POINT
  ++ polygon(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single polygon component defined by a list of
  ++ points, and if so, returns the list of points;  An error is
  ++ signaled otherwise.
polygon?        : % -> B
  ++ polygon?(s) returns true if the \spadtype{ThreeSpace} s contains
  ++ a single polygon component, or false otherwise.
mesh         : (%,L L POINT,L PROP,PROP) -> %
  ++ mesh(s,[[p0],[p1],...,[pn]],[props],prop) adds a surface component,
  ++ defined over a list curves which contains lists of points, to the
  ++ \spadtype{ThreeSpace} s; props is a list which contains the
  ++ subspace component properties for each surface parameter, and
  ++ prop is the subspace component property by which the points are
  ++ defined.
mesh         : (%,L L L R,L PROP,PROP) -> %
  ++ mesh(s,[ [[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]] ],
  ++        [props], prop)
  ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
  ++ defined over a rectangular domain of size WxH where W is the number
  ++ of lists of points from the domain \spad{PointDomain(R)} and H is
  ++ the number of elements in each of those lists; lprops is the list
  ++ of the subspace component properties for each curve list, and
  ++ prop is the subspace component property by which the points are
  ++ defined.
mesh         : (%,L L POINT,B,B) -> %
  ++ mesh(s,[[p0],[p1],...,[pn]], close1, close2) adds a surface
  ++ component to the \spadtype{ThreeSpace}, which is defined over a
  ++ list of curves, in which each of these curves is a list of points.
  ++ The boolean arguments close1 and close2 indicate how the surface
  ++ is to be closed. Argument close1 equal true
  ++ means that each individual list (a curve) is to be closed, i.e. the
  ++ last point of the list is to be connected to the first point.
  ++ Argument close2 equal true
  ++ means that the boundary at one end of the surface is to be
  ++ connected to the boundary at the other end, i.e. the boundaries
  ++ are defined as the first list of points (curve) and
  ++ the last list of points (curve).
mesh         : (%,L L L R,B,B) -> %
  ++ mesh(s,[ [[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]] ],
  ++      close1, close2)
  ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
  ++ defined over a rectangular domain of size WxH where W is the number
  ++ of lists of points from the domain \spad{PointDomain(R)} and H is
  ++ the number of elements in each of those lists; the booleans close1
  ++ and close2 indicate how the surface is to be closed: if close1 is
  ++ true this means that each individual list (a curve) is to be
  ++ closed (i.e.,
  ++ the last point of the list is to be connected to the first point);
  ++ if close2 is true, this means that the boundary at one end of the
  ++ surface is to be connected to the boundary at the other end
  ++ (the boundaries are defined as the first list of points (curve)
  ++ and the last list of points (curve)).
mesh         : L L POINT -> %
  ++ mesh([[p0],[p1],...,[pn]]) creates a surface defined by a list of
  ++ curves which are lists, p0 through pn, of points, and returns a
  ++ \spadtype{ThreeSpace} whose component is the surface.
mesh         : (L L POINT,B,B) -> %
  ++ mesh([[p0],[p1],...,[pn]], close1, close2) creates a surface
  ++ defined over a list of curves, p0 through pn, which are lists of
  ++ points; the booleans close1 and close2 indicate how the surface is
  ++ to be closed: close1 set to true means that each individual list
  ++ (a curve) is to be closed (that is, the last point of the list is
  ++ to be connected to the first point); close2 set to true means
  ++ that the boundary at one end of the surface is to be connected to
  ++ the boundary at the other end (the boundaries are defined as the
  ++ first list of points (curve) and the last list of points (curve));
  ++ the \spadtype{ThreeSpace} containing this surface is returned.
mesh          : % -> L L POINT
  ++ mesh(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single surface component defined by a list curves
  ++ which contain lists of points, and if so, returns the list of
  ++ lists of points;  An error is signaled otherwise.
mesh?         : % -> B
  ++ mesh?(s) returns true if the \spadtype{ThreeSpace} s is composed
  ++ of one component, a mesh comprising a list of curves which are lists
  ++ of points, or returns false if otherwise
lp            : % -> L POINT
  ++ lp(s) returns the list of points component which the
  ++ \spadtype{ThreeSpace}, s, contains; these points are used by
  ++ reference, i.e., the component holds indices referring to the
  ++ points rather than the points themselves. This allows for sharing
  ++ of the points.
lllip         : % -> L L L NNI
  ++ lllip(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of components, which are lists of curves,
  ++ which are lists of indices to points, and if so, returns the list
  ++ of lists of lists;  An error is signaled otherwise.
lllp          : % -> L L L POINT   -- used by view3D
  ++ lllp(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of components, which are lists of curves,
  ++ which are lists of points, and if so, returns the list of
  ++ lists of lists;  An error is signaled otherwise.
llprop        : % -> L L PROP      -- used by view3D
  ++ llprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of curves which are lists of the
  ++ subspace component properties of the curves, and if so, returns the
  ++ list of lists;  An error is signaled otherwise.
lprop         : % -> L PROP        -- used by view3D
  ++ lprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of subspace component properties, and if so,
  ++ returns the list;  An error is signaled otherwise.
objects       : % -> OBJ3D
  ++ objects(s) returns the \spadtype{ThreeSpace}, s, in the form of a
  ++ 3D object record containing information on the number of points,
  ++ curves, polygons and constructs comprising the
  ++ \spadtype{ThreeSpace}..
check         : % -> %             -- used by mesh
  ++ check(s) returns lllpt, list of lists of lists of point information
  ++ about the \spadtype{ThreeSpace} s.
subspace      : % -> SUBSPACE
```

```
    ++ subspace(s) returns the \spadtype{SubSpace} which holds all the
    ++ point information in the \spadtype{ThreeSpace}, s.
  coerce       : % -> O
    ++ coerce(s) returns the \spadtype{ThreeSpace} s to Output format.
```

⟨*SPACEC.dotabb*⟩≡

```
"SPACEC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"SPACEC" -> "SETCAT"
```

⟨*SPACEC.dotfull*⟩≡

```
"ThreeSpaceCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"
```

⟨*SPACEC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ThreeSpaceCategory(a:Ring)" [color=lightblue];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

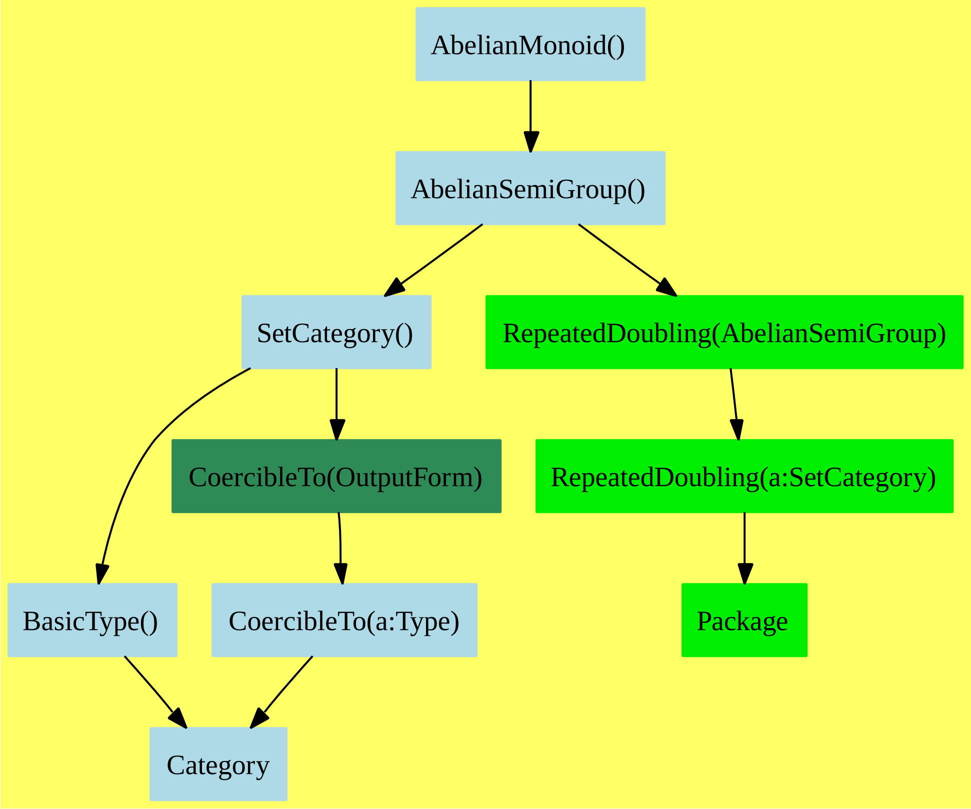
"Category" [color=lightblue];

}
```

Chapter 5

Category Layer 4

5.0.66 AbelianMonoid (ABELMON)


AbelianMonoid()
AbelianSemiGroup()
SetCategory()
RepeatedDoubling(AbelianSemiGroup)
CoercibleTo(OutputForm)
RepeatedDoubling(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*AbelianMonoid.input*⟩≡

```
)set break resume
)sys rm -f AbelianMonoid.output
)spool AbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianMonoid
--R AbelianMonoid  is a category constructor
--R Abbreviation for AbelianMonoid is ABELMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*AbelianMonoid.help*⟩≡

```
====================================================================
AbelianMonoid examples
====================================================================

See Also:
o )show AbelianMonoid
```

See:

⇒ "CancellationAbelianMonoid" (CABMON) 6.0.83 on page 423
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "OrderedAbelianMonoid" (OAMON) 7.0.105 on page 636
⇒ "OrderedAbelianSemiGroup" (OASGP) 6.0.89 on page 507
⇐ "AbelianSemiGroup" (ABELSG) 4.0.36 on page 141

Exports:

0	coerce	hash	latex	sample
zero?	?*?	?+?	?=?	?~=?

These are directly exported but not implemented:

```
0 : () -> %
```

These are implemented by this category:

```
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p141) AbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
```

⟨*category ABELMON AbelianMonoid*⟩≡

```
)abbrev category ABELMON AbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of multiplicative monoids, i.e. semigroups with an
++ additive identity element.
++
++ Axioms\br
++ \tab{5}\spad{leftIdentity("+":(%,%)->%,0)}\tab{5}\spad{ 0+x=x }\br
++ \tab{5}\spad{rightIdentity("+":(%,%)->%,0)}\tab{4}\spad{ x+0=x }
-- following domain must be compiled with subsumption disabled
-- define SourceLevelSubset to be EQUAL

AbelianMonoid(): Category == AbelianSemiGroup with
      0: constant -> %
        ++ 0 is the additive identity element.
      sample: constant -> %
        ++ sample yields a value of type %
      zero?: % -> Boolean
        ++ zero?(x) tests if x is equal to 0.
      "*": (NonNegativeInteger,%) -> %
        ++ n * x is left-multiplication by a non negative integer
    add
      import RepeatedDoubling(%)
      zero? x == x = 0
      n:PositiveInteger * x:% == (n::NonNegativeInteger) * x
      sample() == 0
      if not (% has Ring) then
        n:NonNegativeInteger * x:% ==
          zero? n => 0
          double(n pretend PositiveInteger,x)
```

⟨*ABELMON.dotabb*⟩≡

```
"ABELMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"ABELMON" -> "ABELSG"
```

⟨*ABELMON.dotfull*⟩≡

```
"AbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"AbelianMonoid()" -> "AbelianSemiGroup()"
```

⟨*ABELMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

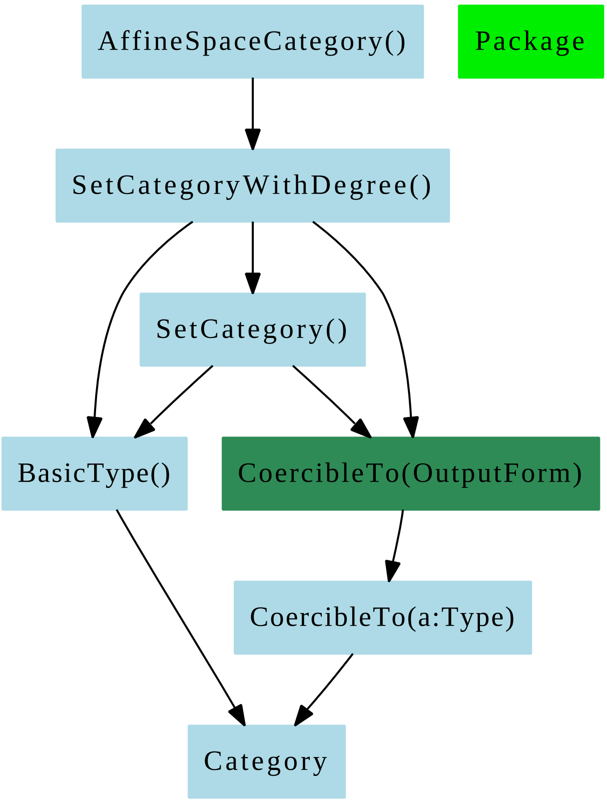
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

5.0.67 AffineSpaceCategory (AFSPCAT)


AffineSpaceCategory()
Package
SetCategoryWithDegree()
SetCategory()
BasicType()
CoercibleTo(OutputForm)
CoercibleTo(a:Type)
Category

⟨*AffineSpaceCategory.input*⟩≡

```
)set break resume
)sys rm -f AffineSpaceCategory.output
)spool AffineSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffineSpaceCategory
--R AffineSpaceCategory K: Field  is a category constructor
--R Abbreviation for AffineSpaceCategory is AFSPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AFSPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                affinePoint : List K -> %
```

```
--R coerce : List K -> %                  coerce : % -> List K
--R coerce : % -> OutputForm              conjugate : % -> %
--R definingField : % -> K                degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K                hash : % -> SingleInteger
--R latex : % -> String                   list : % -> List K
--R orbit : % -> List %                   origin : () -> %
--R pointValue : % -> List K              rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K           ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

⟨*AffineSpaceCategory.help*⟩≡

```
====================================================================
AffineSpaceCategory examples
====================================================================

See Also:
o )show AffineSpaceCategory
```

See:

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.62 on page 274

Exports:

?.?	?=?	?~=?	affinePoint	coerce
conjugate	definingField	degree	hash	latex
list	orbit	origin	pointValue	rational?
removeConjugate	setelt			

These are directly exported but not implemented

```
?.? : (%,Integer) -> K
affinePoint : List K -> %
coerce : % -> List K
coerce : List K -> %
conjugate : % -> %
conjugate : (%,NNI) -> %
definingField : % -> K
list : % -> List K
orbit : % -> List %
orbit : (%,NNI) -> List %
origin : () -> %
pointValue : % -> List K
rational? : % -> Boolean
rational? : (%,NNI) -> Boolean
removeConjugate : (List %,NNI) -> List %
removeConjugate : List % -> List %
setelt : (%,Integer,K) -> K
```

These exports come from (p274) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

⟨*category AFSPCAT AffineSpaceCategory*⟩≡

```
)abbrev category AFSPCAT AffineSpaceCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffineSpaceCategory(K:Field):Category == Implementation where
```

```
LIST ==> List
INT  ==> Integer
NNI  ==> NonNegativeInteger

Implementation ==>
 SetCategoryWithDegree with

  origin: () -> %

  affinePoint: LIST(K) -> %
    ++ affinePoint creates a affine point from a list

  definingField: % -> K

  coerce: % -> List(K)

  coerce: LIST(K) -> %
    ++ coerce a list of K to a affine point.

  orbit: % -> LIST %
    ++ orbit(p) returns the orbit of the point p according to the
    ++ characteristic of K, that is, for q= char K,
    ++ orbit(p) = \{ p, p**q, p**(q**2), p**(q**3), ..... \}

  orbit: (%,NNI) -> LIST %
    ++ orbit(p,n) returns the orbit of the point p according to n, that is
    ++ orbit(p,n) = \{ p, p**n, p**(n**2), p**(n**3), ..... \}

  conjugate: (%,NNI) -> %
    ++ conjugate(p,n) returns p**n, that is all the coordinates of p
    ++ to the power of n

  conjugate: % -> %
    ++ conjugate(p) returns conjugate(p,n) where n is the characteristic
    ++ of the ground field.

  removeConjugate: (LIST(%),NNI) -> LIST(%)
    ++ removeConjugate(lp,n) returns a list of points such that no points
    ++ in the list is the conjugate (according to n) of another point.

  removeConjugate: LIST(%)  -> LIST(%)
    ++ removeConjugate(lp) returns removeConjugate(lp,n)
    ++ where n is the characteristic of the ground field.

  rational?: (%,NNI) -> Boolean
```

```
    ++ rational?(p,n) test if the point is rational according to n.

  rational?: % -> Boolean
    ++ rational?(p) test if the point is rational according to the
    ++ characteristic of the ground field.

  list  : % -> LIST(K)
    ++ list returns the list of the coordinates

  elt: (%,INT) -> K
    ++ elt returns the value of a specified coordinates

  setelt: (%,INT,K) -> K
    ++ setelt sets the value of a specified coordinates

  pointValue: % -> LIST(K)
   ++ pointValue returns the coordinates of the point or of the point
   ++ of origin that represent an infinitly close point
```

⟨*AFSPCAT.dotabb*⟩≡

```
"AFSPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AFSPCAT"];
"AFSPCAT" -> "SETCATD"
```

⟨*AFSPCAT.dotfull*⟩≡

```
"AffineSpaceCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AFSPCAT"];
"AffineSpaceCategory()" -> "SetCategoryWithDegree()"
```

⟨*AFSPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AffineSpaceCategory()" [color=lightblue];
"AffineSpaceCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

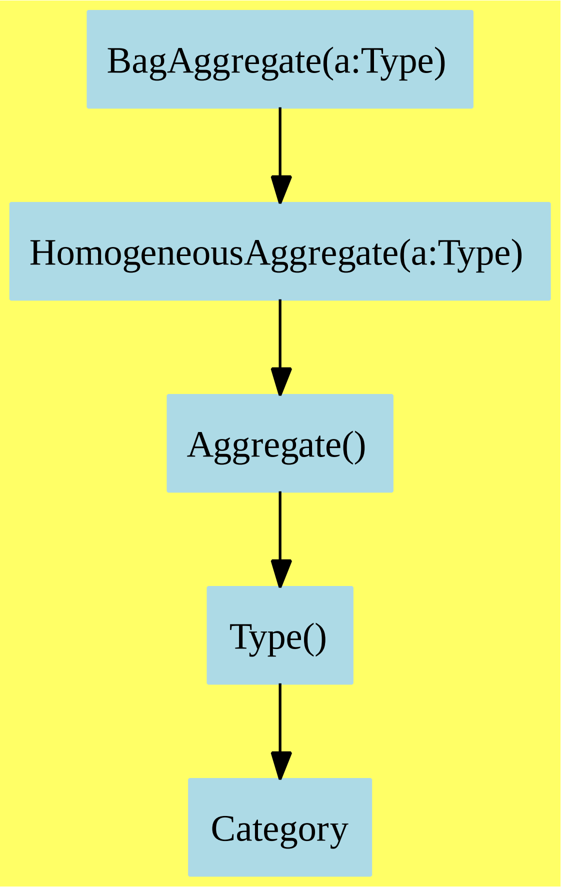
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.68 BagAggregate (BGAGG)


BagAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*BagAggregate.input*⟩≡

```
)set break resume
)sys rm -f BagAggregate.output
)spool BagAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BagAggregate
--R BagAggregate S: Type  is a category constructor
--R Abbreviation for BagAggregate is BGAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BGAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                    copy : % -> %
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               extract! : % -> S
--R insert! : (S,%) -> %                 inspect : % -> S
--R map : ((S -> S),%) -> %              sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*BagAggregate.help*⟩≡

```
====================================================================
BagAggregate examples
====================================================================

See Also:
o )show BagAggregate
```

See:

⇒ “DictionaryOperations” (DIOPS) 6.0.84 on page 428
⇒ “PriorityQueueAggregate” (PRQAGG) 6.0.92 on page 534
⇒ “QueueAggregate” (QUAGG) 6.0.93 on page 540
⇒ “StackAggregate” (SKAGG) 6.0.95 on page 556
⇐ “HomogeneousAggregate” (HOAGG) 4.0.49 on page 203

Exports:

any?
empty
extract!
less?
more?
?=?
bag
empty?
hash
map
parts
?~=?
coerce
eq?
insert!
map!
sample
copy
eval
inspect
member?
size?
count
every?
latex
members
#?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
extract! : % -> S
insert! : (S,%) -> %
inspect : % -> S
```

These are implemented by this category:

```
bag : List S -> %
```

These exports come from (p203) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
```

```
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
\#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category BGAGG BagAggregate*⟩≡

```
)abbrev category BGAGG BagAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A bag aggregate is an aggregate for which one can insert and extract
++ objects, and where the order in which objects are inserted determines
++ the order of extraction.
++ Examples of bags are stacks, queues, and dequeues.

BagAggregate(S:Type): Category == HomogeneousAggregate S with
   shallowlyMutable
     ++ shallowlyMutable means that elements of bags may be
     ++ destructively changed.
   bag: List S -> %
     ++ bag([x,y,...,z]) creates a bag with elements x,y,...,z.
```

```
    extract_!: % -> S
      ++ extract!(u) destructively removes a (random) item from bag u.
    insert_!: (S,%) -> %
      ++ insert!(x,u) inserts item x into bag u.
    inspect: % -> S
      ++ inspect(u) returns an (random) element from a bag.
  add
    bag(l) ==
      x:=empty()
      for s in l repeat x:=insert_!(s,x)
      x
```

⟨*BGAGG.dotabb*⟩≡

```
"BGAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BGAGG" -> "HOAGG"
```

⟨*BGAGG.dotfull*⟩≡

```
"BagAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"BagAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"
```

⟨*BGAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

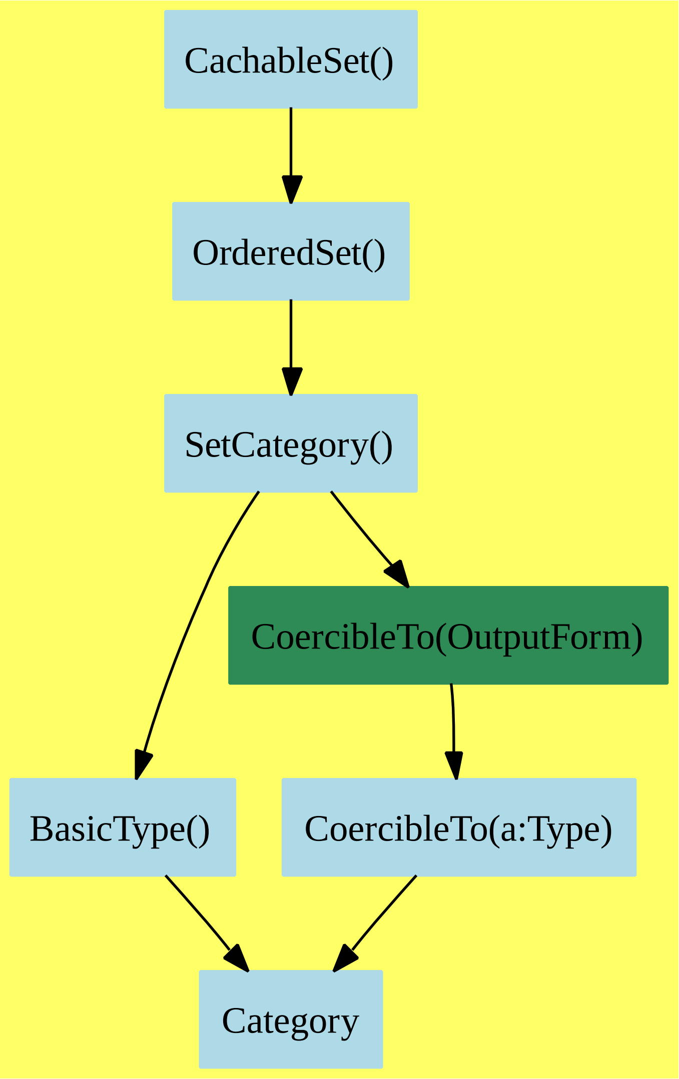
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.0.69 CachableSet (CACHSET)


CachableSet()
OrderedSet()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*CachableSet.input*⟩≡
```
)set break resume
)sys rm -f CachableSet.output
)spool CachableSet.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show CachableSet
--R CachableSet  is a category constructor
--R Abbreviation for CachableSet is CACHSET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CACHSET
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R position : % -> NonNegativeInteger
--R setPosition : (%,NonNegativeInteger) -> Void
--R
--E 1

)spool
)lisp (bye)
```

⟨*CachableSet.help*⟩≡

```
====================================================================
CachableSet examples
====================================================================

See Also:
o )show CachableSet
```

See:

⇐ "OrderedSet" (ORDSET) 4.0.56 on page 242

Exports:

coerce	hash	latex	max	min
position	setPosition	?~=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
position : % -> NonNegativeInteger
setPosition : (%,NonNegativeInteger) -> Void
```

These exports come from (p242) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category CACHSET CachableSet*⟩≡

```
)abbrev category CACHSET CachableSet
++ Sets whose elements can cache an integer
++ Author: Manuel Bronstein
++ Date Created: 31 Oct 1988
++ Date Last Updated: 14 May 1991
++ Description:
++ A cachable set is a set whose elements keep an integer as part
++ of their structure.

CachableSet: Category == OrderedSet with
  position   : % -> NonNegativeInteger
    ++ position(x) returns the integer n associated to x.
  setPosition: (%, NonNegativeInteger) -> Void
    ++ setPosition(x, n) associates the integer n to x.
```

⟨*CACHSET.dotabb*⟩≡
```
"CACHSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CACHSET" -> "ORDSET"
```

⟨*CACHSET.dotfull*⟩≡
```
"CachableSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CachableSet()" -> "OrderedSet()"
```

⟨*CACHSET.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CachableSet()" [color=lightblue];
"CachableSet()" -> "OrderedSet()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

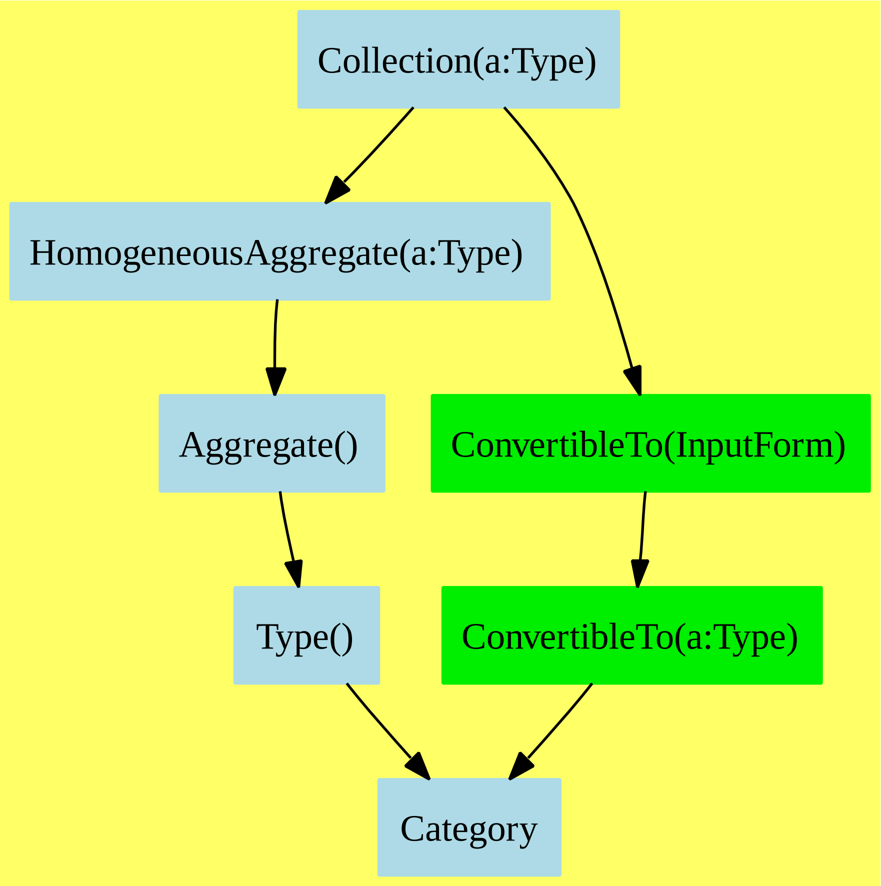
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.70 Collection (CLAGG)


Collection(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
ConvertibleTo(InputForm)
Type()
ConvertibleTo(a:Type)
Category

⟨*Collection.input*⟩≡
```
)set break resume
)sys rm -f Collection.output
)spool Collection.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Collection
--R Collection S: Type  is a category constructor
--R Abbreviation for Collection is CLAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CLAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R construct : List S -> %              copy : % -> %
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               map : ((S -> S),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*Collection.help*⟩≡

```
====================================================================
Collection examples
====================================================================

See Also:
o )show Collection
```

See:

⇒ “DictionaryOperations” (DIOPS) 6.0.84 on page 428
⇒ “LinearAggregate” (LNAGG) 6.0.87 on page 449
⇒ “PolynomialSetCategory” (PSETCAT) 6.0.91 on page 518
⇒ “SetAggregate” (SETAGG) 6.0.94 on page 547
⇐ “ConvertibleTo” (KONVERT) 2.0.8 on page 25
⇐ “HomogeneousAggregate” (HOAGG) 4.0.49 on page 203

Exports:

any?	coerce	construct	copy	convert
count	empty	empty?	eq?	eval
every?	find	hash	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	removeDuplicates	sample
select	size?	#?	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
construct: List S -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p203) HomogeneousAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
            if S has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p25) ConvertibleTo(S:Type):

```
convert : % -> InputForm if S has KONVERT INFORM
```

⟨*category CLAGG Collection*⟩≡

```
)abbrev category CLAGG Collection
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

```
++ A collection is a homogeneous aggregate which can built from
++ list of members. The operation used to build the aggregate is
++ generically named \spadfun{construct}. However, each collection
++ provides its own special function with the same name as the
++ data type, except with an initial lower case letter, e.g.
++ \spadfun{list} for \spadtype{List},
++ \spadfun{flexibleArray} for \spadtype{FlexibleArray}, and so on.

Collection(S:Type): Category == HomogeneousAggregate(S) with
   construct: List S -> %
     ++ \axiom{construct(x,y,...,z)} returns the collection of elements
     ++ \axiom{x,y,...,z} ordered as given. Equivalently written as
     ++ \axiom{[x,y,...,z]$D}, where
     ++ D is the domain. D may be omitted for those of type List.
   find: (S->Boolean, %) -> Union(S, "failed")
     ++ find(p,u) returns the first x in u such that \axiom{p(x)} is true,
     ++ and "failed" otherwise.
   if % has finiteAggregate then
      reduce: ((S,S)->S,%) -> S
        ++ reduce(f,u) reduces the binary operation f across u. For example,
        ++ if u is \axiom{[x,y,...,z]} then \axiom{reduce(f,u)}
        ++ returns \axiom{f(..f(f(x,y),...),z)}.
        ++ Note that if u has one element x, \axiom{reduce(f,u)} returns x.
        ++ Error: if u is empty.
        ++
        ++C )clear all
        ++X reduce(+,[C[i]*x**i for i in 1..5])

      reduce: ((S,S)->S,%,S) -> S
        ++ reduce(f,u,x) reduces the binary operation f across u, where x is
        ++ the identity operation of f.
        ++ Same as \axiom{reduce(f,u)} if u has 2 or more elements.
        ++ Returns \axiom{f(x,y)} if u has one element y,
        ++ x if u is empty.
        ++ For example, \axiom{reduce(+,u,0)} returns the
        ++ sum of the elements of u.
      remove: (S->Boolean,%) -> %
        ++ remove(p,u) returns a copy of u removing all elements x such that
        ++ \axiom{p(x)} is true.
        ++ Note that \axiom{remove(p,u) == [x for x in u | not p(x)]}.
      select: (S->Boolean,%) -> %
        ++ select(p,u) returns a copy of u containing only those elements
        ++ such \axiom{p(x)} is true.
        ++ Note that \axiom{select(p,u) == [x for x in u | p(x)]}.
      if S has SetCategory then
        reduce: ((S,S)->S,%,S,S) -> S
```

```
          ++ reduce(f,u,x,z) reduces the binary operation f across u,
          ++ stopping when an "absorbing element" z is encountered.
          ++ As for \axiom{reduce(f,u,x)}, x is the identity operation of f.
          ++ Same as \axiom{reduce(f,u,x)} when u contains no element z.
          ++ Thus the third argument x is returned when u is empty.
        remove: (S,%) -> %
          ++ remove(x,u) returns a copy of u with all
          ++ elements \axiom{y = x} removed.
          ++ Note that \axiom{remove(y,c) == [x for x in c | x ^= y]}.
        removeDuplicates: % -> %
          ++ removeDuplicates(u) returns a copy of u with all duplicates
          ++ removed.
    if S has ConvertibleTo InputForm then ConvertibleTo InputForm
  add
    if % has finiteAggregate then
      #c                           == # parts c
      count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]
      any?(f, c)                   == _or/[f x for x in parts c]
      every?(f, c)           == _and/[f x for x in parts c]
      find(f:S -> Boolean, c:%) == find(f, parts c)
      reduce(f:(S,S)->S, x:%) == reduce(f, parts x)
      reduce(f:(S,S)->S, x:%, s:S) == reduce(f, parts x, s)
      remove(f:S->Boolean, x:%) ==
        construct remove(f, parts x)
      select(f:S->Boolean, x:%) ==
        construct select(f, parts x)

      if S has SetCategory then
        remove(s:S, x:%) == remove(y +-> y = s, x)
        reduce(f:(S,S)->S, x:%, s1:S, s2:S) == reduce(f, parts x, s1, s2)
        removeDuplicates(x) == construct removeDuplicates parts x
```

⟨*CLAGG.dotabb*⟩≡

```
  "CLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
  "CLAGG" -> "HOAGG"
```

⟨*CLAGG.dotfull*⟩≡
```
"Collection(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
  -> "Collection(a:Type)"
```

⟨*CLAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

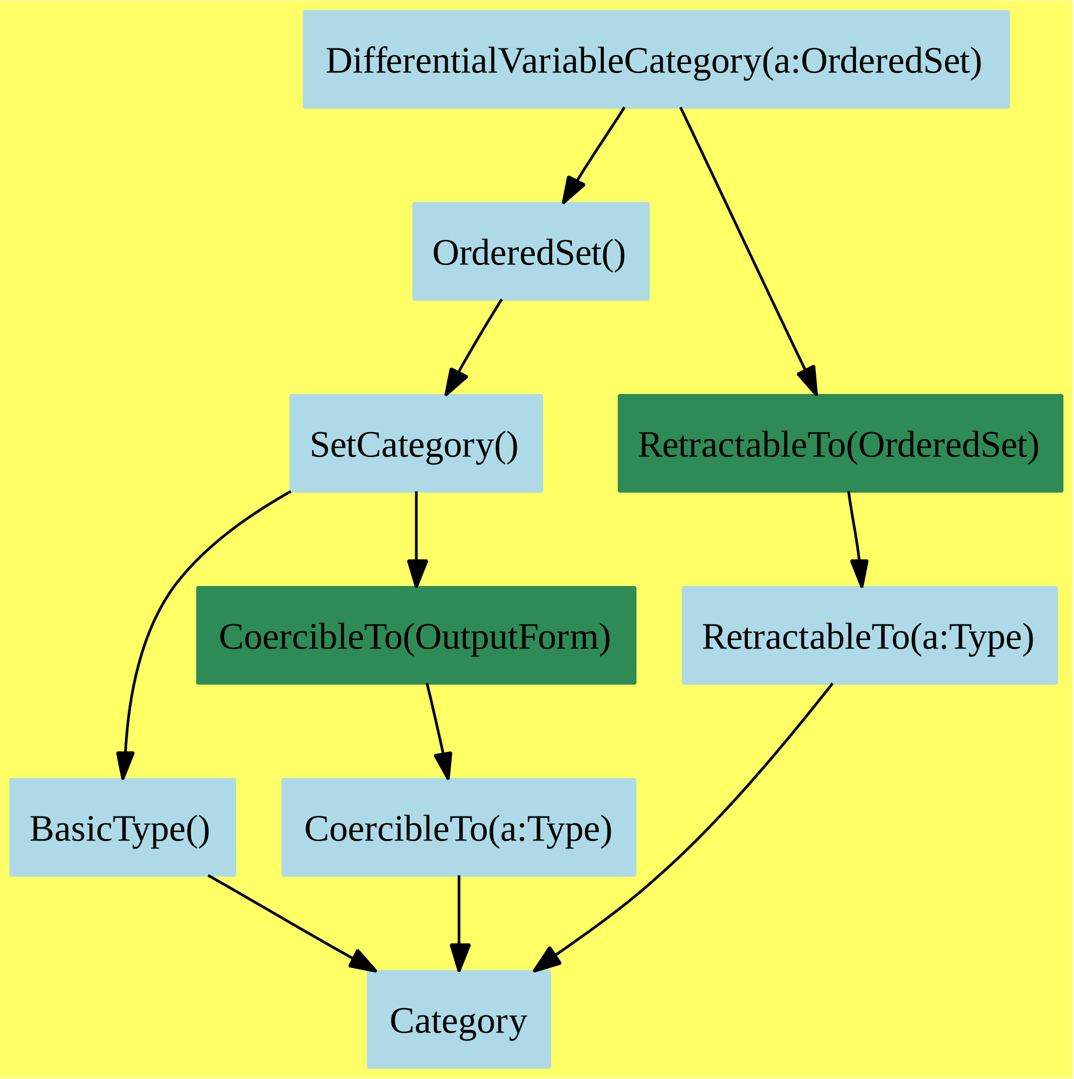
"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.71 DifferentialVariableCategory (DVARCAT)


DifferentialVariableCategory(a:OrderedSet)
OrderedSet()
SetCategory()
RetractableTo(OrderedSet)
CoercibleTo(OutputForm)
RetractableTo(a:Type)
BasicType()
CoercibleTo(a:Type)
Category

⟨*DifferentialVariableCategory.input*⟩≡

```
)set break resume
)sys rm -f DifferentialVariableCategory.output
)spool DifferentialVariableCategory.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show DifferentialVariableCategory
--R DifferentialVariableCategory S: OrderedSet  is a category constructor
--R Abbreviation for DifferentialVariableCategory is DVARCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DVARCAT
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : S -> %
--R coerce : % -> OutputForm              differentiate : % -> %
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R order : % -> NonNegativeInteger       retract : % -> S
--R variable : % -> S                     weight : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*DifferentialVariableCategory.help*⟩≡

```
====================================================================
DifferentialVariableCategory examples
====================================================================

See Also:
o )show DifferentialVariableCategory
```

See:

⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62

Exports:

coerce	differentiate	hash	latex	makeVariable
max	min	order	retract	retractIfCan
variable	weight	?˜=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
makeVariable : (S,NonNegativeInteger) -> %
order : % -> NonNegativeInteger
variable : % -> S
```

These are implemented by this category:

```
coerce : S -> %
coerce : % -> OutputForm
differentiate : % -> %
differentiate : (%,NonNegativeInteger) -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
weight : % -> NonNegativeInteger
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p242) OrderedSet():

```
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p62) RetractableTo(S:OrderedSet):

⟨*category DVARCAT DifferentialVariableCategory*⟩≡

```
)abbrev category DVARCAT DifferentialVariableCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:
++ Related Constructors:DifferentialPolynomialCategory
++ See Also:OrderedDifferentialVariable,
++          SequentialDifferentialVariable,
++          DifferentialSparseMultivariatePolynomial.
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, order, weight
++ References:Ritt, J.F. "Differential Algebra" (Dover, 1950).
++ Description:
++ \spadtype{DifferentialVariableCategory} constructs the
++ set of derivatives of a given set of
++ (ordinary) differential indeterminates.
++ If x,...,y is an ordered set of differential indeterminates,
++ and the prime notation is used for differentiation, then
++ the set of derivatives (including
++ zero-th order) of the differential indeterminates is
++ x,\spad{x'},\spad{x''},..., y,\spad{y'},\spad{y''},...
++ (Note that in the interpreter, the n-th derivative of y is displayed as
++ y with a subscript n.)  This set is
++ viewed as a set of algebraic indeterminates, totally ordered in a
++ way compatible with differentiation and the given order on the
++ differential indeterminates.  Such a total order is called a
++ ranking of the differential indeterminates.
++
++ A domain in this category is needed to construct a differential
++ polynomial domain.  Differential polynomials are ordered
++ by a ranking on the derivatives,  and by an order (extending the
++ ranking) on
++ on the set of differential monomials.  One may thus associate
++ a domain in this category with a ranking of the differential
++ indeterminates, just as one associates a domain in the category
++ \spadtype{OrderedAbelianMonoidSup} with an ordering of the set of
++ monomials in a set of algebraic indeterminates.  The ranking
++ is specified through the binary relation \spadfun{<}.
++ For example, one may define
++ one derivative to be less than another by lexicographically comparing
++ first the \spadfun{order}, then the given order of the differential
++ indeterminates appearing in the derivatives.  This is the default
++ implementation.
++
++ The notion of weight generalizes that of degree.  A
++ polynomial domain may be made into a graded ring
++ if a weight function is given on the set of indeterminates,
++ Very often, a grading is the first step in ordering the set of
++ monomials.  For differential polynomial domains, this
++ constructor provides a function \spadfun{weight}, which
++ allows the assignment of a non-negative number to each derivative of a
++ differential indeterminate.  For example, one may define
++ the weight of a derivative to be simply its \spadfun{order}
++ (this is the default assignment).
++ This weight function can then be extended to the set of
++ all differential polynomials, providing a graded ring structure.

DifferentialVariableCategory(S:OrderedSet): Category ==
  Join(OrderedSet, RetractableTo S) with
    -- Examples:
    -- v:=makeVariable('s,5)
    makeVariable  : (S, NonNegativeInteger) -> $
       ++ makeVariable(s, n) returns the n-th derivative of a
       ++ differential indeterminate s as an algebraic indeterminate.
       -- Example: makeVariable('s, 5)
    order         : $ -> NonNegativeInteger
       ++ order(v) returns n if v is the n-th derivative of any
       ++ differential indeterminate.
       -- Example: order(v)
    variable      : $ -> S
       ++ variable(v) returns s if v is any derivative of the differential
       ++ indeterminate s.
       -- Example: variable(v)
          --  default implementation using above primitives --

    weight        : $ -> NonNegativeInteger
       ++ weight(v) returns the weight of the derivative v.
       -- Example: weight(v)
    differentiate : $ -> $
       ++ differentiate(v) returns the derivative of v.
       -- Example: differentiate(v)
    differentiate : ($, NonNegativeInteger) -> $
       ++ differentiate(v, n) returns the n-th derivative of v.
       -- Example: differentiate(v,2)
    coerce        : S  -> $
       ++ coerce(s) returns s, viewed as the zero-th order derivative of s.
       -- Example: coerce('s); differentiate(%,5)
 add
    import NumberFormats

    coerce (s:S):$ == makeVariable(s, 0)

    differentiate v     == differentiate(v, 1)

    differentiate(v, n) == makeVariable(variable v, n + order v)
    retractIfCan v == (zero?(order v) => variable v; "failed")

    v = u      == (variable v = variable u) and (order v = order u)

    coerce(v:$):OutputForm ==
      a := variable(v)::OutputForm
      zero?(nn := order v) => a
      sub(a, outputForm nn)

    retract v ==
      zero?(order v) => variable v
      error "Not retractable"

    v < u ==
      -- the ranking below is orderly, and is the default --
      order v = order u => variable v < variable u
      order v < order u

    weight v == order v
      --  the default weight is just the order
```

⟨*DVARCAT.dotabb*⟩≡

```
"DVARCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
"DVARCAT" -> "ORDSET"
"DVARCAT" -> "RETRACT"
```

⟨*DVARCAT.dotfull*⟩≡

```
"DifferentialVariableCategory(a:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
"DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
"DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"
```

⟨*DVARCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialVariableCategory(a:OrderedSet)" [color=lightblue];
"DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
"DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"

"RetractableTo(OrderedSet)" [color=seagreen];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

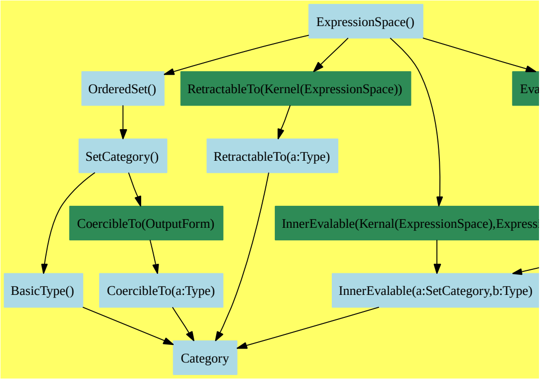
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.72 ExpressionSpace (ES)


ExpressionSpace()
OrderedSet()
RetractableTo(Kernel(ExpressionSpace))
Eva
SetCategory()
RetractableTo(a:Type)
CoercibleTo(OutputForm)
InnerEvalable(Kernal(ExpressionSpace),Express
BasicType()
CoercibleTo(a:Type)
InnerEvalable(a:SetCategory,b:Type)
Category

⟨*ExpressionSpace.input*⟩≡

```
)set break resume
)sys rm -f ExpressionSpace.output
)spool ExpressionSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExpressionSpace
--R ExpressionSpace  is a category constructor
--R Abbreviation for ExpressionSpace is ES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ES
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               belong? : BasicOperator -> Boolean
--R box : List % -> %                     box : % -> %
--R coerce : Kernel % -> %                coerce : % -> OutputForm
--R distribute : (%,%) -> %               distribute : % -> %
--R elt : (BasicOperator,%,%) -> %        elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %            freeOf? : (%,Symbol) -> Boolean
--R freeOf? : (%,%) -> Boolean            hash : % -> SingleInteger
--R height : % -> NonNegativeInteger      is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %       kernels : % -> List Kernel %
--R latex : % -> String                   map : ((% -> %),Kernel %) -> %
```

```
--R max : (%,%) -> %                     min : (%,%) -> %
--R paren : List % -> %                  paren : % -> %
--R retract : % -> Kernel %              subst : (%,Equation %) -> %
--R tower : % -> List Kernel %           ?~=? : (%,%) -> Boolean
--R definingPolynomial : % -> % if $ has RING
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R retractIfCan : % -> Union(Kernel %,"failed")
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ExpressionSpace.help*⟩≡

```
====================================================================
ExpressionSpace examples
====================================================================

See Also:
o )show ExpressionSpace
```

See:

⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇐ “Evalable” (EVALAB) 3.0.25 on page 91
⇐ “InnerEvalable” (IEVALAB) 2.0.12 on page 39
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62

Exports:

belong?	box	coerce	definingPolynomial
distribute	elt	eval	even?
freeOf?	hash	height	is?
kernel	kernels	latex	mainKernel
map	max	min	minPoly
odd?	operator	operators	paren
retract	retractIfCan	subst	tower
?<?	?<=?	?=?	?>?
?>=?	?~=?		

These are directly exported but not implemented:

```
definingPolynomial : % -> % if $ has RING
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List Kernel %,List %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
kernels : % -> List Kernel %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
subst : (%,List Kernel %,List %) -> %
```

These are implemented by this category:

```
belong? : BasicOperator -> Boolean
box : List % -> %
box : % -> %
distribute : % -> %
distribute : (%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
subst : (%,Equation %) -> %
subst : (%,List Equation %) -> %
tower : % -> List Kernel %
```

These exports come from (p242) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p62) RetractableTo(a:Type):

```
coerce : Kernel % -> %
```

These exports come from (p39) InnerEvalable(a:SetCategory,b:Type):

These exports come from (p91) Evalable(a:SetCategory):

⟨*category ES ExpressionSpace*⟩≡

```
)abbrev category ES ExpressionSpace
++ Category for domains on which operators can be applied
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 27 May 1994
++ Keywords: operator, kernel, expression, space.
++ Description:
++ An expression space is a set which is closed under certain operators;

ExpressionSpace(): Category == Defn where
  N   ==> NonNegativeInteger
  K   ==> Kernel %
  OP  ==> BasicOperator
  SY  ==> Symbol
  PAREN  ==> "%paren"::SY
  BOX    ==> "%box"::SY
  DUMMYVAR ==> "%dummyVar"

  Defn ==> Join(OrderedSet, RetractableTo K,
                InnerEvalable(K, %), Evalable %) with
    elt          : (OP, %) -> %
      ++ elt(op,x) or op(x) applies the unary operator op to x.
    elt          : (OP, %, %) -> %
      ++ elt(op,x,y) or op(x, y) applies the binary operator op to x and y.
    elt          : (OP, %, %, %) -> %
      ++ elt(op,x,y,z) or op(x, y, z) applies the ternary operator op
      ++ to x, y and z.
    elt          : (OP, %, %, %, %) -> %
      ++ elt(op,x,y,z,t) or op(x, y, z, t) applies the 4-ary operator op
      ++ to x, y, z and t.
    elt          : (OP, List %) -> %
      ++ elt(op,[x1,...,xn]) or op([x1,...,xn]) applies the n-ary operator
      ++ op to x1,...,xn.
    subst        : (%, Equation %) -> %
      ++ subst(f, k = g) replaces the kernel k by g formally in f.
    subst        : (%, List Equation %) -> %
      ++ subst(f, [k1 = g1,...,kn = gn]) replaces the kernels k1,...,kn
      ++ by g1,...,gn formally in f.
    subst        : (%, List K, List %) -> %
      ++ subst(f, [k1...,kn], [g1,...,gn]) replaces the kernels k1,...,kn
      ++ by g1,...,gn formally in f.
    box          : % -> %
      ++ box(f) returns f with a 'box' around it that prevents f from
      ++ being evaluated when operators are applied to it. For example,
      ++ \spad{log(1)} returns 0, but \spad{log(box 1)}
  ++ returns the formal kernel log(1).
box         : List % -> %
  ++ box([f1,...,fn]) returns \spad{(f1,...,fn)} with a 'box'
  ++ around them that
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(box [x, 2])} returns the formal kernel \spad{atan(x, 2)}.
paren       : % -> %
  ++ paren(f) returns (f). This prevents f from
  ++ being evaluated when operators are applied to it. For example,
  ++ \spad{log(1)} returns 0, but \spad{log(paren 1)} returns the
  ++ formal kernel log((1)).
paren       : List % -> %
  ++ paren([f1,...,fn]) returns \spad{(f1,...,fn)}. This
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(paren [x, 2])} returns the formal
  ++ kernel \spad{atan((x, 2))}.
distribute  : % -> %
  ++ distribute(f) expands all the kernels in f that are
  ++ formally enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or \spadfunFrom{paren}{ExpressionSpace} expression.
distribute  : (%, %) -> %
  ++ distribute(f, g) expands all the kernels in f that contain g in their
  ++ arguments and that are formally
  ++ enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or a \spadfunFrom{paren}{ExpressionSpace} expression.
height      : %  -> N
  ++ height(f) returns the highest nesting level appearing in f.
  ++ Constants have height 0. Symbols have height 1. For any
  ++ operator op and expressions f1,...,fn, \spad{op(f1,...,fn)} has
  ++ height equal to \spad{1 + max(height(f1),...,height(fn))}.
mainKernel  : %  -> Union(K, "failed")
  ++ mainKernel(f) returns a kernel of f with maximum nesting level, or
  ++ if f has no kernels (i.e. f is a constant).
kernels     : %  -> List K
  ++ kernels(f) returns the list of all the top-level kernels
  ++ appearing in f, but not the ones appearing in the arguments
  ++ of the top-level kernels.
tower       : %  -> List K
  ++ tower(f) returns all the kernels appearing in f, no matter
  ++ what their levels are.
operators   : %  -> List OP
  ++ operators(f) returns all the basic operators appearing in f,
  ++ no matter what their levels are.
operator    : OP -> OP
```

```
  ++ operator(op) returns a copy of op with the domain-dependent
  ++ properties appropriate for %.
belong?      : OP -> Boolean
  ++ belong?(op) tests if % accepts op as applicable to its
  ++ elements.
is?          : (%, OP)     -> Boolean
  ++ is?(x, op) tests if x is a kernel and is its operator is op.
is?          : (%, SY) -> Boolean
  ++ is?(x, s) tests if x is a kernel and is the name of its
  ++ operator is s.
kernel       : (OP, %) -> %
  ++ kernel(op, x) constructs op(x) without evaluating it.
kernel       : (OP, List %) -> %
  ++ kernel(op, [f1,...,fn]) constructs \spad{op(f1,...,fn)} without
  ++ evaluating it.
map          : (% -> %, K) -> %
  ++ map(f, k) returns \spad{op(f(x1),...,f(xn))} where
  ++ \spad{k = op(x1,...,xn)}.
freeOf?      : (%, %)  -> Boolean
  ++ freeOf?(x, y) tests if x does not contain any occurrence of y,
  ++ where y is a single kernel.
freeOf?      : (%, SY) -> Boolean
  ++ freeOf?(x, s) tests if x does not contain any operator
  ++ whose name is s.
eval         : (%, List SY, List(% -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.
eval         : (%, List SY, List(List % -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a1,...,an)} in x by
  ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.
eval         : (%, SY, List % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
  ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.
eval         : (%, SY, % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
  ++ for any \spad{a}.
eval         : (%, List OP, List(% -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.
eval         : (%, List OP, List(List % -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a1,...,an)} in x by
  ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.
eval         : (%, OP, List % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
```

```
        ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.
      eval         : (%, OP, % -> %) -> %
        ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
        ++ for any \spad{a}.
      if % has Ring then
        minPoly: K -> SparseUnivariatePolynomial %
          ++ minPoly(k) returns p such that \spad{p(k) = 0}.
        definingPolynomial: % -> %
          ++ definingPolynomial(x) returns an expression p such that
          ++ \spad{p(x) = 0}.
      if % has RetractableTo Integer then
        even?: % -> Boolean
          ++ even? x is true if x is an even integer.
        odd? : % -> Boolean
          ++ odd? x is true if x is an odd integer.

     add

-- the 7 functions not provided are:
--        kernels   minPoly   definingPolynomial
--        coerce:K -> %  eval:(%, List K, List %) -> %
--        subst:(%, List K, List %) -> %
--        eval:(%, List Symbol, List(List % -> %)) -> %

      allKernels: %      -> Set K
      listk     : %      -> List K
      allk      : List % -> Set K
      unwrap    : (List K, %) -> %
      okkernel  : (OP, List %) -> %
      mkKerLists: List Equation % -> Record(lstk: List K, lstv:List %)

      oppren := operator(PAREN)$CommonOperators()
      opbox  := operator(BOX)$CommonOperators()

      box(x:%)     == box [x]
      paren(x:%)   == paren [x]
      belong? op   == op = oppren or op = opbox
      listk f      == parts allKernels f
      tower f      == sort_! listk f
      allk l       == reduce("union", [allKernels f for f in l], {})
      operators f  == [operator k for k in listk f]
      height f     == reduce("max", [height k for k in kernels f], 0)
      freeOf?(x:%, s:SY)       == not member?(s, [name k for k in listk x])
      distribute x == unwrap([k for k in listk x | is?(k, oppren)], x)
      box(l:List %)                  == opbox l
      paren(l:List %)                == oppren l
```

```
    freeOf?(x:%, k:%)              == not member?(retract k, listk x)
    kernel(op:OP, arg:%)           == kernel(op, [arg])
    elt(op:OP, x:%)                == op [x]
    elt(op:OP, x:%, y:%)           == op [x, y]
    elt(op:OP, x:%, y:%, z:%)      == op [x, y, z]
    elt(op:OP, x:%, y:%, z:%, t:%) == op [x, y, z, t]
    eval(x:%, s:SY, f:List % -> %) == eval(x, [s], [f])
    eval(x:%, s:OP, f:List % -> %) == eval(x, [name s], [f])
    eval(x:%, s:SY, f:% -> %)      ==
      eval(x, [s], [(y:List %):% +-> f(first y)])
    eval(x:%, s:OP, f:% -> %)      ==
      eval(x, [s], [(y:List %):% +-> f(first y)])
    subst(x:%, e:Equation %)       == subst(x, [e])

    eval(x:%, ls:List OP, lf:List(% -> %)) ==
      eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

    eval(x:%, ls:List SY, lf:List(% -> %)) ==
      eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

    eval(x:%, ls:List OP, lf:List(List % -> %)) ==
      eval(x, [name s for s in ls]$List(SY), lf)

    map(fn, k) ==
      (l := [fn x for x in argument k]$List(%)) = argument k => k::%
      (operator k) l

    operator op ==
      is?(op, PAREN) => oppren
      is?(op, BOX) => opbox
      error "Unknown operator"

    mainKernel x ==
      empty?(l := kernels x) => "failed"
      n := height(k := first l)
      for kk in rest l repeat
        if height(kk) > n then
          n := height kk
          k := kk
      k

-- takes all the kernels except for the dummy variables, which are second
-- arguments of rootOf's, integrals, sums and products which appear only in
-- their first arguments
    allKernels f ==
      s := brace(l := kernels f)
```

```
  for k in l repeat
      t :=
          (u := property(operator k, DUMMYVAR)) case None =>
              arg := argument k
              s0  := remove_!(retract(second arg)@K, allKernels first arg)
              arg := rest rest arg
              n   := (u::None) pretend N
              if n > 1 then arg := rest arg
              union(s0, allk arg)
          allk argument k
      s := union(s, t)
  s

kernel(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  okkernel(op, args)

okkernel(op, l) ==
  kernel(op, l, 1 + reduce("max", [height f for f in l], 0))$K :: %

elt(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  ((u := arity op) case N) and (#args ^= u::N)
                                  => error "Wrong number of arguments"
  (v := evaluate(op,args)$BasicOperatorFunctions1(%)) case % => v::%
  okkernel(op, args)

retract f ==
  (k := mainKernel f) case "failed" => error "not a kernel"
  k::K::% ^= f => error "not a kernel"
  k::K

retractIfCan f ==
  (k := mainKernel f) case "failed" => "failed"
  k::K::% ^= f => "failed"
  k

is?(f:%, s:SY) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, s)

is?(f:%, op:OP) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, op)

unwrap(l, x) ==
  for k in reverse_! l repeat
    x := eval(x, k, first argument k)
  x

distribute(x, y) ==
  ky := retract y
  unwrap([k for k in listk x |
          is?(k, "%paren"::SY) and member?(ky, listk(k::%))], x)

-- in case of conflicting substitutions e.g. [x = a, x = b],
-- the first one prevails.
-- this is not part of the semantics of the function, but just
-- a feature of this implementation.
eval(f:%, leq:List Equation %) ==
  rec := mkKerLists leq
  eval(f, rec.lstk, rec.lstv)

subst(f:%, leq:List Equation %) ==
  rec := mkKerLists leq
  subst(f, rec.lstk, rec.lstv)

mkKerLists leq ==
  lk := empty()$List(K)
  lv := empty()$List(%)
  for eq in leq repeat
    (k := retractIfCan(lhs eq)@Union(K, "failed")) case "failed" =>
                    error "left hand side must be a single kernel"
    if not member?(k::K, lk) then
      lk := concat(k::K, lk)
      lv := concat(rhs eq, lv)
  [lk, lv]

if % has RetractableTo Integer then
   intpred?: (%, Integer -> Boolean) -> Boolean

   even? x == intpred?(x, even?)
   odd? x  == intpred?(x, odd?)

   intpred?(x, pred?) ==
       (u := retractIfCan(x)@Union(Integer, "failed")) case Integer
              and pred?(u::Integer)
```

⟨*ES.dotabb*⟩≡

```
"ES"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
"ES" -> "ORDSET"
"ES" -> "RETRACT"
"ES" -> "IEVALAB"
"ES" -> "EVALAB"
```

⟨*ES.dotfull*⟩≡

```
"ExpressionSpace()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
"ExpressionSpace()" -> "OrderedSet()"
"ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
"ExpressionSpace()" ->
    "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
"ExpressionSpace()" -> "Evalable(ExpressionSpace)"
```

⟨*ES.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "OrderedSet()"
"ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
"ExpressionSpace()" ->
    "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
"ExpressionSpace()" -> "Evalable(ExpressionSpace)"

"Evalable(ExpressionSpace)" [color=seagreen];
"Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" [color=seagreen];
"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"RetractableTo(Kernel(ExpressionSpace))" [color=seagreen];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

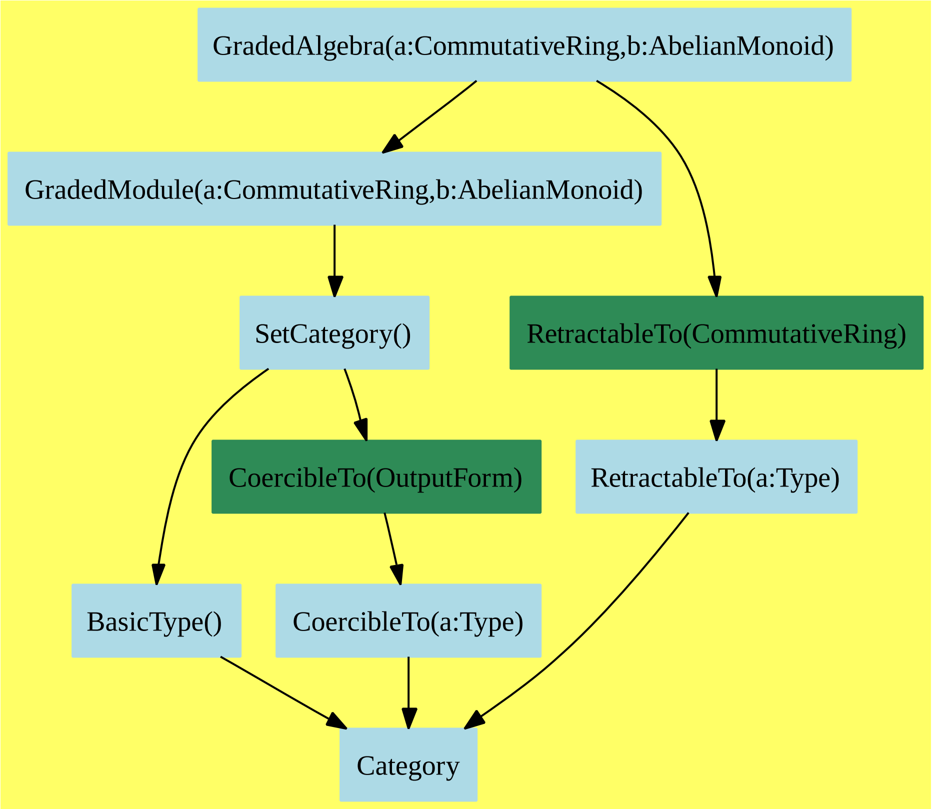
"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.73 GradedAlgebra (GRALG)


GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)
GradedModule(a:CommutativeRing,b:AbelianMonoid)
SetCategory()
RetractableTo(CommutativeRing)
CoercibleTo(OutputForm)
RetractableTo(a:Type)
BasicType()
CoercibleTo(a:Type)
Category

⟨*GradedAlgebra.input*⟩≡

```
)set break resume
)sys rm -f GradedAlgebra.output
)spool GradedAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GradedAlgebra
--R GradedAlgebra(R: CommutativeRing,E: AbelianMonoid)  is a category constructor
--R Abbreviation for GradedAlgebra is GRALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GRALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
```

```
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R coerce : R -> %                      coerce : % -> OutputForm
--R degree : % -> E                      hash : % -> SingleInteger
--R latex : % -> String                  product : (%,%) -> %
--R retract : % -> R                     ?~=? : (%,%) -> Boolean
--R retractIfCan : % -> Union(R,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*GradedAlgebra.help*⟩≡

```
====================================================================
GradedAlgebra examples
====================================================================

See Also:
o )show GradedAlgebra
```

See:

⇐ "GradedModule" (GRMOD) 4.0.48 on page 198
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0	1	coerce	degree	hash
latex	product	retract	retractIfCan	?~=?
?*?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
product : (%,%) -> %
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
```

These exports come from (p198) GradedModule(R, E)
where R:CommutativeRing and E:AbelianMonoid:

```
coerce : % -> OutputForm
degree : % -> E
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?+? : (%,%) -> %
```

These exports come from (p62) RetractableTo(R:CommutativeRing):

```
coerce : R -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

⟨*category GRALG GradedAlgebra*⟩≡

```
)abbrev category GRALG GradedAlgebra
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ Basic Operations: +, *, degree
++ Related Domains: CartesianTensor(n,dim,R)
++ Also See:
```

```
++ AMS Classifications:
++ Keywords: graded module, tensor, multi-linear algebra
++ Examples:
++ References: Encyclopedic Dictionary of Mathematics, MIT Press, 1977
++ Description:
++ GradedAlgebra(R,E) denotes ``E-graded R-algebra''.
++ A graded algebra is a graded module together with a degree preserving
++ R-linear map, called the product.
++
++ The name ``product'' is written out in full so inner and outer products
++ with the same mapping type can be distinguished by name.

GradedAlgebra(R: CommutativeRing, E: AbelianMonoid): Category ==
    Join(GradedModule(R, E),RetractableTo(R)) with
        1: constant -> %
            ++ 1 is the identity for \spad{product}.
        product: (%, %) -> %
            ++ product(a,b) is the degree-preserving R-linear product:
            ++
            ++   \spad{degree product(a,b) = degree a + degree b}
            ++   \spad{product(a1+a2,b) = product(a1,b) + product(a2,b)}
            ++   \spad{product(a,b1+b2) = product(a,b1) + product(a,b2)}
            ++   \spad{product(r*a,b) = product(a,r*b) = r*product(a,b)}
            ++   \spad{product(a,product(b,c)) = product(product(a,b),c)}
  add
        if not (R is %) then
            0: % == (0$R)::%
            1: % == 1$R::%
            (r: R)*(x: %) == product(r::%, x)
            (x: %)*(r: R) == product(x, r::%)
```

⟨*GRALG.dotabb*⟩≡

```
"GRALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
"GRALG" -> "GRMOD"
"GRALG" -> "RETRACT"
```

⟨*GRALG.dotfull*⟩≡

```
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "RetractableTo(CommutativeRing)"
```

⟨*GRALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "RetractableTo(CommutativeRing)"

"RetractableTo(CommutativeRing)" [color=seagreen];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

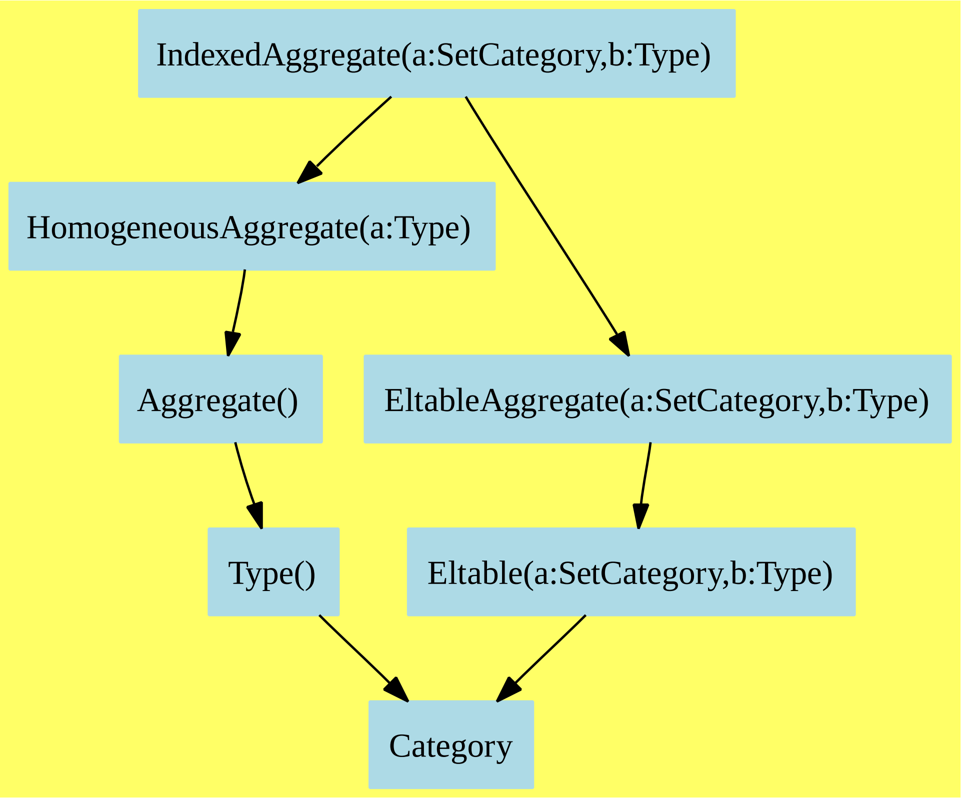
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.0.74 IndexedAggregate (IXAGG)


IndexedAggregate(a:SetCategory,b:Type)
HomogeneousAggregate(a:Type)
Aggregate()
EltableAggregate(a:SetCategory,b:Type)
Type()
Eltable(a:SetCategory,b:Type)
Category

⟨*IndexedAggregate.input*⟩≡

```
)set break resume
)sys rm -f IndexedAggregate.output
)spool IndexedAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedAggregate
--R IndexedAggregate(Index: SetCategory,Entry: Type)  is a category constructor
--R Abbreviation for IndexedAggregate is IXAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IXAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         ?.? : (%,Index) -> Entry
```

```
--R elt : (%,Index,Entry) -> Entry        empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List Entry
--R eq? : (%,%) -> Boolean                index? : (Index,%) -> Boolean
--R indices : % -> List Index             map : ((Entry -> Entry),%) -> %
--R qelt : (%,Index) -> Entry             sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Entry has SETCAT
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteAggregate
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R first : % -> Entry if Index has ORDSET
--R hash : % -> SingleInteger if Entry has SETCAT
--R latex : % -> String if Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Index if Index has ORDSET
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R members : % -> List Entry if $ has finiteAggregate
--R minIndex : % -> Index if Index has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R qsetelt! : (%,Index,Entry) -> Entry if $ has shallowlyMutable
--R setelt : (%,Index,Entry) -> Entry if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Index,Index) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Entry has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*IndexedAggregate.help*⟩≡
```
====================================================================
IndexedAggregate examples
====================================================================

See Also:
o )show IndexedAggregate
```

See:

⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “LinearAggregate” (LNAGG) 6.0.87 on page 449
⇒ “TableAggregate” (TBAGG) 9.0.132 on page 884
⇐ “EltableAggregate” (ELTAGG) 3.0.24 on page 86
⇐ “HomogeneousAggregate” (HOAGG) 4.0.49 on page 203

Exports:

any?	coerce	copy	count	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	first	hash
index?	indices	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	sample
setelt	size?	swap!	?.?	?~=?
#?	?=?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
index? : (Index,%) -> Boolean
indices : % -> List Index
```

These are implemented by this category:

```
elt : (%,Index,Entry) -> Entry
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
          if $ has finiteAggregate and Entry has SETCAT
fill! : (%,Entry) -> % if $ has shallowlyMutable
first : % -> Entry if Index has ORDSET
map : ((Entry -> Entry),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
maxIndex : % -> Index if Index has ORDSET
minIndex : % -> Index if Index has ORDSET
swap! : (%,Index,Index) -> Void if $ has shallowlyMutable
```

These exports come from (p203) HomogeneousAggregate(Entry:Type):

```
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Entry has SETCAT
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT and $ has finiteAggregate
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Entry,List Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Entry,Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,List Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
every? : ((Entry -> Boolean),%) -> Boolean
      if $ has finiteAggregate
hash : % -> SingleInteger if Entry has SETCAT
latex : % -> String if Entry has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (Entry,%) -> Boolean
          if Entry has SETCAT and $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?~=? : (%,%) -> Boolean if Entry has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if Entry has SETCAT
```

These exports come from (p86) EltableAggregate(Index:SetCategory,Entry:Type):

```
qelt : (%,Index) -> Entry
qsetelt! : (%,Index,Entry) -> Entry if $ has shallowlyMutable
setelt : (%,Index,Entry) -> Entry if $ has shallowlyMutable
?.? : (%,Index) -> Entry
```

⟨*category IXAGG IndexedAggregate*⟩≡

```
)abbrev category IXAGG IndexedAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
```

```
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An indexed aggregate is a many-to-one mapping of indices to entries.
++ For example, a one-dimensional-array is an indexed aggregate where
++ the index is an integer.  Also, a table is an indexed aggregate
++ where the indices and entries may have any type.

IndexedAggregate(Index: SetCategory, Entry: Type): Category ==
  Join(HomogeneousAggregate(Entry), EltableAggregate(Index, Entry)) with
   entries: % -> List Entry
      ++ entries(u) returns a list of all the entries of aggregate u
      ++ in no assumed order.
      -- to become entries: % -> Entry* and
      -- entries: % -> Iterator(Entry,Entry)
   index?: (Index,%) -> Boolean
      ++ index?(i,u) tests if i is an index of aggregate u.
   indices: % -> List Index
      ++ indices(u) returns a list of indices of aggregate u in no
      ++ particular order. to become indices:
      --  % -> Index* and indices: % -> Iterator(Index,Index).
-- map: ((Entry,Entry)->Entry,%,%,Entry) -> %
--    ++ exists c = map(f,a,b,x), i:Index where
--    ++    c.i = f(a(i,x),b(i,x)) | index?(i,a) or index?(i,b)
   if Entry has SetCategory and % has finiteAggregate then
      entry?: (Entry,%) -> Boolean
        ++ entry?(x,u) tests if x equals \axiom{u . i} for some index i.
   if Index has OrderedSet then
      maxIndex: % -> Index
        ++ maxIndex(u) returns the maximum index i of aggregate u.
        ++ Note that in general,
        ++ \axiom{maxIndex(u) = reduce(max,[i for i in indices u])};
        ++ if u is a list, \axiom{maxIndex(u) = #u}.
      minIndex: % -> Index
        ++ minIndex(u) returns the minimum index i of aggregate u.
        ++ Note that in general,
        ++ \axiom{minIndex(a) = reduce(min,[i for i in indices a])};
        ++ for lists, \axiom{minIndex(a) = 1}.
      first   : % -> Entry
        ++ first(u) returns the first element x of u.
        ++ Note that for collections, \axiom{first([x,y,...,z]) = x}.
        ++ Error: if u is empty.
```

```
     if % has shallowlyMutable then
        fill_!: (%,Entry) -> %
          ++ fill!(u,x) replaces each entry in aggregate u by x.
          ++ The modified u is returned as value.
        swap_!: (%,Index,Index) -> Void
          ++ swap!(u,i,j) interchanges elements i and j of aggregate u.
          ++ No meaningful value is returned.
   add
    elt(a, i, x) == (index?(i, a) => qelt(a, i); x)

    if % has finiteAggregate then
      entries x == parts x
      if Entry has SetCategory then
        entry?(x, a) == member?(x, a)

    if Index has OrderedSet then
      maxIndex a == "max"/indices(a)
      minIndex a == "min"/indices(a)
      first a    == a minIndex a

    if % has shallowlyMutable then
      map(f, a) == map_!(f, copy a)

      map_!(f, a) ==
        for i in indices a repeat qsetelt_!(a, i, f qelt(a, i))
        a

      fill_!(a, x) ==
        for i in indices a repeat qsetelt_!(a, i, x)
        a

      swap_!(a, i, j) ==
        t := a.i
        qsetelt_!(a, i, a.j)
        qsetelt_!(a, j, t)
        void
```

⟨*IXAGG.dotabb*⟩≡

```
  "IXAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
  "IXAGG" -> "HOAGG"
  "IXAGG" -> "ELTAGG"
```

⟨*IXAGG.dotfull*⟩≡

```
"IndexedAggregate(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "EltableAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(b:Integer,a:Type)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"
```

⟨*IXAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "EltableAggregate(a:SetCategory,b:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

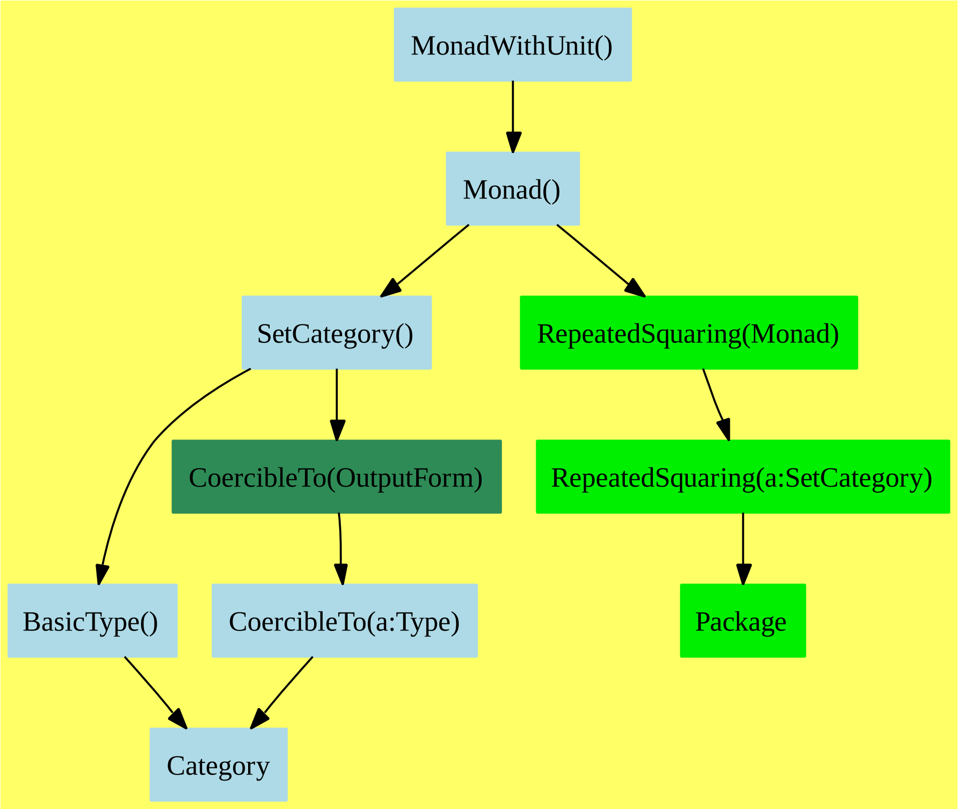
"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.0.75 MonadWithUnit (MONADWU)


MonadWithUnit()
Monad()
SetCategory()
RepeatedSquaring(Monad)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*MonadWithUnit.input*⟩≡

```
)set break resume
)sys rm -f MonadWithUnit.output
)spool MonadWithUnit.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonadWithUnit
--R MonadWithUnit  is a category constructor
--R Abbreviation for MonadWithUnit is MONADWU
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONADWU
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
```

```
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R leftPower : (%,NonNegativeInteger) -> %
--R leftPower : (%,PositiveInteger) -> %
--R leftRecip : % -> Union(%,"failed")
--R rightPower : (%,NonNegativeInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*MonadWithUnit.help*⟩≡

```
====================================================================
MonadWithUnit examples
====================================================================

See Also:
o )show MonadWithUnit
```

See:

⇒ “NonAssociativeRing” (NASRING) 9.0.124 on page 816
⇐ “Monad” (MONAD) 4.0.52 on page 221

Exports:

1	coerce	hash	latex	one?
recip	leftPower	leftRecip	rightPower	rightRecip
?*?	?~=?	?**?	?=?	

These are directly exported but not implemented:

```
1 : () -> %
leftRecip : % -> Union(%,"failed")
recip : % -> Union(%,"failed")
rightRecip : % -> Union(%,"failed")
```

These are implemented by this category:

```
leftPower : (%,NonNegativeInteger) -> %
one? : % -> Boolean
rightPower : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p221) Monad():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category MONADWU MonadWithUnit*⟩≡

```
)abbrev category MONADWU MonadWithUnit
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: *, **, 1
++ Related Constructors: SemiGroup, Monoid, Monad
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: Monad with unit, binary operation
++ Reference:
```

```
++ N. Jacobson: Structure and Representations of Jordan Algebras
++ AMS, Providence, 1968
++ Description:
++ MonadWithUnit is the class of multiplicative monads with unit,
++ i.e. sets with a binary operation and a unit element.
++
++ Axioms\br
++ \tab{5}leftIdentity("*":(%,%)->%,1) e.g. 1*x=x\br
++ \tab{5}rightIdentity("*":(%,%)->%,1) e.g x*1=x
++
++ Common Additional Axioms\br
++ \tab{5}unitsKnown - if "recip" says "failed", it PROVES input wasn't a unit

MonadWithUnit(): Category == Monad with
      1: constant ->  %
        ++ 1 returns the unit element, denoted by 1.
      one?: % -> Boolean
        ++ one?(a) tests whether \spad{a} is the unit 1.
      rightPower: (%,NonNegativeInteger) -> %
        ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
        ++ i.e. \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
        ++ \spad{rightPower(a,0) := 1}.
      leftPower: (%,NonNegativeInteger) -> %
        ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
        ++ i.e. \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
        ++ \spad{leftPower(a,0) := 1}.
      "**": (%,NonNegativeInteger) -> %
        ++ \spad{a**n} returns the \spad{n}-th power of \spad{a},
        ++ defined by repeated squaring.
      recip: % -> Union(%,"failed")
        ++ recip(a) returns an element, which is both a left and a right
        ++ inverse of \spad{a},
        ++ or \spad{"failed"} if such an element doesn't exist or cannot
        ++ be determined (see unitsKnown).
      leftRecip: % -> Union(%,"failed")
        ++ leftRecip(a) returns an element, which is a left inverse of
        ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
        ++ or cannot be determined (see unitsKnown).
      rightRecip: % -> Union(%,"failed")
        ++ rightRecip(a) returns an element, which is a right inverse of
        ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
        ++ or cannot be determined (see unitsKnown).
    add
      import RepeatedSquaring(%)
      one? x == x = 1
      x:% ** n:NonNegativeInteger ==
          zero? n => 1
          expt(x,n pretend PositiveInteger)
       rightPower(a,n) ==
         zero? n => 1
         res := 1
         for i in 1..n repeat res := res * a
         res
       leftPower(a,n) ==
         zero? n => 1
         res := 1
         for i in 1..n repeat res := a * res
         res
```

⟨*MONADWU.dotabb*⟩≡

```
"MONADWU"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MONADWU" -> "MONAD"
```

⟨*MONADWU.dotfull*⟩≡

```
"MonadWithUnit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MonadWithUnit()" -> "Monad()"
```

⟨*MONADWU.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

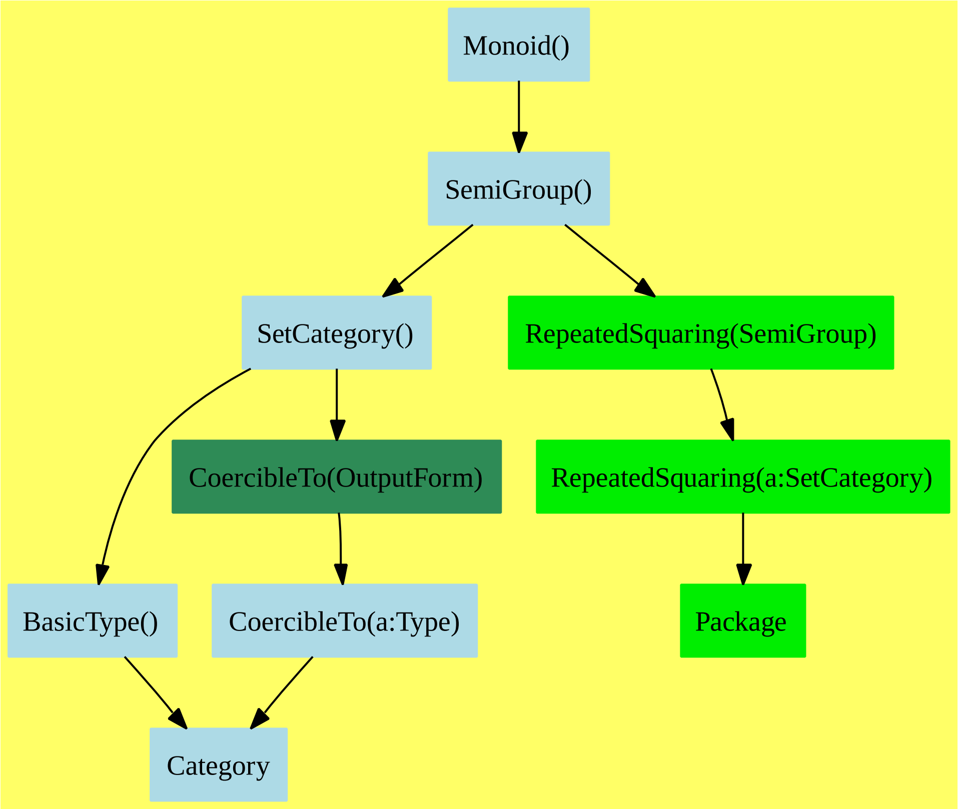
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.0.76 Monoid (MONOID)


Monoid()
SemiGroup()
SetCategory()
RepeatedSquaring(SemiGroup)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*Monoid.input*⟩≡

```
)set break resume
)sys rm -f Monoid.output
)spool Monoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Monoid
--R Monoid  is a category constructor
--R Abbreviation for Monoid is MONOID
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONOID
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
```

```
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*Monoid.help*⟩≡

```
====================================================================
Monoid examples
====================================================================

See Also:
o )show Monoid
```

See:

⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "Group" (GROUP) 6.0.86 on page 443
⇒ "OrderedMonoid" (ORDMON) 6.0.90 on page 512
⇒ "OrderedRing" (ORDRING) 10.0.144 on page 971
⇒ "Ring" (RING) 9.0.129 on page 854
⇐ "SemiGroup" (SGROUP) 4.0.61 on page 269

Exports:

1	coerce	hash	latex	one?
recip	sample	?*?	?=?	?~=?
?**?	?^?			

These are directly exported but not implemented:

```
1 : () -> %
```

These are implemented by this category:

```
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p269) SemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category MONOID Monoid*⟩≡

```
)abbrev category MONOID Monoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ The class of multiplicative monoids, i.e. semigroups with a
++ multiplicative identity element.
++
++ Axioms\br
++ \tab{5}\spad{leftIdentity("*":(%,%)->%,1)}\tab{5}\spad{1*x=x}\br
++ \tab{5}\spad{rightIdentity("*":(%,%)->%,1)}\tab{4}\spad{x*1=x}
++
++ Conditional attributes\br
++ \tab{5}unitsKnown - \spadfun{recip} only returns "failed" on non-units

Monoid(): Category == SemiGroup with
      1: constant ->  %
        ++ 1 is the multiplicative identity.
      sample: constant -> %
        ++ sample yields a value of type %
      one?: % -> Boolean
        ++ one?(x) tests if x is equal to 1.
      "**": (%,NonNegativeInteger) -> %
        ++ x**n returns the repeated product
        ++ of x n times, i.e. exponentiation.
      "^" : (%,NonNegativeInteger) -> %
        ++ x^n returns the repeated product
        ++ of x n times, i.e. exponentiation.
      recip: % -> Union(%,"failed")
        ++ recip(x) tries to compute the multiplicative inverse for x
        ++ or "failed" if it cannot find the inverse (see unitsKnown).
    add
      import RepeatedSquaring(%)
      _^(x:%, n:NonNegativeInteger):% == x ** n
      one? x == x = 1
      sample() == 1
      recip x ==
        (x = 1) => x
        "failed"
      x:% ** n:NonNegativeInteger ==
         zero? n => 1
         expt(x,n pretend PositiveInteger)
```

⟨*MONOID.dotabb*⟩≡

```
"MONOID"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"MONOID" -> "SGROUP"
```

⟨*MONOID.dotfull*⟩≡
```
"Monoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"Monoid()" -> "SemiGroup()"
```

⟨*MONOID.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

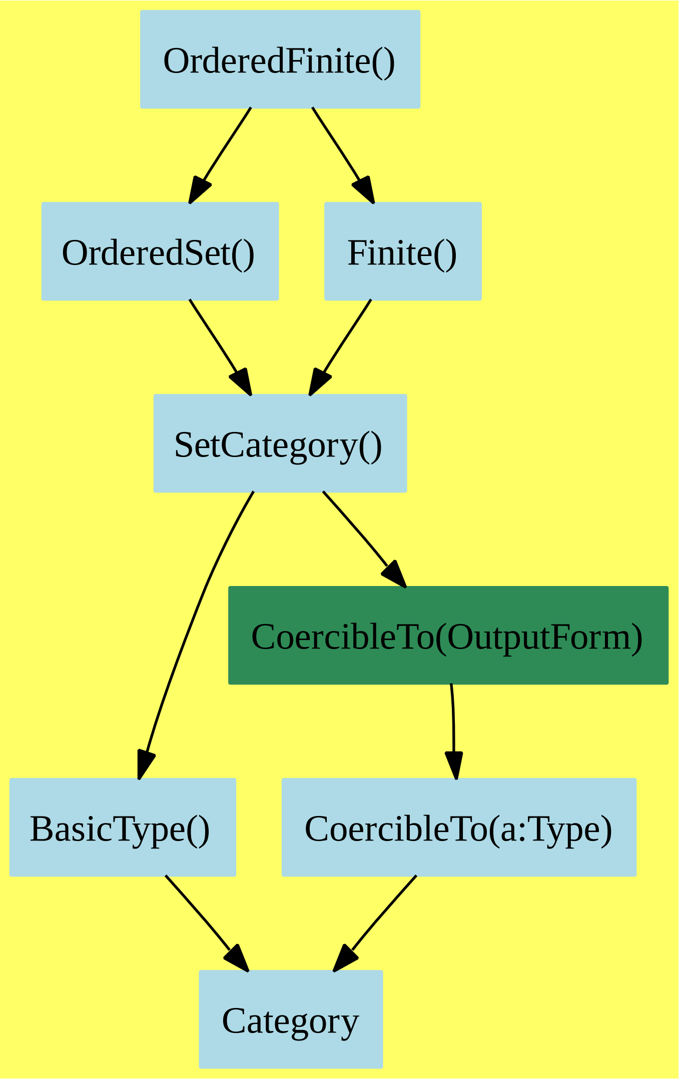
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.77 OrderedFinite (ORDFIN)


OrderedFinite()
OrderedSet()
Finite()
SetCategory()
CoercibleTo(OutputForm)
BasicType()
CoercibleTo(a:Type)
Category

⟨*OrderedFinite.input*⟩≡

```
)set break resume
)sys rm -f OrderedFinite.output
)spool OrderedFinite.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show OrderedFinite
--R OrderedFinite  is a category constructor
--R Abbreviation for OrderedFinite is ORDFIN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDFIN
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger             index : PositiveInteger -> %
--R latex : % -> String                   lookup : % -> PositiveInteger
--R max : (%,%) -> %                      min : (%,%) -> %
--R random : () -> %                      size : () -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedFinite.help*⟩≡

```
====================================================================
OrderedFinite examples
====================================================================

See Also:
o )show OrderedFinite
```

See:

⇐ “Finite” (FINITE) 4.0.46 on page 188
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242

Exports:

coerce hash index latex lookup
max min random size ?~=?
?<? ?<=? ?=? ?>? ?>=?

These exports come from (p242) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p188) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

⟨*category ORDFIN OrderedFinite*⟩≡

```
)abbrev category ORDFIN OrderedFinite
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered finite sets.

OrderedFinite(): Category == Join(OrderedSet, Finite)
```

⟨*ORDFIN.dotabb*⟩≡

```
"ORDFIN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"ORDFIN" -> "ORDSET"
"ORDFIN" -> "FINITE"
```

⟨*ORDFIN.dotfull*⟩≡

```
"OrderedFinite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"
```

⟨*ORDFIN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedFinite()" [color=lightblue];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

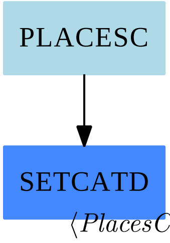
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.0.78 PlacesCategory (PLACESC)



⟨*PlacesCategory.input*⟩≡

```
)set break resume
)sys rm -f PlacesCategory.output
)spool PlacesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlacesCategory
--R PlacesCategory(K: Field,PCS: LocalPowerSeriesCategory t#1)  is a category con
--R Abbreviation for PlacesCategory is PLACESC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PLACESC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> Divisor %        ?+? : (%,%) -> Divisor %
--R -? : % -> Divisor %                   ?-? : (%,%) -> Divisor %
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R create : Symbol -> %                  create : List K -> %
--R degree : % -> PositiveInteger         ?.? : (%,Integer) -> K
--R foundPlaces : () -> List %            hash : % -> SingleInteger
--R itsALeaf! : % -> Void                 latex : % -> String
--R leaf? : % -> Boolean                  localParam : % -> List PCS
--R reduce : List % -> Divisor %          setParam! : (%,List PCS) -> Void
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*PlacesCategory.help*⟩≡

```
====================================================================
PlacesCategory examples
====================================================================

See Also:
o )show PlacesCategory
```

See:

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.62 on page 274

Exports:

-?	?*?	?+?	?-?
?.?	?=?	?~=?	coerce
create	degree	foundPlaces	hash
itsALeaf!	latex	leaf?	localParam
reduce	setDegree!	setFoundPlacesToEmpty	setParam!

These are directly exported but not implemented:

```
-? : % -> Divisor %
?*? : (Integer,%) -> Divisor %
?+? : (%,%) -> Divisor %
?+? : (%,Divisor %) -> Divisor %
?+? : (Divisor %,%) -> Divisor %
?-? : (%,%) -> Divisor %
?-? : (%,Divisor %) -> Divisor %
?-? : (Divisor %,%) -> Divisor %
?.? : (%,Integer) -> K
create : List K -> %
create : Symbol -> %
foundPlaces : () -> List %
itsALeaf! : % -> Void
leaf? : % -> Boolean
localParam : % -> List PCS
reduce : List % -> Divisor %
setDegree! : (%,PositiveInteger) -> Void
setFoundPlacesToEmpty : () -> List %
setParam! : (%,List PCS) -> Void
```

These exports come from (p274) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

⟨*category PLACESC PlacesCategory*⟩≡

```
)abbrev category PLACESC PlacesCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
```

```
PlacesCategory(K:Field,PCS:LocalPowerSeriesCategory(K)):Category
  == SetCategoryWithDegree with

    "+": (%,%)       -> Divisor(%)

    "+": (Divisor(%) , % ) -> Divisor(%)

    "+": (%, Divisor(%)) -> Divisor(%)

    "-": (%,%)       -> Divisor(%)

    "-": (Divisor(%) , % ) -> Divisor(%)

    "-": (%, Divisor(%)) -> Divisor(%)

    "-": % -> Divisor(%)

    "*": (Integer,%) -> Divisor(%)

    reduce: List %   -> Divisor(%)

    create: List K -> %

    create: Symbol -> %

    localParam: % -> List PCS
      ++ localParam(pl) returns the local parametrization associated
      ++ to the place pl.

    setParam_!: (%,List PCS)  -> Void
      ++ setParam!(pl,ls) set the local parametrization of pl to ls.

    setDegree_!: (%,PositiveInteger)  -> Void
      ++ setDegree!(pl,ls) set the degree.

    leaf?: % -> Boolean
      ++ leaf?(pl) test if the place pl correspond to a leaf of a
      ++ desingularisation tree.

    itsALeaf_!: % -> Void

    foundPlaces: () -> List %
      ++ foundPlaces() returns the list of all "created" places up to now.

    setFoundPlacesToEmpty: () -> List %
      ++ setFoundPlacesToEmpty() does what it says.
```

```
        ++ (this should not be used)!!!

      elt: (%,Integer) -> K
        ++ elt returns the value of a specified coordinates if the places
        ++ correspnd to a simple point
```

⟨PLACESC.dotabb⟩≡

```
"PLACESC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PLACESC"];
"SETCATD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SETCATD"]
"PLACESC" -> "SETCATD"
```

⟨PLACESC.dotfull⟩≡

```
"PlacesCategory()" [color=lightblue,href="bookvol10.2.pdf#nameddest=PLACESC"];
"PlacesCategory()" -> "SetCategoryWithDegree()"
```

⟨*PLACESC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PlacesCategory()" [color=lightblue];
"PlacesCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

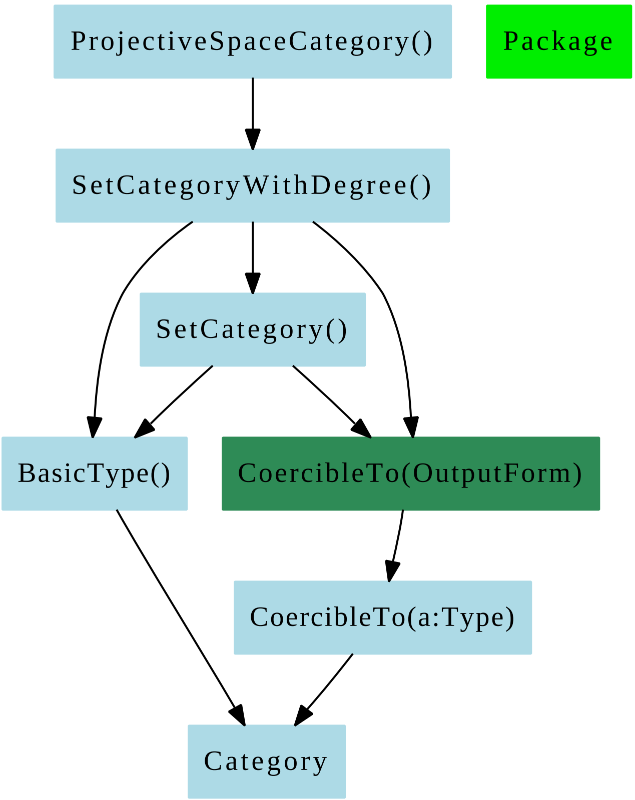
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.79 ProjectiveSpaceCategory (PRSPCAT)


ProjectiveSpaceCategory()
Package
SetCategoryWithDegree()
SetCategory()
BasicType()
CoercibleTo(OutputForm)
CoercibleTo(a:Type)
Category

⟨*ProjectiveSpaceCategory.input*⟩≡

```
)set break resume
)sys rm -f ProjectiveSpaceCategory.output
)spool ProjectiveSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectiveSpaceCategory
--R ProjectiveSpaceCategory K: Field  is a category constructor
--R Abbreviation for ProjectiveSpaceCategory is PRSPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRSPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : List K -> %
```

```
--R coerce : % -> List K                     coerce : % -> OutputForm
--R conjugate : % -> %                       definingField : % -> K
--R degree : % -> PositiveInteger            ?.? : (%,Integer) -> K
--R hash : % -> SingleInteger                homogenize : % -> %
--R homogenize : (%,Integer) -> %            lastNonNul : % -> Integer
--R lastNonNull : % -> Integer               latex : % -> String
--R list : % -> List K                       orbit : % -> List %
--R pointValue : % -> List K                 projectivePoint : List K -> %
--R rational? : % -> Boolean                 setelt : (%,Integer,K) -> K
--R ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

⟨*ProjectiveSpaceCategory.help*⟩≡

```
====================================================================
ProjectiveSpaceCategory examples
====================================================================

See Also:
o )show ProjectiveSpaceCategory
```

See:

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.62 on page 274

Exports:

?.?	?=?	? =?	coerce
conjugate	definingField	degree	hash
homogenize	lastNonNul	lastNonNull	latex
list	orbit	orbit	pointValue
projectivePoint	rational?	removeConjugate	setelt

These are directly exported but not implemented:

```
?.? : (%,Integer) -> K
coerce : % -> List K
coerce : List K -> %
conjugate : % -> %
conjugate : (%,NonNegativeInteger) -> %
definingField : % -> K
homogenize : % -> %
homogenize : (%,Integer) -> %
lastNonNul : % -> Integer
lastNonNull : % -> Integer
list : % -> List K
orbit : % -> List %
orbit : (%,NonNegativeInteger) -> List %
pointValue : % -> List K
projectivePoint : List K -> %
rational? : % -> Boolean
rational? : (%,NonNegativeInteger) -> Boolean
removeConjugate : (List %,NonNegativeInteger) -> List %
removeConjugate : List % -> List %
setelt : (%,Integer,K) -> K
```

These exports come from (p274) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

⟨*category PRSPCAT ProjectiveSpaceCategory*⟩≡

```
)abbrev category PRSPCAT ProjectiveSpaceCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
```

```
++ This is part of the PAFF package, related to projective space.
ProjectiveSpaceCategory(K:Field):Category == Implementation where

 LIST ==> List
 INT  ==> Integer
 NNI  ==> NonNegativeInteger

 Implementation ==>
  SetCategoryWithDegree with

    lastNonNul: % -> INT

    homogenize: (%,INT) -> %
      ++ homogenize the point according to the coordinate specified
      ++ by the integer

    projectivePoint: LIST(K) -> %
      ++ projectivePoint creates a projective point from a list

    definingField: % -> K

    coerce: % -> List(K)
      ++ coerce a a projective point list of K

    coerce: LIST(K) -> %
      ++ coerce a list of K to a projective point.

    orbit: % -> LIST %
      ++ orbit(p) returns the orbit of the point p according to the
      ++ characteristic of K, that is, for q= char K,
      ++ orbit(p) = \{ p, p**q, p**(q**2), p**(q**3), ..... \}

    orbit: (%,NNI) -> LIST %
      ++ orbit(p,n) returns the orbit of the point p according to n, that is
      ++ orbit(p,n) = \{ p, p**n, p**(n**2), p**(n**3), ..... \}

    conjugate: (%,NNI) -> %
      ++ conjugate(p,n) returns p**n, that is all the coordinates of p
      ++ to the power of n

    conjugate: % -> %
      ++ conjugate(p) returns conjugate(p,n) where n is the
      ++ characteristic of the ground field.

    removeConjugate: (LIST(%),NNI) -> LIST(%)
      ++ removeConjugate(lp,n) returns a list of points such that no points
```

```
     ++ in the list is the conjugate (according to n) of another point.

   removeConjugate: LIST(%)  -> LIST(%)
     ++ removeConjugate(lp) returns removeConjugate(lp,n)
     ++ where n is the characteristic of the ground field.

   rational?: (%,NNI) -> Boolean
     ++ rational?(p,n) test if the point is rational according to n.

   rational?: % -> Boolean
     ++ rational?(p) test if the point is rational according to the
     ++ characteristic of the ground field.

   homogenize: % -> %
     ++ homogenize(pt) the point according to the coordinate
     ++ which is the last non null.

   lastNonNull: % -> INT
     ++ lastNonNull returns the integer corresponding to the last
     ++ non null coordinates.

   list  : % -> LIST(K)
     ++ list returns the list of the coordinates

   elt: (%,INT) -> K
     ++ elt returns the value of a specified coordinates

   setelt: (%,INT,K) -> K
     ++ setelt sets the value of a specified coordinates

   pointValue: % -> LIST(K)
    ++ pointValue returns the coordinates of the point or of the point
    ++ of origin that represent an infinitly close point
```

⟨*PRSPCAT.dotabb*⟩≡

```
"PRSPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRSPCAT"];
"PRSPCAT" -> "SETCATD"
```

⟨*PRSPCAT.dotfull*⟩≡

```
"ProjectiveSpaceCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRSPCAT"];
"ProjectiveSpaceCategory()" -> "SetCategoryWithDegree()"
```

⟨*PRSPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ProjectiveSpaceCategory()" [color=lightblue];
"ProjectiveSpaceCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

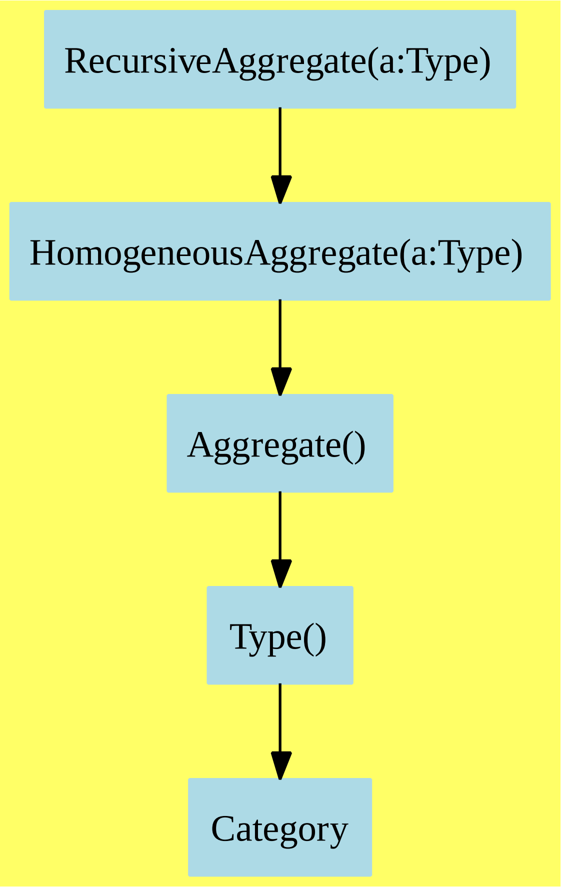
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.80 RecursiveAggregate (RCAGG)


RecursiveAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*RecursiveAggregate.input*⟩≡

```
)set break resume
)sys rm -f RecursiveAggregate.output
)spool RecursiveAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RecursiveAggregate
--R RecursiveAggregate S: Type  is a category constructor
--R Abbreviation for RecursiveAggregate is RCAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RCAGG
```

```
--R
--R-------------------------------- Operations --------------------------------
--R children : % -> List %                 copy : % -> %
--R cyclic? : % -> Boolean                 distance : (%,%) -> Integer
--R ?.value : (%,value) -> S               empty : () -> %
--R empty? : % -> Boolean                  eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                   leaves : % -> List S
--R map : ((S -> S),%) -> %                nodes : % -> List %
--R sample : () -> %                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*RecursiveAggregate.help*⟩≡

```
====================================================================
RecursiveAggregate examples
====================================================================

See Also:
o )show RecursiveAggregate
```

See:

⇒ "BinaryRecursiveAggregate" (BRAGG) 6.0.82 on page 414
⇒ "DoublyLinkedAggregate" (DLAGG) 6.0.85 on page 436
⇒ "UnaryRecursiveAggregate" (URAGG) 6.0.96 on page 563
⇐ "HomogeneousAggregate" (HOAGG) 4.0.49 on page 203

Exports:

any?
child?
children
coerce
copy
count
cyclic?
distance
empty
empty?
eq?
eval
every?
hash
latex
leaf?
leaves
less?
map
map!
member?
members
more?
nodes
node?
parts
sample
setchildren!
setelt
setvalue!
size?
value
?.value
?~=?
#?
?=?

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
children : % -> List %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
nodes : % -> List %
leaf? : % -> Boolean
leaves : % -> List S
node? : (%,%) -> Boolean if S has SETCAT
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
value : % -> S
```

These are implemented by this category:

```
child? : (%,%) -> Boolean if S has SETCAT
setelt : (%,value,S) -> S if $ has shallowlyMutable
?.value : (%,value) -> S
```

These exports come from (p203) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
      if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?~=? : (%,%) -> Boolean if S has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category RCAGG RecursiveAggregate*⟩≡

```
)abbrev category RCAGG RecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A recursive aggregate over a type S is a model for a
++ a directed graph containing values of type S.
++ Recursively, a recursive aggregate is a node
++ consisting of a \spadfun{value} from S and 0 or more \spadfun{children}
++ which are recursive aggregates.
++ A node with no children is called a \spadfun{leaf} node.
++ A recursive aggregate may be cyclic for which some operations as noted
++ may go into an infinite loop.

RecursiveAggregate(S:Type): Category == HomogeneousAggregate(S) with
   children: % -> List %
     ++ children(u) returns a list of the children of aggregate u.
   -- should be % -> %* and also needs children: % -> Iterator(S,S)
   nodes: % -> List %
     ++ nodes(u) returns a list of all of the nodes of aggregate u.
   -- to become % -> %* and also nodes: % -> Iterator(S,S)
   leaf?: % -> Boolean
     ++ leaf?(u) tests if u is a terminal node.
   value: % -> S
     ++ value(u) returns the value of the node u.
   elt: (%,"value") -> S
     ++ elt(u,"value") (also written: \axiom{a. value}) is
     ++ equivalent to \axiom{value(a)}.
   cyclic?: % -> Boolean
     ++ cyclic?(u) tests if u has a cycle.
   leaves: % -> List S
     ++ leaves(t) returns the list of values in obtained by visiting the
     ++ nodes of tree \axiom{t} in left-to-right order.
   distance: (%,%) -> Integer
     ++ distance(u,v) returns the path length (an integer) from node u to v.
   if S has SetCategory then
      child?: (%,%) -> Boolean
        ++ child?(u,v) tests if node u is a child of node v.
      node?: (%,%) -> Boolean
        ++ node?(u,v) tests if node u is contained in node v
        ++ (either as a child, a child of a child, etc.).
   if % has shallowlyMutable then
      setchildren_!: (%,List %)->%
        ++ setchildren!(u,v) replaces the current children of node u
        ++ with the members of v in left-to-right order.
      setelt: (%,"value",S) -> S
        ++ setelt(a,"value",x) (also written \axiom{a . value := x})
        ++ is equivalent to \axiom{setvalue!(a,x)}
      setvalue_!: (%,S) -> S
        ++ setvalue!(u,x) sets the value of node u to x.
 add
   elt(x,"value") == value x
```

```
    if % has shallowlyMutable then
      setelt(x,"value",y) == setvalue_!(x,y)
    if S has SetCategory then
      child?(x,l) == member?(x,children(l))
```

⟨*RCAGG.dotabb*⟩≡

```
"RCAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RCAGG" -> "HOAGG"
```

⟨*RCAGG.dotfull*⟩≡

```
"RecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"
```

⟨*RCAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

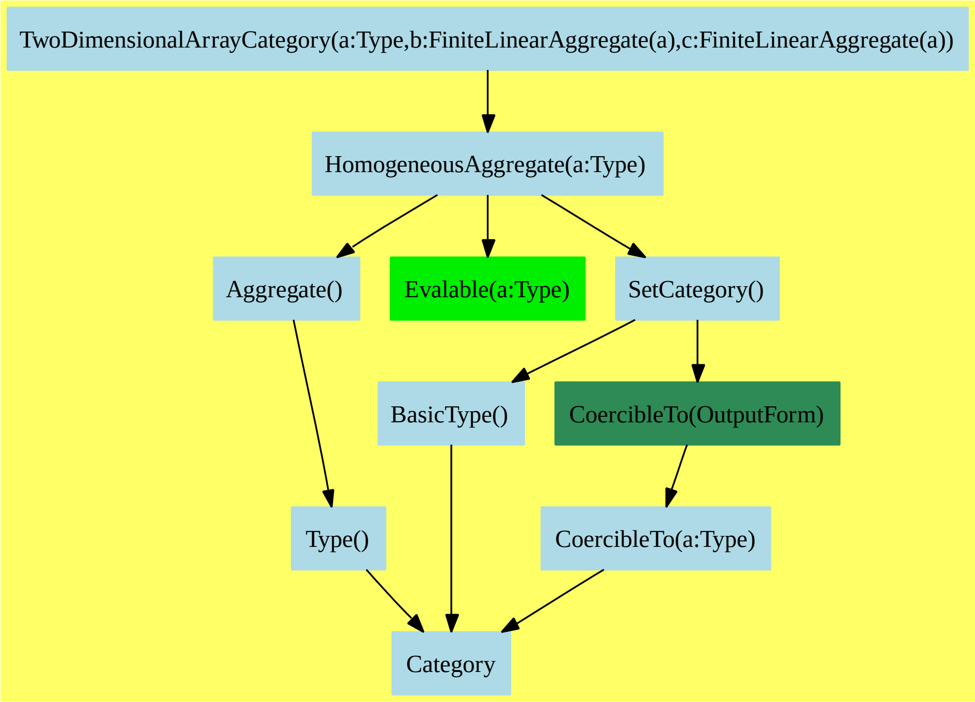
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.0.81 TwoDimensionalArrayCategory (ARR2CAT)


TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))
HomogeneousAggregate(a:Type)
Aggregate()
Evalable(a:Type)
SetCategory()
BasicType()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

TwoDimensionalArrayCategory is a general array category which allows different representations and indexing schemes. Rows and columns may be extracted with rows returned as objects of type Row and columns returned as objects of type Col. The index of the 'first' row may be obtained by calling the function 'minRowIndex'. The index of the 'first' column may be obtained by calling the function 'minColIndex'. The index of the first element of a 'Row' is the same as the index of the first column in an array and vice versa.

⟨*TwoDimensionalArrayCategory.input*⟩≡
```
  )set break resume
  )sys rm -f TwoDimensionalArrayCategory.output
  )spool TwoDimensionalArrayCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show TwoDimensionalArrayCategory
  --R TwoDimensionalArrayCategory(R: Type,Row: FiniteLinearAggregate t#1,Col: FiniteLinearAggr
  --R Abbreviation for TwoDimensionalArrayCategory is ARR2CAT
  --R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ARR2CAT
--R
--R------------------------------- Operations --------------------------------
--R column : (%,Integer) -> Col           copy : % -> %
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                fill! : (%,R) -> %
--R map : (((R,R) -> R),%,%,R) -> %       map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %               map! : ((R -> R),%) -> %
--R maxColIndex : % -> Integer            maxRowIndex : % -> Integer
--R minColIndex : % -> Integer            minRowIndex : % -> Integer
--R ncols : % -> NonNegativeInteger       nrows : % -> NonNegativeInteger
--R parts : % -> List R                   qelt : (%,Integer,Integer) -> R
--R row : (%,Integer) -> Row              sample : () -> %
--R setRow! : (%,Integer,Row) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (%,Integer,Integer,R) -> R
--R setColumn! : (%,Integer,Col) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*TwoDimensionalArrayCategory.help*⟩≡

```
====================================================================
TwoDimensionalArrayCategory examples
====================================================================

See Also:
o )show TwoDimensionalArrayCategory
```

See:

⇒ "MatrixCategory" (MATCAT) 6.0.88 on page 458
⇐ "HomogeneousAggregate" (HOAGG) 4.0.49 on page 203

Exports:

any?	column	coerce	copy	count
elt	empty	empty?	eq?	eval
every?	fill!	hash	latex	less?
map	map!	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
new	nrows	parts	qelt	qsetelt!
row	sample	setColumn!	setRow!	setelt
size?	#?	?=?	?~=?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
elt : (%,Integer,Integer) -> R
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
new : (NonNegativeInteger,NonNegativeInteger,R) -> %
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
qelt : (%,Integer,Integer) -> R
qsetelt! : (%,Integer,Integer,R) -> R
setelt : (%,Integer,Integer,R) -> R
```

These are implemented by this category:

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map : (((R,R) -> R),%,%,R) -> %
map! : ((R -> R),%) -> %
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R
row : (%,Integer) -> Row
setColumn! : (%,Integer,Col) -> %
setRow! : (%,Integer,Row) -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p203) HomogeneousAggregate(R:Type)

```
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
members : % -> List R if $ has finiteAggregate
sample : () -> %
?~=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category ARR2CAT TwoDimensionalArrayCategory*⟩≡

```
)abbrev category ARR2CAT TwoDimensionalArrayCategory
++ Author:
++ Date Created: 27 October 1989
++ Date Last Updated: 27 June 1990
++ Keywords: array, data structure
++ Examples:
++ References:
++ Description:
++ Two dimensional array categories and domains
```

```
TwoDimensionalArrayCategory(R,Row,Col): Category == Definition where
 R   : Type
 Row : FiniteLinearAggregate R
 Col : FiniteLinearAggregate R

 Definition == HomogeneousAggregate(R) with

   shallowlyMutable
    ++ one may destructively alter arrays

   finiteAggregate
    ++ two-dimensional arrays are finite

--% Array creation

   new: (NonNegativeInteger,NonNegativeInteger,R) -> %
    ++ new(m,n,r) is an m-by-n array all of whose entries are r
    ++
    ++X arr : ARRAY2 INT := new(5,4,0)

   fill_!: (%,R) -> %
    ++ fill!(m,r) fills m with r's
    ++
    ++X arr : ARRAY2 INT := new(5,4,0)
    ++X fill!(arr,10)

--% Size inquiries

   minRowIndex : % -> Integer
    ++ minRowIndex(m) returns the index of the 'first' row of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X minRowIndex(arr)

   maxRowIndex : % -> Integer
    ++ maxRowIndex(m) returns the index of the 'last' row of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X maxRowIndex(arr)

   minColIndex : % -> Integer
    ++ minColIndex(m) returns the index of the 'first' column of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X minColIndex(arr)
   maxColIndex : % -> Integer
    ++ maxColIndex(m) returns the index of the 'last' column of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X maxColIndex(arr)

   nrows : % -> NonNegativeInteger
    ++ nrows(m) returns the number of rows in the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X nrows(arr)

   ncols : % -> NonNegativeInteger
    ++ ncols(m) returns the number of columns in the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X ncols(arr)

--% Part extractions

   elt: (%,Integer,Integer) -> R
    ++ elt(m,i,j) returns the element in the ith row and jth
    ++ column of the array m
    ++ error check to determine if indices are in proper ranges
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X elt(arr,1,1)

   qelt: (%,Integer,Integer) -> R
    ++ qelt(m,i,j) returns the element in the ith row and jth
    ++ column of the array m
    ++ NO error check to determine if indices are in proper ranges
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X qelt(arr,1,1)

   elt: (%,Integer,Integer,R) -> R
    ++ elt(m,i,j,r) returns the element in the ith row and jth
    ++ column of the array m, if m has an ith row and a jth column,
    ++ and returns r otherwise
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X elt(arr,1,1,6)
    ++X elt(arr,1,10,6)
    row: (%,Integer) -> Row
     ++ row(m,i) returns the ith row of m
     ++ error check to determine if index is in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X row(arr,1)

    column: (%,Integer) -> Col
     ++ column(m,j) returns the jth column of m
     ++ error check to determine if index is in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X column(arr,1)

    parts: % -> List R
     ++ parts(m) returns a list of the elements of m in row major order
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X parts(arr)

--% Part assignments

    setelt: (%,Integer,Integer,R) -> R
     -- will become setelt_!
     ++ setelt(m,i,j,r) sets the element in the ith row and jth
     ++ column of m to r
     ++ error check to determine if indices are in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)
     ++X setelt(arr,1,1,17)

    qsetelt_!: (%,Integer,Integer,R) -> R
     ++ qsetelt!(m,i,j,r) sets the element in the ith row and jth
     ++ column of m to r
     ++ NO error check to determine if indices are in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)
     ++X qsetelt!(arr,1,1,17)

    setRow_!: (%,Integer,Row) -> %
     ++ setRow!(m,i,v) sets to ith row of m to v
     ++
     ++X T1:=TwoDimensionalArray Integer
     ++X arr:T1:= new(5,4,0)
     ++X T2:=OneDimensionalArray Integer
     ++X arow:=construct([1,2,3,4]::List(INT))$T2
     ++X setRow!(arr,1,arow)$T1

    setColumn_!: (%,Integer,Col) -> %
     ++ setColumn!(m,j,v) sets to jth column of m to v
     ++
     ++X T1:=TwoDimensionalArray Integer
     ++X arr:T1:= new(5,4,0)
     ++X T2:=OneDimensionalArray Integer
     ++X acol:=construct([1,2,3,4,5]::List(INT))$T2
     ++X setColumn!(arr,1,acol)$T1

--% Map and Zip

    map: (R -> R,%) -> %
     ++ map(f,a) returns \spad{b}, where \spad{b(i,j) = f(a(i,j))}
     ++ for all \spad{i, j}
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X map(-,arr)
     ++X map((x +-> x + x),arr)

    map_!: (R -> R,%) -> %
     ++ map!(f,a)  assign \spad{a(i,j)} to \spad{f(a(i,j))}
     ++ for all \spad{i, j}
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X map!(-,arr)

    map:((R,R) -> R,%,%) -> %
     ++ map(f,a,b) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
     ++ for all \spad{i, j}
     ++
     ++X adder(a:Integer,b:Integer):Integer == a+b
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X map(adder,arr,arr)

    map:((R,R) -> R,%,%,R) -> %
     ++ map(f,a,b,r) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
     ++ when both \spad{a(i,j)} and \spad{b(i,j)} exist;
     ++ else \spad{c(i,j) = f(r, b(i,j))} when \spad{a(i,j)} does not exist;
     ++ else \spad{c(i,j) = f(a(i,j),r)} when \spad{b(i,j)} does not exist;
     ++ otherwise \spad{c(i,j) = f(r,r)}.
     ++
     ++X adder(a:Integer,b:Integer):Integer == a+b
     ++X arr1 : ARRAY2 INT := new(5,4,10)
     ++X arr2 : ARRAY2 INT := new(3,3,10)
     ++X map(adder,arr1,arr2,17)
```

```
  add

--% Predicates

    any?(f,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          f(qelt(m,i,j)) => return true
      false

    every?(f,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          not f(qelt(m,i,j)) => return false
      true

    size?(m,n) == nrows(m) * ncols(m) = n
    less?(m,n) == nrows(m) * ncols(m) < n
    more?(m,n) == nrows(m) * ncols(m) > n

--% Size inquiries

    # m == nrows(m) * ncols(m)

--% Part extractions

    elt(m,i,j,r) ==
      i < minRowIndex(m) or i > maxRowIndex(m) => r
      j < minColIndex(m) or j > maxColIndex(m) => r
      qelt(m,i,j)

    count(f:R -> Boolean,m:%) ==
      num : NonNegativeInteger := 0
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          if f(qelt(m,i,j)) then num := num + 1
      num

    parts m ==
      entryList : List R := nil()
      for i in maxRowIndex(m)..minRowIndex(m) by -1 repeat
        for j in maxColIndex(m)..minColIndex(m) by -1 repeat
          entryList := concat(qelt(m,i,j),entryList)
      entryList
```

```
--% Creation

    copy m ==
      ans := new(nrows m,ncols m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(ans,i,j,qelt(m,i,j))
      ans

    fill_!(m,r) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(m,i,j,r)
      m

    map(f,m) ==
      ans := new(nrows m,ncols m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(ans,i,j,f(qelt(m,i,j)))
      ans

    map_!(f,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(m,i,j,f(qelt(m,i,j)))
      m

    map(f,m,n) ==
      (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) =>
        error "map: arguments must have same dimensions"
      ans := new(nrows m,ncols m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(ans,i,j,f(qelt(m,i,j),qelt(n,i,j)))
      ans

    map(f,m,n,r) ==
      maxRow := max(maxRowIndex m,maxRowIndex n)
      maxCol := max(maxColIndex m,maxColIndex n)
      ans := new(max(nrows m,nrows n),max(ncols m,ncols n),NIL$Lisp)
      for i in minRowIndex(m)..maxRow repeat
        for j in minColIndex(m)..maxCol repeat
          qsetelt_!(ans,i,j,f(elt(m,i,j,r),elt(n,i,j,r)))
      ans
```

```
setRow_!(m,i,v) ==
  i < minRowIndex(m) or i > maxRowIndex(m) =>
    error "setRow!: index out of range"
  for j in minColIndex(m)..maxColIndex(m) _
    for k in minIndex(v)..maxIndex(v) repeat
      qsetelt_!(m,i,j,v.k)
  m

setColumn_!(m,j,v) ==
  j < minColIndex(m) or j > maxColIndex(m) =>
    error "setColumn!: index out of range"
  for i in minRowIndex(m)..maxRowIndex(m) _
    for k in minIndex(v)..maxIndex(v) repeat
      qsetelt_!(m,i,j,v.k)
  m

if R has _= : (R,R) -> Boolean then

  m = n ==
    eq?(m,n) => true
    (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) => false
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        not (qelt(m,i,j) = qelt(n,i,j)) => return false
    true

  member?(r,m) ==
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        qelt(m,i,j) = r => return true
    false

  count(r:R,m:%) == count(x +-> x = r,m)

if Row has shallowlyMutable then

  row(m,i) ==
    i < minRowIndex(m) or i > maxRowIndex(m) =>
      error "row: index out of range"
    v : Row := new(ncols m,NIL$Lisp)
    for j in minColIndex(m)..maxColIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(v,k,qelt(m,i,j))
    v

if Col has shallowlyMutable then
```

```
    column(m,j) ==
      j < minColIndex(m) or j > maxColIndex(m) =>
        error "column: index out of range"
      v : Col := new(nrows m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) _
        for k in minIndex(v)..maxIndex(v) repeat
          qsetelt_!(v,k,qelt(m,i,j))
      v

  if R has CoercibleTo(OutputForm) then

    coerce(m:%) ==
      l : List List OutputForm
      l := [[qelt(m,i,j) :: OutputForm _
                for j in minColIndex(m)..maxColIndex(m)] _
                for i in minRowIndex(m)..maxRowIndex(m)]
      matrix l
```

⟨*ARR2CAT.dotabb*⟩≡

```
"ARR2CAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"ARR2CAT" -> "HOAGG"
```

⟨*ARR2CAT.dotfull*⟩≡

```
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOneDimensiona
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOneDimensiona
-> "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a
```

⟨*ARR2CAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

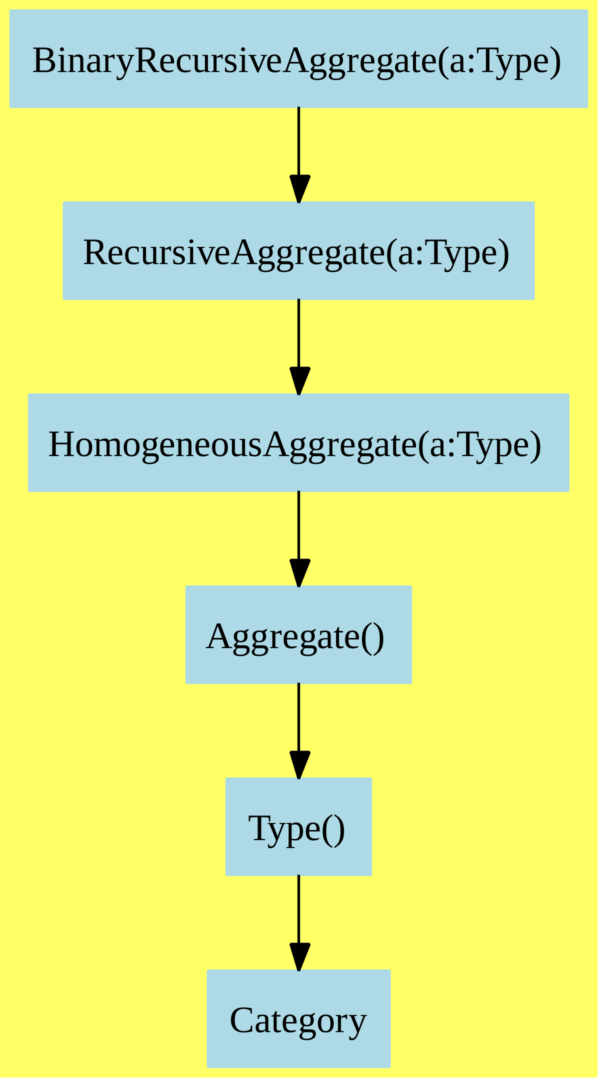
"Category" [color=lightblue];

}
```

Chapter 6

Category Layer 5

6.0.82 BinaryRecursiveAggregate (BRAGG)


BinaryRecursiveAggregate(a:Type)
RecursiveAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*BinaryRecursiveAggregate.input*⟩≡

```
)set break resume
)sys rm -f BinaryRecursiveAggregate.output
)spool BinaryRecursiveAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryRecursiveAggregate
--R BinaryRecursiveAggregate S: Type  is a category constructor
--R Abbreviation for BinaryRecursiveAggregate is BRAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BRAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.right : (%,right) -> %             ?.left : (%,left) -> %
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                 leaves : % -> List S
--R left : % -> %                        map : ((S -> S),%) -> %
--R nodes : % -> List %                  right : % -> %
--R sample : () -> %                     value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
```

```
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*BinaryRecursiveAggregate.help*⟩≡

```
====================================================================
BinaryRecursiveAggregate examples
====================================================================

See Also:
o )show BinaryRecursiveAggregate
```

See:

⇒ “BinaryTreeCategory” (BTCAT) 7.0.98 on page 582
⇐ “RecursiveAggregate” (RCAGG) 5.0.80 on page 392

Exports:

any? children child? coerce copy
count cyclic? distance empty empty?
eq? eval every? hash latex
leaf? leaves left less? map
map! member? members more? nodes
node? parts right sample setchildren!
setelt setleft! setright! setvalue! size?
value #? ?=? ?~=? ?.right
?.left ?.value

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
left : % -> %
right : % -> %
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
cyclic? : % -> Boolean
leaf? : % -> Boolean
leaves : % -> List S
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
```

```
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

These exports come from (p392) RecursiveAggregate(S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

⟨*category BRAGG BinaryRecursiveAggregate*⟩≡

```
)abbrev category BRAGG BinaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A binary-recursive aggregate has 0, 1 or 2 children and serves
++ as a model for a binary tree or a doubly-linked aggregate structure

BinaryRecursiveAggregate(S:Type):Category == RecursiveAggregate S with
   -- needs preorder, inorder and postorder iterators
   left: % -> %
     ++ left(u) returns the left child.
   elt: (%,"left") -> %
     ++ elt(u,"left") (also written: \axiom{a . left}) is
     ++ equivalent to \axiom{left(a)}.
   right: % -> %
     ++ right(a) returns the right child.
   elt: (%,"right") -> %
     ++ elt(a,"right") (also written: \axiom{a . right})
     ++ is equivalent to \axiom{right(a)}.
   if % has shallowlyMutable then
      setelt: (%,"left",%) -> %
        ++ setelt(a,"left",b) (also written \axiom{a . left := b}) is
        ++ equivalent to \axiom{setleft!(a,b)}.
      setleft_!: (%,%) -> %
         ++ setleft!(a,b) sets the left child of \axiom{a} to be b.
      setelt: (%,"right",%) -> %
         ++ setelt(a,"right",b) (also written \axiom{b . right := b})
         ++ is equivalent to \axiom{setright!(a,b)}.
      setright_!: (%,%) -> %
         ++ setright!(a,x) sets the right child of t to be x.
 add
   cycleMax ==> 1000

   elt(x,"left")  == left x
   elt(x,"right") == right x
   leaf? x == empty? x or empty? left x and empty? right x
   leaves t ==
     empty? t => empty()$List(S)
     leaf? t => [value t]
     concat(leaves left t,leaves right t)
   nodes x ==
    l := empty()$List(%)
    empty? x => l
    concat(nodes left x,concat([x],nodes right x))
  children x ==
    l := empty()$List(%)
    empty? x => l
    empty? left x  => [right x]
    empty? right x => [left x]
    [left x, right x]
  if % has SetAggregate(S) and S has SetCategory then
    node?(u,v) ==
      empty? v => false
      u = v => true
      for y in children v repeat node?(u,y) => return true
      false
    x = y ==
      empty?(x) => empty?(y)
      empty?(y) => false
      value x = value y and left x = left y and right x = right y
    if % has finiteAggregate then
      member?(x,u) ==
        empty? u => false
        x = value u => true
        member?(x,left u) or member?(x,right u)

  if S has SetCategory then
    coerce(t:%): OutputForm ==
      empty? t =>  "[]"::OutputForm
      v := value(t):: OutputForm
      empty? left t =>
        empty? right t => v
        r := coerce(right t)@OutputForm
        bracket ["."::OutputForm, v, r]
      l := coerce(left t)@OutputForm
      r :=
        empty? right t => "."::OutputForm
        coerce(right t)@OutputForm
      bracket [l, v, r]

  if % has finiteAggregate then
    aggCount: (%,NonNegativeInteger) -> NonNegativeInteger
    #x == aggCount(x,0)
    aggCount(x,k) ==
      empty? x => 0
      k := k + 1
      k = cycleMax and cyclic? x => error "cyclic tree"
```

```
        for y in children x repeat k := aggCount(y,k)
        k

    isCycle?:  (%, List %) -> Boolean
    eqMember?: (%, List %) -> Boolean
    cyclic? x         == not empty? x and isCycle?(x,empty()$(List %))
    isCycle?(x,acc) ==
      empty? x => false
      eqMember?(x,acc) => true
      for y in children x | not empty? y repeat
        isCycle?(y,acc) => return true
      false
    eqMember?(y,l) ==
      for x in l repeat eq?(x,y) => return true
      false
    if % has shallowlyMutable then
      setelt(x,"left",b)  == setleft_!(x,b)
      setelt(x,"right",b) == setright_!(x,b)
```

⟨*BRAGG.dotabb*⟩≡

```
"BRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BRAGG" -> "RCAGG"
```

⟨*BRAGG.dotfull*⟩≡

```
"BinaryRecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"BinaryRecursiveAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:Type)"
```

⟨*BRAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

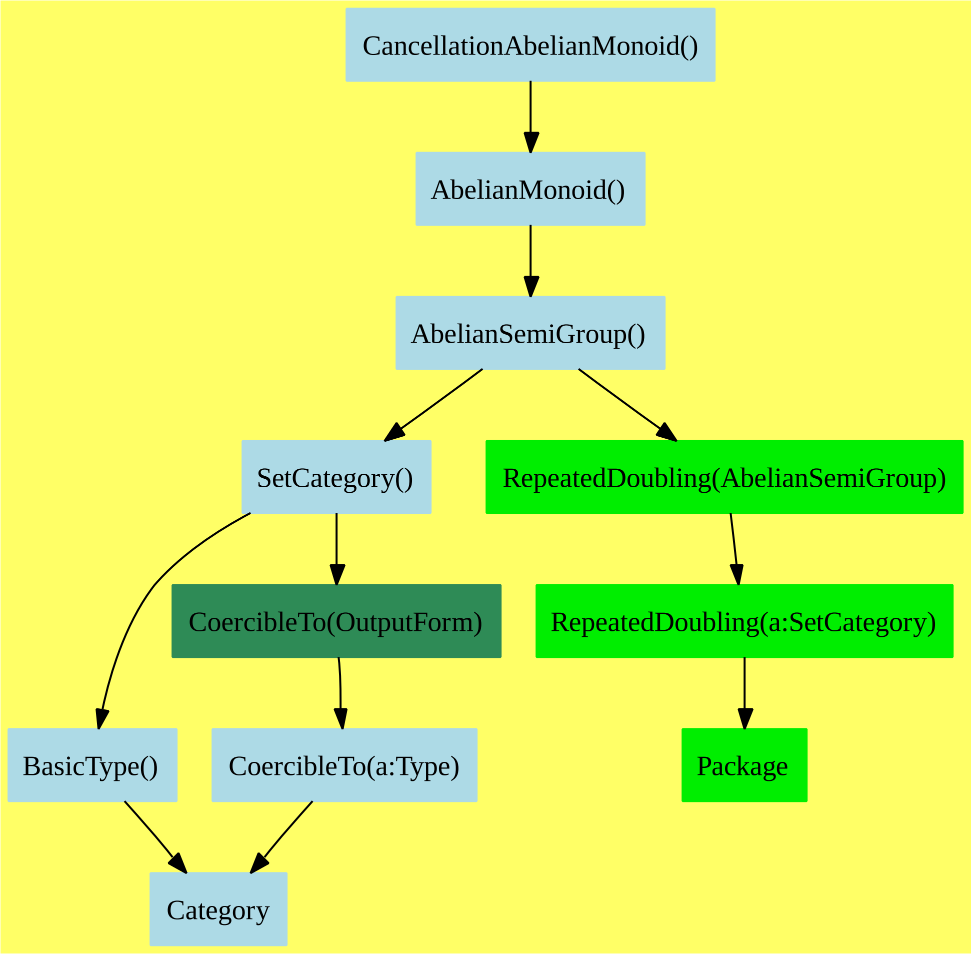
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.83 CancellationAbelianMonoid (CABMON)


CancellationAbelianMonoid()
AbelianMonoid()
AbelianSemiGroup()
SetCategory()
RepeatedDoubling(AbelianSemiGroup)
CoercibleTo(OutputForm)
RepeatedDoubling(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*CancellationAbelianMonoid.input*⟩≡

```
)set break resume
)sys rm -f CancellationAbelianMonoid.output
)spool CancellationAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CancellationAbelianMonoid
--R CancellationAbelianMonoid  is a category constructor
--R Abbreviation for CancellationAbelianMonoid is CABMON
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CABMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*CancellationAbelianMonoid.help*⟩≡

```
====================================================================
CancellationAbelianMonoid examples
====================================================================

See Also:
o )show CancellationAbelianMonoid
```

See:

⇒ “AbelianGroup” (ABELGRP) 7.0.97 on page 576
⇒ “FreeAbelianMonoidCategory” (FAMONC) 7.0.103 on page 622
⇒ “OrderedCancellationAbelianMonoid” (OCAMON) 8.0.118 on page 767
⇐ “AbelianMonoid” (ABELMON) 5.0.66 on page 301

Exports:

0 coerce hash latex sample
subtractIfCan zero? ?~=? ?*? ?+?
?=?

These are directly exported but not implemented:

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

These exports come from (p301) AbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

⟨*category CABMON CancellationAbelianMonoid*⟩≡

```
)abbrev category CABMON CancellationAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: Davenport & Trager I
++ Description:
++ This is an \spadtype{AbelianMonoid} with the cancellation property, i.e.\br
++ \tab{5}\spad{ a+b = a+c => b=c }.\br
++ This is formalised by the partial subtraction operator,
++ which satisfies the  Axioms\br
++ \tab{5}\spad{c = a+b <=> c-b = a}
```

```
CancellationAbelianMonoid(): Category == AbelianMonoid with
      subtractIfCan: (%,%) -> Union(%,"failed")
         ++ subtractIfCan(x, y) returns an element z such that \spad{z+y=x}
         ++ or "failed" if no such element exists.
```

⟨*CABMON.dotabb*⟩≡

```
"CABMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
"CABMON" -> "ABELMON"
```

⟨*CABMON.dotfull*⟩≡

```
"CancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"
```

⟨*CABMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

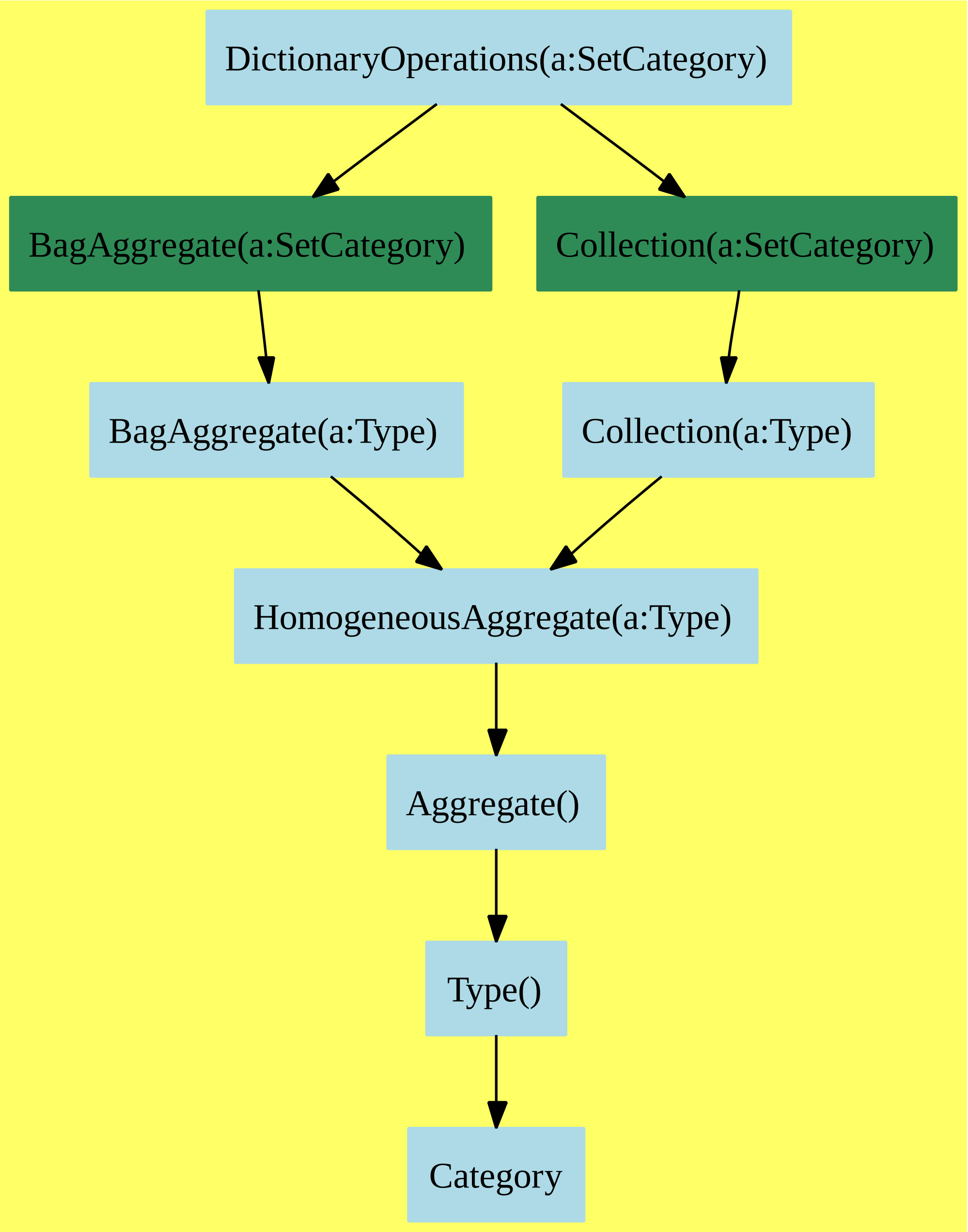
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

6.0.84 DictionaryOperations (DIOPS)


DictionaryOperations(a:SetCategory)
BagAggregate(a:SetCategory)
Collection(a:SetCategory)
BagAggregate(a:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*DictionaryOperations.input*⟩≡

```
)set break resume
```

```
)sys rm -f DictionaryOperations.output
)spool DictionaryOperations.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DictionaryOperations
--R DictionaryOperations S: SetCategory  is a category constructor
--R Abbreviation for DictionaryOperations is DIOPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIOPS
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                     construct : List S -> %
--R copy : % -> %                         dictionary : List S -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      map : ((S -> S),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

```
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*DictionaryOperations.help*⟩≡

```
====================================================================
DictionaryOperations examples
====================================================================

See Also:
o )show DictionaryOperations
```

See:

⇒ “Dictionary” (DIAGG) 7.0.99 on page 589
⇒ “MultiDictionary” (MDAGG) 7.0.104 on page 629
⇐ “BagAggregate” (BGAGG) 5.0.68 on page 312
⇐ “Collection” (CLAGG) 5.0.70 on page 322

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	size?	#?
?=?	?~=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
dictionary : List S -> %
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These are implemented by this category:

```
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
copy : % -> %
dictionary : () -> %
```

These exports come from (p312) BagAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
```

```
         if $ has finiteAggregate
bag : List S -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p322) Collection(S:SetCategory)

```
convert : % -> InputForm if S has KONVERT INFORM
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
                if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

⟨*category DIOPS DictionaryOperations*⟩≡

```
)abbrev category DIOPS DictionaryOperations
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category is a collection of operations common to both
++ categories \spadtype{Dictionary} and \spadtype{MultiDictionary}

DictionaryOperations(S:SetCategory): Category ==
  Join(BagAggregate S, Collection(S)) with
   dictionary: () -> %
     ++ dictionary()$D creates an empty dictionary of type D.
   dictionary: List S -> %
     ++ dictionary([x,y,...,z]) creates a dictionary consisting of
     ++ entries \axiom{x,y,...,z}.
-- insert: (S,%) -> S                     ++ insert an entry
-- member?: (S,%) -> Boolean                     ++ search for an entry
-- remove_!: (S,%,NonNegativeInteger) -> %
--   ++ remove!(x,d,n) destructively changes dictionary d by removing
--   ++ up to n entries y such that \axiom{y = x}.
-- remove_!: (S->Boolean,%,NonNegativeInteger) -> %
--   ++ remove!(p,d,n) destructively changes dictionary d by removing
--   ++ up to n entries x such that \axiom{p(x)} is true.
   if % has finiteAggregate then
     remove_!: (S,%) -> %
       ++ remove!(x,d) destructively changes dictionary d by removing
       ++ all entries y such that \axiom{y = x}.
     remove_!: (S->Boolean,%) -> %
       ++ remove!(p,d) destructively changes dictionary d by removeing
       ++ all entries x such that \axiom{p(x)} is true.
     select_!: (S->Boolean,%) -> %
       ++ select!(p,d) destructively changes dictionary d by removing
       ++ all entries x such that \axiom{p(x)} is not true.
 add
   construct l == dictionary l
   dictionary() == empty()
   if % has finiteAggregate then
     copy d == dictionary parts d
     coerce(s:%):OutputForm ==
        prefix("dictionary"@String :: OutputForm,
                                    [x::OutputForm for x in parts s])
```

⟨*DIOPS.dotabb*⟩≡

```
  "DIOPS" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
  "DIOPS" -> "BGAGG"
  "DIOPS" -> "CLAGG"
```

⟨*DIOPS.dotfull*⟩≡

```
  "DictionaryOperations(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
  "DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
  "DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"
```

⟨*DIOPS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

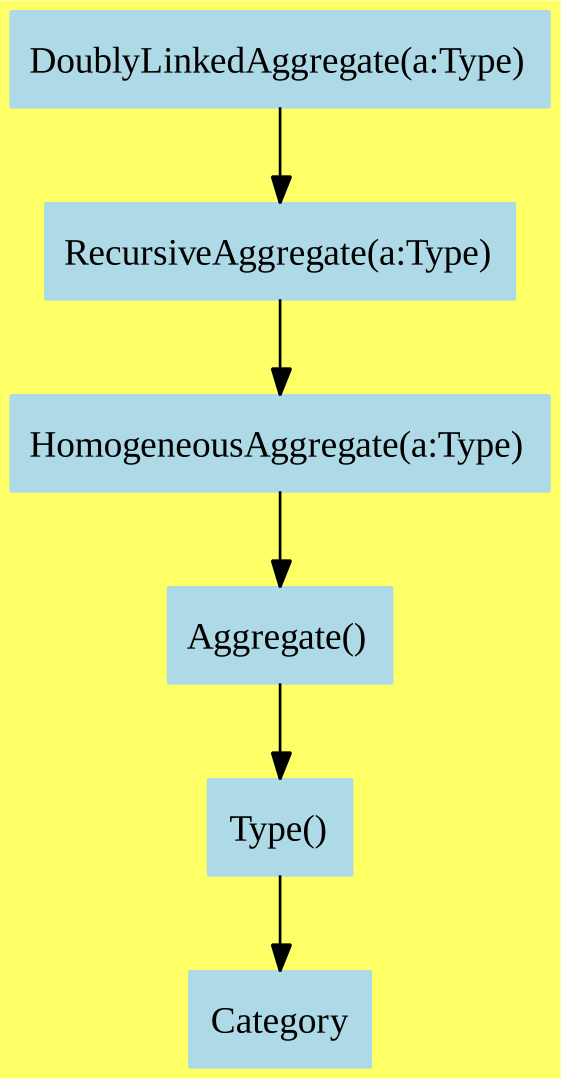
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.85 DoublyLinkedAggregate (DLAGG)


DoublyLinkedAggregate(a:Type)
RecursiveAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*DoublyLinkedAggregate.input*⟩≡

```
)set break resume
)sys rm -f DoublyLinkedAggregate.output
)spool DoublyLinkedAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show DoublyLinkedAggregate
--R DoublyLinkedAggregate S: Type  is a category constructor
--R Abbreviation for DoublyLinkedAggregate is DLAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DLAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R head : % -> %                        last : % -> S
--R leaf? : % -> Boolean                 leaves : % -> List S
--R map : ((S -> S),%) -> %              next : % -> %
--R nodes : % -> List %                  previous : % -> %
--R sample : () -> %                     tail : % -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setnext! : (%,%) -> % if $ has shallowlyMutable
--R setprevious! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
```

```
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*DoublyLinkedAggregate.help*⟩≡

```
====================================================================
DoublyLinkedAggregate examples
====================================================================

See Also:
o )show DoublyLinkedAggregate
```

See:

⇐ "RecursiveAggregate" (RCAGG) 5.0.80 on page 392

Exports:

any? children child? coerce concat!
copy count cyclic? distance empty
empty? eq? eval every? hash
head last latex leaf? leaves
less? map map! member? members
more? next nodes node? parts
previous sample setchildren! setelt setnext!
setprevious! setvalue! size? tail value
#? ?=? ?˜=? ?.value

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat! : (%,%) -> % if $ has shallowlyMutable
head : % -> %
last : % -> S
next : % -> %
previous : % -> %
setnext! : (%,%) -> % if $ has shallowlyMutable
setprevious! : (%,%) -> % if $ has shallowlyMutable
tail : % -> %
```

These exports come from (p392) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
```

```
         if $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

⟨*category DLAGG DoublyLinkedAggregate*⟩≡

```
)abbrev category DLAGG DoublyLinkedAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ A doubly-linked aggregate serves as a model for a doubly-linked
++ list, that is, a list which can has links to both next and previous
++ nodes and thus can be efficiently traversed in both directions.

DoublyLinkedAggregate(S:Type): Category == RecursiveAggregate S with
   last: % -> S
     ++ last(l) returns the last element of a doubly-linked aggregate l.
     ++ Error: if l is empty.
   head: % -> %
     ++ head(l) returns the first element of a doubly-linked aggregate l.
     ++ Error: if l is empty.
   tail: % -> %
     ++ tail(l) returns the doubly-linked aggregate l starting at
     ++ its second element.
     ++ Error: if l is empty.
   previous: % -> %
     ++ previous(l) returns the doubly-link list beginning with its previous
     ++ element.
     ++ Error: if l has no previous element.
     ++ Note that \axiom{next(previous(l)) = l}.
   next: % -> %
     ++ next(l) returns the doubly-linked aggregate beginning with its next
     ++ element.
     ++ Error: if l has no next element.
     ++ Note that \axiom{next(l) = rest(l)} and \axiom{previous(next(l)) = l}.
   if % has shallowlyMutable then
      concat_!: (%,%) -> %
        ++ concat!(u,v) destructively concatenates doubly-linked aggregate v
        ++ to the end of doubly-linked aggregate u.
      setprevious_!: (%,%) -> %
        ++ setprevious!(u,v) destructively sets the previous node of
        ++ doubly-linked aggregate u to v, returning v.
      setnext_!: (%,%) -> %
        ++ setnext!(u,v) destructively sets the next node of doubly-linked
        ++ aggregate u to v, returning v.
```

⟨*DLAGG.dotabb*⟩≡

```
"DLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DLAGG" -> "RCAGG"
```

⟨*DLAGG.dotfull*⟩≡

```
"DoublyLinkedAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

⟨*DLAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DoublyLinkedAggregate(a:Type)" [color=lightblue];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

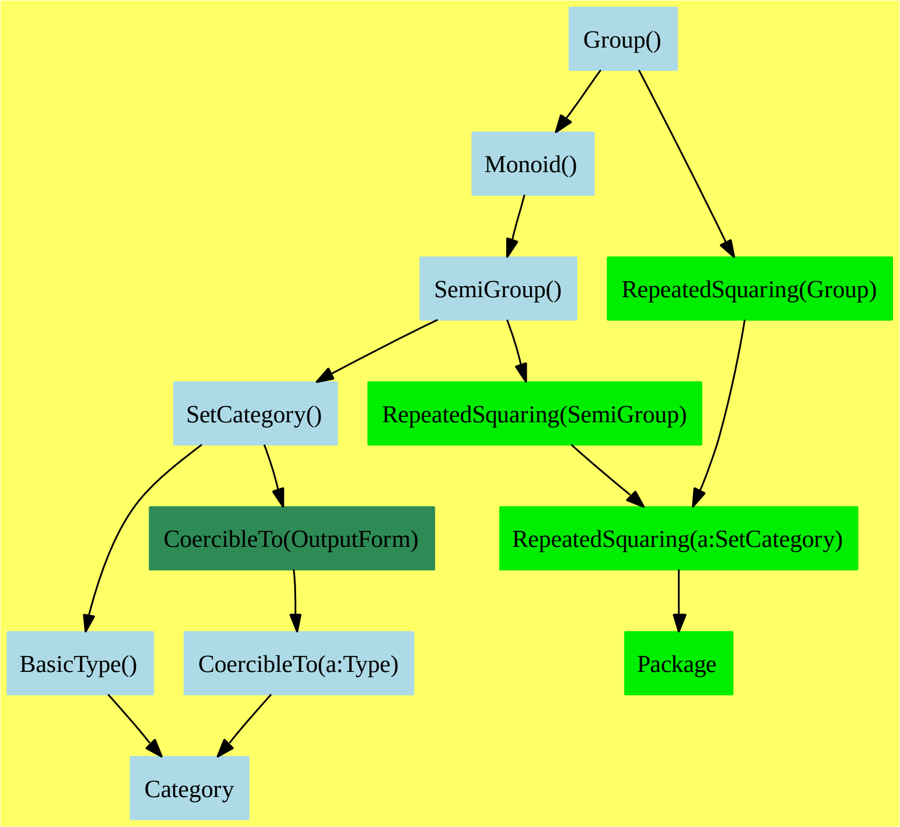
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.86 Group (GROUP)


Group()
Monoid()
SemiGroup()
RepeatedSquaring(Group)
SetCategory()
RepeatedSquaring(SemiGroup)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*Group.input*⟩≡

```
)set break resume
)sys rm -f Group.output
)spool Group.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Group
--R Group  is a category constructor
--R Abbreviation for Group is GROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GROUP
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (%,%) -> %                      ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R conjugate : (%,%) -> %                hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*Group.help*⟩≡

```
====================================================================
Group examples
====================================================================

See Also:
o )show Group
```

See:

⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "PermutationCategory" (PERMCAT) 7.0.106 on page 640
⇐ "Monoid" (MONOID) 5.0.76 on page 371

Exports:

1	coerce	commutator	conjugate	hash
inv	latex	one?	recip	sample
?~=?	?*?	?**?	?/?	?=?
?^?				

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
commutator : (%,%) -> %
conjugate : (%,%) -> %
recip : % -> Union(%,"failed")
?/? : (%,%) -> %
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p371) Monoid():

```
1 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category GROUP Group*⟩≡

```
)abbrev category GROUP Group
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of multiplicative groups, i.e. monoids with
++ multiplicative inverses.
++
++ Axioms\br
++ \tab{5}\spad{leftInverse("*":(%,%)->%,inv)}\tab{5}\spad{ inv(x)*x = 1 }\br
++ \tab{5}\spad{rightInverse("*":(%,%)->%,inv)}\tab{4}\spad{ x*inv(x) = 1 }

Group(): Category == Monoid with
      inv: % -> %
        ++ inv(x) returns the inverse of x.
      "/": (%,%) -> %
        ++ x/y is the same as x times the inverse of y.
      "**": (%,Integer) -> %
        ++ x**n returns x raised to the integer power n.
      "^": (%,Integer) -> %
        ++ x^n returns x raised to the integer power n.
      unitsKnown
        ++ unitsKnown asserts that recip only returns
        ++ "failed" for non-units.
      conjugate: (%,%) -> %
        ++ conjugate(p,q) computes \spad{inv(q) * p * q}; this is
        ++ 'right action by conjugation'.
      commutator: (%,%) -> %
        ++ commutator(p,q) computes \spad{inv(p) * inv(q) * p * q}.
    add
      import RepeatedSquaring(%)
      x:% / y:% == x*inv(y)
      recip(x:%) == inv(x)
      _^(x:%, n:Integer):% == x ** n
      x:% ** n:Integer ==
         zero? n => 1
         n<0 => expt(inv(x),(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)
      conjugate(p,q) == inv(q) * p * q
```

```
        commutator(p,q) == inv(p) * inv(q) * p * q
```

⟨*GROUP.dotabb*⟩≡

```
"GROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"GROUP" -> "MONOID"
```

⟨*GROUP.dotfull*⟩≡

```
"Group()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"Group()" -> "Monoid()"
```

⟨*GROUP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

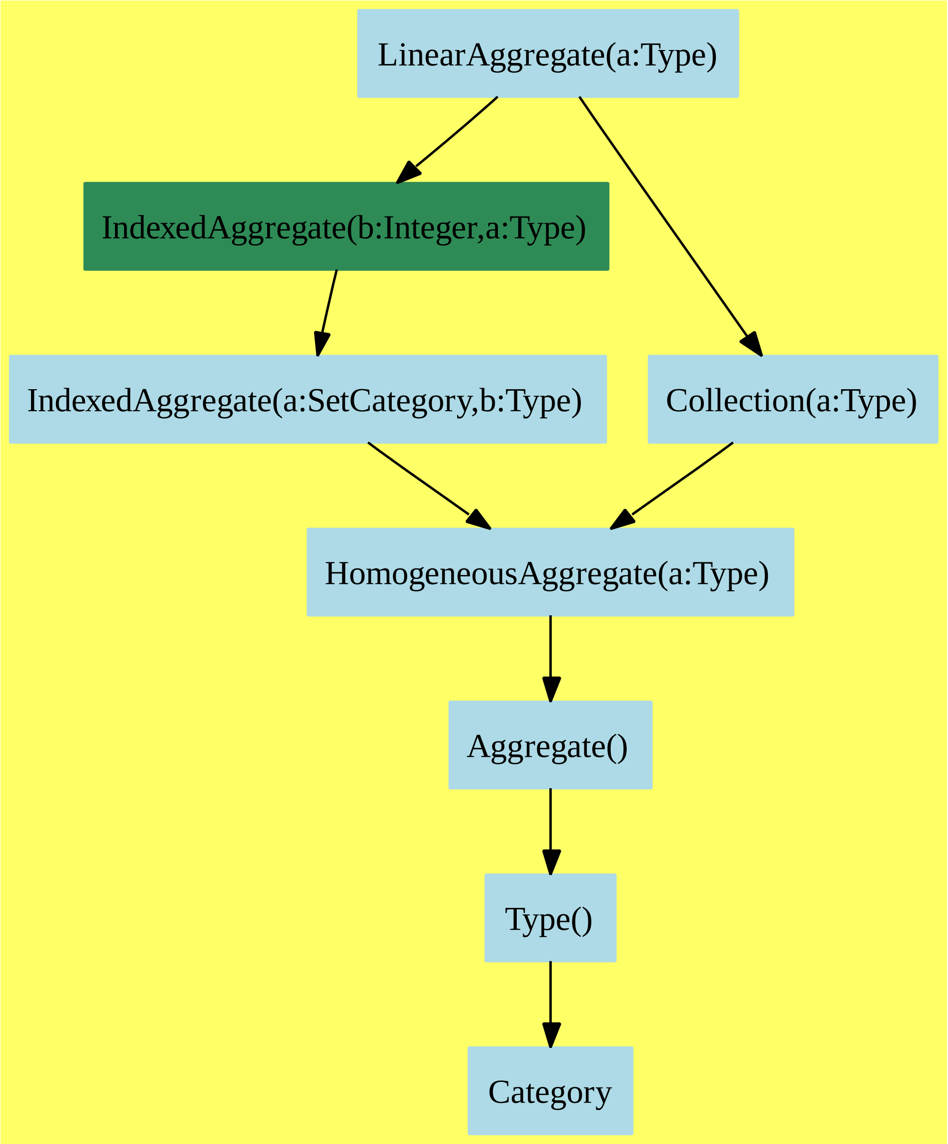
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

6.0.87 LinearAggregate (LNAGG)


LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
IndexedAggregate(a:SetCategory,b:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*LinearAggregate.input*⟩≡

```
)set break resume
)sys rm -f LinearAggregate.output
)spool LinearAggregate.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show LinearAggregate
--R LinearAggregate S: Type  is a category constructor
--R Abbreviation for LinearAggregate is LNAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LNAGG
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (S,%) -> %                   concat : (%,S) -> %
--R construct : List S -> %               copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %           map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S               sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
```

```
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*LinearAggregate.help*⟩≡

```
====================================================================
LinearAggregate examples
====================================================================

See Also:
o )show LinearAggregate
```

See:

⇒ "ExtensibleLinearAggregate" (ELAGG) 7.0.101 on page 604
⇒ "FiniteLinearAggregate" (FLAGG) 7.0.102 on page 613
⇒ "StreamAggregate" (STAGG) 7.0.107 on page 647
⇐ "Collection" (CLAGG) 5.0.70 on page 322
⇐ "IndexedAggregate" (IXAGG) 5.0.74 on page 356

Exports:

any?	coerce	concat	construct	convert
copy	count	delete	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	find	first	hash
index?	indices	insert	latex	less?
map	map!	maxIndex	member?	members
minIndex	more?	new	parts	qelt
qsetelt!	reduce	remove	removeDuplicates	sample
setelt	size?	swap!	?~=?	#?
?=?	?.?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (%,%) -> %
concat : List % -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
new : (NonNegativeInteger,S) -> %
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
?.? : (%,UniversalSegment Integer) -> %
```

These are implemented by this category:

```
concat : (%,S) -> %
concat : (S,%) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
```

These exports come from (p356) IndexedAggregate(Integer,S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
```

```
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p322) Collection(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
                if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

⟨*category LNAGG LinearAggregate*⟩≡

```
)abbrev category LNAGG LinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A linear aggregate is an aggregate whose elements are indexed by integers.
++ Examples of linear aggregates are strings, lists, and
++ arrays.
++ Most of the exported operations for linear aggregates are non-destructive
++ but are not always efficient for a particular aggregate.
++ For example, \spadfun{concat} of two lists needs only to copy its first
++ argument, whereas \spadfun{concat} of two arrays needs to copy both
++ arguments. Most of the operations exported here apply to infinite
++ objects (e.g. streams) as well to finite ones.
++ For finite linear aggregates, see \spadtype{FiniteLinearAggregate}.

LinearAggregate(S:Type): Category ==
  Join(IndexedAggregate(Integer, S), Collection(S)) with
   new         : (NonNegativeInteger,S) -> %
  ++ new(n,x) returns \axiom{fill!(new n,x)}.
concat: (%,S) -> %
  ++ concat(u,x) returns aggregate u with additional element x at the end.
  ++ Note that for lists, \axiom{concat(u,x) == concat(u,[x])}
concat: (S,%) -> %
  ++ concat(x,u) returns aggregate u with additional element at the front.
  ++ Note that for lists: \axiom{concat(x,u) == concat([x],u)}.
concat: (%,%) -> %
   ++ concat(u,v) returns an aggregate consisting of the elements of u
   ++ followed by the elements of v.
   ++ Note that if \axiom{w = concat(u,v)} then
   ++ \axiom{w.i = u.i for i in indices u}
   ++ and \axiom{w.(j + maxIndex u) = v.j for j in indices v}.
concat: List % -> %
   ++ concat(u), where u is a lists of aggregates \axiom{[a,b,...,c]},
   ++ returns a single aggregate consisting of the elements of \axiom{a}
   ++ followed by those
   ++ of b followed ... by the elements of c.
   ++ Note that \axiom{concat(a,b,...,c) = concat(a,concat(b,...,c))}.
map: ((S,S)->S,%,%) -> %
  ++ map(f,u,v) returns a new collection w with elements
  ++ \axiom{z = f(x,y)} for corresponding elements x and y from u and v.
  ++ Note that for linear aggregates, \axiom{w.i = f(u.i,v.i)}.
elt: (%,UniversalSegment(Integer)) -> %
   ++ elt(u,i..j) (also written: \axiom{a(i..j)}) returns the aggregate of
   ++ elements \axiom{u} for k from i to j in that order.
   ++ Note that in general, \axiom{a.s = [a.k for i in s]}.
delete: (%,Integer) -> %
   ++ delete(u,i) returns a copy of u with the \axiom{i}th
   ++ element deleted. Note that for lists,
   ++ \axiom{delete(a,i) == concat(a(0..i - 1),a(i + 1,..))}.
delete: (%,UniversalSegment(Integer)) -> %
   ++ delete(u,i..j) returns a copy of u with the \axiom{i}th through
   ++ \axiom{j}th element deleted.
   ++ Note that \axiom{delete(a,i..j) = concat(a(0..i-1),a(j+1..))}.
insert: (S,%,Integer) -> %
   ++ insert(x,u,i) returns a copy of u having x as its
   ++ \axiom{i}th element.
   ++ Note that \axiom{insert(x,a,k) = concat(concat(a(0..k-1),x),a(k..))}.
insert: (%,%,Integer) -> %
   ++ insert(v,u,k) returns a copy of u having v inserted beginning at the
   ++ \axiom{i}th element.
   ++ Note that \axiom{insert(v,u,k) = concat( u(0..k-1), v, u(k..) )}.
if % has shallowlyMutable then
 setelt: (%,UniversalSegment(Integer),S) -> S
   ++ setelt(u,i..j,x) (also written: \axiom{u(i..j) := x}) destructively
```

```
        ++ replaces each element in the segment \axiom{u(i..j)} by x.
        ++ The value x is returned.
        ++ Note that u is destructively change so
        ++ that \axiom{u.k := x for k in i..j};
        ++ its length remains unchanged.
   add
    indices a         == [i for i in minIndex a .. maxIndex a]
    index?(i, a)         == i >= minIndex a and i <= maxIndex a
    concat(a:%, x:S)        == concat(a, new(1, x))
    concat(x:S, y:%)        == concat(new(1, x), y)
    insert(x:S, a:%, i:Integer) == insert(new(1, x), a, i)
    if % has finiteAggregate then
      maxIndex l == #l - 1 + minIndex l

  --if % has shallowlyMutable then new(n, s)  == fill_!(new n, s)
```

⟨*LNAGG.dotabb*⟩≡

```
  "LNAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
  "LNAGG" -> "IXAGG"
  "LNAGG" -> "CLAGG"
```

⟨*LNAGG.dotfull*⟩≡

```
  "LinearAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
  "LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
  "LinearAggregate(a:Type)" -> "Collection(a:Type)"
```

⟨*LNAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

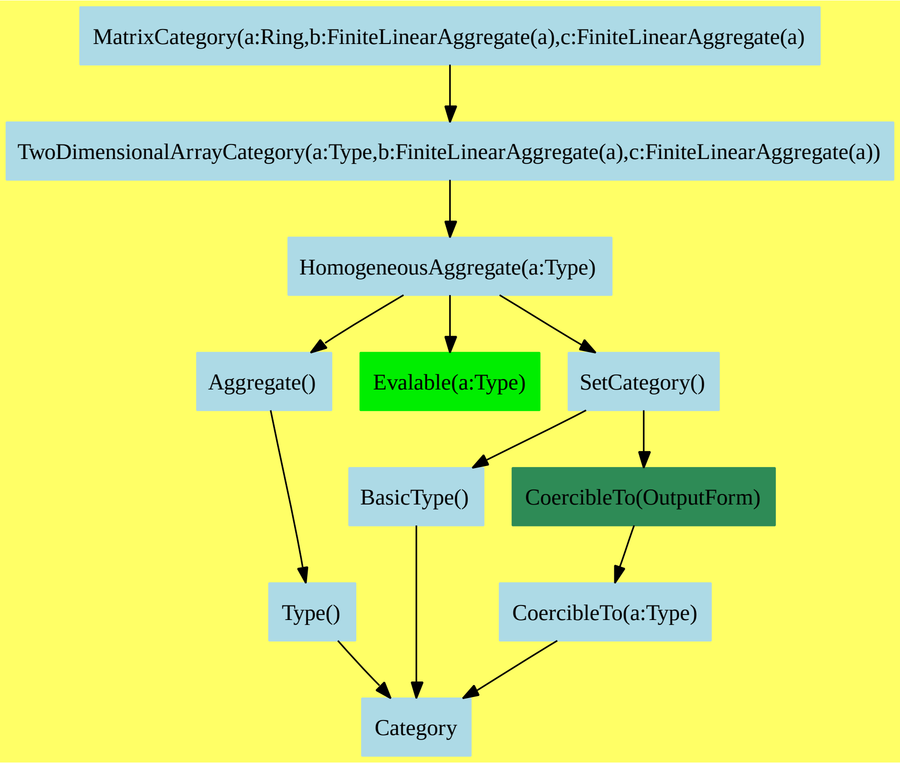
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.0.88 MatrixCategory (MATCAT)


MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)
TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))
HomogeneousAggregate(a:Type)
Aggregate()
Evalable(a:Type)
SetCategory()
BasicType()
CoercibleTo(OutputForm)
Type()
CoercibleTo(a:Type)
Category

⟨*MatrixCategory.input*⟩≡

```
)set break resume
)sys rm -f MatrixCategory.output
)spool MatrixCategory.output
)set message test on
)set message auto off
)clear all


--S 1 of 59
square? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (1)  true
--R                                                                   Type: Boolean
--E 1

--S 2 of 59
```

```
diagonal? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (2)  false
--R                                                            Type: Boolean
--E 2

--S 3 of 59
symmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (3)  false
--R                                                            Type: Boolean
--E 3

--S 4 of 59
antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (4)  false
--R                                                            Type: Boolean
--E 4

--S 5 of 59
z:Matrix(INT):=zero(3,3)
--R
--R
--R        +0  0  0+
--R        |       |
--R   (5)  |0  0  0|
--R        |       |
--R        +0  0  0+
--R                                                     Type: Matrix Integer
--E 5

--S 6 of 59
matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R        +1  2  3+
--R        |       |
--R        |4  5  6|
--R   (6)  |       |
--R        |7  8  9|
--R        |       |
--R        +1  1  1+
--R                                                   Type: Matrix Integer
--E 6

--S 7 of 59
z:Matrix(INT):=scalarMatrix(3,5)
--R
--R
--R         +5  0  0+
--R         |       |
--R   (7)   |0  5  0|
--R         |       |
--R         +0  0  5+
--R                                                   Type: Matrix Integer
--E 7

--S 8 of 59
diagonalMatrix [1,2,3]
--R
--R
--R         +1  0  0+
--R         |       |
--R   (8)   |0  2  0|
--R         |       |
--R         +0  0  3+
--R                                                   Type: Matrix Integer
--E 8

--S 9 of 59
diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]
--R
--R
--R         +1  2  0  0+
--R         |          |
--R         |3  4  0  0|
--R   (9)   |          |
--R         |0  0  4  5|
--R         |          |
--R         +0  0  6  7+
--R                                                   Type: Matrix Integer
--E 9

--S 10 of 59
coerce([1,2,3])@Matrix(INT)
--R
--R
--R          +1+
--R         | |
--R   (10)  |2|
--R         | |
--R         +3+
--R                                                         Type: Matrix Integer
--E 10

--S 11 of 59
transpose([1,2,3])@Matrix(INT)
--R
--R
--R   (11)  [1  2  3]
--R                                                         Type: Matrix Integer
--E 11

--S 12 of 59
transpose matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1   1    1    1 +
--R         |                   |
--R         |1  2   3    4    5 |
--R         |                   |
--R   (12)  |1  4   9   16   25 |
--R         |                   |
--R         |1  8   27  64   125|
--R         |                   |
--R         +1  16  81  256  625+
--R                                                         Type: Matrix Integer
--E 12

--S 13 of 59
squareTop matrix [[j**i for i in 0..2] for j in 1..5]
--R
--R
--R         +1  1  1+
--R         |       |
--R   (13)  |1  2  4|
--R         |       |
--R         +1  3  9+
--R                                                         Type: Matrix Integer
--E 13

--S 14 of 59
t1:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
```

```
--R
--R        +1  1  1    1    1 +
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R   (14) |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
--R        |                  |
--R        +1  5  25  125  625+
--R                                                 Type: Matrix Integer
--E 14

--S 15 of 59
horizConcat(t1,t1)
--R
--R
--R        +1  1  1    1    1   1  1  1    1    1 +
--R        |                                      |
--R        |1  2  4    8   16   1  2  4    8   16 |
--R        |                                      |
--R   (15) |1  3  9   27   81   1  3  9   27   81 |
--R        |                                      |
--R        |1  4  16  64   256  1  4  16  64   256|
--R        |                                      |
--R        +1  5  25  125  625  1  5  25  125  625+
--R                                                 Type: Matrix Integer
--E 15

--S 16 of 59
t2:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R        +1  1  1    1    1 +
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R   (16) |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
--R        |                  |
--R        +1  5  25  125  625+
--R                                                 Type: Matrix Integer
--E 16

--S 17 of 59
```

```
vertConcat(t2,t2)
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         |1  5  25  125  625|
--R   (17)  |                  |
--R         |1  1  1    1    1 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 17

--S 18 of 59
t3:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (18)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 18

--S 19 of 59
listOfLists t3
--R
--R
--R   (19)
```

```
--R   [[1,1,1,1,1],[1,2,4,8,16],[1,3,9,27,81],[1,4,16,64,256],[1,5,25,125,625]]
--R                                                        Type: List List Integer
--E 19

--S 20 of 59
t4:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (20)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                          Type: Matrix Integer
--E 20

--S 21 of 59
elt(t4,3,3)
--R
--R
--R   (21)  9
--R                                                         Type: PositiveInteger
--E 21

--S 22 of 59
t5:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (22)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                          Type: Matrix Integer
--E 22

--S 23 of 59
setelt(t5,3,3,10)
```

```
--R
--R
--R   (23)  10
--R                                                      Type: PositiveInteger
--E 23

--S 24 of 59
t6:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (24)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                       Type: Matrix Integer
--E 24

--S 25 of 59
swapRows!(t6,2,4)
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R   (25)  |1  3  9   27   81 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         +1  5  25  125  625+
--R                                                       Type: Matrix Integer
--E 25

--S 26 of 59
t7:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (26)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                         Type: Matrix Integer
--E 26

--S 27 of 59
swapColumns!(t7,2,4)
--R
--R
--R         +1   1   1   1   1 +
--R         |                  |
--R         |1   8   4   2  16 |
--R         |                  |
--R   (27)  |1  27   9   3  81 |
--R         |                  |
--R         |1  64   16  4  256|
--R         |                  |
--R         +1  125  25  5  625+
--R                                                         Type: Matrix Integer
--E 27

--S 28 of 59
t8:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (28)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                         Type: Matrix Integer
--E 28

--S 29 of 59
subMatrix(t8,1,3,2,4)
--R
--R
--R         +1  1  1 +
--R         |        |
--R   (29)  |2  4  8 |
--R         |        |
--R         +3  9  27+
--R                                                     Type: Matrix Integer
--E 29

--S 30 of 59
t9:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (30)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 30

--S 31 of 59
setsubMatrix!(t9,2,2,matrix [[3,3],[3,3]])
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  3  3    8   16 |
--R         |                  |
--R   (31)  |1  3  3   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 31

--S 32 of 59
t0:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (32)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                         Type: Matrix Integer
--E 32

--S 33 of 59
t0+t0
--R
--R
--R         +2  2   2    2    2  +
--R         |                    |
--R         |2  4   8   16    32 |
--R         |                    |
--R   (33)  |2  6   18  54   162 |
--R         |                    |
--R         |2  8   32  128  512 |
--R         |                    |
--R         +2  10  50  250  1250+
--R                                                         Type: Matrix Integer
--E 33

--S 34 of 59
t0-t0
--R
--R
--R         +0  0  0  0  0+
--R         |             |
--R         |0  0  0  0  0|
--R         |             |
--R   (34)  |0  0  0  0  0|
--R         |             |
--R         |0  0  0  0  0|
--R         |             |
--R         +0  0  0  0  0+
--R                                                         Type: Matrix Integer
--E 34

--S 35 of 59
-t0
--R
--R
--R         +- 1  - 1  - 1    - 1    - 1 +
--R         |                            |
--R         |- 1  - 2  - 4    - 8   - 16 |
--R         |                            |
--R   (35)  |- 1  - 3  - 9   - 27   - 81 |
--R         |                            |
--R         |- 1  - 4  - 16  - 64   - 256|
--R         |                            |
--R         +- 1  - 5  - 25  - 125  - 625+
--R                                                         Type: Matrix Integer
--E 35

--S 36 of 59
t0*t0
--R
--R
--R         + 5    15    55     225    979  +
--R         |                               |
--R         |31   129    573   2637   12405 |
--R         |                               |
--R   (36)  |121  547   2551   12121  58315 |
--R         |                               |
--R         |341  1593  7585   36561  177745|
--R         |                               |
--R         +781  3711  17871  86841  424731+
--R                                                         Type: Matrix Integer
--E 36

--S 37 of 59
1/3*t0
--R
--R
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1                 |
--R   (37)  |-  1  3    9   27 |
--R         |3                 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                  |
--R         |1  5  25  125  625|
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                                Type: Matrix Fraction Integer
--E 37

--S 38 of 59
m:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (38)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                         Type: Matrix Integer
--E 38

--S 39 of 59
t0*1/3
--R
--R
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1                 |
--R   (39)  |-  1  3    9   27 |
--R         |3                 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                  |
--R         |1  5  25  125  625|
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                                Type: Matrix Fraction Integer
```

```
--E 39

--S 40 of 59
3*t0
--R
--R
--R         +3  3   3    3    3  +
--R         |                    |
--R         |3  6   12  24    48 |
--R         |                    |
--R   (40)  |3  9   27  81   243 |
--R         |                    |
--R         |3  12  48  192  768 |
--R         |                    |
--R         +3  15  75  375  1875+
--R                                                     Type: Matrix Integer
--E 40

--S 41 of 59
c:=coerce([1,2,3,4,5])@Matrix(INT)
--R
--R
--R         +1+
--R         | |
--R         |2|
--R         | |
--R   (41)  |3|
--R         | |
--R         |4|
--R         | |
--R         +5+
--R                                                     Type: Matrix Integer
--E 41

--S 42 of 59
t0*c
--R
--R
--R         + 15 +
--R         |    |
--R         |129 |
--R         |    |
--R   (42)  |547 |
--R         |    |
--R         |1593|
--R         |    |
--R         +3711+
--R                                                         Type: Matrix Integer
--E 42

--S 43 of 59
r:=transpose([1,2,3,4,5])@Matrix(INT)
--R
--R
--R   (43)  [1  2  3  4  5]
--R                                                         Type: Matrix Integer
--E 43

--S 44 of 59
r*t0
--R
--R
--R   (44)  [15  55  225  979  4425]
--R                                                         Type: Matrix Integer
--E 44

--S 45 of 59
t0**3
--R
--R
--R         + 1279    5995     28635     138385    674175  +
--R         |                                              |
--R         |15775    74581    358021   1735927    8476705 |
--R         |                                              |
--R   (45)  |73655   348927   1677079   8138493   39765355 |
--R         |                                              |
--R         |223825  1061251  5103579   24775909  121090455|
--R         |                                              |
--R         +533935  2532835  12184195  59162185  289195879+
--R                                                         Type: Matrix Integer
--E 45

--S 46 of 59
t10:=matrix [[2**i for i in 2..4] for j in 1..5]
--R
--R
--R         +4  8  16+
--R         |        |
--R         |4  8  16|
--R         |        |
--R   (46)  |4  8  16|
--R         |        |
--R         |4  8  16|
--R         |        |
--R         +4  8  16+
--R                                                        Type: Matrix Integer
--E 46

--S 47 of 59
exquo(t10,2)
--R
--R
--R         +2  4  8+
--R         |       |
--R         |2  4  8|
--R         |       |
--R   (47)  |2  4  8|
--R         |       |
--R         |2  4  8|
--R         |       |
--R         +2  4  8+
--R                                             Type: Union(Matrix Integer,...)
--E 47

--S 48 of 59
t10/4
--R
--R
--R         +1  2  4+
--R         |       |
--R         |1  2  4|
--R         |       |
--R   (48)  |1  2  4|
--R         |       |
--R         |1  2  4|
--R         |       |
--R         +1  2  4+
--R                                               Type: Matrix Fraction Integer
--E 48

--S 49 of 59
rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  0  0  0  0 +
--R         |              |
--R         |0  1  1  1  1 |
--R         |              |
--R   (49)  |0  0  2  0  2 |
--R         |              |
--R         |0  0  0  6  12|
--R         |              |
--R         +0  0  0  0  24+
--R                                                      Type: Matrix Integer
--E 49

--S 50 of 59
columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R   (50)  [[1,4,7,1],[2,5,8,1]]
--R                                                 Type: List Vector Integer
--E 50

--S 51 of 59
rank matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (51)  2
--R                                                     Type: PositiveInteger
--E 51

--S 52 of 59
nullity matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (52)  1
--R                                                     Type: PositiveInteger
--E 52

--S 53 of 59
nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (53)  [[1,- 2,1]]
--R                                                 Type: List Vector Integer
--E 53

--S 54 of 59
determinant matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (54)  288
--R                                                     Type: PositiveInteger
```

```
--E 54

--S 55 of 59
minordet matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (55)  288
--R                                                        Type: PositiveInteger
--E 55

--S 56 of 59
pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
--R
--R
--R   (56)  1
--R                                                        Type: PositiveInteger
--E 56

--S 57 of 59
inverse matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         + 5    - 10   10   - 5    1  +
--R         |                            |
--R         |  77  107     39   61     25|
--R         |- --  ---   - --   --   - --|
--R         |  12   6       2    6     12|
--R         |                            |
--R         | 71     59   49     41   35 |
--R         | --   - --   --   - --   -- |
--R   (57)  | 24      6    4      6   24 |
--R         |                            |
--R         |   7   13          11      5|
--R         |- --   --   - 3    --   - --|
--R         |  12    6           6     12|
--R         |                            |
--R         |  1     1    1      1     1 |
--R         | --   - -    -    - -    -- |
--R         + 24     6    4      6   24 +
--R                                     Type: Union(Matrix Fraction Integer,...)
--E 57

--S 58 of 59
(matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
--R
--R
```

```
--R          + 5    15    55     225    979  +
--R          |                               |
--R          |31   129    573   2637   12405 |
--R          |                               |
--R   (58)   |121  547   2551   12121  58315 |
--R          |                               |
--R          |341  1593  7585   36561  177745|
--R          |                               |
--R          +781  3711  17871  86841  424731+
--R                                                         Type: Matrix Integer
--E 58

--S 59 of 59
)show MatrixCategory
--R
--R MatrixCategory(R: Ring,Row: FiniteLinearAggregate t#1,Col: FiniteLinearAggreg
--R Abbreviation for MatrixCategory is MATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Row,%) -> Row                 ?*? : (%,Col) -> Col
--R ?*? : (Integer,%) -> %               ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                     ?*? : (%,%) -> %
--R ?+? : (%,%) -> %                     -? : % -> %
--R ?-? : (%,%) -> %                     antisymmetric? : % -> Boolean
--R coerce : Col -> %                    column : (%,Integer) -> Col
--R copy : % -> %                        diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %         diagonalMatrix : List R -> %
--R elt : (%,Integer,Integer,R) -> R     elt : (%,Integer,Integer) -> R
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               fill! : (%,R) -> %
--R horizConcat : (%,%) -> %             listOfLists : % -> List List R
--R map : (((R,R) -> R),%,%,R) -> %      map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %              map! : ((R -> R),%) -> %
--R matrix : List List R -> %            maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer           minColIndex : % -> Integer
--R minRowIndex : % -> Integer           ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger      parts : % -> List R
--R qelt : (%,Integer,Integer) -> R      row : (%,Integer) -> Row
--R sample : () -> %                     setRow! : (%,Integer,Row) -> %
--R square? : % -> Boolean               squareTop : % -> %
--R symmetric? : % -> Boolean            transpose : % -> %
--R transpose : Row -> %                 vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?**? : (%,Integer) -> % if R has FIELD
```

```
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R columnSpace : % -> List Col if R has EUCDOM
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R elt : (%,List Integer,List Integer) -> %
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R hash : % -> SingleInteger if R has SETCAT
--R inverse : % -> Union(%,"failed") if R has FIELD
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R nullSpace : % -> List Col if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R pfaffian : % -> R if R has COMRING
--R qsetelt! : (%,Integer,Integer,R) -> R
--R rank : % -> NonNegativeInteger if R has INTDOM
--R rowEchelon : % -> % if R has EUCDOM
--R scalarMatrix : (NonNegativeInteger,R) -> %
--R setColumn! : (%,Integer,Col) -> %
--R setelt : (%,List Integer,List Integer,%) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 59

)spool
```

```
)lisp (bye)
```

⟨*MatrixCategory.help*⟩≡
```
====================================================================
MatrixCategory examples
====================================================================

Predicates:

square?(m) returns true if m is a square matrix
(if m has the same number of rows as columns) and false otherwise.

  square matrix [[j**i for i in 0..4] for j in 1..5]

diagonal?(m) returns true if the matrix m is square and
diagonal (i.e. all entries of m not on the diagonal are zero) and
false otherwise.

  diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

symmetric?(m) returns true if the matrix m is square and
symmetric (i.e. \spad{m[i,j] = m[j,i]} for all i and j) and false
otherwise.

  symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

antisymmetric?(m) returns true if the matrix m is square and
antisymmetric (i.e. m[i,j] = -m[j,i] for all i and j)
and false otherwise.

  antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]


Creation

zero(m,n) returns an m-by-n zero matrix.

  z:Matrix(INT):=zero(3,3)

matrix(l) converts the list of lists l to a matrix, where the
list of lists is viewed as a list of the rows of the matrix.

  matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

scalarMatrix(n,r) returns an n-by-n matrix with r's on the
diagonal and zeroes elsewhere.

  z:Matrix(INT):=scalarMatrix(3,5)
```

diagonalMatrix(l) returns a diagonal matrix with the elements of l on the diagonal.

```
  diagonalMatrix [1,2,3]
```

diagonalMatrix([m1,...,mk]) creates a block diagonal matrix M with block matrices m1,...,mk down the diagonal, with 0 block matrices elsewhere.

```
More precisly: if ri := nrows mi, ci := ncols mi,
then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1)),
if (r1+..+r(l-1)) < i <= r1+..+rl and
   (c1+..+c(l-1)) < i <= c1+..+cl,
    m.i.j = 0  otherwise.
```

```
  diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]
```

coerce(col) converts the column col to a column matrix.

```
  coerce([1,2,3])@Matrix(INT)
```

transpose(r) converts the row r to a row matrix.

```
  transpose([1,2,3])@Matrix(INT)
```

Creation of new matrices from old

transpose(m) returns the transpose of the matrix m.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  transpose m
```

squareTop(m) returns an n-by-n matrix consisting of the first n rows of the m-by-n matrix m. Error: if m < n.

```
  m:=matrix [[j**i for i in 0..2] for j in 1..5]
  squareTop m
```

horizConcat(x,y) horizontally concatenates two matrices with an equal number of rows. The entries of y appear to the right of the entries of x. Error: if the matrices do not have the same number of rows.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  horizConcat(m,m)

vertConcat(x,y) vertically concatenates two matrices with an
equal number of columns. The entries of y appear below
of the entries of x.  Error: if the matrices
do not have the same number of columns.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  vertConcat(m,m)

Part extractions/assignments

listOfLists(m) returns the rows of the matrix m as a list of lists

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  listOfLists m

elt(x,rowList,colList) returns an m-by-n matrix consisting
of elements of x, where m = # rowList and n = # colList
If rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then the (k,l)-th entry of elt(x,rowList,colList) is x(i<k>,j<l>).

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  elt(m,3,3)

setelt(x,rowList,colList,y) destructively alters the matrix x.
If y is m-by-n,
   rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then x(i<k>,j<l>)
 is set to y(k,l) for k = 1,...,m and l = 1,...,n

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setelt(m,3,3,10)

swapRows!(m,i,j) interchanges the i-th and j-th
rows of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  swapRows!(m,2,4)

swapColumns!(m,i,j) interchanges the i-th and j-th
columns of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  swapColumns!(m,2,4)
```

subMatrix(x,i1,i2,j1,j2) extracts the submatrix [x(i,j)] where the index i ranges from i1 to i2 and the index j ranges from j1 to j2.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  subMatrix(m,1,3,2,4)
```

setsubMatrix(x,i1,j1,y) destructively alters the matrix x. Here x(i,j) is set to y(i-i1+1,j-j1+1) for i = i1,...,i1-1+nrows y and j = j1,...,j1-1+ncols y.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])
```

Arithmetic

x + y is the sum of the matrices x and y. It is an error if the dimensions are incompatible.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m+m
```

x - y is the difference of the matrices x and y. It is an error if the dimensions are incompatible.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m-m
```

-x returns the negative of the matrix x.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  -m
```

x * y is the product of the matrices x and y. It is an error if the dimensions are incompatible.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*m
```

r*x is the left scalar multiple of the scalar r and the matrix x.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  1/3*m
```

x * r is the right scalar multiple of the scalar r and the matrix x.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*1/3
```

n * x is an integer multiple.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  3*m
```

x * c is the product of the matrix x and the column vector c.
It is an error if the dimensions are incompatible.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  c:=coerce([1,2,3,4,5])@Matrix(INT)
  m *c
```

r * x is the product of the row vector r and the matrix x.
It is an error if the dimensions are incompatible.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  r:=transpose([1,2,3,4,5])@Matrix(INT)
  r*m
```

x ** n computes a non-negative integral power of the matrix x.
It is an error if the matrix is not square.

```
  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m**3
```

exquo(m,r) computes the exact quotient of the elements
of m by r, returning "failed" if this is not possible.

```
  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  exquo(m,2)
```

m/r divides the elements of m by r, r must be non-zero.

```
  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  m/4
```

Linear algebra

rowEchelon(m) returns the row echelon form of the matrix m.

```
  rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]
```

columnSpace(m) returns a sublist of columns of the matrix m

```
  columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
```

rank(m) returns the rank of the matrix m.

```
  rank matrix [[1,2,3],[4,5,6],[7,8,9]]
```

nullity(m) returns the nullity of the matrix m. This is
the dimension of the null space of the matrix m.

```
  nullity matrix [[1,2,3],[4,5,6],[7,8,9]]
```

nullSpace(m) returns a basis for the null space of the matrix m.

```
  nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]
```

determinant(m) returns the determinant of the matrix m.
It is an error if the matrix is not square.

```
  determinant matrix [[j**i for i in 0..4] for j in 1..5]
```

minordet(m) computes the determinant of the matrix m using minors.
It is an error if the matrix is not square.

```
  minordet matrix [[j**i for i in 0..4] for j in 1..5]
```

pfaffian(m) returns the Pfaffian of the matrix m.
It is an error if the matrix is not antisymmetric

```
  pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
```

inverse(m) returns the inverse of the matrix m.
If the matrix is not invertible, "failed" is returned.
It is an error if the matrix is not square.

```
  inverse matrix [[j**i for i in 0..4] for j in 1..5]
```

`m**n` computes an integral power of the matrix m.
It is an error if matrix is not square or
if the matrix is square but not invertible.

```
  (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
```

We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The Matrix domain is the domain of all matrices.

All three domains share the same representation, inherited from Matrix. Most algorithms are only implemented for Matrix but implemented in separate packages.

- MatrixLinearAlgebraFunctions is the top-level package that calls the other packages
- InnerMatrixLinearAlgebraFunctions contains implementations that work over a Field
- InnerMatrixQuotientFieldFunctions contain implementations that work over a quotient field

Implementations that rely on the representation of matrices used in Matrix should be put into these packages.

See:

⇐ "TwoDimensionalArrayCategory" (ARR2CAT) 5.0.81 on page 399
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.147 on page 994
⇒ "SquareMatrixCategory" (SMATCAT) 12.0.168 on page 1192

Exports:

antisymmetric?
any?
coerce
column
columnSpace
copy
count
determinant
diagonal?
diagonalMatrix
elt
empty
empty?
eq?
eval
every?
exquo
fill!
hash
horizConcat
inverse
latex
less?
listOfLists
map
map!
matrix
maxColIndex
maxRowIndex
member?
members
minColIndex
minordet
minRowIndex
more?
ncols
new
nrows
nullSpace
nullity
parts
pfaffian
qelt
qsetelt!
rank
row
rowEchelon
sample
scalarMatrix
setColumn!
setelt
setRow!
setsubMatrix!
size?
square?
squareTop
subMatrix
swapColumns!
swapRows!
symmetric?
transpose
vertConcat
zero
#?
?**?
?/?
?=?
?~=?
?*?
?+?
-?
?-?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
minordet : % -> R if R has commutative *
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
rowEchelon : % -> % if R has EUCDOM
rank : % -> NonNegativeInteger if R has INTDOM
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
coerce : Col -> %
columnSpace: % -> List Col if R has EUCDOM
diagonal? : % -> Boolean
diagonalMatrix : List % -> %
diagonalMatrix : List R -> %
elt : (%,List Integer,List Integer) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
horizConcat : (%,%) -> %
listOfLists : % -> List List R
pfaffian: % -> R if R has COMRING
matrix : List List R -> %
scalarMatrix : (NonNegativeInteger,R) -> %
setelt : (%,List Integer,List Integer,%) -> %
setsubMatrix! : (%,Integer,Integer,%) -> %
square? : % -> Boolean
squareTop : % -> %
subMatrix : (%,Integer,Integer,Integer,Integer) -> %
swapColumns! : (%,Integer,Integer) -> %
swapRows! : (%,Integer,Integer) -> %
symmetric? : % -> Boolean
transpose : Row -> %
transpose : % -> %
vertConcat : (%,%) -> %
zero : (NonNegativeInteger,NonNegativeInteger) -> %
?/? : (%,R) -> % if R has FIELD
```

```
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,Integer) -> % if R has FIELD
?*? : (%,Col) -> Col
?*? : (Row,%) -> Row
?*? : (%,%) -> %
```

These exports come from (p399) TwoDimensionalArrayCategory(R,Row,Col) where R:Ring, Row:FiniteLinearAggregate(R), Col:FiniteLinearAggregate(R):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
elt : (%,Integer,Integer) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((R,R) -> R),%,%,R) -> %
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
minColIndex : % -> Integer
minRowIndex : % -> Integer
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,NonNegativeInteger,R) -> %
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
```

```
parts : % -> List R
qelt : (%,Integer,Integer) -> R
qsetelt! : (%,Integer,Integer,R) -> R
row : (%,Integer) -> Row
sample : () -> %
setColumn! : (%,Integer,Col) -> %
setelt : (%,Integer,Integer,R) -> R
setRow! : (%,Integer,Row) -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean if R has SETCAT
?=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category MATCAT MatrixCategory*⟩≡

```
)abbrev category MATCAT MatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: Matrix(R)
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{MatrixCategory} is a general matrix category which allows
++ different representations and indexing schemes.  Rows and
++ columns may be extracted with rows returned as objects of
++ type Row and colums returned as objects of type Col.
++ A domain belonging to this category will be shallowly mutable.
++ The index of the 'first' row may be obtained by calling the
++ function \spadfun{minRowIndex}.  The index of the 'first' column may
++ be obtained by calling the function \spadfun{minColIndex}.  The index of
++ the first element of a Row is the same as the index of the
++ first column in a matrix and vice versa.

MatrixCategory(R,Row,Col): Category == Definition where
  R   : Ring
  Row : FiniteLinearAggregate R
  Col : FiniteLinearAggregate R

  Definition ==> TwoDimensionalArrayCategory(R,Row,Col) with
     shallowlyMutable
       ++ One may destructively alter matrices

     finiteAggregate
```

```
    ++ matrices are finite

--% Predicates

     square?  : % -> Boolean
       ++ \spad{square?(m)} returns true if m is a square matrix
       ++ (if m has the same number of rows as columns) and false otherwise.
       ++
       ++X square matrix [[j**i for i in 0..4] for j in 1..5]

     diagonal?: % -> Boolean
       ++ \spad{diagonal?(m)} returns true if the matrix m is square and
       ++ diagonal (i.e. all entries of m not on the diagonal are zero) and
       ++ false otherwise.
       ++
       ++X diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

     symmetric?: % -> Boolean
       ++ \spad{symmetric?(m)} returns true if the matrix m is square and
       ++ symmetric (i.e. \spad{m[i,j] = m[j,i]} for all i and j) and false
       ++ otherwise.
       ++
       ++X symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

     antisymmetric?: % -> Boolean
       ++ \spad{antisymmetric?(m)} returns true if the matrix m is square and
       ++ antisymmetric (i.e. \spad{m[i,j] = -m[j,i]} for all i and j)
       ++ and false otherwise.
       ++
       ++X antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]


--% Creation

     zero: (NonNegativeInteger,NonNegativeInteger) -> %
       ++ \spad{zero(m,n)} returns an m-by-n zero matrix.
       ++
       ++X z:Matrix(INT):=zero(3,3)

     matrix: List List R -> %
       ++ \spad{matrix(l)} converts the list of lists l to a matrix, where the
       ++ list of lists is viewed as a list of the rows of the matrix.
       ++
       ++X matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

     scalarMatrix: (NonNegativeInteger,R) -> %
```

```
        ++ \spad{scalarMatrix(n,r)} returns an n-by-n matrix with r's on the
        ++ diagonal and zeroes elsewhere.
        ++
        ++X z:Matrix(INT):=scalarMatrix(3,5)

      diagonalMatrix: List R -> %
        ++ \spad{diagonalMatrix(l)} returns a diagonal matrix with the elements
        ++ of l on the diagonal.
        ++
        ++X diagonalMatrix [1,2,3]

      diagonalMatrix: List % -> %
        ++ \spad{diagonalMatrix([m1,...,mk])} creates a block diagonal matrix
        ++ M with block matrices m1,...,mk down the diagonal,
        ++ with 0 block matrices elsewhere.
        ++ More precisly: if \spad{ri := nrows mi}, \spad{ci := ncols mi},
        ++ then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
        ++ \spad{m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1))}, if
        ++ \spad{(r1+..+r(l-1)) < i <= r1+..+rl} and
        ++ \spad{(c1+..+c(l-1)) < i <= c1+..+cl},
        ++ \spad{m.i.j} = 0  otherwise.
        ++
        ++X diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]

      coerce: Col -> %
        ++ \spad{coerce(col)} converts the column col to a column matrix.
        ++
        ++X coerce([1,2,3])@Matrix(INT)

      transpose: Row -> %
        ++ \spad{transpose(r)} converts the row r to a row matrix.
        ++
        ++X transpose([1,2,3])@Matrix(INT)

--% Creation of new matrices from old

      transpose: % -> %
        ++ \spad{transpose(m)} returns the transpose of the matrix m.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X transpose m

      squareTop: % -> %
        ++ \spad{squareTop(m)} returns an n-by-n matrix consisting of the first
        ++ n rows of the m-by-n matrix m. Error: if
        ++ \spad{m < n}.
      ++
      ++X m:=matrix [[j**i for i in 0..2] for j in 1..5]
      ++X squareTop m

    horizConcat: (%,%) -> %
      ++ \spad{horizConcat(x,y)} horizontally concatenates two matrices with
      ++ an equal number of rows. The entries of y appear to the right
      ++ of the entries of x.  Error: if the matrices
      ++ do not have the same number of rows.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X horizConcat(m,m)

    vertConcat: (%,%) -> %
      ++ \spad{vertConcat(x,y)} vertically concatenates two matrices with an
      ++ equal number of columns. The entries of y appear below
      ++ of the entries of x.  Error: if the matrices
      ++ do not have the same number of columns.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X vertConcat(m,m)

--% Part extractions/assignments

    listOfLists: % -> List List R
      ++ \spad{listOfLists(m)} returns the rows of the matrix m as a list
      ++ of lists.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X listOfLists m

    elt: (%,List Integer,List Integer) -> %
      ++ \spad{elt(x,rowList,colList)} returns an m-by-n matrix consisting
      ++ of elements of x, where \spad{m = # rowList} and \spad{n = # colList}
      ++ If \spad{rowList = [i<1>,i<2>,...,i<m>]} and \spad{colList =
      ++ [j<1>,j<2>,...,j<n>]}, then the \spad{(k,l)}th entry of
      ++ \spad{elt(x,rowList,colList)} is \spad{x(i<k>,j<l>)}.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X elt(m,3,3)

    setelt: (%,List Integer,List Integer, %) -> %
      ++ \spad{setelt(x,rowList,colList,y)} destructively alters the matrix x.
      ++ If y is \spad{m}-by-\spad{n}, \spad{rowList = [i<1>,i<2>,...,i<m>]}
      ++ and \spad{colList = [j<1>,j<2>,...,j<n>]}, then \spad{x(i<k>,j<l>)}
      ++ is set to \spad{y(k,l)} for \spad{k = 1,...,m} and \spad{l = 1,...,n}
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X setelt(m,3,3,10)

  swapRows_!: (%,Integer,Integer) -> %
    ++ \spad{swapRows!(m,i,j)} interchanges the \spad{i}th and \spad{j}th
    ++ rows of m. This destructively alters the matrix.
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X swapRows!(m,2,4)

  swapColumns_!: (%,Integer,Integer) -> %
    ++ \spad{swapColumns!(m,i,j)} interchanges the \spad{i}th and \spad{j}th
    ++ columns of m. This destructively alters the matrix.
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X swapColumns!(m,2,4)

  subMatrix: (%,Integer,Integer,Integer,Integer) -> %
    ++ \spad{subMatrix(x,i1,i2,j1,j2)} extracts the submatrix
    ++ \spad{[x(i,j)]} where the index i ranges from \spad{i1} to \spad{i2}
    ++ and the index j ranges from \spad{j1} to \spad{j2}.
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X subMatrix(m,1,3,2,4)

  setsubMatrix_!: (%,Integer,Integer,%) -> %
    ++ \spad{setsubMatrix(x,i1,j1,y)} destructively alters the
    ++ matrix x. Here \spad{x(i,j)} is set to \spad{y(i-i1+1,j-j1+1)} for
    ++ \spad{i = i1,...,i1-1+nrows y} and \spad{j = j1,...,j1-1+ncols y}.
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])


--% Arithmetic

  "+": (%,%) -> %
    ++ \spad{x + y} is the sum of the matrices x and y.
    ++ Error: if the dimensions are incompatible.
    ++
    ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
    ++X m+m

  "-": (%,%) -> %
    ++ \spad{x - y} is the difference of the matrices x and y.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m-m

"-":  %    -> %
  ++ \spad{-x} returns the negative of the matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X -m

"*": (%,%) -> %
  ++ \spad{x * y} is the product of the matrices x and y.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m*m

"*": (R,%) -> %
  ++ \spad{r*x} is the left scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 1/3*m

"*": (%,R) -> %
  ++ \spad{x * r} is the right scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m*1/3

"*": (Integer,%) -> %
  ++ \spad{n * x} is an integer multiple.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 3*m

"*": (%,Col) -> Col
  ++ \spad{x * c} is the product of the matrix x and the column vector c.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X c:=coerce([1,2,3,4,5])@Matrix(INT)
  ++X m*c
```

```
    "*": (Row,%) -> Row
      ++ \spad{r * x} is the product of the row vector r and the matrix x.
      ++ Error: if the dimensions are incompatible.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X r:=transpose([1,2,3,4,5])@Matrix(INT)
      ++X r*m

    "**": (%,NonNegativeInteger) -> %
      ++ \spad{x ** n} computes a non-negative integral power of the matrix x.
      ++ Error: if the matrix is not square.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X m**3

    if R has IntegralDomain then
      "exquo": (%,R) -> Union(%,"failed")
        ++ \spad{exquo(m,r)} computes the exact quotient of the elements
        ++ of m by r, returning \axiom{"failed"} if this is not possible.
        ++
        ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
        ++X exquo(m,2)

    if R has Field then
      "/": (%,R) -> %
        ++ \spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.
        ++
        ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
        ++X m/4

--% Linear algebra

    if R has EuclideanDomain then
      rowEchelon: % -> %
        ++ \spad{rowEchelon(m)} returns the row echelon form of the matrix m.
        ++
        ++X rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]

      columnSpace: % -> List Col
        ++ \spad{columnSpace(m)} returns a sublist of columns of the matrix m
        ++ forming a basis of its column space
        ++
        ++X columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

    if R has IntegralDomain then
      rank: % -> NonNegativeInteger
    ++ \spad{rank(m)} returns the rank of the matrix m.
    ++
    ++X rank matrix [[1,2,3],[4,5,6],[7,8,9]]

  nullity: % -> NonNegativeInteger
    ++ \spad{nullity(m)} returns the nullity of the matrix m. This is
    ++ the dimension of the null space of the matrix m.
    ++
    ++X nullity matrix [[1,2,3],[4,5,6],[7,8,9]]

  nullSpace: % -> List Col
    ++ \spad{nullSpace(m)} returns a basis for the null space of
    ++ the matrix m.
    ++
    ++X nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]

if R has commutative("*") then
  determinant: % -> R
    ++ \spad{determinant(m)} returns the determinant of the matrix m.
    ++ Error: if the matrix is not square.
    ++
    ++X determinant matrix [[j**i for i in 0..4] for j in 1..5]

  minordet: % -> R
    ++ \spad{minordet(m)} computes the determinant of the matrix m using
    ++ minors. Error: if the matrix is not square.
    ++
    ++X minordet matrix [[j**i for i in 0..4] for j in 1..5]

if R has CommutativeRing then
  pfaffian: % -> R
    ++ \spad{pfaffian(m)} returns the Pfaffian of the matrix m.
    ++ Error if the matrix is not antisymmetric
    ++
    ++X pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
if R has Field then
  inverse: % -> Union(%,"failed")
    ++ \spad{inverse(m)} returns the inverse of the matrix m.
    ++ If the matrix is not invertible, "failed" is returned.
    ++ Error: if the matrix is not square.
    ++
    ++X inverse matrix [[j**i for i in 0..4] for j in 1..5]

  "**": (%,Integer) -> %
    ++ \spad{m**n} computes an integral power of the matrix m.
    ++ Error: if matrix is not square or if the matrix
```

```
          ++ is square but not invertible.
          ++
          ++X (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2

    add
      minr ==> minRowIndex
      maxr ==> maxRowIndex
      minc ==> minColIndex
      maxc ==> maxColIndex
      mini ==> minIndex
      maxi ==> maxIndex

--% Predicates

      square? x == nrows x = ncols x

      diagonal? x ==
        not square? x => false
        for i in minr x .. maxr x repeat
          for j in minc x .. maxc x | (j - minc x) ^= (i - minr x) repeat
            not zero? qelt(x, i, j) => return false
        true

      symmetric? x ==
        (nRows := nrows x) ^= ncols x => false
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(nRows - 1) repeat
          for j in (i + 1)..(nRows - 1) repeat
            qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
        true

      antisymmetric? x ==
        (nRows := nrows x) ^= ncols x => false
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(nRows - 1) repeat
          for j in i..(nRows - 1) repeat
            qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
              return false
        true

--% Creation of matrices

      zero(rows,cols) == new(rows,cols,0)

      matrix(l: List List R) ==
        null l => new(0,0,0)
```

```
  -- error check: this is a top level function
  rows : NonNegativeInteger := 1; cols := # first l
  cols = 0 => error "matrices with zero columns are not supported"
  for ll in rest l repeat
    cols ^= # ll => error "matrix: rows of different lengths"
    rows := rows + 1
  ans := new(rows,cols,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans

scalarMatrix(n,r) ==
  ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) repeat
    qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix(l: List R) ==
  n := #l; ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
      for r in l repeat qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix(list: List %) ==
  rows : NonNegativeInteger := 0
  cols : NonNegativeInteger := 0
  for mat in list repeat
    rows := rows + nrows mat
    cols := cols + ncols mat
  ans := zero(rows,cols)
  loR := minr ans; loC := minc ans
  for mat in list repeat
    hiR := loR + nrows(mat) - 1; hiC := loC + nrows(mat) - 1
    for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
      for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
        qsetelt_!(ans,i,j,qelt(mat,k,l))
    loR := hiR + 1; loC := hiC + 1
  ans

coerce(v:Col) ==
  x := new(#v,1,0)
  one := minc(x)
  for i in minr(x)..maxr(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,i,one,qelt(v,k))
  x
```

```
transpose(v:Row) ==
  x := new(1,#v,0)
  one := minr(x)
  for j in minc(x)..maxc(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,one,j,qelt(v,k))
  x

transpose(x:%) ==
  ans := new(ncols x,nrows x,0)
  for i in minr(ans)..maxr(ans) repeat
    for j in minc(ans)..maxc(ans) repeat
      qsetelt_!(ans,i,j,qelt(x,j,i))
  ans

squareTop x ==
  nrows x < (cols := ncols x) =>
    error "squareTop: number of columns exceeds number of rows"
  ans := new(cols,cols,0)
  for i in minr(x)..(minr(x) + cols - 1) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j))
  ans

horizConcat(x,y) ==
  (rows := nrows x) ^= nrows y =>
    error "HConcat: matrices must have same number of rows"
  ans := new(rows,(cols := ncols x) + ncols y,0)
  for i in minr(x)..maxr(x) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j))
  for i in minr(y)..maxr(y) repeat
    for j in minc(y)..maxc(y) repeat
      qsetelt_!(ans,i,j + cols,qelt(y,i,j))
  ans

vertConcat(x,y) ==
  (cols := ncols x) ^= ncols y =>
    error "HConcat: matrices must have same number of columns"
  ans := new((rows := nrows x) + nrows y,cols,0)
  for i in minr(x)..maxr(x) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j))
  for i in minr(y)..maxr(y) repeat
    for j in minc(y)..maxc(y) repeat
      qsetelt_!(ans,i + rows,j,qelt(y,i,j))
    ans

--% Part extraction/assignment

   listOfLists x ==
     ll : List List R := nil()
     for i in maxr(x)..minr(x) by -1 repeat
       l : List R := nil()
       for j in maxc(x)..minc(x) by -1 repeat
         l := cons(qelt(x,i,j),l)
       ll := cons(l,ll)
     ll

   swapRows_!(x,i1,i2) ==
     (i1 < minr(x)) or (i1 > maxr(x)) or (i2 < minr(x)) or _
         (i2 > maxr(x)) => error "swapRows!: index out of range"
     i1 = i2 => x
     for j in minc(x)..maxc(x) repeat
       r := qelt(x,i1,j)
       qsetelt_!(x,i1,j,qelt(x,i2,j))
       qsetelt_!(x,i2,j,r)
     x

   swapColumns_!(x,j1,j2) ==
     (j1 < minc(x)) or (j1 > maxc(x)) or (j2 < minc(x)) or _
         (j2 > maxc(x)) => error "swapColumns!: index out of range"
     j1 = j2 => x
     for i in minr(x)..maxr(x) repeat
       r := qelt(x,i,j1)
       qsetelt_!(x,i,j1,qelt(x,i,j2))
       qsetelt_!(x,i,j2,r)
     x

   elt(x:%,rowList:List Integer,colList:List Integer) ==
     for ei in rowList repeat
       (ei < minr(x)) or (ei > maxr(x)) =>
         error "elt: index out of range"
     for ej in colList repeat
       (ej < minc(x)) or (ej > maxc(x)) =>
         error "elt: index out of range"
     y := new(# rowList,# colList,0)
     for ei in rowList for i in minr(y)..maxr(y) repeat
       for ej in colList for j in minc(y)..maxc(y) repeat
         qsetelt_!(y,i,j,qelt(x,ei,ej))
     y
    setelt(x:%,rowList:List Integer,colList:List Integer,y:%) ==
      for ei in rowList repeat
        (ei < minr(x)) or (ei > maxr(x)) =>
          error "setelt: index out of range"
      for ej in colList repeat
        (ej < minc(x)) or (ej > maxc(x)) =>
          error "setelt: index out of range"
      ((# rowList) ^= (nrows y)) or ((# colList) ^= (ncols y)) =>
        error "setelt: matrix has bad dimensions"
      for ei in rowList for i in minr(y)..maxr(y) repeat
        for ej in colList for j in minc(y)..maxc(y) repeat
          qsetelt_!(x,ei,ej,qelt(y,i,j))
      y

    subMatrix(x,i1,i2,j1,j2) ==
      (i2 < i1) => error "subMatrix: bad row indices"
      (j2 < j1) => error "subMatrix: bad column indices"
      (i1 < minr(x)) or (i2 > maxr(x)) =>
        error "subMatrix: index out of range"
      (j1 < minc(x)) or (j2 > maxc(x)) =>
        error "subMatrix: index out of range"
      rows := (i2 - i1 + 1) pretend NonNegativeInteger
      cols := (j2 - j1 + 1) pretend NonNegativeInteger
      y := new(rows,cols,0)
      for i in minr(y)..maxr(y) for k in i1..i2 repeat
        for j in minc(y)..maxc(y) for l in j1..j2 repeat
          qsetelt_!(y,i,j,qelt(x,k,l))
      y

    setsubMatrix_!(x,i1,j1,y) ==
      i2 := i1 + nrows(y) -1
      j2 := j1 + ncols(y) -1
      (i1 < minr(x)) or (i2 > maxr(x)) =>
       error _
        "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
      (j1 < minc(x)) or (j2 > maxc(x)) =>
       error _
        "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
      for i in minr(y)..maxr(y) for k in i1..i2 repeat
        for j in minc(y)..maxc(y) for l in j1..j2 repeat
          qsetelt_!(x,k,l,qelt(y,i,j))
      x

--% Arithmetic

    x + y ==
      ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
        error "can't add matrices of different dimensions"
      ans := new(r,c,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j) + qelt(y,i,j))
      ans

    x - y ==
      ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
        error "can't subtract matrices of different dimensions"
      ans := new(r,c,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j) - qelt(y,i,j))
      ans

    - x == map((r1:R):R +-> - r1,x)

    a:R * x:% == map((r1:R):R +-> a * r1,x)

    x:% * a:R == map((r1:R):R +-> r1 * a,x)

    m:Integer * x:% == map((r1:R):R +-> m * r1,x)

    x:% * y:% ==
      (ncols x ^= nrows y) =>
        error "can't multiply matrices of incompatible dimensions"
      ans := new(nrows x,ncols y,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(y)..maxc(y) repeat
          entry :=
            sum : R := 0
            for k in minr(y)..maxr(y) for l in minc(x)..maxc(x) repeat
              sum := sum + qelt(x,i,l) * qelt(y,k,j)
            sum
          qsetelt_!(ans,i,j,entry)
      ans

    positivePower:(%,Integer) -> %
    positivePower(x,n) ==
--      one? n => x
      (n = 1) => x
      odd? n => x * positivePower(x,n - 1)
      y := positivePower(x,n quo 2)
      y * y
```

```
x:% ** n:NonNegativeInteger ==
  not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
  zero? n => scalarMatrix(nn,1)
  positivePower(x,n)

--if R has ConvertibleTo InputForm then
  --convert(x:%):InputForm ==
    --convert [convert("matrix"::Symbol)@InputForm,
             --convert listOfLists x]$List(InputForm)

if Col has shallowlyMutable then

  x:% * v:Col ==
    ncols(x) ^= #v =>
      error "can't multiply matrix A and vector v if #cols A ^= #v"
    w : Col := new(nrows x,0)
    for i in minr(x)..maxr(x) for k in mini(w)..maxi(w) repeat
      w.k :=
        sum : R := 0
        for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
          sum := sum + qelt(x,i,j) * v(l)
        sum
    w

if Row has shallowlyMutable then

  v:Row * x:% ==
    nrows(x) ^= #v =>
      error "can't multiply vector v and matrix A if #rows A ^= #v"
    w : Row := new(ncols x,0)
    for j in minc(x)..maxc(x) for k in mini(w)..maxi(w) repeat
      w.k :=
        sum : R := 0
        for i in minr(x)..maxr(x) for l in mini(v)..maxi(v) repeat
          sum := sum + qelt(x,i,j) * v(l)
        sum
    w

if R has EuclideanDomain then
  columnSpace M ==
    M2 := rowEchelon M
    basis: List Col := []
    n: Integer := ncols M
    m: Integer := nrows M
    indRow: Integer := 1
    for k in 1..n while indRow <= m repeat
      if not zero?(M2.(indRow,k)) then
        basis := cons(column(M,k),basis)
        indRow := indRow + 1
    reverse! basis

if R has CommutativeRing then
  skewSymmetricUnitMatrix(n:PositiveInteger):% ==
    matrix [[(if i=j+1 and odd? j
               then -1
               else if i=j-1 and odd? i
                      then 1
                      else 0) for j in 1..n] for i in 1..n]

  SUPR ==> SparseUnivariatePolynomial R

  PfChar(A:%):SUPR ==
    n := nrows A
    (n = 2) => monomial(1$R,2)$SUPR + qelt(A,1,2)::SUPR
    M:=subMatrix(A,3,n,3,n)
    r:=subMatrix(A,1,1,3,n)
    s:=subMatrix(A,3,n,2,2)
    p:=PfChar(M)
    d:=degree(p)$SUPR
    B:=skewSymmetricUnitMatrix((n-2)::PositiveInteger)
    C:=r*B
    g:List R := [qelt(C*s,1,1), qelt(A,1,2), 1]
    if d >= 4 then
      B:=M*B
      for i in 4..d by 2 repeat
        C:=C*B
        g:=cons(qelt(C*s,1,1),g)
    g:=reverse! g
    res:SUPR := 0
    for i in 0..d by 2 for j in 2..d+2 repeat
      c:=coefficient(p,i)
      for e in first(g,j) for k in 2..-d by -2 repeat
        res:=res+monomial(c*e,(k+i)::NonNegativeInteger)$SUPR
    res

  pfaffian a ==
    if antisymmetric? a
      then if odd? nrows a
             then 0
             else PfChar(a).0
      else
```

```
              error "pfaffian: only defined for antisymmetric square matrices"

       if R has IntegralDomain then
         x exquo a ==
           ans := new(nrows x,ncols x,0)
           for i in minr(x)..maxr(x) repeat
             for j in minc(x)..maxc(x) repeat
               entry :=
                 (r := (qelt(x,i,j) exquo a)) case "failed" =>
                   return "failed"
                 r :: R
               qsetelt_!(ans,i,j,entry)
           ans

       if R has Field then
         x / r == map((r1:R):R +-> r1 / r,x)

         x:% ** n:Integer ==
           not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
           zero? n => scalarMatrix(nn,1)
           positive? n => positivePower(x,n)
           (xInv := inverse x) case "failed" =>
             error "**: matrix must be invertible"
           positivePower(xInv :: %,-n)
```

⟨*MATCAT.dotabb*⟩≡

```
  "MATCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
  "MATCAT" -> "ARR2CAT"
```

⟨*MATCAT.dotfull*⟩≡

```
  "MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
  "MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
   ->
  "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
```

⟨*MATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 [color=lightblue];
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 ->
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

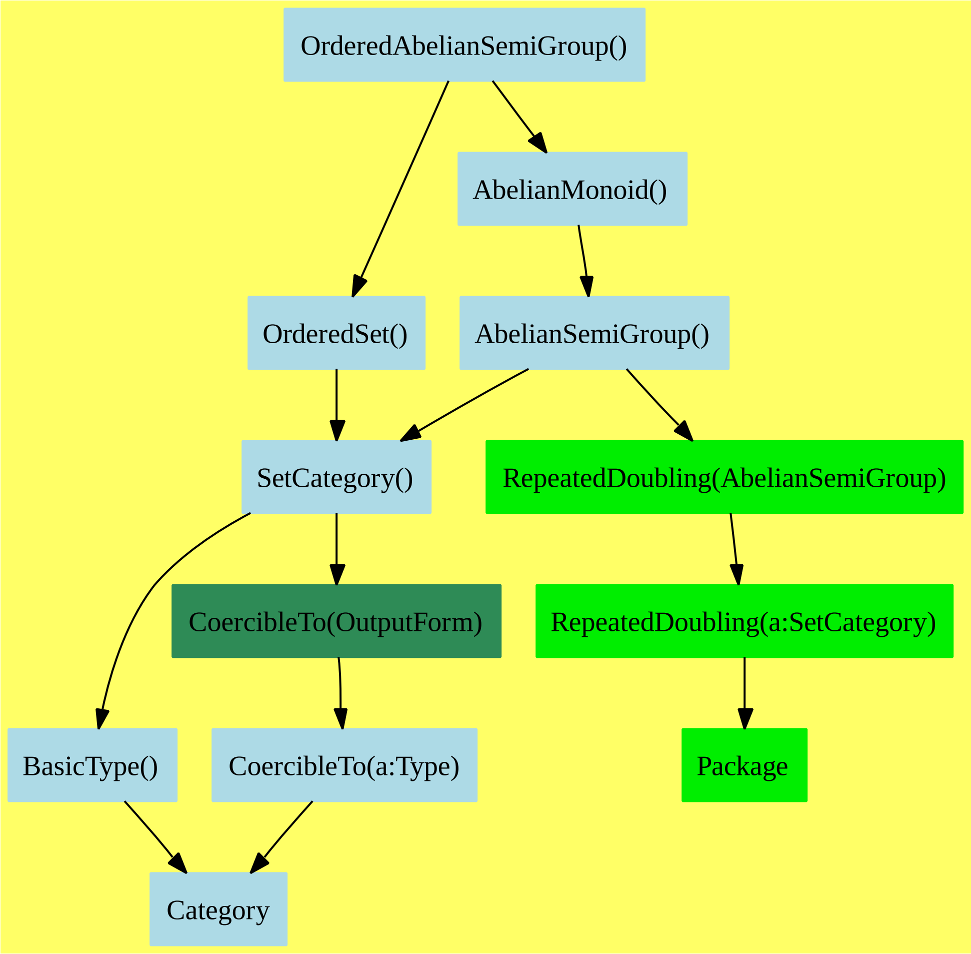
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.0.89 OrderedAbelianSemiGroup (OASGP)


OrderedAbelianSemiGroup()
AbelianMonoid()
OrderedSet()
AbelianSemiGroup()
SetCategory()
RepeatedDoubling(AbelianSemiGroup)
CoercibleTo(OutputForm)
RepeatedDoubling(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*OrderedAbelianSemiGroup.input*⟩≡

```
)set break resume
)sys rm -f OrderedAbelianSemiGroup.output
)spool OrderedAbelianSemiGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianSemiGroup
--R OrderedAbelianSemiGroup  is a category constructor
--R Abbreviation for OrderedAbelianSemiGroup is OASGP
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OASGP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedAbelianSemiGroup.help*⟩≡

```
====================================================================
OrderedAbelianSemiGroup examples
====================================================================

See Also:
o )show OrderedAbelianSemiGroup
```

See:

⇒ “OrderedAbelianMonoid” (OAMON) 7.0.105 on page 636
⇐ “AbelianMonoid” (ABELMON) 5.0.66 on page 301
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?~=?	?*?
?+?	?<?	?<=?	?=?	?>?
?>=?				

These exports come from (p242) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p301) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

⟨*category OASGP OrderedAbelianSemiGroup*⟩≡

```
)abbrev category OASGP OrderedAbelianSemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian semigroups, such that the addition
++ preserves the ordering.\br
++
++ Axiom\br
++ \tab{5} x < y => x+z < y+z

OrderedAbelianSemiGroup(): Category == Join(OrderedSet, AbelianMonoid)
```

⟨*OASGP.dotabb*⟩≡

```
"OASGP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
"OASGP" -> "ORDSET"
"OASGP" -> "ABELMON"
```

⟨*OASGP.dotfull*⟩≡

```
"OrderedAbelianSemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"
```

⟨OASGP.dotpic⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

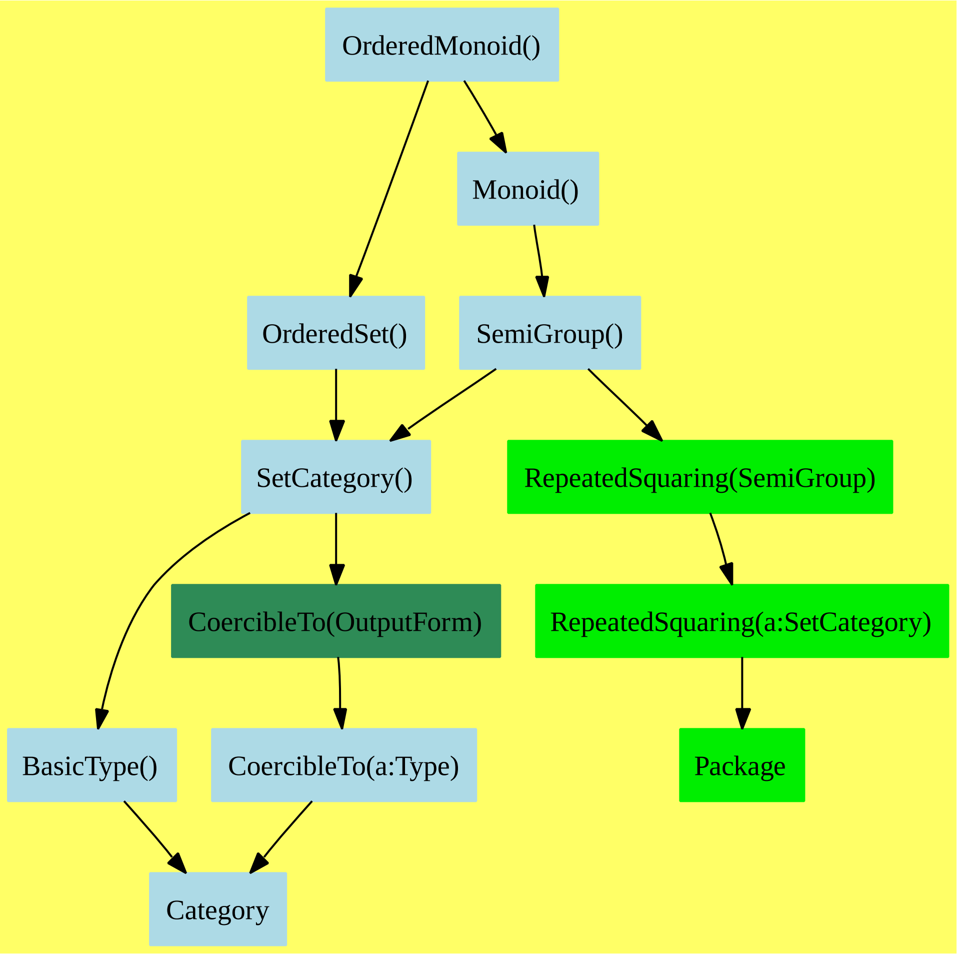
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

6.0.90 OrderedMonoid (ORDMON)


OrderedMonoid()
Monoid()
OrderedSet()
SemiGroup()
SetCategory()
RepeatedSquaring(SemiGroup)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*OrderedMonoid.input*⟩≡

```
)set break resume
)sys rm -f OrderedMonoid.output
)spool OrderedMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedMonoid
--R OrderedMonoid  is a category constructor
--R Abbreviation for OrderedMonoid is ORDMON
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,PositiveInteger) -> %
--R ?<? : (%,%) -> Boolean                 ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                 ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedMonoid.help*⟩≡

```
====================================================================
OrderedMonoid examples
====================================================================

See Also:
o )show OrderedMonoid
```

See:

⇐ “Monoid” (MONOID) 5.0.76 on page 371
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242

Exports:

1	coerce	hash	latex	max
min	one?	recip	sample	?*?
?**?	?<?	?<=?	?=?	?>?
?>=?	?~=?	?^?		

These exports come from (p371) Monoid():

```
1 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?*? : (%,%) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category ORDMON OrderedMonoid*⟩≡

```
)abbrev category ORDMON OrderedMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ Ordered sets which are also monoids, such that multiplication
++ preserves the ordering.
++
++ Axioms\br
++ \tab{5}\spad{x < y => x*z < y*z}\br
++ \tab{5}\spad{x < y => z*x < z*y}

OrderedMonoid(): Category == Join(OrderedSet, Monoid)
```

⟨*ORDMON.dotabb*⟩≡

```
"ORDMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"ORDMON" -> "ORDSET"
"ORDMON" -> "MONOID"
```

⟨*ORDMON.dotfull*⟩≡

```
"OrderedMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"OrderedMonoid()" -> "OrderedSet()"
"OrderedMonoid()" -> "Monoid()"
```

⟨*ORDMON.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"OrderedMonoid()" -> "OrderedSet()"
"OrderedMonoid()" -> "Monoid()"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

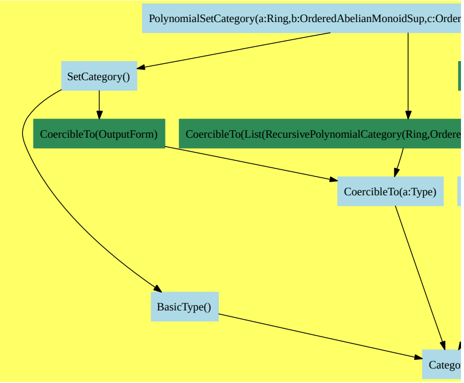
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

6.0.91 PolynomialSetCategory (PSETCAT)


PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:Order
SetCategory()
CoercibleTo(OutputForm)
CoercibleTo(List(RecursivePolynomialCategory(Ring,Ordere
CoercibleTo(a:Type)
BasicType()
Catego

⟨*PolynomialSetCategory.input*⟩≡

```
)set break resume
)sys rm -f PolynomialSetCategory.output
)spool PolynomialSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialSetCategory
--R PolynomialSetCategory(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet,P
--R Abbreviation for PolynomialSetCategory is PSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> List P
--R coerce : % -> OutputForm              collect : (%,VarSet) -> %
--R collectUnder : (%,VarSet) -> %        collectUpper : (%,VarSet) -> %
--R construct : List P -> %               copy : % -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   mainVariables : % -> List VarSet
--R map : ((P -> P),%) -> %               mvar : % -> VarSet
--R retract : List P -> %                 sample : () -> %
--R trivialIdeal? : % -> Boolean          variables : % -> List VarSet
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if P has KONVERT INFORM
```

```
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R mainVariable? : (VarSet,%) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
--R triangular? : % -> Boolean if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*PolynomialSetCategory.help*⟩≡

```
====================================================================
PolynomialSetCategory examples
====================================================================

See Also:
o )show PolynomialSetCategory
```

See:

⇒ “TriangularSetCategory” (TSETCAT) 7.0.108 on page 657
⇐ “CoercibleTo” (KOERCE) 2.0.6 on page 19
⇐ “Collection” (CLAGG) 5.0.70 on page 322
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130

Exports:

any?
coerce
collect
collectUnder
collectUpper
construct
convert
copy
count
empty
empty?
eq?
eval
every?
find
hash
headRemainder
latex
less?
mainVariables
mainVariable?
map
map!
member?
members
more?
mvar
parts
reduce
remainder
remove
removeDuplicates
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
triangular?
trivialIdeal?
variables
#?
?~=?
?=?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
mvar : % -> VarSet
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
```

These are implemented by this category:

```
collect : (%,VarSet) -> %
collectUnder : (%,VarSet) -> %
collectUpper : (%,VarSet) -> %
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
mainVariables : % -> List VarSet
mainVariable? : (VarSet,%) -> Boolean
```

```
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List VarSet
?=? : (%,%) -> Boolean
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
```

These exports come from (p322) Collection(P)
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
construct : List P -> %
copy : % -> %
count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p19) CoercibleTo(List(P))
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
coerce : % -> List P
convert : % -> InputForm if P has KONVERT INFORM
```

These exports come from (p1153) IntegralDomain():

⟨*category PSETCAT PolynomialSetCategory*⟩≡

```
)abbrev category PSETCAT PolynomialSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References:
++ Description:
++ A category for finite subsets of a polynomial ring.
++ Such a set is only regarded as a set of polynomials and not
++ identified to the ideal it generates. So two distinct sets may
++ generate the same the ideal. Furthermore, for \spad{R} being an
++ integral domain, a set of polynomials may be viewed as a representation
++ of the ideal it generates in the polynomial ring \spad{(R)^(-1) P},
++ or the set of its zeros (described for instance by the radical of the
++ previous ideal, or a split of the associated affine variety) and so on.
++ So this category provides operations about those different notions.

PolynomialSetCategory(R:Ring, E:OrderedAbelianMonoidSup,_
  VarSet:OrderedSet, P:RecursivePolynomialCategory(R,E,VarSet)): Category ==
    Join(SetCategory,Collection(P),CoercibleTo(List(P))) with
     finiteAggregate
     retractIfCan : List(P) -> Union($,"failed")
         ++ \axiom{retractIfCan(lp)} returns an element of the domain
         ++ whose elements are the members of \axiom{lp} if such an element
```

```
        ++ exists, otherwise \axiom{"failed"} is returned.
    retract : List(P) -> $
        ++ \axiom{retract(lp)} returns an element of the domain whose elements
        ++ are the members of \axiom{lp} if such an element exists, otherwise
        ++ an error is produced.
    mvar : $ -> VarSet
        ++  \axiom{mvar(ps)} returns the main variable of the non constant
        ++ polynomial with the greatest main variable, if any, else an
        ++ error is returned.
    variables : $ -> List VarSet
        ++  \axiom{variables(ps)} returns the decreasingly sorted list of the
        ++  variables which are variables of some polynomial in \axiom{ps}.
    mainVariables : $  -> List VarSet
        ++ \axiom{mainVariables(ps)} returns the decreasingly sorted list
        ++ of the variables which are main variables of some polynomial
        ++ in \axiom{ps}.
    mainVariable? : (VarSet,$) -> Boolean
        ++ \axiom{mainVariable?(v,ps)} returns true iff \axiom{v} is the
        ++ main variable of some polynomial in \axiom{ps}.
    collectUnder : ($,VarSet) -> $
        ++ \axiom{collectUnder(ps,v)} returns the set consisting of the
        ++ polynomials of \axiom{ps} with main variable less than \axiom{v}.
    collect : ($,VarSet) -> $
        ++ \axiom{collect(ps,v)}  returns the set consisting of the
        ++ polynomials of \axiom{ps} with \axiom{v} as main variable.
    collectUpper : ($,VarSet) -> $
        ++ \axiom{collectUpper(ps,v)} returns the set consisting of the
        ++ polynomials of \axiom{ps} with main variable greater
        ++ than \axiom{v}.
    sort : ($,VarSet) -> Record(under:$,floor:$,upper:$)
        ++ \axiom{sort(v,ps)} returns \axiom{us,vs,ws} such that \axiom{us}
        ++ is \axiom{collectUnder(ps,v)}, \axiom{vs} is \axiom{collect(ps,v)}
        ++ and \axiom{ws} is \axiom{collectUpper(ps,v)}.
    trivialIdeal?: $ -> Boolean
        ++ \axiom{trivialIdeal?(ps)} returns true iff \axiom{ps} does
        ++ not contain non-zero elements.
    if R has IntegralDomain
    then
      roughBase? : $ -> Boolean
          ++ \axiom{roughBase?(ps)} returns true iff for every pair
          ++ \axiom{{p,q}} of polynomials in \axiom{ps} their leading
          ++ monomials are relatively prime.
      roughSubIdeal?  : ($,$) -> Boolean
          ++  \axiom{roughSubIdeal?(ps1,ps2)} returns true iff it can proved
          ++ that all polynomials in  \axiom{ps1} lie in the ideal generated
          ++ by \axiom{ps2} in \axiom{\axiom{(R)^(-1) P}} without computing
        ++ Groebner bases.
    roughEqualIdeals? : ($,$) -> Boolean
        ++ \axiom{roughEqualIdeals?(ps1,ps2)} returns true iff it can
        ++ proved that \axiom{ps1} and \axiom{ps2} generate the same ideal
        ++ in \axiom{(R)^(-1) P} without computing Groebner bases.
    roughUnitIdeal? : $ -> Boolean
        ++ \axiom{roughUnitIdeal?(ps)} returns true iff \axiom{ps} contains
        ++  some non null element lying in the base ring \axiom{R}.
    headRemainder : (P,$) -> Record(num:P,den:R)
        ++ \axiom{headRemainder(a,ps)} returns \axiom{[b,r]} such that the
        ++ leading monomial of \axiom{b} is reduced in the sense of
        ++ Groebner bases w.r.t. \axiom{ps} and \axiom{r*a - b} lies in
        ++ the ideal generated by \axiom{ps}.
    remainder : (P,$) -> Record(rnum:R,polnum:P,den:R)
        ++ \axiom{remainder(a,ps)} returns \axiom{[c,b,r]} such that
        ++ \axiom{b} is fully reduced in the sense of Groebner bases
        ++ w.r.t. \axiom{ps}, \axiom{r*a - c*b} lies in the ideal
        ++ generated by \axiom{ps}. Furthermore, if \axiom{R} is a
        ++ gcd-domain, \axiom{b} is primitive.
    rewriteIdealWithHeadRemainder : (List(P),$) -> List(P)
        ++ \axiom{rewriteIdealWithHeadRemainder(lp,cs)} returns \axiom{lr}
        ++ such that the leading monomial of every polynomial in \axiom{lr}
        ++ is reduced in the sense of Groebner bases w.r.t. \axiom{cs}
        ++ and \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same
        ++ ideal in \axiom{(R)^(-1) P}.
    rewriteIdealWithRemainder : (List(P),$) -> List(P)
        ++ \axiom{rewriteIdealWithRemainder(lp,cs)} returns \axiom{lr}
        ++ such that every polynomial in \axiom{lr} is fully reduced in
        ++ the sense of Groebner bases w.r.t. \axiom{cs} and
        ++ \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same ideal
        ++ in \axiom{(R)^(-1) P}.
    triangular? : $ -> Boolean
        ++ \axiom{triangular?(ps)} returns true iff \axiom{ps} is a
        ++ triangular set, i.e. two distinct polynomials have distinct
        ++ main variables and no constant lies in \axiom{ps}.

add

   NNI ==> NonNegativeInteger
   B ==> Boolean

   elements: $ -> List(P)

   elements(ps:$):List(P) ==
     lp : List(P) := members(ps)$$
```

```
variables1(lp:List(P)):(List VarSet) ==
  lvars : List(List(VarSet)) := [variables(p)$P for p in lp]
  sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
        removeDuplicates(concat(lvars)$List(VarSet)))

variables2(lp:List(P)):(List VarSet) ==
  lvars : List(VarSet) := [mvar(p)$P for p in lp]
  sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
        removeDuplicates(lvars)$List(VarSet))

variables (ps:$) ==
  variables1(elements(ps))

mainVariables (ps:$) ==
  variables2(remove(ground?,elements(ps)))

mainVariable? (v,ps) ==
  lp : List(P) := remove(ground?,elements(ps))
  while (not empty? lp) and (not (mvar(first(lp)) = v)) repeat
    lp := rest lp
  (not empty? lp)

collectUnder (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collectUpper (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) > v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collect (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) = v)
      then
        lq := cons(p,lq)
  construct(lq)$$

sort (ps,v) ==
  lp : List P := elements(ps)
  us : List P := []
  vs : List P := []
  ws : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        us := cons(p,us)
      else
        if (mvar(p) = v)
          then
            vs := cons(p,vs)
          else
            ws := cons(p,ws)
  [construct(us)$$,_
   construct(vs)$$,_
   construct(ws)$$]$Record(under:$,floor:$,upper:$)

ps1 = ps2 ==
  {p for p in elements(ps1)} =$(Set P) {p for p in elements(ps2)}

exactQuo : (R,R) -> R

localInf? (p:P,q:P):B ==
  degree(p) <$E degree(q)

localTriangular? (lp:List(P)):B ==
  lp := remove(zero?, lp)
  empty? lp => true
  any? (ground?, lp) => false
  lp := sort((z1:P,z2:P):Boolean +-> mvar(z1)$P > mvar(z2)$P, lp)
  p,q : P
  p := first lp
  lp := rest lp
  while (not empty? lp) and (mvar(p) > mvar((q := first(lp)))) repeat
    p := q
    lp := rest lp
  empty? lp

triangular? ps ==
  localTriangular? elements ps

trivialIdeal? ps ==
  empty?(remove(zero?,elements(ps))$(List(P)))$(List(P))

if R has IntegralDomain
then

  roughUnitIdeal? ps ==
    any?(ground?,remove(zero?,elements(ps))$(List(P)))$(List P)

  relativelyPrimeLeadingMonomials? (p:P,q:P):B ==
    dp : E := degree(p)
    dq : E := degree(q)
    (sup(dp,dq)$E =$E dp +$E dq)@B

  roughBase? ps ==
    lp := remove(zero?,elements(ps))$(List(P))
    empty? lp => true
    rB? : B := true
    while (not empty? lp) and rB? repeat
      p := first lp
      lp := rest lp
      copylp := lp
      while (not empty? copylp) and rB? repeat
        rB? := relativelyPrimeLeadingMonomials?(p,first(copylp))
        copylp := rest copylp
    rB?

  roughSubIdeal?(ps1,ps2) ==
    lp: List(P) := rewriteIdealWithRemainder(elements(ps1),ps2)
    empty? (remove(zero?,lp))

  roughEqualIdeals? (ps1,ps2) ==
    ps1 =$$ ps2 => true
    roughSubIdeal?(ps1,ps2) and roughSubIdeal?(ps2,ps1)

if (R has GcdDomain) and (VarSet has ConvertibleTo (Symbol))
then
```

```
        LPR ==> List Polynomial R
        LS ==> List Symbol

        if R has EuclideanDomain
          then
            exactQuo(r:R,s:R):R ==
              r quo$R s
          else
            exactQuo(r:R,s:R):R ==
              (r exquo$R s)::R

        headRemainder (a,ps) ==
          lp1 : List(P) := remove(zero?, elements(ps))$(List(P))
          empty? lp1 => [a,1$R]
          any?(ground?,lp1) => [reductum(a),1$R]
          r : R := 1$R
          lp1 := sort(localInf?, reverse elements(ps))
          lp2 := lp1
          e : Union(E, "failed")
          while (not zero? a) and (not empty? lp2) repeat
            p := first lp2
            if ((e:= subtractIfCan(degree(a),degree(p))) case E)
              then
                g := gcd((lca := leadingCoefficient(a)),_
                         (lcp := leadingCoefficient(p)))$R
                (lca,lcp) := (exactQuo(lca,g),exactQuo(lcp,g))
                a := lcp * reductum(a) - monomial(lca, e::E)$P * reductum(p)
                r := r * lcp
                lp2 := lp1
              else
                lp2 := rest lp2
          [a,r]

        makeIrreducible! (frac:Record(num:P,den:R)):Record(num:P,den:R) ==
          g := gcd(frac.den,frac.num)$P
--          one? g => frac
          (g = 1) => frac
          frac.num := exactQuotient!(frac.num,g)
          frac.den := exactQuo(frac.den,g)
          frac

        remainder (a,ps) ==
          hRa := makeIrreducible! headRemainder (a,ps)
          a := hRa.num
          r : R := hRa.den
          zero? a => [1$R,a,r]
```

```
      b : P := monomial(1$R,degree(a))$P
      c : R := leadingCoefficient(a)
      while not zero?(a := reductum a) repeat
        hRa := makeIrreducible!  headRemainder (a,ps)
        a := hRa.num
        r := r * hRa.den
        g := gcd(c,(lca := leadingCoefficient(a)))$R
        b := ((hRa.den) * exactQuo(c,g)) * b + _
              monomial(exactQuo(lca,g),degree(a))$P
        c := g
      [c,b,r]

    rewriteIdealWithHeadRemainder(ps,cs) ==
      trivialIdeal? cs => ps
      roughUnitIdeal? cs => [0$P]
      ps := remove(zero?,ps)
      empty? ps => ps
      any?(ground?,ps) => [1$P]
      rs : List P := []
      while not empty? ps repeat
        p := first ps
        ps := rest ps
        p := (headRemainder(p,cs)).num
        if not zero? p
          then
            if ground? p
              then
                ps := []
                rs := [1$P]
              else
                primitivePart! p
                rs := cons(p,rs)
      removeDuplicates rs

    rewriteIdealWithRemainder(ps,cs) ==
      trivialIdeal? cs => ps
      roughUnitIdeal? cs => [0$P]
      ps := remove(zero?,ps)
      empty? ps => ps
      any?(ground?,ps) => [1$P]
      rs : List P := []
      while not empty? ps repeat
        p := first ps
        ps := rest ps
        p := (remainder(p,cs)).polnum
        if not zero? p
```

```
            then
              if ground? p
                then
                  ps := []
                  rs := [1$P]
                else
                  rs := cons(unitCanonical(p),rs)
        removeDuplicates rs
```

⟨*PSETCAT.dotabb*⟩≡

```
"PSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PSETCAT" -> "KOERCE"
"PSETCAT" -> "CLAGG"
"PSETCAT" -> "SETCAT"
```

⟨*PSETCAT.dotfull*⟩≡

```
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet(

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
```

⟨*PSETCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Ordere
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

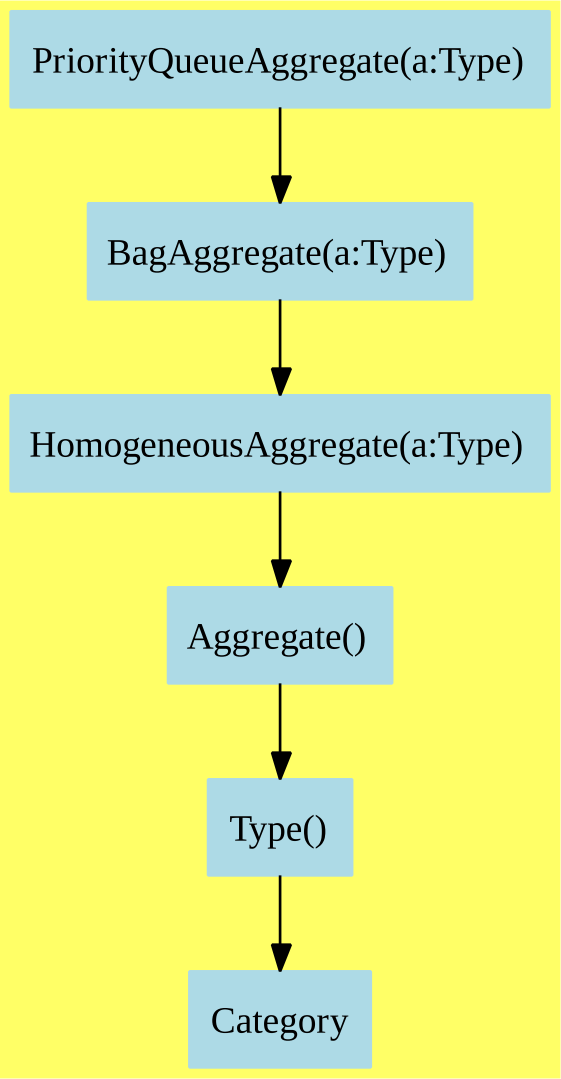
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"
"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

6.0.92 PriorityQueueAggregate (PRQAGG)


PriorityQueueAggregate(a:Type)
BagAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*PriorityQueueAggregate.input*⟩≡

```
)set break resume
)sys rm -f PriorityQueueAggregate.output
)spool PriorityQueueAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show PriorityQueueAggregate
--R PriorityQueueAggregate S: OrderedSet  is a category constructor
--R Abbreviation for PriorityQueueAggregate is PRQAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRQAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                     copy : % -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R insert! : (S,%) -> %                  inspect : % -> S
--R map : ((S -> S),%) -> %               max : % -> S
--R merge : (%,%) -> %                    merge! : (%,%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*PriorityQueueAggregate.help*⟩≡

```
====================================================================
PriorityQueueAggregate examples
====================================================================

See Also:
o )show PriorityQueueAggregate
```

See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.0.128 on page 846
⇐ "BagAggregate" (BGAGG) 5.0.68 on page 312

Exports:

any? bag copy coerce count
empty empty? eq? eval every?
extract! hash insert! inspect latex
less? map map! max member?
members merge merge! more? parts
sample size? #? ?=? ?~=?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

These exports come from (p312) BagAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

```
eval : (%,List Equation S) -> %
        if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
        if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category PRQAGG PriorityQueueAggregate*⟩≡

```
)abbrev category PRQAGG PriorityQueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A priority queue is a bag of items from an ordered set where the item
++ extracted is always the maximum element.

PriorityQueueAggregate(S:OrderedSet): Category == BagAggregate S with
   finiteAggregate
   max: % -> S
     ++ max(q) returns the maximum element of priority queue q.
   merge: (%,%) -> %
     ++ merge(q1,q2) returns combines priority queues q1 and q2 to return
     ++ a single priority queue q.
   merge_!: (%,%) -> %
     ++ merge!(q,q1) destructively changes priority queue q to include the
     ++ values from priority queue q1.
```

⟨*PRQAGG.dotabb*⟩≡
```
"PRQAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PRQAGG" -> "BGAGG"
```

⟨*PRQAGG.dotfull*⟩≡
```
"PriorityQueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"PriorityQueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:OrderedSet)" ->
   "PriorityQueueAggregate(a:SetCategory)"
```

⟨*PRQAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

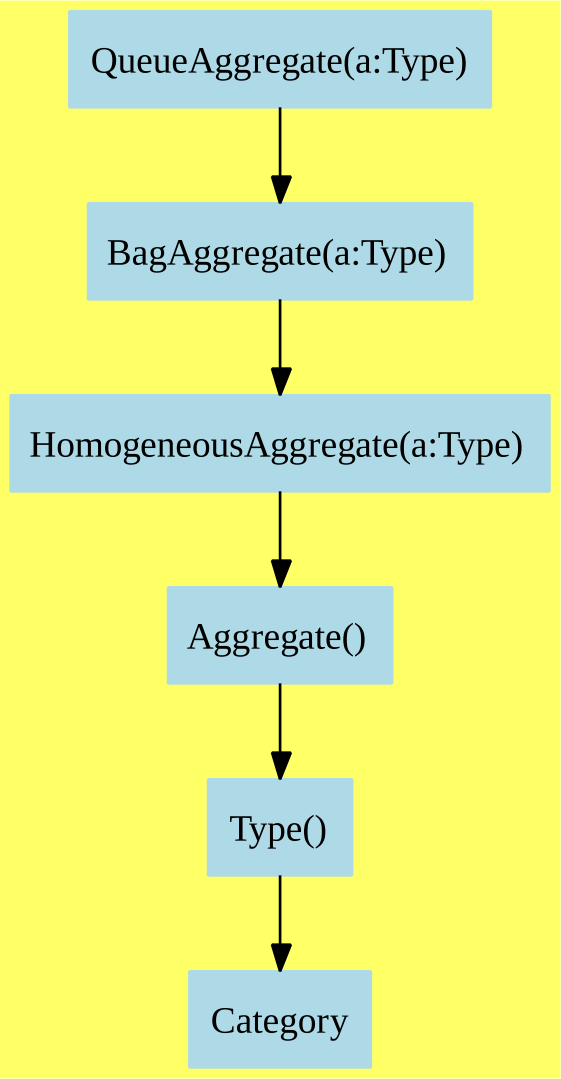
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.93 QueueAggregate (QUAGG)


QueueAggregate(a:Type)
BagAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*QueueAggregate.input*⟩≡

```
)set break resume
)sys rm -f QueueAggregate.output
)spool QueueAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show QueueAggregate
--R QueueAggregate S: Type  is a category constructor
--R Abbreviation for QueueAggregate is QUAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QUAGG
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                         bag : List S -> %
--R copy : % -> %                         dequeue! : % -> S
--R empty : () -> %                       empty? : % -> Boolean
--R enqueue! : (S,%) -> S                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     front : % -> S
--R insert! : (S,%) -> %                  inspect : % -> S
--R length : % -> NonNegativeInteger      map : ((S -> S),%) -> %
--R rotate! : % -> %                      sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*QueueAggregate.help*⟩≡

```
====================================================================
QueueAggregate examples
====================================================================

See Also:
o )show QueueAggregate
```

See:

⇒ “DequeueAggregate” (DQAGG) 7.0.100 on page 597
⇐ “BagAggregate” (BGAGG) 5.0.68 on page 312

Exports:

any?	bag	back	coerce	copy
count	dequeue!	empty	empty?	enqueue!
eq?	eval	every?	extract!	front
hash	insert!	inspect	latex	length
less?	map	map!	member?	members
more?	parts	rotate!	sample	size?
#?	?=?	?~=?		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

These exports come from (p312) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
```

```
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category QUAGG QueueAggregate*⟩≡

```
)abbrev category QUAGG QueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A queue is a bag where the first item inserted is the first
++ item extracted.

QueueAggregate(S:Type): Category == BagAggregate S with
   finiteAggregate
   enqueue_!: (S, %) -> S
     ++ enqueue!(x,q) inserts x into the queue q at the back end.
   dequeue_!: % -> S
     ++ dequeue! s destructively extracts the first (top) element
     ++ from queue q. The element previously second in the queue becomes
```

```
      ++ the first element. Error: if q is empty.
    rotate_!: % -> %
      ++ rotate! q rotates queue q so that the element at the front of
      ++ the queue goes to the back of the queue.
      ++ Note that rotate! q is equivalent to enqueue!(dequeue!(q)).
    length: % -> NonNegativeInteger
      ++ length(q) returns the number of elements in the queue.
      ++ Note that \axiom{length(q) = #q}.
    front: % -> S
      ++ front(q) returns the element at the front of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.
    back: % -> S
      ++ back(q) returns the element at the back of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.
```

⟨*QUAGG.dotabb*⟩≡

```
"QUAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QUAGG" -> "BGAGG"
```

⟨*QUAGG.dotfull*⟩≡

```
"QueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"QueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:SetCategory)" -> "QueueAggregate(a:Type)"
```

⟨*QUAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QueueAggregate(a:Type)" [color=lightblue];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

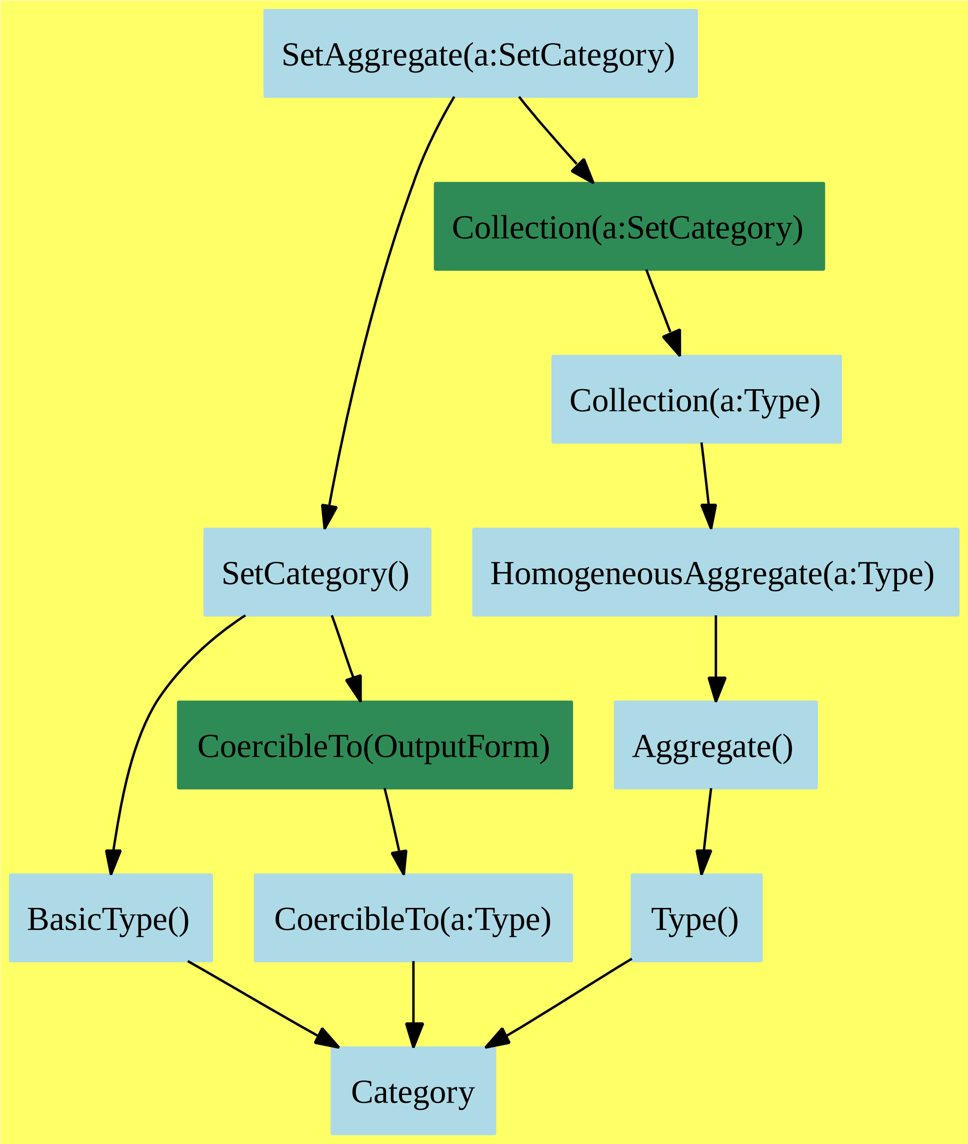
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.94 SetAggregate (SETAGG)


SetAggregate(a:SetCategory)
Collection(a:SetCategory)
Collection(a:Type)
SetCategory()
HomogeneousAggregate(a:Type)
CoercibleTo(OutputForm)
Aggregate()
BasicType()
CoercibleTo(a:Type)
Type()
Category

⟨*SetAggregate.input*⟩≡

```
)set break resume
)sys rm -f SetAggregate.output
)spool SetAggregate.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show SetAggregate
--R SetAggregate S: SetCategory  is a category constructor
--R Abbreviation for SetAggregate is SETAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R brace : List S -> %                   brace : () -> %
--R coerce : % -> OutputForm              construct : List S -> %
--R copy : % -> %                         difference : (%,S) -> %
--R difference : (%,%) -> %               empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             intersect : (%,%) -> %
--R latex : % -> String                   map : ((S -> S),%) -> %
--R sample : () -> %                      set : List S -> %
--R set : () -> %                         subset? : (%,%) -> Boolean
--R union : (S,%) -> %                    union : (%,S) -> %
--R union : (%,%) -> %                    ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

```
--R size? : (%,NonNegativeInteger) -> Boolean
--R symmetricDifference : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*SetAggregate.help*⟩≡

```
====================================================================
SetAggregate examples
====================================================================

See Also:
o )show SetAggregate
```

See:

⇒ "FiniteSetAggregate" (FSAGG) 8.0.110 on page 685
⇒ "MultisetAggregate" (MSETAGG) 8.0.115 on page 742
⇐ "Collection" (CLAGG) 5.0.70 on page 322
⇐ "SetCategory" (SETCAT) 3.0.34 on page 130

Exports:

any?	brace	coerce	construct	convert
copy	count	difference	empty	empty?
eq?	eval	every?	find	hash
intersect	latex	less?	map	map!
member?	members	more?	parts	reduce
remove	removeDuplicates	sample	select	set
size?	subset?	symmetricDifference	union	#?
?<?	?=?	?~=?		

Attributes Exported:

- **partiallyOrderedSet** is true if a set with < which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **nil**

These are directly exported but not implemented:

```
brace : List S -> %
brace : () -> %
difference : (%,%) -> %
intersect : (%,%) -> %
set : List S -> %
set : () -> %
subset? : (%,%) -> Boolean
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
difference : (%,S) -> %
symmetricDifference : (%,%) -> %
union : (S,%) -> %
union : (%,S) -> %
```

These exports come from (p130) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
```

```
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p322) Collection(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
```

```
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT
    and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
    if $ has finiteAggregate
```

⟨*category SETAGG SetAggregate*⟩≡

```
)abbrev category SETAGG SetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A set category lists a collection of set-theoretic operations
++ useful for both finite sets and multisets.
++ Note however that finite sets are distinct from multisets.
++ Although the operations defined for set categories are
++ common to both, the relationship between the two cannot
++ be described by inclusion or inheritance.

SetAggregate(S:SetCategory):
  Category == Join(SetCategory, Collection(S)) with
   partiallyOrderedSet
   "<"         : (%, %) -> Boolean
     ++ s < t returns true if all elements of set aggregate s are also
     ++ elements of set aggregate t.
   brace       : () -> %
     ++ brace()$D (otherwise written {}$D)
     ++ creates an empty set aggregate of type D.
     ++ This form is considered obsolete. Use \axiomFun{set} instead.
   brace       : List S -> %
     ++ brace([x,y,...,z])
     ++ creates a set aggregate containing items x,y,...,z.
     ++ This form is considered obsolete. Use \axiomFun{set} instead.
   set               : () -> %
     ++ set()$D creates an empty set aggregate of type D.
   set               : List S -> %
     ++ set([x,y,...,z]) creates a set aggregate containing items x,y,...,z.
   intersect: (%, %) -> %
     ++ intersect(u,v) returns the set aggregate w consisting of
     ++ elements common to both set aggregates u and v.
     ++ Note that equivalent to the notation (not currently supported)
     ++ {x for x in u | member?(x,v)}.
   difference  : (%, %) -> %
     ++ difference(u,v) returns the set aggregate w consisting of
     ++ elements in set aggregate u but not in set aggregate v.
     ++ If u and v have no elements in common, \axiom{difference(u,v)}
     ++ returns a copy of u.
     ++ Note that equivalent to the notation (not currently supported)
     ++ \axiom{{x for x in u | not member?(x,v)}}.
   difference  : (%, S) -> %
     ++ difference(u,x) returns the set aggregate u with element x removed.
     ++ If u does not contain x, a copy of u is returned.
     ++ Note that \axiom{difference(s, x) = difference(s, {x})}.
   symmetricDifference : (%, %) -> %
     ++ symmetricDifference(u,v) returns the set aggregate of elements x
     ++ which are members of set aggregate u or set aggregate v but
     ++ not both. If u and v have no elements in common,
     ++ \axiom{symmetricDifference(u,v)} returns a copy of u.
     ++ Note that \axiom{symmetricDifference(u,v) =
     ++  union(difference(u,v),difference(v,u))}
   subset?     : (%, %) -> Boolean
     ++ subset?(u,v) tests if u is a subset of v.
     ++ Note that equivalent to
     ++ \axiom{reduce(and,{member?(x,v) for x in u},true,false)}.
   union       : (%, %) -> %
     ++ union(u,v) returns the set aggregate of elements which are members
     ++ of either set aggregate u or v.
   union       : (%, S) -> %
     ++ union(u,x) returns the set aggregate u with the element x added.
     ++ If u already contains x, \axiom{union(u,x)} returns a copy of u.
   union       : (S, %) -> %
     ++ union(x,u) returns the set aggregate u with the element x added.
     ++ If u already contains x, \axiom{union(x,u)} returns a copy of u.
 add
  symmetricDifference(x, y)    == union(difference(x, y), difference(y, x))
  union(s:%, x:S) == union(s, {x})
  union(x:S, s:%) == union(s, {x})
  difference(s:%, x:S) == difference(s, {x})
```

⟨*SETAGG.dotabb*⟩≡
```
"SETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SETAGG" -> "SETCAT"
"SETAGG" -> "CLAGG"
```

⟨*SETAGG.dotfull*⟩≡
```
"SetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"
```

⟨*SETAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

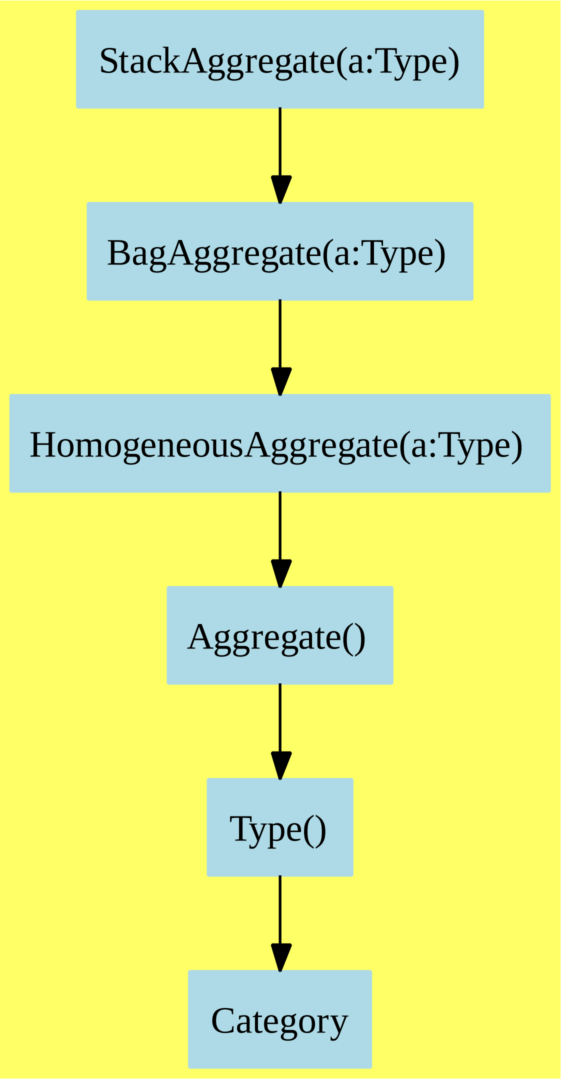
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.0.95 StackAggregate (SKAGG)


StackAggregate(a:Type)
BagAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*StackAggregate.input*⟩≡

```
)set break resume
)sys rm -f StackAggregate.output
)spool StackAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show StackAggregate
--R StackAggregate S: Type  is a category constructor
--R Abbreviation for StackAggregate is SKAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SKAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                      copy : % -> %
--R depth : % -> NonNegativeInteger       empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      map : ((S -> S),%) -> %
--R pop! : % -> S                         push! : (S,%) -> S
--R sample : () -> %                      top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*StackAggregate.help*⟩≡

```
====================================================================
StackAggregate examples
====================================================================

See Also:
o )show StackAggregate
```

See:

⇒ “DequeueAggregate” (DQAGG) 7.0.100 on page 597
⇐ “BagAggregate” (BGAGG) 5.0.68 on page 312

Exports:

any?	bag	coerce	copy	count
depth	empty	empty?	eq?	eval
every?	extract!	hash	insert!	inspect
latex	less?	map	map!	member?
members	more?	parts	pop!	push!
sample	size?	top	#?	?=?
?~=?				

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
depth : % -> NonNegativeInteger
pop! : % -> S
push! : (S,%) -> S
top : % -> S
```

These exports come from (p312) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
```

```
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category SKAGG StackAggregate*⟩≡

```
)abbrev category SKAGG StackAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A stack is a bag where the last item inserted is the first item extracted.

StackAggregate(S:Type): Category == BagAggregate S with
   finiteAggregate
   push_!: (S,%) -> S
     ++ push!(x,s) pushes x onto stack s, i.e. destructively changing s
     ++ so as to have a new first (top) element x.
     ++ Afterwards, pop!(s) produces x and pop!(s) produces the original s.
     ++
     ++X a:Stack INT:= stack [1,2,3,4,5]
     ++X push! a
     ++X a
```

```
    pop_!: % -> S
      ++ pop!(s) returns the top element x, destructively removing x from s.
      ++ Note that Use \axiom{top(s)} to obtain x without removing it from s.
      ++ Error: if s is empty.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X pop! a
      ++X a
    top: % -> S
      ++ top(s) returns the top element x from s; s remains unchanged.
      ++ Note that Use \axiom{pop!(s)} to obtain x and remove it from s.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X top a
    depth: % -> NonNegativeInteger
      ++ depth(s) returns the number of elements of stack s.
      ++ Note that \axiom{depth(s) = #s}.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X depth a
```

⟨*SKAGG.dotabb*⟩≡

```
"SKAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"SKAGG" -> "BGAGG"
```

⟨*SKAGG.dotfull*⟩≡

```
"StackAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"StackAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:SetCategory)" -> "StackAggregate(a:Type)"
```

⟨*SKAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StackAggregate(a:Type)" [color=lightblue];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

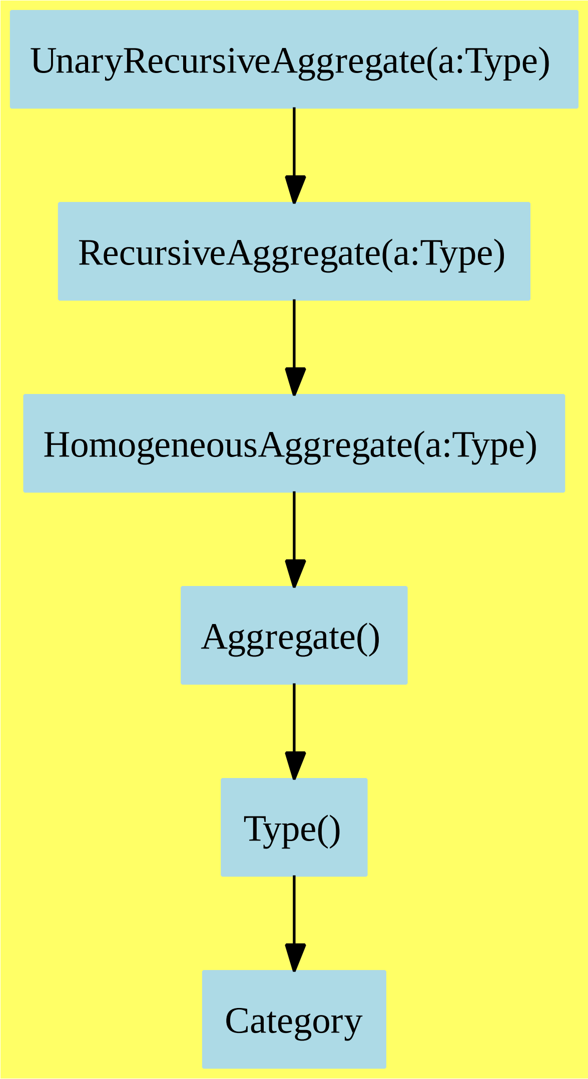
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.96 UnaryRecursiveAggregate (URAGG)


UnaryRecursiveAggregate(a:Type)
RecursiveAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*UnaryRecursiveAggregate.input*⟩≡

```
)set break resume
)sys rm -f UnaryRecursiveAggregate.output
)spool UnaryRecursiveAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show UnaryRecursiveAggregate
--R UnaryRecursiveAggregate S: Type  is a category constructor
--R Abbreviation for UnaryRecursiveAggregate is URAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for URAGG
--R
--R------------------------------- Operations ---------------------------------
--R children : % -> List %                concat : (S,%) -> %
--R concat : (%,%) -> %                   copy : % -> %
--R cycleEntry : % -> %                   cycleTail : % -> %
--R cyclic? : % -> Boolean                distance : (%,%) -> Integer
--R ?.last : (%,last) -> S                ?.rest : (%,rest) -> %
--R ?.first : (%,first) -> S              ?.value : (%,value) -> S
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                first : % -> S
--R last : % -> S                         leaf? : % -> Boolean
--R leaves : % -> List S                  map : ((S -> S),%) -> %
--R nodes : % -> List %                   rest : % -> %
--R sample : () -> %                      second : % -> S
--R tail : % -> %                         third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
```

```
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R rest : (%,NonNegativeInteger) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnaryRecursiveAggregate.help*⟩≡

```
====================================================================
UnaryRecursiveAggregate examples
====================================================================

See Also:
o )show UnaryRecursiveAggregate
```

See:

⇒ "StreamAggregate" (STAGG) 7.0.107 on page 647
⇐ "RecursiveAggregate" (RCAGG) 5.0.80 on page 392

Exports:

any?	children	child?	coerce	concat
concat!	copy	count	cycleEntry	cycleLength
cycleSplit!	cycleTail	cyclic?	distance	empty
empty?	eq?	eval	every?	first
hash	last	latex	leaf?	leaves
less?	map	map!	member?	members
more?	nodes	node?	parts	rest
sample	second	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?	split!
tail	third	value	#?	?.last
?.rest	?.first	?.value	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (S,%) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
concat! : (%,%) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
rest : % -> %
setfirst! : (%,S) -> S if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
concat : (%,%) -> %
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
```

```
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
rest : (%,NonNegativeInteger) -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p392) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

```
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setelt : (%,value,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
?.value : (%,value) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category URAGG UnaryRecursiveAggregate*⟩≡

```
)abbrev category URAGG UnaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A unary-recursive aggregate is a one where nodes may have either
++ 0 or 1 children.
++ This aggregate models, though not precisely, a linked
++ list possibly with a single cycle.
++ A node with one children models a non-empty list, with the
++ \spadfun{value} of the list designating the head, or \spadfun{first},
++ of the list, and the child designating the tail, or \spadfun{rest},
++ of the list. A node with no child then designates the empty list.
++ Since these aggregates are recursive aggregates, they may be cyclic.

UnaryRecursiveAggregate(S:Type): Category == RecursiveAggregate S with
   concat: (%,%) -> %
      ++ concat(u,v) returns an aggregate w consisting of the elements of u
      ++ followed by the elements of v.
      ++ Note that \axiom{v = rest(w,#a)}.
   concat: (S,%) -> %
      ++ concat(x,u) returns aggregate consisting of x followed by
      ++ the elements of u.
   ++ Note that if \axiom{v = concat(x,u)} then \axiom{x = first v}
   ++ and \axiom{u = rest v}.
first: % -> S
   ++ first(u) returns the first element of u
   ++ (equivalently, the value at the current node).
elt: (%,"first") -> S
   ++ elt(u,"first") (also written: \axiom{u . first})
   ++ is equivalent to first u.
first: (%,NonNegativeInteger) -> %
   ++ first(u,n) returns a copy of the first n (\axiom{n >= 0})
   ++ elements of u.
rest: % -> %
   ++ rest(u) returns an aggregate consisting of all but the first
   ++ element of u
   ++ (equivalently, the next node of u).
elt: (%,"rest") -> %
   ++ elt(%,"rest") (also written: \axiom{u.rest}) is
   ++ equivalent to \axiom{rest u}.
rest: (%,NonNegativeInteger) -> %
   ++ rest(u,n) returns the \axiom{n}th (n >= 0) node of u.
   ++ Note that \axiom{rest(u,0) = u}.
last: % -> S
   ++ last(u) resturn the last element of u.
   ++ Note that for lists, \axiom{last(u)=u . (maxIndex u)=u . (# u - 1)}.
elt: (%,"last") -> S
   ++ elt(u,"last") (also written: \axiom{u . last}) is equivalent
   ++ to last u.
last: (%,NonNegativeInteger) -> %
   ++ last(u,n) returns a copy of the last n (\axiom{n >= 0}) nodes of u.
   ++ Note that \axiom{last(u,n)} is a list of n elements.
tail: % -> %
   ++ tail(u) returns the last node of u.
   ++ Note that if u is \axiom{shallowlyMutable},
   ++ \axiom{setrest(tail(u),v) = concat(u,v)}.
second: % -> S
   ++ second(u) returns the second element of u.
   ++ Note that \axiom{second(u) = first(rest(u))}.
third: % -> S
   ++ third(u) returns the third element of u.
   ++ Note that \axiom{third(u) = first(rest(rest(u)))}.
cycleEntry: % -> %
   ++ cycleEntry(u) returns the head of a top-level cycle contained in
   ++ aggregate u, or \axiom{empty()} if none exists.
cycleLength: % -> NonNegativeInteger
   ++ cycleLength(u) returns the length of a top-level cycle
   ++ contained  in aggregate u, or 0 is u has no such cycle.
```

```
  cycleTail: % -> %
     ++ cycleTail(u) returns the last node in the cycle, or
     ++ empty if none exists.
  if % has shallowlyMutable then
     concat_!: (%,%) -> %
       ++ concat!(u,v) destructively concatenates v to the end of u.
       ++ Note that \axiom{concat!(u,v) = setlast_!(u,v)}.
     concat_!: (%,S) -> %
       ++ concat!(u,x) destructively adds element x to the end of u.
       ++ Note that \axiom{concat!(a,x) = setlast!(a,[x])}.
     cycleSplit_!: % -> %
       ++ cycleSplit!(u) splits the aggregate by dropping off the cycle.
       ++ The value returned is the cycle entry, or nil if none exists.
       ++ For example, if \axiom{w = concat(u,v)} is the cyclic list where
       ++ v is the head of the cycle, \axiom{cycleSplit!(w)} will drop v
       ++ off w thus destructively changing w to u, and returning v.
     setfirst_!: (%,S) -> S
       ++ setfirst!(u,x) destructively changes the first element of a to x.
     setelt: (%,"first",S) -> S
       ++ setelt(u,"first",x) (also written: \axiom{u.first := x}) is
       ++ equivalent to \axiom{setfirst!(u,x)}.
     setrest_!: (%,%) -> %
       ++ setrest!(u,v) destructively changes the rest of u to v.
     setelt: (%,"rest",%) -> %
       ++ setelt(u,"rest",v) (also written: \axiom{u.rest := v}) is
       ++ equivalent to \axiom{setrest!(u,v)}.
     setlast_!: (%,S) -> S
       ++ setlast!(u,x) destructively changes the last element of u to x.
     setelt: (%,"last",S) -> S
       ++ setelt(u,"last",x) (also written: \axiom{u.last := b})
       ++ is equivalent to \axiom{setlast!(u,v)}.
     split_!: (%,Integer) -> %
        ++ split!(u,n) splits u into two aggregates: \axiom{v = rest(u,n)}
        ++ and \axiom{w = first(u,n)}, returning \axiom{v}.
        ++ Note that afterwards \axiom{rest(u,n)} returns \axiom{empty()}.
add
 cycleMax ==> 1000

 findCycle: % -> %

 elt(x, "first") == first x
 elt(x,  "last") == last x
 elt(x,  "rest") == rest x
 second x          == first rest x
 third x          == first rest rest x
 cyclic? x          == not empty? x and not empty? findCycle x
```

```
last x          == first tail x

nodes x ==
  l := empty()$List(%)
  while not empty? x repeat
    l := concat(x, l)
    x := rest x
  reverse_! l

children x ==
  l := empty()$List(%)
  empty? x => l
  concat(rest x,l)

leaf? x == empty? x

value x ==
  empty? x => error "value of empty object"
  first x

less?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  i > 0

more?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  zero?(i) and not empty? l

size?(l, n) ==
  i := n::Integer
  while not empty? l and i > 0 repeat (l := rest l; i := i - 1)
  empty? l and zero? i

#x ==
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    x := rest x
  k

tail x ==
  empty? x => error "empty list"
  y := rest x
  for k in 0.. while not empty? y repeat
    k = cycleMax and cyclic? x => error "cyclic list"
```

```
      y := rest(x := y)
    x

  findCycle x ==
    y := rest x
    while not empty? y repeat
      if eq?(x, y) then return x
      x := rest x
      y := rest y
      if empty? y then return y
      if eq?(x, y) then return y
      y := rest y
    y

  cycleTail x ==
    empty?(y := x := cycleEntry x) => x
    z := rest x
    while not eq?(x,z) repeat (y := z; z := rest z)
    y

  cycleEntry x ==
    empty? x => x
    empty?(y := findCycle x) => y
    z := rest y
    for l in 1.. while not eq?(y,z) repeat z := rest z
    y := x
    for k in 1..l repeat y := rest y
    while not eq?(x,y) repeat (x := rest x; y := rest y)
    x

  cycleLength x ==
    empty? x => 0
    empty?(x := findCycle x) => 0
    y := rest x
    for k in 1.. while not eq?(x,y) repeat y := rest y
    k

  rest(x, n) ==
    for i in 1..n repeat
      empty? x => error "Index out of range"
      x := rest x
    x

  if % has finiteAggregate then
    last(x, n) ==
      n > (m := #x) => error "index out of range"
    copy rest(x, (m - n)::NonNegativeInteger)

if S has SetCategory then
  x = y ==
    eq?(x, y) => true
    for k in 0.. while not empty? x and not empty? y repeat
      k = cycleMax and cyclic? x => error "cyclic list"
      first x ^= first y => return false
      x := rest x
      y := rest y
    empty? x and empty? y

  node?(u, v) ==
    for k in 0.. while not empty? v repeat
      u = v => return true
      k = cycleMax and cyclic? v => error "cyclic list"
      v := rest v
    u=v

if % has shallowlyMutable then
  setelt(x, "first", a) == setfirst_!(x, a)
  setelt(x,  "last", a) == setlast_!(x, a)
  setelt(x,  "rest", a) == setrest_!(x, a)
  concat(x:%, y:%)          == concat_!(copy x, y)

  setlast_!(x, s) ==
    empty? x => error "setlast: empty list"
    setfirst_!(tail x, s)
    s

  setchildren_!(u,lv) ==
    #lv=1 => setrest_!(u, first lv)
    error "wrong number of children specified"

  setvalue_!(u,s) == setfirst_!(u,s)

  split_!(p, n) ==
    n < 1 => error "index out of range"
    p := rest(p, (n - 1)::NonNegativeInteger)
    q := rest p
    setrest_!(p, empty())
    q

  cycleSplit_! x ==
    empty?(y := cycleEntry x) or eq?(x, y) => y
    z := rest x
```

```
        while not eq?(z, y) repeat (x := z; z := rest z)
        setrest_!(x, empty())
        y
```

⟨URAGG.dotabb⟩≡

```
  "URAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
  "URAGG" -> "RCAGG"
```

⟨URAGG.dotfull⟩≡

```
  "UnaryRecursiveAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
  "UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

⟨URAGG.dotpic⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "UnaryRecursiveAggregate(a:Type)" [color=lightblue];
  "UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

  "RecursiveAggregate(a:Type)" [color=lightblue];
  "RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

  "HomogeneousAggregate(a:Type)" [color=lightblue];
  "HomogeneousAggregate(a:Type)" -> "Aggregate()"

  "Aggregate()" [color=lightblue];
  "Aggregate()" -> "Type()"

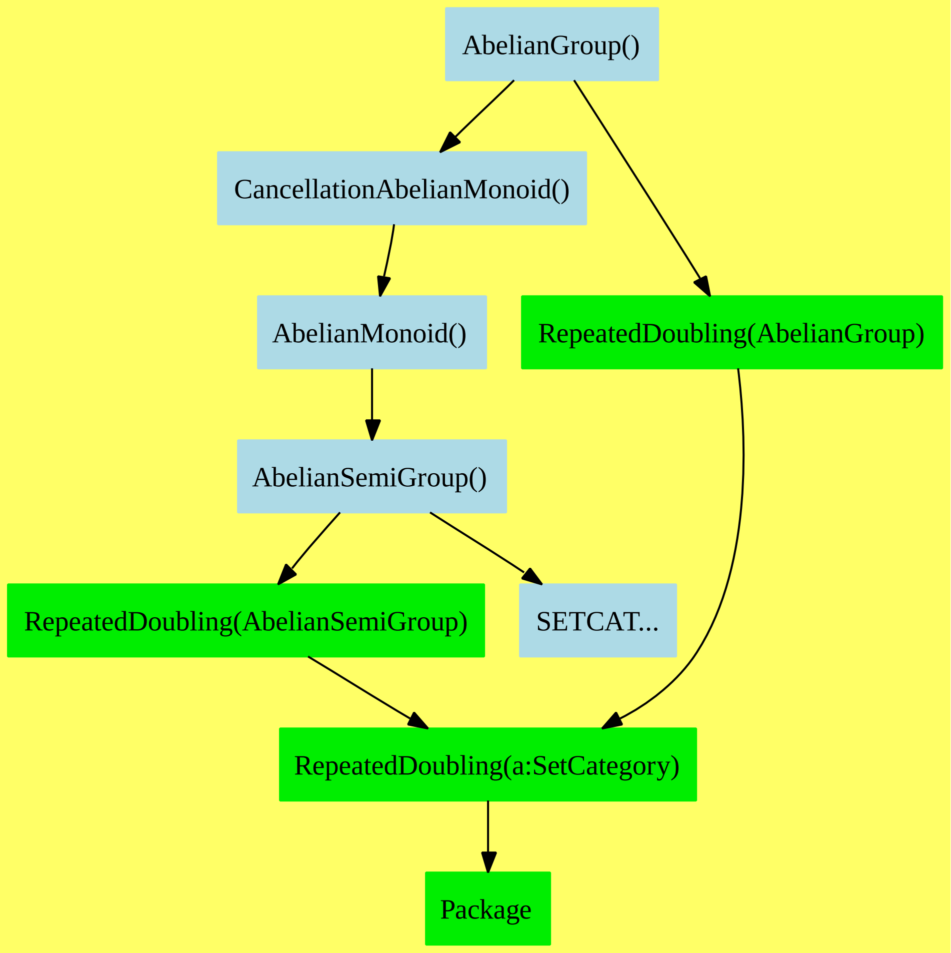
  "Type()" [color=lightblue];
  "Type()" -> "Category"

  "Category" [color=lightblue];
  }
```

Chapter 7

Category Layer 6

7.0.97 AbelianGroup (ABELGRP)


AbelianGroup()
CancellationAbelianMonoid()
AbelianMonoid()
RepeatedDoubling(AbelianGroup)
AbelianSemiGroup()
RepeatedDoubling(AbelianSemiGroup)
SETCAT...
RepeatedDoubling(a:SetCategory)
Package

⟨*AbelianGroup.input*⟩≡
```
)set break resume
)sys rm -f AbelianGroup.output
)spool AbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianGroup
--R AbelianGroup  is a category constructor
--R Abbreviation for AbelianGroup is ABELGRP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELGRP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 0 : () -> %                          coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*AbelianGroup.help*⟩≡
```
====================================================================
AbelianGroup examples
====================================================================

See Also:
o )show AbelianGroup
```

See:

⇒ "FiniteDivisorCategory" (FDIVCAT) 8.0.109 on page 678
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "LeftModule" (LMODULE) 8.0.113 on page 722
⇒ "NonAssociativeRng" (NARNG) 8.0.116 on page 749
⇒ "OrderedAbelianGroup" (OAGROUP) 9.0.126 on page 836
⇒ "RightModule" (RMODULE) 8.0.120 on page 788
⇒ "Rng" (RNG) 8.0.121 on page 793
⇐ "CancellationAbelianMonoid" (CABMON) 6.0.83 on page 423

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
-? : % -> %
```

These are implemented by this category:

```
subtractIfCan : (%,%) -> Union(%,"failed")
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

These exports come from (p423) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

⟨*category ABELGRP AbelianGroup*⟩≡

```
)abbrev category ABELGRP AbelianGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of abelian groups, i.e. additive monoids where
++ each element has an additive inverse.
++
++ Axioms\br
++ \tab{5}\spad{-(-x) = x}\br
++ \tab{5}\spad{x+(-x) = 0}
-- following domain must be compiled with subsumption disabled

AbelianGroup(): Category == CancellationAbelianMonoid with
      "-": % -> %
        ++ -x is the additive inverse of x.
      "-": (%,%) -> %
        ++ x-y is the difference of x and y
        ++ i.e. \spad{x + (-y)}.
        -- subsumes the partial subtraction from previous
      "*": (Integer,%) -> %
        ++ n*x is the product of x by the integer n.
    add
      (x:% - y:%):% == x+(-y)
      subtractIfCan(x:%, y:%):Union(%, "failed") == (x-y)::Union(%,"failed")
      n:NonNegativeInteger * x:% == (n::Integer) * x
      import RepeatedDoubling(%)
      if not (% has Ring) then
        n:Integer * x:% ==
          zero? n => 0
          n>0 => double(n pretend PositiveInteger,x)
          double((-n) pretend PositiveInteger,-x)
```

⟨*ABELGRP.dotabb*⟩≡

```
"ABELGRP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"ABELGRP" -> "CABMON"
```

⟨*ABELGRP.dotfull*⟩≡

```
"AbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"
```

⟨*ABELGRP.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

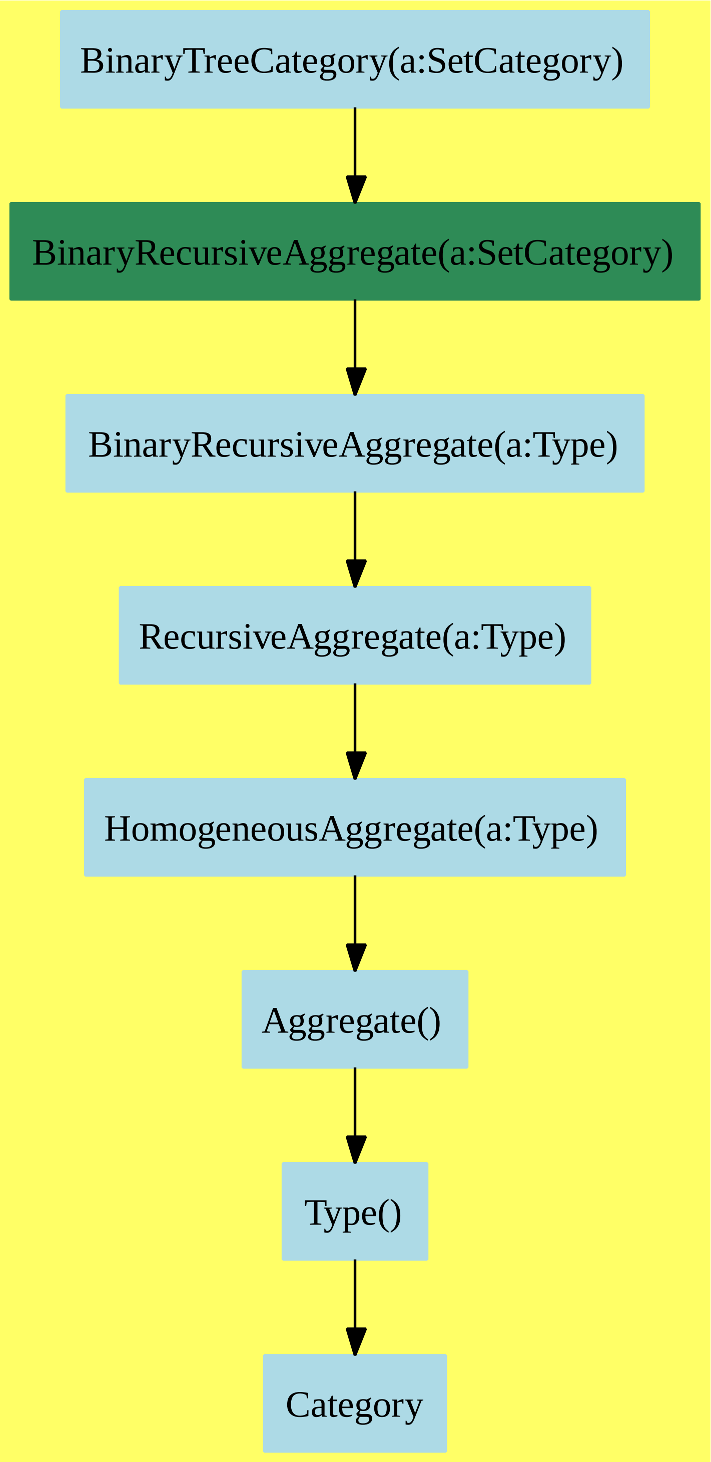
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

7.0.98 BinaryTreeCategory (BTCAT)


BinaryTreeCategory(a:SetCategory)
BinaryRecursiveAggregate(a:SetCategory)
BinaryRecursiveAggregate(a:Type)
RecursiveAggregate(a:Type)
HomogeneousAggregate(a:Type)
Aggregate()
Type()
Category

⟨*BinaryTreeCategory.input*⟩≡

```
)set break resume
)sys rm -f BinaryTreeCategory.output
)spool BinaryTreeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryTreeCategory
--R BinaryTreeCategory S: SetCategory  is a category constructor
--R Abbreviation for BinaryTreeCategory is BTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BTCAT
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %                copy : % -> %
--R cyclic? : % -> Boolean                distance : (%,%) -> Integer
--R ?.right : (%,right) -> %              ?.left : (%,left) -> %
--R ?.value : (%,value) -> S              empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                  leaves : % -> List S
--R left : % -> %                         map : ((S -> S),%) -> %
--R node : (%,S,%) -> %                   nodes : % -> List %
--R right : % -> %                        sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
```

```
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*BinaryTreeCategory.help*⟩≡

```
====================================================================
BinaryTreeCategory examples
====================================================================

See Also:
o )show BinaryTreeCategory
```

See:

⇐ “BinaryRecursiveAggregate” (BRAGG) 6.0.82 on page 414

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	left	map
map!	member?	members	more?	node
node?	nodes	parts	right	sample
setchildren!	setelt	setleft!	setright!	setvalue!
size?	value	#?	?=?	?~=?
?.right	?.left	?.value		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
node : (%,S,%) -> %
```

These are implemented by this category:

```
copy : % -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p414) BinaryRecursiveAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
```

```
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
left : % -> %
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
right : % -> %
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

⟨*category BTCAT BinaryTreeCategory*⟩≡

```
)abbrev category BTCAT BinaryTreeCategory
++ Author:W. H. Burge
++ Date Created:17 Feb 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{BinaryTreeCategory(S)} is the category of
++ binary trees: a tree which is either empty or else is a
++ \spadfun{node} consisting of a value and a \spadfun{left} and
++ \spadfun{right}, both binary trees.

BinaryTreeCategory(S: SetCategory): Category == _
      BinaryRecursiveAggregate(S) with
   shallowlyMutable
    ++ Binary trees have updateable components
   finiteAggregate
    ++ Binary trees have a finite number of components
   node: (%,S,%) -> %
    ++ node(left,v,right) creates a binary tree with value \spad{v}, a binary
    ++ tree \spad{left}, and a binary tree \spad{right}.
    ++
 add
     cycleTreeMax ==> 5

     copy t ==
       empty? t => empty()
       node(copy left t, value t, copy right t)

     if % has shallowlyMutable then
       map_!(f,t) ==
         empty? t => t
         t.value := f(t.value)
         map_!(f,left t)
         map_!(f,right t)
         t

     if % has finiteAggregate then
       treeCount : (%, NonNegativeInteger) -> NonNegativeInteger

       #t == treeCount(t,0)

       treeCount(t,k) ==
         empty? t => k
         k := k + 1
         k = cycleTreeMax and cyclic? t => error "cyclic binary tree"
         k := treeCount(left t,k)
         treeCount(right t,k)
```

⟨*BTCAT.dotabb*⟩≡

```
"BTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BTCAT" -> "BRAGG"
```

⟨*BTCAT.dotfull*⟩≡

```
"BinaryTreeCategory(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"
```

⟨*BTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BinaryTreeCategory(a:SetCategory)" [color=lightblue];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"

"BinaryRecursiveAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:Type)"

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

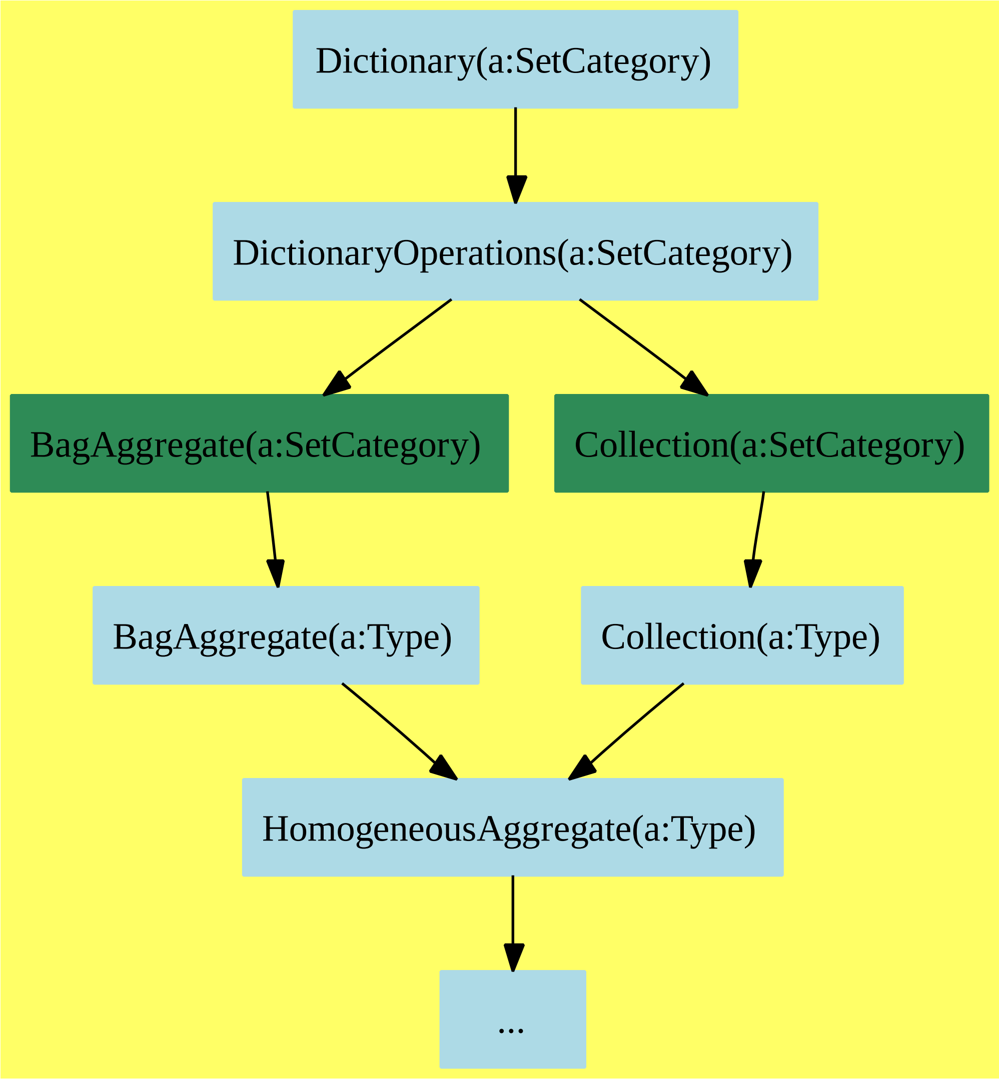
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

7.0.99 Dictionary (DIAGG)


Dictionary(a:SetCategory)
DictionaryOperations(a:SetCategory)
BagAggregate(a:SetCategory)
Collection(a:SetCategory)
BagAggregate(a:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
...

⟨*Dictionary.input*⟩≡

```
)set break resume
)sys rm -f Dictionary.output
)spool Dictionary.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show Dictionary
--R Dictionary S: SetCategory  is a category constructor
--R Abbreviation for Dictionary is DIAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                     construct : List S -> %
--R copy : % -> %                         dictionary : List S -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      map : ((S -> S),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

```
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*Dictionary.help*⟩≡

```
====================================================================
Dictionary examples
====================================================================

See Also:
o )show Dictionary
```

See:

⇒ "FiniteSetAggregate" (FSAGG) 8.0.110 on page 685
⇒ "KeyedDictionary" (KDAGG) 8.0.111 on page 694
⇐ "DictionaryOperations" (DIOPS) 6.0.84 on page 428

Exports:

any? copy eq? hash map parts sample ?~=?
bag count eval insert! map! reduce select ?=?
coerce dictionary every? inspect member? remove select!
construct empty extract! latex members remove! size?
convert empty? find less? more? removeDuplicates #?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
dictionary : List S -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p428) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
```

```
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
      if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category DIAGG Dictionary*⟩≡

```
)abbrev category DIAGG Dictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
```

```
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A dictionary is an aggregate in which entries can be inserted,
++ searched for and removed. Duplicates are thrown away on insertion.
++ This category models the usual notion of dictionary which involves
++ large amounts of data where copying is impractical.
++ Principal operations are thus destructive (non-copying) ones.

Dictionary(S:SetCategory): Category ==
 DictionaryOperations S add
   dictionary l ==
     d := dictionary()
     for x in l repeat insert_!(x, d)
     d

   if % has finiteAggregate then
    -- remove(f:S->Boolean,t:%)  == remove_!(f, copy t)
    -- select(f, t)           == select_!(f, copy t)
     select_!(f, t)         == remove_!((x:S):Boolean +-> not f(x), t)

     --extract_! d ==
     --         empty? d => error "empty dictionary"
     --         remove_!(x := first parts d, d, 1)
     --         x

     s = t ==
       eq?(s,t) => true
       #s ^= #t => false
       _and/[member?(x, t) for x in parts s]

     remove_!(f:S->Boolean, t:%) ==
       for m in parts t repeat if f m then remove_!(m, t)
       t
```

⟨*DIAGG.dotabb*⟩≡

```
"DIAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"DIAGG" -> "DIOPS"
```

⟨DIAGG.dotfull⟩≡

```
"Dictionary(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"Dictionary(Record(a:SetCategory,b:SetCategory))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"
```

⟨*DIAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

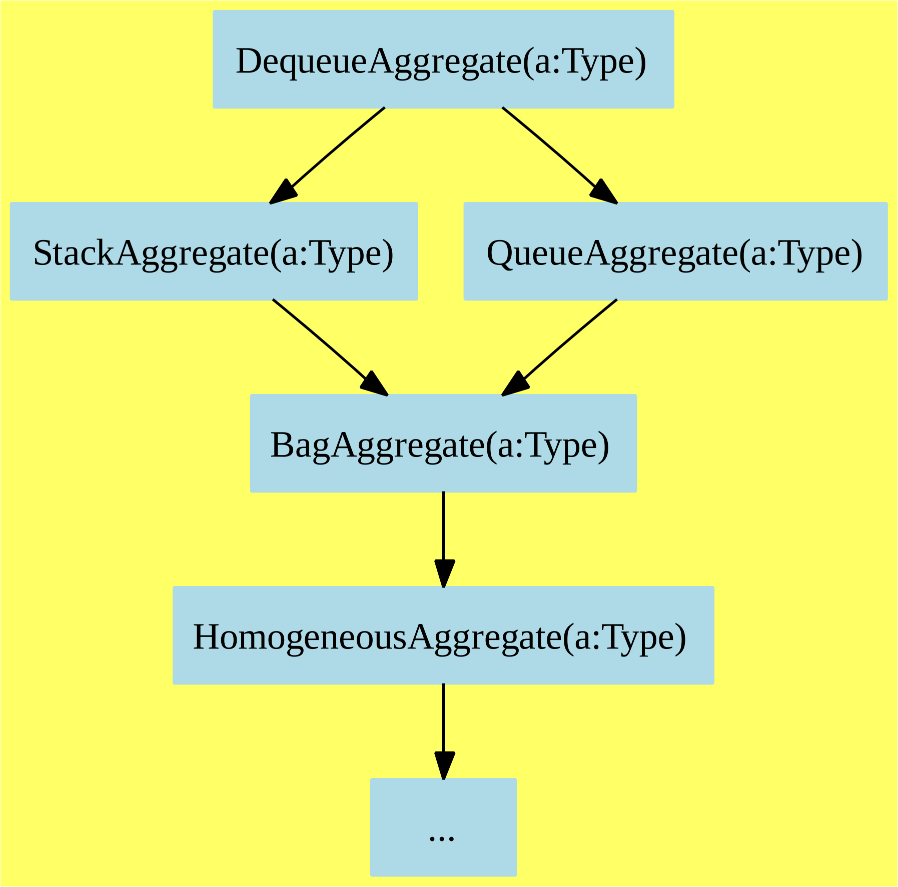
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

7.0.100 DequeueAggregate (DQAGG)


DequeueAggregate(a:Type)
StackAggregate(a:Type)
QueueAggregate(a:Type)
BagAggregate(a:Type)
HomogeneousAggregate(a:Type)
...

⟨*DequeueAggregate.input*⟩≡

```
)set break resume
)sys rm -f DequeueAggregate.output
)spool DequeueAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DequeueAggregate
--R DequeueAggregate S: Type  is a category constructor
--R Abbreviation for DequeueAggregate is DQAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DQAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                        bag : List S -> %
--R bottom! : % -> S                     copy : % -> %
--R depth : % -> NonNegativeInteger      dequeue : List S -> %
--R dequeue : () -> %                    dequeue! : % -> S
--R empty : () -> %                      empty? : % -> Boolean
--R enqueue! : (S,%) -> S                eq? : (%,%) -> Boolean
--R extract! : % -> S                    extractBottom! : % -> S
--R extractTop! : % -> S                 front : % -> S
--R height : % -> NonNegativeInteger     insert! : (S,%) -> %
--R insertBottom! : (S,%) -> S           insertTop! : (S,%) -> S
--R inspect : % -> S                     length : % -> NonNegativeInteger
--R map : ((S -> S),%) -> %              pop! : % -> S
--R push! : (S,%) -> S                   reverse! : % -> %
--R rotate! : % -> %                     sample : () -> %
--R top : % -> S                         top! : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*DequeueAggregate.help*⟩≡

```
====================================================================
DequeueAggregate examples
====================================================================

See Also:
o )show DequeueAggregate
```

See:

⇐ “QueueAggregate” (QUAGG) 6.0.93 on page 540
⇐ “StackAggregate” (SKAGG) 6.0.95 on page 556

Exports:

any?	back	bag	bottom!	coerce
copy	count	depth	dequeue	dequeue!
empty	empty?	enqueue!	eq?	eval
every?	extract!	extractBottom!	extractTop!	front
hash	height	insert!	insertBottom!	insertTop!
inspect	latex	length	less?	map
map!	members	member?	more?	parts
pop!	push!	reverse!	rotate!	sample
size?	top	top!	#?	?=?
?~=?				

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
bottom! : % -> S
dequeue : () -> %
dequeue : List S -> %
extractBottom! : % -> S
extractTop! : % -> S
height : % -> NonNegativeInteger
insertBottom! : (S,%) -> S
insertTop! : (S,%) -> S
reverse! : % -> %
top! : % -> S
```

These exports come from (p556) StackAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
```

```
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
depth : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
pop! : % -> S
push! : (S,%) -> S
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
top : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p540) QueueAggregate(S:Type):

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

⟨*category DQAGG DequeueAggregate*⟩≡

```
)abbrev category DQAGG DequeueAggregate
```

```
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A dequeue is a doubly ended stack, that is, a bag where first items
++ inserted are the first items extracted, at either the front or
++ the back end of the data structure.

DequeueAggregate(S:Type):
 Category == Join(StackAggregate S,QueueAggregate S) with
   dequeue: () -> %
     ++ dequeue()$D creates an empty dequeue of type D.
   dequeue: List S -> %
     ++ dequeue([x,y,...,z]) creates a dequeue with first (top or front)
     ++ element x, second element y,...,and last (bottom or back) element z.
   height: % -> NonNegativeInteger
     ++ height(d) returns the number of elements in dequeue d.
     ++ Note that \axiom{height(d) = # d}.
   top_!: % -> S
     ++ top!(d) returns the element at the top (front) of the dequeue.
   bottom_!: % -> S
     ++ bottom!(d) returns the element at the bottom (back) of the dequeue.
   insertTop_!: (S,%) -> S
     ++ insertTop!(x,d) destructively inserts x into the dequeue d, that is,
     ++ at the top (front) of the dequeue.
     ++ The element previously at the top of the dequeue becomes the
     ++ second in the dequeue, and so on.
   insertBottom_!: (S,%) -> S
     ++ insertBottom!(x,d) destructively inserts x into the dequeue d
     ++ at the bottom (back) of the dequeue.
   extractTop_!: % -> S
     ++ extractTop!(d) destructively extracts the top (front) element
     ++ from the dequeue d.
     ++ Error: if d is empty.
   extractBottom_!: % -> S
     ++ extractBottom!(d) destructively extracts the bottom (back) element
     ++ from the dequeue d.
     ++ Error: if d is empty.
   reverse_!: % -> %
     ++ reverse!(d) destructively replaces d by its reverse dequeue, i.e.
```

```
    ++ the top (front) element is now the bottom (back) element, and so on.
```

⟨*DQAGG.dotabb*⟩≡

```
"DQAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DQAGG" -> "SKAGG"
"DQAGG" -> "QUAGG"
```

⟨*DQAGG.dotfull*⟩≡

```
"DequeueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
"DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

"DequeueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DequeueAggregate(a:SetCategory)" -> "DequeueAggregate(a:Type)"
```

⟨*DQAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DequeueAggregate(a:Type)" [color=lightblue];
"DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
"DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

"StackAggregate(a:Type)" [color=lightblue];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

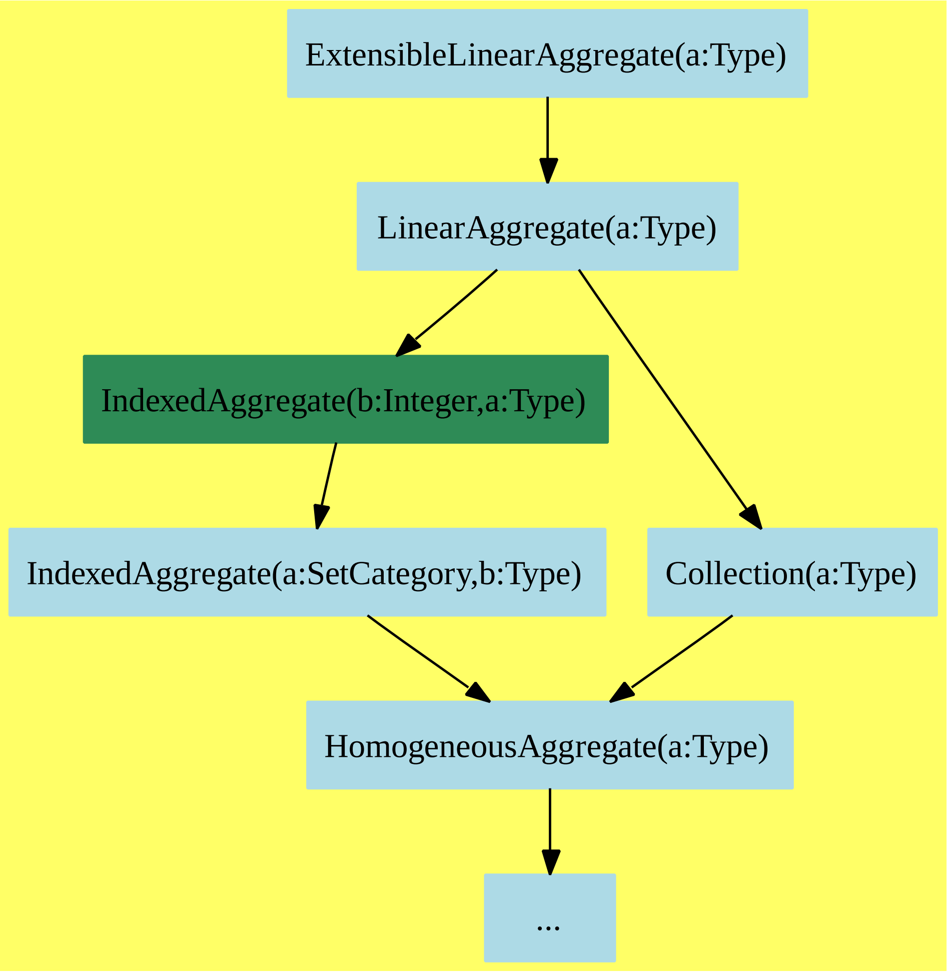
"QueueAggregate(a:Type)" [color=lightblue];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.101 ExtensibleLinearAggregate (ELAGG)


ExtensibleLinearAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
IndexedAggregate(a:SetCategory,b:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
...

⟨*ExtensibleLinearAggregate.input*⟩≡

```
)set break resume
)sys rm -f ExtensibleLinearAggregate.output
)spool ExtensibleLinearAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExtensibleLinearAggregate
--R ExtensibleLinearAggregate S: Type  is a category constructor
--R Abbreviation for ExtensibleLinearAggregate is ELAGG
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELAGG
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (S,%) -> %                   concat : (%,S) -> %
--R concat! : (%,%) -> %                  concat! : (%,S) -> %
--R construct : List S -> %               copy : % -> %
--R delete : (%,Integer) -> %             delete! : (%,Integer) -> %
--R ?.? : (%,Integer) -> S                elt : (%,Integer,S) -> S
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List S                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           insert : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %          insert! : (S,%,Integer) -> %
--R map : (((S,S) -> S),%,%) -> %         map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %     qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R merge! : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
```

```
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : (S,%) -> % if S has SETCAT
--R remove! : ((S -> Boolean),%) -> %
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates! : % -> % if S has SETCAT
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*ExtensibleLinearAggregate.help*⟩≡

```
====================================================================
ExtensibleLinearAggregate examples
====================================================================

See Also:
o )show ExtensibleLinearAggregate
```

See:

⇒ “ListAggregate” (LSAGG) 8.0.114 on page 727
⇐ “LinearAggregate” (LNAGG) 6.0.87 on page 449

Exports:

any?	coerce	concat	concat!	construct
copy	convert	count	delete	delete!
elt	empty	empty?	entries	entry?
eval	every?	eq?	fill!	find
first	hash	index?	indices	insert
insert!	latex	less?	map	map!
maxIndex	member?	members	merge!	minIndex
more?	new	parts	qelt	qsetelt!
reduce	remove	remove!	removeDuplicates	removeDuplicates!
sample	select	select!	setelt	size?
swap!	#?	?=?	?.?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
concat! : (%,S) -> %
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (S,%,Integer) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
concat : (%,%) -> %
concat : (%,S) -> %
concat! : (%,%) -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
insert : (S,%,Integer) -> %
merge! : (%,%) -> % if S has ORDSET
```

```
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove! : (S,%) -> % if S has SETCAT
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p449) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (S,%) -> %
construct : List S -> %
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
convert : % -> InputForm if S has KONVERT INFORM
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
index? : (Integer,%) -> Boolean
indices : % -> List Integer
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
```

```
         if S has SETCAT and $ has finiteAggregate
maxIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
sample : () -> %
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
```

⟨*category ELAGG ExtensibleLinearAggregate*⟩≡

```
)abbrev category ELAGG ExtensibleLinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An extensible aggregate is one which allows insertion and deletion of
++ entries. These aggregates are models of lists and streams which are
++ represented by linked structures so as to make insertion, deletion, and
++ concatenation efficient. However, access to elements of these
++ extensible aggregates is generally slow since access is made from the end.
++ See \spadtype{FlexibleArray} for an exception.

ExtensibleLinearAggregate(S:Type):Category == LinearAggregate S with
```

```
    shallowlyMutable
    concat_!: (%,S) -> %
      ++ concat!(u,x) destructively adds element x to the end of u.
    concat_!: (%,%) -> %
      ++ concat!(u,v) destructively appends v to the end of u.
      ++ v is unchanged
    delete_!: (%,Integer) -> %
      ++ delete!(u,i) destructively deletes the \axiom{i}th element of u.
      ++
      ++E Data:=Record(age:Integer,gender:String)
      ++E a1:AssociationList(String,Data):=table()
      ++E a1."tim":=[55,"male"]$Data
      ++E delete!(a1,1)

    delete_!: (%,UniversalSegment(Integer)) -> %
      ++ delete!(u,i..j) destructively deletes elements u.i through u.j.
    remove_!: (S->Boolean,%) -> %
      ++ remove!(p,u) destructively removes all elements x of
      ++ u such that \axiom{p(x)} is true.
    insert_!: (S,%,Integer) -> %
      ++ insert!(x,u,i) destructively inserts x into u at position i.
    insert_!: (%,%,Integer) -> %
      ++ insert!(v,u,i) destructively inserts aggregate v into u
      ++ at position i.
    merge_!: ((S,S)->Boolean,%,%) -> %
      ++ merge!(p,u,v) destructively merges u and v using predicate p.
    select_!: (S->Boolean,%) -> %
      ++ select!(p,u) destructively changes u by keeping only values
      ++ x such that \axiom{p(x)}.
    if S has SetCategory then
      remove_!: (S,%) -> %
        ++ remove!(x,u) destructively removes all values x from u.
      removeDuplicates_!: % -> %
        ++ removeDuplicates!(u) destructively removes duplicates from u.
    if S has OrderedSet then merge_!: (%,%) -> %
        ++ merge!(u,v) destructively merges u and v in ascending order.
  add
    delete(x:%, i:Integer)          == delete_!(copy x, i)
    delete(x:%, i:UniversalSegment(Integer))          == delete_!(copy x, i)
    remove(f:S -> Boolean, x:%)   == remove_!(f, copy x)
    insert(s:S, x:%, i:Integer)   == insert_!(s, copy x, i)
    insert(w:%, x:%, i:Integer)   == insert_!(copy w, copy x, i)
    select(f, x)                   == select_!(f, copy x)
    concat(x:%, y:%)          == concat_!(copy x, y)
    concat(x:%, y:S)          == concat_!(copy x, new(1, y))
    concat_!(x:%, y:S)          == concat_!(x, new(1, y))
    if S has SetCategory then
      remove(s:S, x:%)             == remove_!(s, copy x)
      remove_!(s:S, x:%)             == remove_!(y +-> y = s, x)
      removeDuplicates(x:%)   == removeDuplicates_!(copy x)

    if S has OrderedSet then
      merge_!(x, y) == merge_!(_<$S, x, y)
```

⟨*ELAGG.dotabb*⟩≡

```
"ELAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ELAGG" -> "LNAGG"
```

⟨*ELAGG.dotfull*⟩≡

```
"ExtensibleLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*ELAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

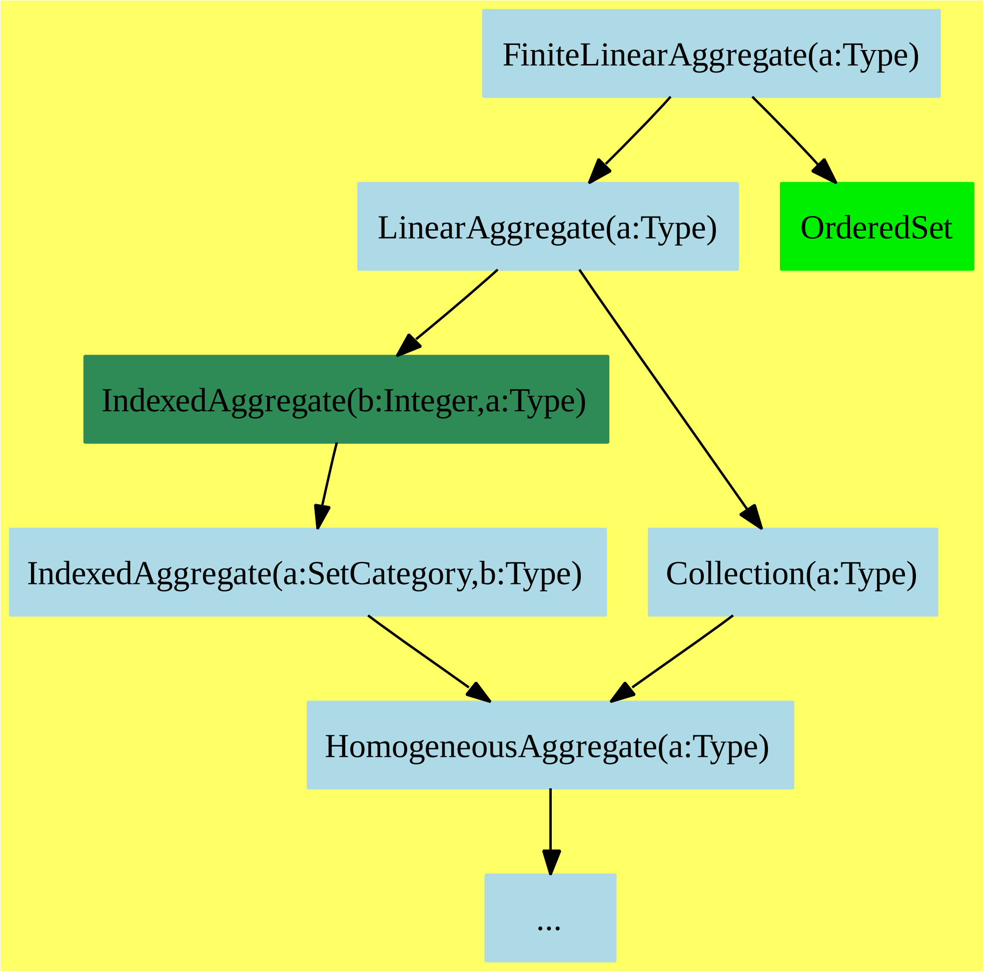
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.102 FiniteLinearAggregate (FLAGG)


FiniteLinearAggregate(a:Type)
LinearAggregate(a:Type)
OrderedSet
IndexedAggregate(b:Integer,a:Type)
IndexedAggregate(a:SetCategory,b:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
...

⟨*FiniteLinearAggregate.input*⟩≡

```
)set break resume
)sys rm -f FiniteLinearAggregate.output
)spool FiniteLinearAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteLinearAggregate
--R FiniteLinearAggregate S: Type  is a category constructor
--R Abbreviation for FiniteLinearAggregate is FLAGG
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLAGG
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (S,%) -> %                   concat : (%,S) -> %
--R construct : List S -> %               copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %           map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S               reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
```

```
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteLinearAggregate.help*⟩≡

```
====================================================================
FiniteLinearAggregate examples
====================================================================

See Also:
o )show FiniteLinearAggregate
```

See:

⇒ “OneDimensionalArrayAggregate” (A1AGG) 8.0.117 on page 754
⇒ “ListAggregate” (LSAGG) 8.0.114 on page 727
⇐ “LinearAggregate” (LNAGG) 6.0.87 on page 449
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242

Exports:

any? coerce concat construct convert
copy copyInto! count delete elt
empty empty? entries entry? eq?
eval every? fill! find first
hash index? indices insert latex
less? map map! max maxIndex
member? members merge min minIndex
more? new parts position qelt
qsetelt! reduce remove removeDuplicates reverse
reverse! sample select setelt size?
sort sort! sorted? swap! #?
?.? ?<? ?<=? ?=? ?>?
?>=? ?~=?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
sorted? : (((S,S) -> Boolean),%) -> Boolean
position : (S,%,Integer) -> Integer if S has SETCAT
position : ((S -> Boolean),%) -> Integer
reverse! : % -> % if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
```

These are implemented by this category:

```
merge : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : % -> % if S has ORDSET
sort : (((S,S) -> Boolean),%) -> %
sorted? : % -> Boolean if S has ORDSET
sort! : % -> %
         if S has ORDSET and $ has shallowlyMutable
```

These exports come from (p449) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (S,%) -> %
concat : (%,S) -> %
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
```

```
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,S) -> %
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,Integer,S) -> S
         if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger
         if $ has finiteAggregate
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
?~=? : (%,%) -> Boolean if S has SETCAT
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p242) OrderedSet:

```
max : (%,%) -> % if S has ORDSET
```

```
min : (%,%) -> % if S has ORDSET
?<? : (%,%) -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

⟨*category FLAGG FiniteLinearAggregate*⟩≡

```
)abbrev category FLAGG FiniteLinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A finite linear aggregate is a linear aggregate of finite length.
++ The finite property of the aggregate adds several exports to the
++ list of exports from \spadtype{LinearAggregate} such as
++ \spadfun{reverse}, \spadfun{sort}, and so on.

FiniteLinearAggregate(S:Type): Category == LinearAggregate S with
   finiteAggregate
   merge: ((S,S)->Boolean,%,%) -> %
      ++ merge(p,a,b) returns an aggregate c which merges \axiom{a} and b.
      ++ The result is produced by examining each element x of \axiom{a}
      ++ and y of b successively. If \axiom{p(x,y)} is true, then x is
      ++ inserted into the result; otherwise y is inserted. If x is
      ++ chosen, the next element of \axiom{a} is examined, and so on.
      ++ When all the elements of one aggregate are examined, the
      ++ remaining elements of the other are appended.
      ++ For example, \axiom{merge(<,[1,3],[2,7,5])} returns
      ++ \axiom{[1,2,3,7,5]}.
   reverse: % -> %
      ++ reverse(a) returns a copy of \axiom{a} with elements
      ++ in reverse order.
   sort: ((S,S)->Boolean,%) -> %
      ++ sort(p,a) returns a copy of \axiom{a} sorted using total ordering
      ++ predicate p.
   sorted?: ((S,S)->Boolean,%) -> Boolean
      ++ sorted?(p,a) tests if \axiom{a} is sorted according to predicate p.
   position: (S->Boolean, %) -> Integer
      ++ position(p,a) returns the index i of the first x in \axiom{a}
      ++ such that \axiom{p(x)} is true, and \axiom{minIndex(a) - 1}
      ++ if there is no such x.
```

```
     if S has SetCategory then
        position: (S, %)        -> Integer
          ++ position(x,a) returns the index i of the first occurrence of
          ++ x in a, and \axiom{minIndex(a) - 1} if there is no such x.
        position: (S,%,Integer) -> Integer
          ++ position(x,a,n) returns the index i of the first occurrence of
          ++ x in \axiom{a} where \axiom{i >= n}, and \axiom{minIndex(a) - 1}
          ++ if no such x is found.
     if S has OrderedSet then
        OrderedSet
        merge: (%,%) -> %
          ++ merge(u,v) merges u and v in ascending order.
          ++ Note that \axiom{merge(u,v) = merge(<=,u,v)}.
        sort: % -> %
          ++ sort(u) returns an u with elements in ascending order.
          ++ Note that \axiom{sort(u) = sort(<=,u)}.
        sorted?: % -> Boolean
          ++ sorted?(u) tests if the elements of u are in ascending order.
     if % has shallowlyMutable then
        copyInto_!: (%,%,Integer) -> %
          ++ copyInto!(u,v,i) returns aggregate u containing a copy of
          ++ v inserted at element i.
        reverse_!: % -> %
          ++ reverse!(u) returns u with its elements in reverse order.
        sort_!: ((S,S)->Boolean,%) -> %
          ++ sort!(p,u) returns u with its elements ordered by p.
        if S has OrderedSet then sort_!: % -> %
          ++ sort!(u) returns u with its elements in ascending order.
  add
      if S has SetCategory then
        position(x:S, t:%) == position(x, t, minIndex t)

      if S has OrderedSet then
--      sorted? l          == sorted?(_<$S, l)
        sorted? l          == sorted?((x,y) +-> x < y or x = y, l)
        merge(x, y) == merge(_<$S, x, y)
        sort l          == sort(_<$S, l)

      if % has shallowlyMutable then
        reverse x         == reverse_! copy x
        sort(f, l) == sort_!(f, copy l)

        if S has OrderedSet then
          sort_! l == sort_!(_<$S, l)
```

⟨*FLAGG.dotabb*⟩≡

```
"FLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FLAGG" -> "LNAGG"
```

⟨*FLAGG.dotfull*⟩≡

```
"FiniteLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*FLAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
"FiniteLinearAggregate(a:Type)" -> "OrderedSet"

"OrderedSet" [color="#00EE00"];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

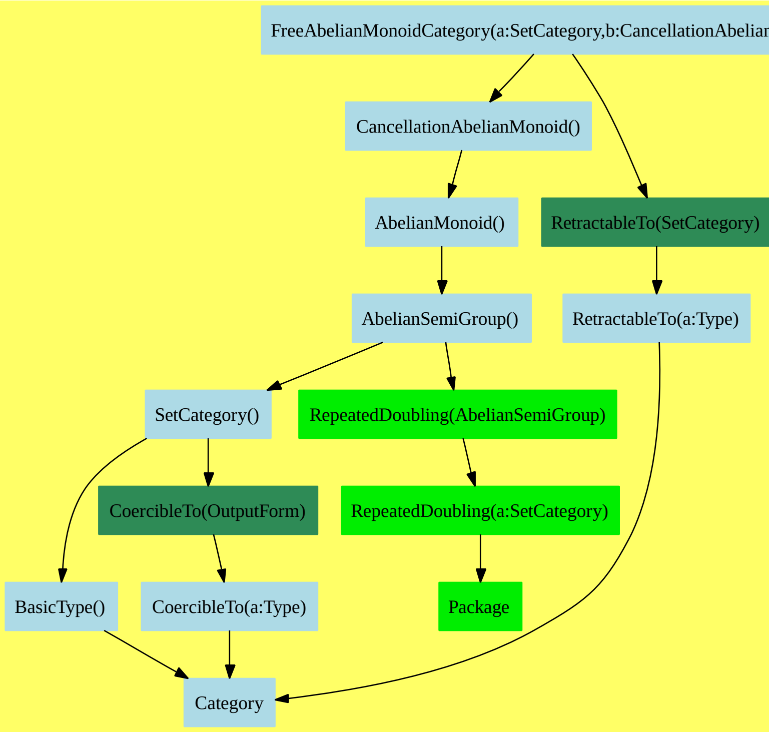
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.103 FreeAbelianMonoidCategory (FAMONC)


FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelian
CancellationAbelianMonoid()
AbelianMonoid()
RetractableTo(SetCategory)
AbelianSemiGroup()
RetractableTo(a:Type)
SetCategory()
RepeatedDoubling(AbelianSemiGroup)
CoercibleTo(OutputForm)
RepeatedDoubling(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*FreeAbelianMonoidCategory.input*⟩≡
```
)set break resume
)sys rm -f FreeAbelianMonoidCategory.output
)spool FreeAbelianMonoidCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianMonoidCategory
--R FreeAbelianMonoidCategory(S: SetCategory,E: CancellationAbelianMonoid)  is a
--R Abbreviation for FreeAbelianMonoidCategory is FAMONC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAMONC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (E,S) -> %                      ?*? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                      ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
```

```
--R coefficient : (S,%) -> E             coerce : S -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  mapCoef : ((E -> E),%) -> %
--R mapGen : ((S -> S),%) -> %           nthCoef : (%,Integer) -> E
--R nthFactor : (%,Integer) -> S         retract : % -> S
--R sample : () -> %                     size : % -> NonNegativeInteger
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R highCommonTerms : (%,%) -> % if E has OAMON
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: E)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FreeAbelianMonoidCategory.help*⟩≡

```
====================================================================
FreeAbelianMonoidCategory examples
====================================================================

See Also:
o )show FreeAbelianMonoidCategory
```

See:

⇐ "CancellationAbelianMonoid" (CABMON) 6.0.83 on page 423
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size	subtractIfCan
terms	zero?	?*?	?+?	?=?
?~=?				

These are directly exported but not implemented:

```
coefficient : (S,%) -> E
highCommonTerms : (%,%) -> % if E has OAMON
mapCoef : ((E -> E),%) -> %
mapGen : ((S -> S),%) -> %
nthCoef : (%,Integer) -> E
nthFactor : (%,Integer) -> S
size : % -> NonNegativeInteger
terms : % -> List Record(gen: S,exp: E)
?+? : (S,%) -> %
?*? : (E,S) -> %
```

These exports come from (p423) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p62) RetractableTo(SetCategory):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

⟨*category FAMONC FreeAbelianMonoidCategory*⟩≡

```
)abbrev category FAMONC FreeAbelianMonoidCategory
++ Category for free abelian monoid on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ A free abelian monoid on a set S is the monoid of finite sums of
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are in a given abelian monoid. The operation is commutative.

FreeAbelianMonoidCategory(S: SetCategory, E:CancellationAbelianMonoid): _
  Category == Join(CancellationAbelianMonoid, RetractableTo S) with
        "+"        : (S, $) -> $
          ++ s + x returns the sum of s and x.
        "*"        : (E, S) -> $
          ++ e * s returns e times s.
        size       : $ -> NonNegativeInteger
          ++ size(x) returns the number of terms in x.
          ++ mapGen(f, a1\^e1 ... an\^en) returns
          ++\spad{f(a1)\^e1 ... f(an)\^en}.
        terms      : $ -> List Record(gen: S, exp: E)
          ++ terms(e1 a1 + ... + en an) returns \spad{[[a1, e1],...,[an, en]]}.
        nthCoef    : ($, Integer) -> E
          ++ nthCoef(x, n) returns the coefficient of the n^th term of x.
        nthFactor  : ($, Integer) -> S
          ++ nthFactor(x, n) returns the factor of the n^th term of x.
        coefficient: (S, $) -> E
          ++ coefficient(s, e1 a1 + ... + en an) returns ei such that
          ++ ai = s, or 0 if s is not one of the ai's.
        mapCoef    : (E -> E, $) -> $
          ++ mapCoef(f, e1 a1 +...+ en an) returns
          ++ \spad{f(e1) a1 +...+ f(en) an}.
        mapGen     : (S -> S, $) -> $
          ++ mapGen(f, e1 a1 +...+ en an) returns
          ++ \spad{e1 f(a1) +...+ en f(an)}.
        if E has OrderedAbelianMonoid then
          highCommonTerms: ($, $) -> $
            ++ highCommonTerms(e1 a1 + ... + en an, f1 b1 + ... + fm bm)
            ++ returns \spad{reduce(+,[max(ei, fi) ci])}
            ++ where ci ranges in the intersection
            ++ of \spad{{a1,...,an}} and \spad{{b1,...,bm}}.
```

⟨*FAMONC.dotabb*⟩≡
```
"FAMONC"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FAMONC" -> "CABMON"
"FAMONC" -> "RETRACT"
```

⟨*FAMONC.dotfull*⟩≡
```
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"
```

⟨*FAMONC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

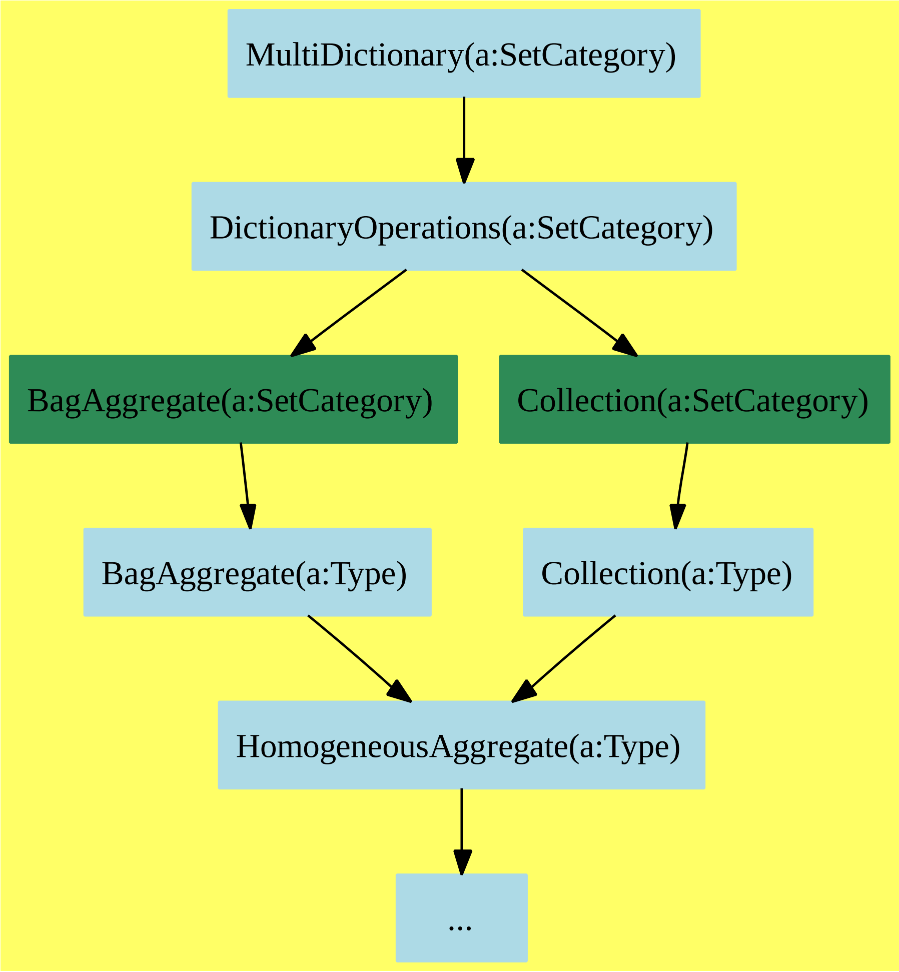
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

7.0.104 MultiDictionary (MDAGG)


MultiDictionary(a:SetCategory)
DictionaryOperations(a:SetCategory)
BagAggregate(a:SetCategory)
Collection(a:SetCategory)
BagAggregate(a:Type)
Collection(a:Type)
HomogeneousAggregate(a:Type)
...

⟨*MultiDictionary.input*⟩≡

```
)set break resume
)sys rm -f MultiDictionary.output
)spool MultiDictionary.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MultiDictionary
--R MultiDictionary S: SetCategory  is a category constructor
--R Abbreviation for MultiDictionary is MDAGG
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MDAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                     construct : List S -> %
--R copy : % -> %                         dictionary : List S -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      map : ((S -> S),%) -> %
--R removeDuplicates! : % -> %            sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List Record(entry: S,count: NonNegativeInteger)
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R insert! : (S,%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
```

```
--R
--E 1

)spool
)lisp (bye)
```

⟨MultiDictionary.help⟩≡

```
====================================================================
MultiDictionary examples
====================================================================

See Also:
o )show MultiDictionary
```

See:

⇒ “MultisetAggregate” (MSETAGG) 8.0.115 on page 742
⇐ “DictionaryOperations” (DIOPS) 6.0.84 on page 428

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	duplicates	empty
empty?	eq?	eval	every?	extract!
find	hash	insert!	inspect	latex
less?	map	map!	member?	members
more?	parts	reduce	remove	remove!
removeDuplicates	removeDuplicates!	sample	select	select!
size?	#?	?=?	?~=?	

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
insert! : (S,%,NonNegativeInteger) -> %
removeDuplicates! : % -> %
duplicates : % -> List Record(entry:S,count:NonNegativeInteger)
```

These exports come from (p428) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
```

```
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
sample : () -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
         if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category MDAGG MultiDictionary*⟩≡

```
)abbrev category MDAGG MultiDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
```

```
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A multi-dictionary is a dictionary which may contain duplicates.
++ As for any dictionary, its size is assumed large so that
++ copying (non-destructive) operations are generally to be avoided.

MultiDictionary(S:SetCategory): Category == DictionaryOperations S with
-- count: (S,%) -> NonNegativeInteger              ++ multiplicity count
   insert_!: (S,%,NonNegativeInteger) -> %
     ++ insert!(x,d,n) destructively inserts n copies of x into dictionary d.
-- remove_!: (S,%,NonNegativeInteger) -> %
--   ++ remove!(x,d,n) destructively removes (up to) n copies of x from
--   ++ dictionary d.
   removeDuplicates_!: % -> %
     ++ removeDuplicates!(d) destructively removes any duplicate values
     ++ in dictionary d.
   duplicates: % -> List Record(entry:S,count:NonNegativeInteger)
     ++ duplicates(d) returns a list of values which have duplicates in d
--   ++ duplicates(d) returns a list of            ++ duplicates iterator
-- to become duplicates: % -> Iterator(D,D)
```

⟨*MDAGG.dotabb*⟩≡

```
"MDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
"MDAGG" -> "DIOPS"
```

⟨*MDAGG.dotfull*⟩≡

```
"MultiDictionary(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"
```

⟨*MDAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

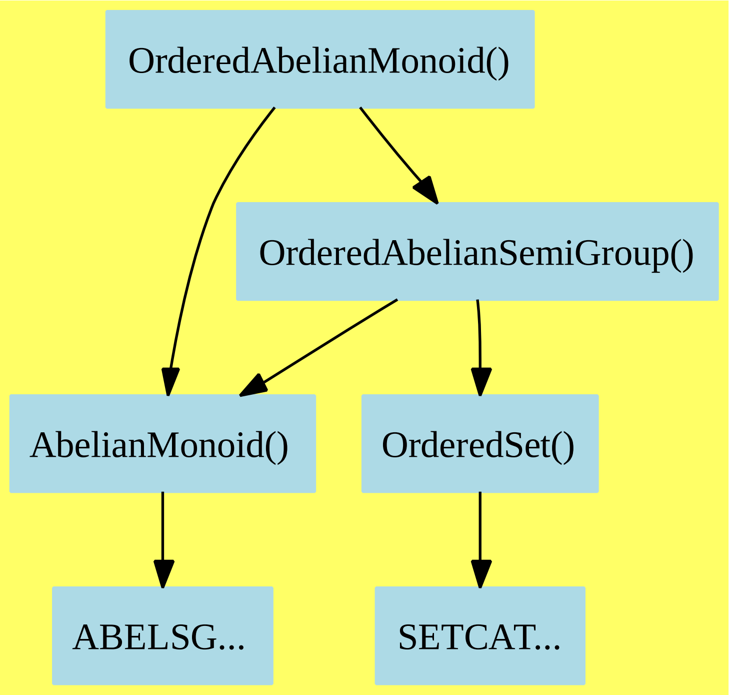
"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.105 OrderedAbelianMonoid (OAMON)


OrderedAbelianMonoid()
OrderedAbelianSemiGroup()
AbelianMonoid()
OrderedSet()
ABELSG...
SETCAT...

⟨*OrderedAbelianMonoid.input*⟩≡
```
)set break resume
)sys rm -f OrderedAbelianMonoid.output
)spool OrderedAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianMonoid
--R OrderedAbelianMonoid  is a category constructor
--R Abbreviation for OrderedAbelianMonoid is OAMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               0 : () -> %
```

```
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedAbelianMonoid.help*⟩≡

```
====================================================================
OrderedAbelianMonoid examples
====================================================================

See Also:
o )show OrderedAbelianMonoid
```

See:

⇒ "OrderedCancellationAbelianMonoid" (OCAMON) 8.0.118 on page 767
⇐ "AbelianMonoid" (ABELMON) 5.0.66 on page 301
⇐ "OrderedAbelianSemiGroup" (OASGP) 6.0.89 on page 507

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?*?	?+?
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These exports come from (p507) OrderedAbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p301) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

⟨*category OAMON OrderedAbelianMonoid*⟩≡

```
)abbrev category OAMON OrderedAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian monoids, such that the addition
++ preserves the ordering.

OrderedAbelianMonoid(): Category ==
        Join(OrderedAbelianSemiGroup, AbelianMonoid)
```

⟨OAMON.dotabb⟩≡

```
"OAMON" [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OAMON" -> "OASGP"
"OAMON" -> "ABELMON"
```

⟨OAMON.dotfull⟩≡

```
"OrderedAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"
```

⟨OAMON.dotpic⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

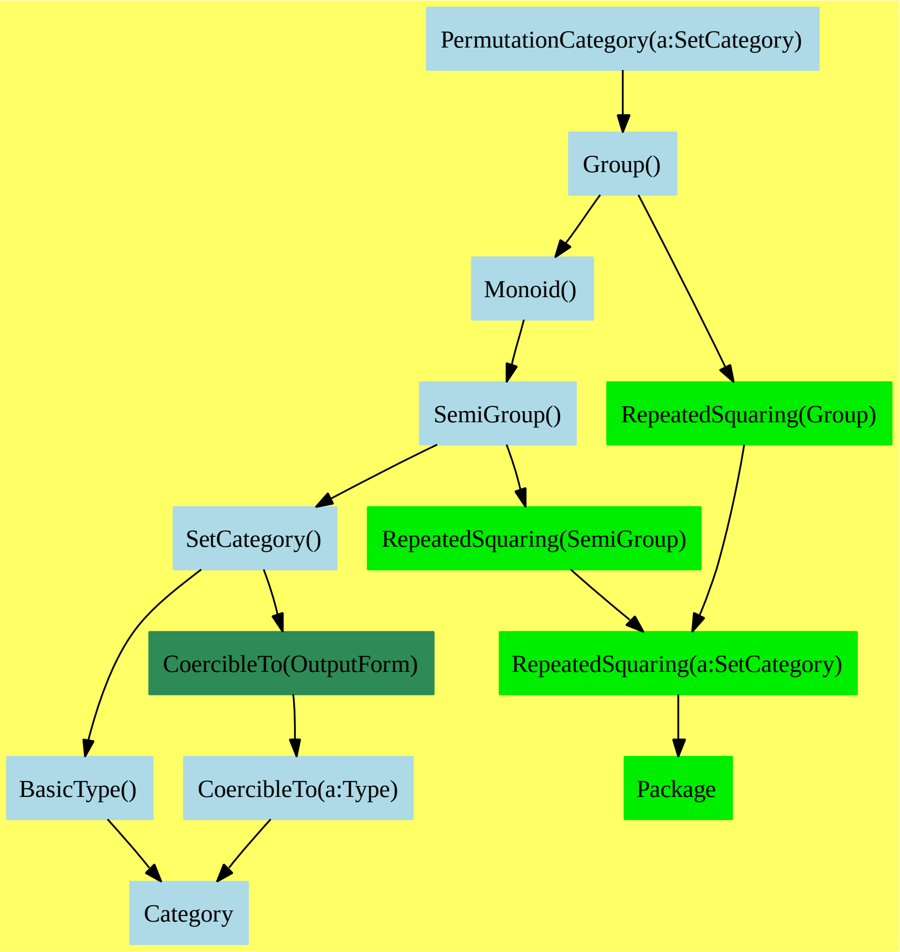
"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

7.0.106 PermutationCategory (PERMCAT)


PermutationCategory(a:SetCategory)
Group()
Monoid()
SemiGroup()
RepeatedSquaring(Group)
SetCategory()
RepeatedSquaring(SemiGroup)
CoercibleTo(OutputForm)
RepeatedSquaring(a:SetCategory)
BasicType()
CoercibleTo(a:Type)
Package
Category

⟨*PermutationCategory.input*⟩≡

```
)set break resume
)sys rm -f PermutationCategory.output
)spool PermutationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PermutationCategory
--R PermutationCategory S: SetCategory  is a category constructor
--R Abbreviation for PermutationCategory is PERMCAT
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PERMCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R 1 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> OutputForm
--R commutator : (%,%) -> %               conjugate : (%,%) -> %
--R cycle : List S -> %                   cycles : List List S -> %
--R ?.? : (%,S) -> S                      eval : (%,S) -> S
--R hash : % -> SingleInteger             inv : % -> %
--R latex : % -> String                   one? : % -> Boolean
--R orbit : (%,S) -> Set S                recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R ?>? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R ?>=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R max : (%,%) -> % if S has ORDSET or S has FINITE
--R min : (%,%) -> % if S has ORDSET or S has FINITE
--R
--E 1

)spool
)lisp (bye)
```

⟨*PermutationCategory.help*⟩≡

```
===================================================================
PermutationCategory examples
===================================================================

See Also:
o )show PermutationCategory
```

See:

⇐ "Group" (GROUP) 6.0.86 on page 443

Exports:

1 coerce commutator conjugate cycle
cycles eval hash inv latex
max min one? orbit recip
sample ?ˆ? ?.? ?˜=? ?**?
?<? ?<=? ?>? ?>=? ?*?
?/? ?=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
cycle : List S -> %
cycles : List List S -> %
eval : (%,S) -> S
orbit : (%,S) -> Set S
?<? : (%,%) -> Boolean
?.? : (%,S) -> S
```

These exports come from (p443) Group():

```
1 : () -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
conjugate : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?/? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if S has ORDSET or S has FINITE
min : (%,%) -> % if S has ORDSET or S has FINITE
?>? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?<=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?>=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
```

⟨*category PERMCAT PermutationCategory*⟩≡

```
)abbrev category PERMCAT PermutationCategory
++ Authors:  Holger Gollan, Johannes Grabmeier, Gerhard Schneider
++ Date Created: 27 July 1989
++ Date Last Updated: 29 March 1990
++ Basic Operations: cycle, cycles, eval, orbit
++ Related Constructors: PermutationGroup, PermutationGroupExamples
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords: permutation, symmetric group
++ References:
++ Description:
++ PermutationCategory provides a categorial environment
++ for subgroups of bijections of a set (i.e. permutations)

PermutationCategory(S:SetCategory): Category  ==  Group with
    cycle   :  List S       ->  %
      ++ cycle(ls) coerces a cycle ls, i.e. a list with not
      ++ repetitions to a permutation, which maps ls.i to
      ++ ls.i+1, indices modulo the length of the list.
      ++ Error: if repetitions occur.
    cycles  :  List List S  ->  %
      ++ cycles(lls) coerces a list list of cycles lls
      ++ to a permutation, each cycle being a list with not
      ++ repetitions, is coerced to the permutation, which maps
      ++ ls.i to ls.i+1, indices modulo the length of the list,
      ++ then these permutations are mutiplied.
      ++ Error: if repetitions occur in one cycle.
    eval  :  (%,S)          ->  S
      ++ eval(p, el) returns the image of el under the
      ++ permutation p.
    elt  :  (%,S)          ->  S
      ++ elt(p, el) returns the image of el under the
      ++ permutation p.
    orbit :  (%,S)          ->  Set S
      ++ orbit(p, el) returns the orbit of el under the
      ++ permutation p, i.e. the set which is given by applications of
      ++ the powers of p to el.
    "<" : (%,%)             ->  Boolean
```

```
        ++ p < q is an order relation on permutations.
        ++ Note that this order is only total if and only if S is totally ordered
        ++ or S is finite.
      if S has OrderedSet then OrderedSet
      if S has Finite then OrderedSet
```

⟨*PERMCAT.dotabb*⟩≡

```
  "PERMCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
  "PERMCAT" -> "GROUP"
```

⟨*PERMCAT.dotfull*⟩≡

```
  "PermutationCategory(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
  "PermutationCategory(a:SetCategory)" -> "Group()"
```

⟨*PERMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PermutationCategory(a:SetCategory)" [color=lightblue];
"PermutationCategory(a:SetCategory)" -> "Group()"

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

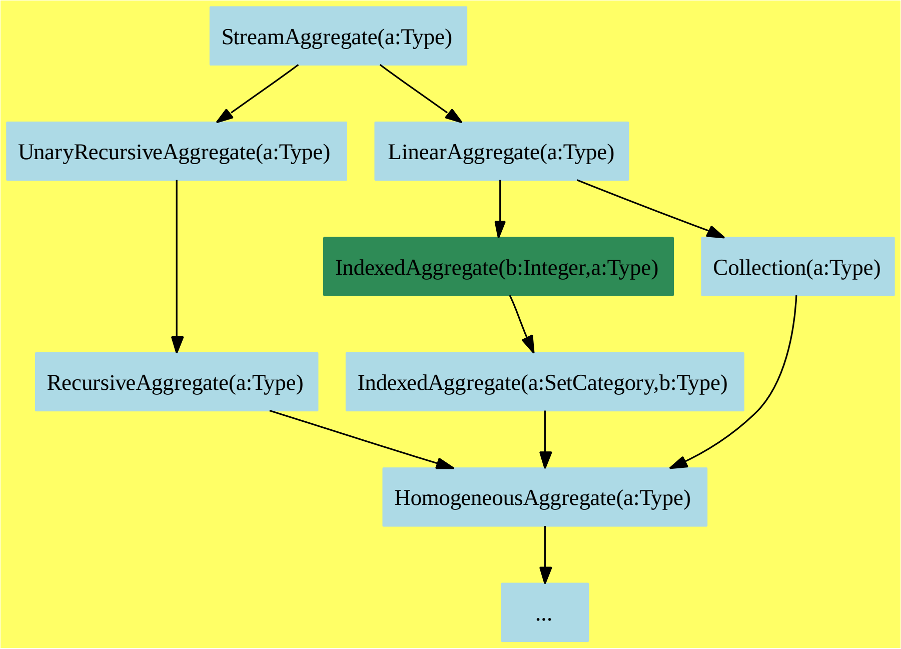
"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

7.0.107 StreamAggregate (STAGG)


StreamAggregate(a:Type)
UnaryRecursiveAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
Collection(a:Type)
RecursiveAggregate(a:Type)
IndexedAggregate(a:SetCategory,b:Type)
HomogeneousAggregate(a:Type)
...

⟨*StreamAggregate.input*⟩≡

```
)set break resume
)sys rm -f StreamAggregate.output
)spool StreamAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StreamAggregate
--R StreamAggregate S: Type  is a category constructor
--R Abbreviation for StreamAggregate is STAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %                concat : (%,S) -> %
--R concat : List % -> %                  concat : (S,%) -> %
--R concat : (%,%) -> %                   construct : List S -> %
--R copy : % -> %                         cycleEntry : % -> %
--R cycleTail : % -> %                    cyclic? : % -> Boolean
--R delete : (%,Integer) -> %             distance : (%,%) -> Integer
```

```
--R elt : (%,Integer,S) -> S              ?.? : (%,Integer) -> S
--R ?.last : (%,last) -> S                ?.rest : (%,rest) -> %
--R ?.first : (%,first) -> S              ?.value : (%,value) -> S
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List S                 eq? : (%,%) -> Boolean
--R explicitlyFinite? : % -> Boolean      first : % -> S
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (S,%,Integer) -> %           insert : (%,%,Integer) -> %
--R last : % -> S                         leaf? : % -> Boolean
--R leaves : % -> List S                  map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R nodes : % -> List %                   possiblyInfinite? : % -> Boolean
--R qelt : (%,Integer) -> S               rest : % -> %
--R sample : () -> %                      second : % -> S
--R tail : % -> %                         third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
```

```
--R members : % -> List S if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R rest : (%,NonNegativeInteger) -> %
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*StreamAggregate.help*⟩≡

```
====================================================================
StreamAggregate examples
====================================================================

See Also:
o )show StreamAggregate
```

See:

⇒ "LazyStreamAggregate" (LZSTAGG) 8.0.112 on page 702
⇐ "LinearAggregate" (LNAGG) 6.0.87 on page 449
⇐ "UnaryRecursiveAggregate" (URAGG) 6.0.96 on page 563

Exports:

any?
children
child?
coerce
concat
concat!
construct
convert
copy
count
cycleEntry
cycleLength
cycleSplit!
cycleTail
cyclic?
delete
distance
elt
empty
empty?
entries
entry?
eq?
eval
every?
explicitlyFinite?
fill!
find
first
hash
index?
indices
insert
last
latex
leaf?
leaves
less?
map
map!
maxIndex
member?
members
minIndex
more?
new
nodes
node?
parts
possiblyInfinite?
qelt
qsetelt!
reduce
remove
removeDuplicates
rest
sample
second
select
setchildren!
setelt
setfirst!
setlast!
setrest!
setvalue!
size?
split!
swap!
tail
third
value
#?
?=?
?.?
?.first
?.last
?.rest
?.value
?~=?

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
concat : (%,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
fill! : (%,S) -> % if $ has shallowlyMutable
first : (%,NonNegativeInteger) -> %
explicitlyFinite? : % -> Boolean
map! : ((S -> S),%) -> % if $ has shallowlyMutable
possiblyInfinite? : % -> Boolean
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
```

```
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
```

These exports come from (p563) UnaryRecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (S,%) -> %
concat : (%,S) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
```

```
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

These exports come from (p449) LinearAggregate(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,S) -> S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
find : ((S -> Boolean),%) -> Union(S,"failed")
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
map : (((S,S) -> S),%,%) -> %
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT
         and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT
         and $ has finiteAggregate
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
```

⟨*category STAGG StreamAggregate*⟩≡

```
)abbrev category STAGG StreamAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A stream aggregate is a linear aggregate which possibly has an infinite
++ number of elements. A basic domain constructor which builds stream
++ aggregates is \spadtype{Stream}. From streams, a number of infinite
++ structures such power series can be built. A stream aggregate may
++ also be infinite since it may be cyclic.
++ For example, see \spadtype{DecimalExpansion}.

StreamAggregate(S:Type): Category ==
  Join(UnaryRecursiveAggregate S, LinearAggregate S) with
     explicitlyFinite?: % -> Boolean
       ++ explicitlyFinite?(s) tests if the stream has a finite
       ++ number of elements, and false otherwise.
       ++ Note that for many datatypes,
       ++ \axiom{explicitlyFinite?(s) = not possiblyInfinite?(s)}.
     possiblyInfinite?: % -> Boolean
       ++ possiblyInfinite?(s) tests if the stream s could possibly
       ++ have an infinite number of elements.
       ++ Note that for many datatypes,
       ++ \axiom{possiblyInfinite?(s) = not explictlyFinite?(s)}.
add
  c2: (%, %) -> S

  explicitlyFinite? x == not cyclic? x
  possiblyInfinite? x == cyclic? x
  first(x, n)              == construct [c2(x, x := rest x) for i in 1..n]

  c2(x, r) ==
    empty? x => error "Index out of range"
    first x

  elt(x:%, i:Integer) ==
    i := i - minIndex x
    (i < 0) or empty?(x := rest(x, i::NonNegativeInteger)) => _
        error "index out of range"
    first x

  elt(x:%, i:UniversalSegment(Integer)) ==
    l := lo(i) - minIndex x
    l < 0 => error "index out of range"
    not hasHi i => copy(rest(x, l::NonNegativeInteger))
    (h := hi(i) - minIndex x) < l => empty()
    first(rest(x, l::NonNegativeInteger), (h - l + 1)::NonNegativeInteger)

  if % has shallowlyMutable then
    concat(x:%, y:%) == concat_!(copy x, y)

    concat l ==
      empty? l => empty()
      concat_!(copy first l, concat rest l)

    map_!(f, l) ==
      y := l
      while not empty? l repeat
        setfirst_!(l, f first l)
        l := rest l
      y

    fill_!(x, s) ==
      y := x
      while not empty? y repeat (setfirst_!(y, s); y := rest y)
      x

    setelt(x:%, i:Integer, s:S) ==
      i := i - minIndex x
      (i < 0) or empty?(x := rest(x,i::NonNegativeInteger)) => _
          error "index out of range"
      setfirst_!(x, s)

    setelt(x:%, i:UniversalSegment(Integer), s:S) ==
      (l := lo(i) - minIndex x) < 0 => error "index out of range"
      h := if hasHi i then hi(i) - minIndex x else maxIndex x
      h < l => s
      y := rest(x, l::NonNegativeInteger)
      z := rest(y, (h - l + 1)::NonNegativeInteger)
      while not eq?(y, z) repeat (setfirst_!(y, s); y := rest y)
      s

    concat_!(x:%, y:%) ==
      empty? x => y
      setrest_!(tail x, y)
      x
```

⟨*STAGG.dotabb*⟩≡

```
"STAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
"STAGG" -> "RCAGG"
"STAGG" -> "LNAGG"
```

⟨*STAGG.dotfull*⟩≡

```
"StreamAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*STAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

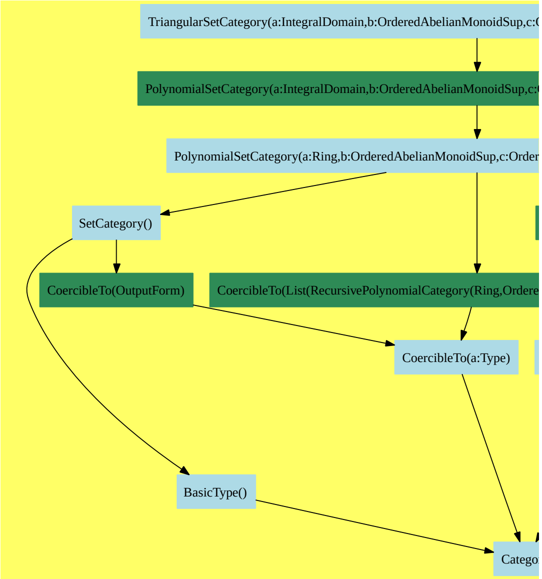
"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];

}
```

7.0.108 TriangularSetCategory (TSETCAT)


TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:
PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:
PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:Order
SetCategory()
CoercibleTo(OutputForm)
CoercibleTo(List(RecursivePolynomialCategory(Ring,Ordere
CoercibleTo(a:Type)
BasicType()
Catego

⟨*TriangularSetCategory.input*⟩≡

```
)set break resume
)sys rm -f TriangularSetCategory.output
)spool TriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TriangularSetCategory
--R TriangularSetCategory(R: IntegralDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: Recu
--R Abbreviation for TriangularSetCategory is TSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List V      coerce : % -> List P
--R coerce : % -> OutputForm              collect : (%,V) -> %
--R collectQuasiMonic : % -> %            collectUnder : (%,V) -> %
--R collectUpper : (%,V) -> %             construct : List P -> %
--R copy : % -> %                         degree : % -> NonNegativeInteger
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extend : (%,P) -> %
```

```
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List P                last : % -> Union(P,"failed")
--R latex : % -> String                   mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List V           map : ((P -> P),%) -> %
--R mvar : % -> V                         normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean        reduceByQuasiMonic : (P,%) -> P
--R removeZero : (P,%) -> P               rest : % -> Union(%,"failed")
--R retract : List P -> %                 sample : () -> %
--R stronglyReduce : (P,%) -> P           stronglyReduced? : % -> Boolean
--R trivialIdeal? : % -> Boolean          variables : % -> List V
--R zeroSetSplit : List P -> List %       ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"f
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List P)
--R
--E 1

)spool
)lisp (bye)
```

⟨*TriangularSetCategory.help*⟩≡

```
====================================================================
TriangularSetCategory examples
====================================================================

See Also:
o )show TriangularSetCategory
```

See:

⇒ “RegularTriangularSetCategory” (RSETCAT) 8.0.119 on page 772
⇐ “PolynomialSetCategory” (PSETCAT) 6.0.91 on page 518

Exports:

algebraic?
algebraicVariables
any?
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
convert
copy
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
last
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
quasiComponent
reduce
reduced?
reduceByQuasiMonic
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?˜=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
extendIfCan : (%,P) -> Union(%,"failed")
zeroSetSplit : List P -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
```

These are implemented by this category:

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet : (List P,((P,P) -> Boolean)) ->
  Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coHeight : % -> NonNegativeInteger if V has FINITE
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
degree : % -> NonNegativeInteger
extend : (%,P) -> %
first : % -> Union(P,"failed")
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
mvar : % -> V
normalized? : % -> Boolean
```

```
normalized? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retractIfCan : List P -> Union(%,"failed")
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
select : (%,V) -> Union(P,"failed")
stronglyReduce : (P,%) -> P
stronglyReduced? : % -> Boolean
stronglyReduced? : (P,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p518) PolynomialSetCategory(R,E,V,P)
where R:IntegralDomain, E:OrderedAbelianMonoidSup,
V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm
collect : (%,V) -> %
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
hash : % -> SingleInteger
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean
  if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
```

```
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
  if P has SETCAT
  and $ has finiteAggregate
retract : List P -> %
rewriteIdealWithHeadRemainder : (List P,%) -> List P
  if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P
  if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
```

⟨*category TSETCAT TriangularSetCategory*⟩≡

```
)abbrev category TSETCAT TriangularSetCategory
++ Author: Marc Moreno Maza (marc@nag.co.uk)
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References :
++  [1] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++ Description:
++ The category of triangular sets of multivariate polynomials
++ with coefficients in an integral domain.
++ Let \axiom{R} be an integral domain and \axiom{V} a finite ordered set of
++ variables, say \axiom{X1 < X2 < ... < Xn}.
++ A set \axiom{S} of polynomials in \axiom{R[X1,X2,...,Xn]} is triangular
++ if no elements of \axiom{S} lies in \axiom{R}, and if two distinct
++ elements of \axiom{S} have distinct main variables.
++ Note that the empty set is a triangular set. A triangular set is not
++ necessarily a (lexicographical) Groebner basis and the notion of
++ reduction related to triangular sets is based on the recursive view
++ of polynomials. We recall this notion here and refer to [1] for more
++ details.
++ A polynomial \axiom{P} is reduced w.r.t a non-constant polynomial
++ \axiom{Q} if the degree of \axiom{P} in the main variable of \axiom{Q}
++ is less than the main degree of \axiom{Q}.
++ A polynomial \axiom{P} is reduced w.r.t a triangular set \axiom{T}
++ if it is reduced w.r.t. every polynomial of \axiom{T}.

TriangularSetCategory(R:IntegralDomain,E:OrderedAbelianMonoidSup,_
 V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
        Category ==
   PolynomialSetCategory(R,E,V,P) with
     finiteAggregate
     shallowlyMutable

     infRittWu? : ($,$) -> Boolean
       ++ \axiom{infRittWu?(ts1,ts2)} returns true iff \axiom{ts2} has
       ++ higher rank than \axiom{ts1} in Wu Wen Tsun sense.
     basicSet : (List P,((P,P)->Boolean)) -> _
      Union(Record(bas:$,top:List P),"failed")
       ++ \axiom{basicSet(ps,redOp?)} returns \axiom{[bs,ts]} where
       ++ \axiom{concat(bs,ts)} is \axiom{ps} and \axiom{bs}
       ++ is a basic set in Wu Wen Tsun sense of \axiom{ps} w.r.t
       ++ the reduction-test \axiom{redOp?}, if no non-zero constant
       ++ polynomial lie in \axiom{ps}, otherwise \axiom{"failed"} is returned.
     basicSet : (List P,(P->Boolean),((P,P)->Boolean)) -> _
      Union(Record(bas:$,top:List P),"failed")
       ++ \axiom{basicSet(ps,pred?,redOp?)} returns the same as
       ++ \axiom{basicSet(qs,redOp?)}
       ++ where \axiom{qs} consists of the polynomials of \axiom{ps}
       ++ satisfying property \axiom{pred?}.
     initials : $ -> List P
       ++ \axiom{initials(ts)} returns the list of the non-constant initials
       ++ of the members of \axiom{ts}.
     degree : $ -> NonNegativeInteger
       ++ \axiom{degree(ts)} returns the product of main degrees of the
       ++ members of \axiom{ts}.
     quasiComponent : $ -> Record(close:List P,open:List P)
       ++ \axiom{quasiComponent(ts)} returns \axiom{[lp,lq]} where \axiom{lp}
       ++ is the list of the members of \axiom{ts} and \axiom{lq}is
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  ++ \axiom{initials(ts)}.
normalized? : (P,$) -> Boolean
  ++ \axiom{normalized?(p,ts)} returns true iff \axiom{p} and all
  ++ its iterated initials have degree zero w.r.t. the main variables
  ++ of the polynomials of \axiom{ts}
normalized? : $  -> Boolean
  ++ \axiom{normalized?(ts)} returns true iff for every axiom{p} in
  ++ \axiom{ts} we have \axiom{normalized?(p,us)} where \axiom{us}
  ++ is \axiom{collectUnder(ts,mvar(p))}.
reduced? : (P,$,((P,P) -> Boolean)) -> Boolean
  ++ \axiom{reduced?(p,ts,redOp?)} returns true iff \axiom{p} is reduced
  ++ w.r.t.in the sense of the operation \axiom{redOp?}, that is if for
  ++ every \axiom{t} in \axiom{ts} \axiom{redOp?(p,t)} holds.
stronglyReduced? : (P,$) -> Boolean
  ++ \axiom{stronglyReduced?(p,ts)} returns true iff \axiom{p}
  ++ is reduced w.r.t. \axiom{ts}.
headReduced? : (P,$) -> Boolean
  ++ \axiom{headReduced?(p,ts)} returns true iff the head of \axiom{p} is
  ++ reduced w.r.t. \axiom{ts}.
initiallyReduced? : (P,$) -> Boolean
  ++ \axiom{initiallyReduced?(p,ts)} returns true iff \axiom{p} and all
  ++ its iterated initials are reduced w.r.t. to the elements of
  ++ \axiom{ts} with the same main variable.
autoReduced? : ($,((P,List(P)) -> Boolean)) -> Boolean
  ++ \axiom{autoReduced?(ts,redOp?)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to every other in the sense of
  ++ \axiom{redOp?}
stronglyReduced? : $ -> Boolean
  ++ \axiom{stronglyReduced?(ts)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.
headReduced? : $ -> Boolean
  ++ headReduced?(ts) returns true iff the head of every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.
initiallyReduced? : $ -> Boolean
  ++ initiallyReduced?(ts) returns true iff for every element \axiom{p}
  ++ of \axiom{ts}. \axiom{p} and all its iterated initials are reduced
  ++ w.r.t. to the other elements of \axiom{ts} with the same main
  ++ variable.
reduce : (P,$,((P,P) -> P),((P,P) -> Boolean) ) -> P
  ++ \axiom{reduce(p,ts,redOp,redOp?)} returns a polynomial \axiom{r}
  ++ such that \axiom{redOp?(r,p)} holds for every \axiom{p} of
  ++ \axiom{ts} and there exists some product \axiom{h} of the initials
  ++ of the members of \axiom{ts} such that \axiom{h*p - r} lies in the
  ++ ideal generated by \axiom{ts}. The operation \axiom{redOp} must
  ++ satisfy the following conditions. For every \axiom{p} and \axiom{q}
  ++ we have  \axiom{redOp?(redOp(p,q),q)} and there exists an integer
  ++ \axiom{e} and a polynomial \axiom{f} such that
  ++ \axiom{init(q)^e*p = f*q + redOp(p,q)}.
rewriteSetWithReduction : (List P,$,((P,P) -> P),((P,P) -> Boolean) ) ->_
 List P
  ++ \axiom{rewriteSetWithReduction(lp,ts,redOp,redOp?)} returns a list
  ++ \axiom{lq} of polynomials such that
  ++ \axiom{[reduce(p,ts,redOp,redOp?) for p in lp]} and \axiom{lp}
  ++ have the same zeros inside the regular zero set of \axiom{ts}.
  ++ Moreover, for every polynomial \axiom{q} in \axiom{lq} and every
  ++ polynomial \axiom{t} in \axiom{ts}
  ++ \axiom{redOp?(q,t)} holds and there exists a polynomial \axiom{p}
  ++ in the ideal generated by \axiom{lp} and a product \axiom{h} of
  ++ \axiom{initials(ts)} such that \axiom{h*p - r} lies in the ideal
  ++ generated by \axiom{ts}.
  ++ The operation \axiom{redOp} must satisfy the following conditions.
  ++ For every \axiom{p} and \axiom{q} we have
  ++ \axiom{redOp?(redOp(p,q),q)}
  ++ and there exists an integer \axiom{e} and a polynomial \axiom{f}
  ++ such that \axiom{init(q)^e*p = f*q + redOp(p,q)}.
stronglyReduce : (P,$) -> P
  ++ \axiom{stronglyReduce(p,ts)} returns a polynomial \axiom{r} such that
  ++ \axiom{stronglyReduced?(r,ts)} holds and there exists some product
  ++ \axiom{h} of \axiom{initials(ts)}
  ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.
headReduce : (P,$) -> P
  ++ \axiom{headReduce(p,ts)} returns a polynomial \axiom{r}  such that
  ++ \axiom{headReduce?(r,ts)} holds and there exists some product
  ++ \axiom{h} of \axiom{initials(ts)} such that \axiom{h*p - r} lies
  ++ in the ideal generated by \axiom{ts}.
initiallyReduce : (P,$) -> P
  ++ \axiom{initiallyReduce(p,ts)} returns a polynomial \axiom{r}
  ++ such that  \axiom{initiallyReduced?(r,ts)}
  ++ holds and there exists some product \axiom{h} of \axiom{initials(ts)}
  ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.
removeZero: (P, $) -> P
  ++ \axiom{removeZero(p,ts)} returns \axiom{0} if \axiom{p} reduces
  ++ to \axiom{0} by pseudo-division w.r.t \axiom{ts} otherwise
  ++ returns a polynomial \axiom{q} computed from \axiom{p}
  ++ by removing any coefficient in \axiom{p} reducing to \axiom{0}.
collectQuasiMonic: $ -> $
  ++ \axiom{collectQuasiMonic(ts)} returns the subset of \axiom{ts}
  ++ consisting of the polynomials with initial in \axiom{R}.
reduceByQuasiMonic: (P, $) -> P
  ++ \axiom{reduceByQuasiMonic(p,ts)} returns the same as
  ++ \axiom{remainder(p,collectQuasiMonic(ts)).polnum}.
zeroSetSplit : List P -> List $
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  ++ \axiom{zeroSetSplit(lp)} returns a list \axiom{lts} of triangular
  ++ sets such that the zero set of \axiom{lp} is the union of the
  ++ closures of the regular zero sets  of the members of \axiom{lts}.
zeroSetSplitIntoTriangularSystems : List P -> _
 List Record(close:$,open:List P)
  ++ \axiom{zeroSetSplitIntoTriangularSystems(lp)} returns a list of
  ++ triangular systems \axiom{[[ts1,qs1],...,[tsn,qsn]]} such that the
  ++ zero set of \axiom{lp} is the union of the closures of the
  ++ \axiom{W_i} where \axiom{W_i} consists of the zeros of \axiom{ts}
  ++ which do not cancel any polynomial in \axiom{qsi}.
first : $ -> Union(P,"failed")
  ++ \axiom{first(ts)} returns the polynomial of \axiom{ts} with
  ++ greatest main variable if \axiom{ts} is not empty, otherwise
  ++ returns \axiom{"failed"}.
last : $ -> Union(P,"failed")
  ++ \axiom{last(ts)} returns the polynomial of \axiom{ts} with
  ++ smallest main variable if \axiom{ts} is not empty, otherwise
  ++ returns \axiom{"failed"}.
rest : $ -> Union($,"failed")
  ++ \axiom{rest(ts)} returns the polynomials of \axiom{ts} with smaller
  ++ main variable than \axiom{mvar(ts)} if \axiom{ts} is not empty,
  ++ otherwise returns "failed"
algebraicVariables : $ -> List(V)
  ++ \axiom{algebraicVariables(ts)} returns the decreasingly sorted
  ++ list of the main variables of the polynomials of \axiom{ts}.
algebraic? : (V,$) -> Boolean
  ++ \axiom{algebraic?(v,ts)} returns true iff \axiom{v} is the
  ++ main variable of some polynomial in \axiom{ts}.
select : ($,V) -> Union(P,"failed")
  ++ \axiom{select(ts,v)} returns the polynomial of \axiom{ts} with
  ++ \axiom{v} as main variable, if any.
extendIfCan : ($,P) -> Union($,"failed")
  ++ \axiom{extendIfCan(ts,p)} returns a triangular set which encodes
  ++ the simple extension by \axiom{p} of the extension of the base
  ++ field defined by \axiom{ts}, according
  ++ to the properties of triangular sets of the current domain.
  ++ If the required properties do not hold then "failed" is returned.
  ++ This operation encodes in some sense the properties of the
  ++ triangular sets of the current category. Is is used to implement
  ++ the \axiom{construct} operation to guarantee that every triangular
  ++ set build from a list of polynomials has the required properties.
extend : ($,P) -> $
  ++ \axiom{extend(ts,p)} returns a triangular set which encodes the
  ++ simple extension by \axiom{p} of the extension of the base field
  ++ defined by \axiom{ts}, according to the properties of triangular
  ++ sets of the current category. If the required properties do not
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        ++ hold an error is returned.
      if V has Finite
      then
        coHeight : $ -> NonNegativeInteger
          ++ \axiom{coHeight(ts)} returns \axiom{size()\$V} minus \axiom{\#ts}.
   add

      GPS ==> GeneralPolynomialSet(R,E,V,P)
      B ==> Boolean
      RBT ==> Record(bas:$,top:List P)

      ts:$ = us:$ ==
        empty?(ts)$$ => empty?(us)$$
        empty?(us)$$ => false
        first(ts)::P =$P first(us)::P => rest(ts)::$ =$$ rest(us)::$
        false

      infRittWu?(ts,us) ==
        empty?(us)$$ => not empty?(ts)$$
        empty?(ts)$$ => false
        p : P := (last(ts))::P
        q : P := (last(us))::P
        infRittWu?(p,q)$P => true
        supRittWu?(p,q)$P => false
        v : V := mvar(p)
        infRittWu?(collectUpper(ts,v),collectUpper(us,v))$$

      reduced?(p,ts,redOp?) ==
        lp : List P := members(ts)
        while (not empty? lp) and (redOp?(p,first(lp))) repeat
          lp := rest lp
        empty? lp

      basicSet(ps,redOp?) ==
        ps := remove(zero?,ps)
        any?(ground?,ps) => "failed"::Union(RBT,"failed")
        ps := sort(infRittWu?,ps)
        p,b : P
        bs := empty()$$
        ts : List P := []
        while not empty? ps repeat
          b := first(ps)
          bs := extend(bs,b)$$
          ps := rest ps
          while (not empty? ps) and _
                (not reduced?((p := first(ps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      ps := rest ps
  ([bs,ts]$RBT)::Union(RBT,"failed")

basicSet(ps,pred?,redOp?) ==
  ps := remove(zero?,ps)
  any?(ground?,ps) => "failed"::Union(RBT,"failed")
  gps : List P := []
  bps : List P := []
  while not empty? ps repeat
    p := first ps
    ps := rest ps
    if pred?(p)
      then
        gps := cons(p,gps)
      else
        bps := cons(p,bps)
  gps := sort(infRittWu?,gps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? gps repeat
    b := first(gps)
    bs := extend(bs,b)$$
    gps := rest gps
    while (not empty? gps) and _
          (not reduced?((p := first(gps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      gps := rest gps
  ts := sort(infRittWu?,concat(ts,bps))
  ([bs,ts]$RBT)::Union(RBT,"failed")

initials ts ==
  lip : List P := []
  empty? ts => lip
  lp := members(ts)
  while not empty? lp repeat
     p := first(lp)
     if not ground?((ip := init(p)))
       then
         lip := cons(primPartElseUnitCanonical(ip),lip)
     lp := rest lp
  removeDuplicates lip

degree ts ==
  empty? ts => 0$NonNegativeInteger
  lp := members ts
  d : NonNegativeInteger := mdeg(first lp)
  while not empty? (lp := rest lp) repeat
    d := d * mdeg(first lp)
  d

quasiComponent ts ==
  [members(ts),initials(ts)]

normalized?(p,ts) ==
  normalized?(p,members(ts))$P

stronglyReduced? (p,ts) ==
  reduced?(p,members(ts))$P

headReduced? (p,ts) ==
  stronglyReduced?(head(p),ts)

initiallyReduced? (p,ts) ==
  lp : List (P) := members(ts)
  red : Boolean := true
  while (not empty? lp) and (not ground?(p)$P) and red repeat
    while (not empty? lp) and (mvar(first(lp)) > mvar(p)) repeat
      lp := rest lp
    if (not empty? lp)
      then
        if  (mvar(first(lp)) = mvar(p))
          then
            if reduced?(p,first(lp))
              then
                lp := rest lp
                p := init(p)
              else
                red := false
          else
            p := init(p)
  red

reduce(p,ts,redOp,redOp?) ==
  (empty? ts) or (ground? p) => p
  ts0 := ts
  while (not empty? ts) and (not ground? p) repeat
     reductor := (first ts)::P
     ts := (rest ts)::$
     if not redOp?(p,reductor)
       then
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         p := redOp(p,reductor)
         ts := ts0
  p

rewriteSetWithReduction(lp,ts,redOp,redOp?) ==
  trivialIdeal? ts => lp
  lp := remove(zero?,lp)
  empty? lp => lp
  any?(ground?,lp) => [1$P]
  rs : List P := []
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    p := primPartElseUnitCanonical reduce(p,ts,redOp,redOp?)
    if not zero? p
      then
        if ground? p
          then
            lp := []
            rs := [1$P]
          else
            rs := cons(p,rs)
  removeDuplicates rs

stronglyReduce(p,ts) ==
  reduce (p,ts,lazyPrem,reduced?)

headReduce(p,ts) ==
  reduce (p,ts,headReduce,headReduced?)

initiallyReduce(p,ts) ==
  reduce (p,ts,initiallyReduce,initiallyReduced?)

removeZero(p,ts) ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  if algebraic?(v,ts)
    then
      q := lazyPrem(p,select(ts,v)::P)
      zero? q => return q
      zero? removeZero(q,ts_v_-) => return 0
  empty? ts_v_- => p
  q: P := 0
  while positive? degree(p,v) repeat
     q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
```

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      p := tail(p)
   q + removeZero(p,ts_v_-)

 reduceByQuasiMonic(p, ts) ==
   (ground? p) or (empty? ts) => p
   remainder(p,collectQuasiMonic(ts)).polnum

 autoReduced?(ts : $,redOp? : ((P,List(P)) -> Boolean)) ==
   empty? ts => true
   lp : List (P) := members(ts)
   p : P := first(lp)
   lp := rest lp
   while (not empty? lp) and redOp?(p,lp) repeat
      p := first lp
      lp := rest lp
   empty? lp

 stronglyReduced? ts ==
   autoReduced? (ts, reduced?)

 normalized? ts ==
   autoReduced? (ts,normalized?)

 headReduced? ts ==
   autoReduced? (ts,headReduced?)

 initiallyReduced?  ts ==
   autoReduced? (ts,initiallyReduced?)

 mvar ts ==
   empty? ts => error"Error from TSETCAT in mvar : #1 is empty"
   mvar((first(ts))::P)$P

 first ts ==
   empty? ts => "failed"::Union(P,"failed")
   lp : List(P) := sort(supRittWu?,members(ts))$(List P)
   first(lp)::Union(P,"failed")

 last ts ==
   empty? ts => "failed"::Union(P,"failed")
   lp : List(P) := sort(infRittWu?,members(ts))$(List P)
   first(lp)::Union(P,"failed")

 rest ts ==
   empty? ts => "failed"::Union($,"failed")
   lp : List(P) := sort(supRittWu?,members(ts))$(List P)
```

```
  construct(rest(lp))::Union($,"failed")

coerce (ts:$) : List(P) ==
  sort(supRittWu?,members(ts))$(List P)

algebraicVariables ts ==
  [mvar(p) for p in members(ts)]

algebraic? (v,ts) ==
  member?(v,algebraicVariables(ts))

select  (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v = mvar(first lp))) repeat
    lp := rest lp
  empty? lp => "failed"::Union(P,"failed")
  (first lp)::Union(P,"failed")

collectQuasiMonic ts ==
  lp: List(P) := members(ts)
  newlp: List(P) := []
  while (not empty? lp) repeat
    if ground? init(first(lp)) then newlp := cons(first(lp),newlp)
    lp := rest lp
  construct(newlp)

collectUnder (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v > mvar(first lp))) repeat
    lp := rest lp
  construct(lp)

collectUpper  (ts,v) ==
  lp1 : List(P) := sort(supRittWu?,members(ts))$(List P)
  lp2 : List(P) := []
  while (not empty? lp1) and  (mvar(first lp1) > v) repeat
    lp2 := cons(first(lp1),lp2)
    lp1 := rest lp1
  construct(reverse lp2)

construct(lp:List(P)) ==
  rif := retractIfCan(lp)@Union($,"failed")
  not (rif case $) => error"in construct : LP -> $ from TSETCAT : bad arg"
  rif::$

retractIfCan(lp:List(P)) ==
          empty? lp => (empty()$$)::Union($,"failed")
          lp := sort(supRittWu?,lp)
          rif := retractIfCan(rest(lp))@Union($,"failed")
          not (rif case $) => _
           error "in retractIfCan : LP -> ... from TSETCAT : bad arg"
          extendIfCan(rif::$,first(lp))@Union($,"failed")

        extend(ts:$,p:P):$ ==
          eif := extendIfCan(ts,p)@Union($,"failed")
          not (eif case $) => error"in extend : ($,P) -> $ from TSETCAT : bad ars"
          eif::$

        if V has Finite
        then

          coHeight ts ==
            n := size()$V
            m := #(members ts)
            subtractIfCan(n,m)$NonNegativeInteger::NonNegativeInteger
```

⟨*TSETCAT.dotabb*⟩≡

```
  "TSETCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
  "TSETCAT" -> "PSETCAT"
```

⟨*TSETCAT.dotfull*⟩≡

```
  "TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
   [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
  "TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
    -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedS

  "TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
   [color=seagreen,href="bookvol10.2.pdf#nameddest=TSETCAT"];
  "TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
   ->
  "TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
```

⟨*TSETCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet(

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())" [color
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

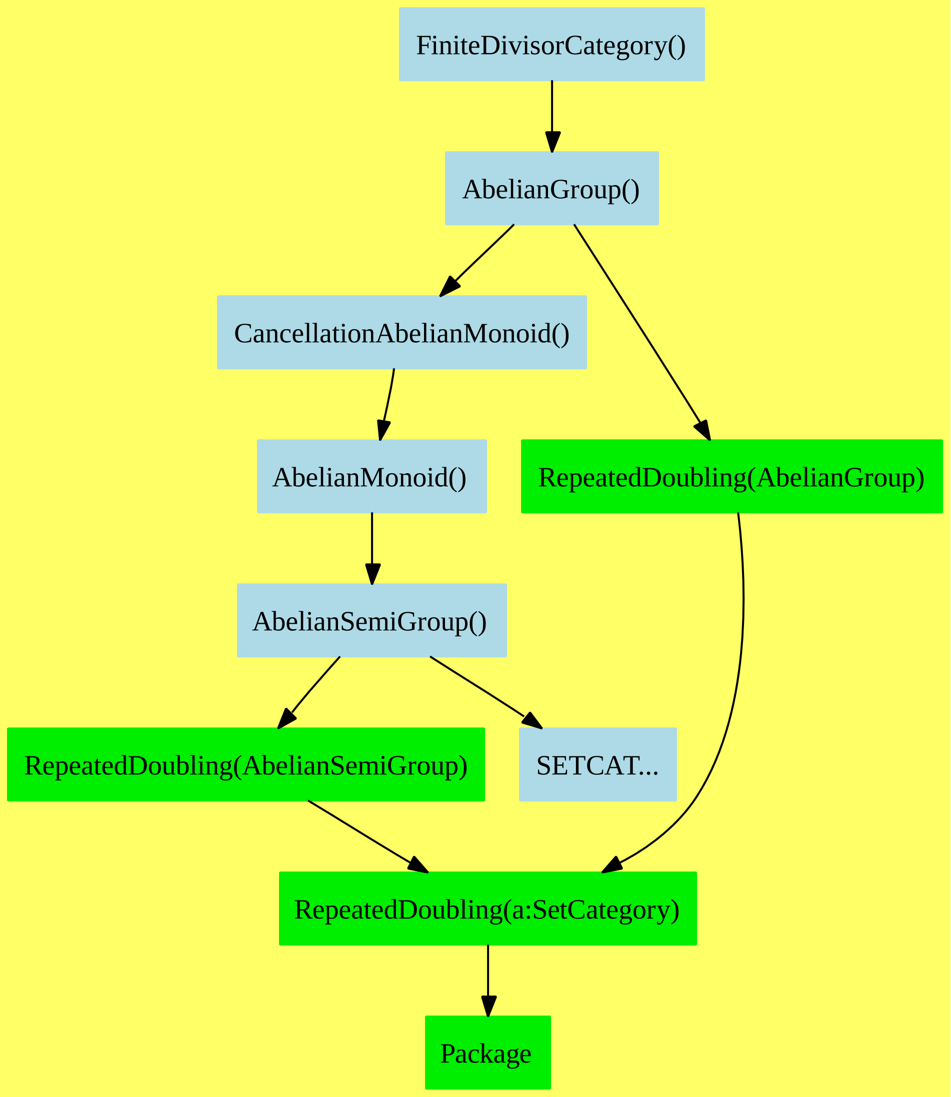
"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 8

Category Layer 7

8.0.109 FiniteDivisorCategory (FDIVCAT)


FiniteDivisorCategory()
AbelianGroup()
CancellationAbelianMonoid()
AbelianMonoid()
RepeatedDoubling(AbelianGroup)
AbelianSemiGroup()
RepeatedDoubling(AbelianSemiGroup)
SETCAT...
RepeatedDoubling(a:SetCategory)
Package

⟨*FiniteDivisorCategory.input*⟩≡

```
)set break resume
)sys rm -f FiniteDivisorCategory.output
)spool FiniteDivisorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteDivisorCategory
--R FiniteDivisorCategory(F: Field,UP: UnivariatePolynomialCategory t#1,UPUP: UnivariatePoly
--R Abbreviation for FiniteDivisorCategory is FDIVCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FDIVCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R divisor : (R,UP,UP,UP,F) -> %         divisor : (F,F,Integer) -> %
--R divisor : (F,F) -> %                  divisor : R -> %
--R hash : % -> SingleInteger             latex : % -> String
--R principal? : % -> Boolean             reduce : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
--R generator : % -> Union(R,"failed")
--R ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteDivisorCategory.help*⟩≡

```
====================================================================
FiniteDivisorCategory examples
====================================================================

See Also:
o )show FiniteDivisorCategory
```

See:

⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576

Exports:

0	coerce	decompose	divisor	generator
hash	ideal	latex	principal?	reduce
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?=?	

These are directly exported but not implemented:

```
decompose : % ->
  Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
divisor : R -> %
divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
divisor : (F,F) -> %
divisor : (F,F,Integer) -> %
divisor : (R,UP,UP,UP,F) -> %
generator : % -> Union(R,"failed")
ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
reduce : % -> %
```

These are implemented by this category:

```
principal? : % -> Boolean
```

These exports come from (p576) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
-? : % -> %
?-? : (%,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

⟨*category FDIVCAT FiniteDivisorCategory*⟩≡

```
)abbrev category FDIVCAT FiniteDivisorCategory
++ Category for finite rational divisors on a curve
++ Author: Manuel Bronstein
++ Date Created: 19 May 1993
++ Date Last Updated: 19 May 1993
++ Keywords: divisor, algebraic, curve.
++ Description:
++ This category describes finite rational divisors on a curve, that
++ is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.

FiniteDivisorCategory(F, UP, UPUP, R): Category == Result where
  F   : Field
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP
  R   : FunctionFieldCategory(F, UP, UPUP)

  ID  ==> FractionalIdeal(UP, Fraction UP, UPUP, R)

  Result ==> AbelianGroup with
    ideal      : % -> ID
      ++ ideal(D) returns the ideal corresponding to a divisor D.
    divisor    : ID -> %
      ++ divisor(I) makes a divisor D from an ideal I.
    divisor    : R -> %
      ++ divisor(g) returns the divisor of the function g.
    divisor    : (F, F) -> %
      ++ divisor(a, b) makes the divisor P: \spad{(x = a, y = b)}.
      ++ Error: if P is singular.
    divisor    : (F, F, Integer) -> %
      ++ divisor(a, b, n) makes the divisor
      ++ \spad{nP} where P: \spad{(x = a, y = b)}.
      ++ P is allowed to be singular if n is a multiple of the rank.
    decompose  : % -> Record(id:ID, principalPart: R)
      ++ decompose(d) returns \spad{[id, f]} where \spad{d = (id) + div(f)}.
    reduce     : % -> %
      ++ reduce(D) converts D to some reduced form (the reduced forms can
      ++ be differents in different implementations).
    principal? : % -> Boolean
      ++ principal?(D) tests if the argument is the divisor of a function.
    generator  : % -> Union(R, "failed")
      ++ generator(d) returns f if \spad{(f) = d},
      ++ "failed" if d is not principal.
    divisor    : (R, UP, UP, UP, F) -> %
      ++ divisor(h, d, d', g, r) returns the sum of all the finite points
      ++ where \spad{h/d} has residue \spad{r}.
      ++ \spad{h} must be integral.
      ++ \spad{d} must be squarefree.
      ++ \spad{d'} is some derivative of \spad{d} (not necessarily dd/dx).
      ++ \spad{g = gcd(d,discriminant)} contains the ramified zeros of \spad{d}
   add
    principal? d == generator(d) case R
```

⟨*FDIVCAT.dotabb*⟩≡

```
"FDIVCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
"FDIVCAT" -> "ABELGRP"
```

⟨*FDIVCAT.dotfull*⟩≡

```
"FiniteDivisorCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
"FiniteDivisorCategory()" -> "AbelianGroup()"
```

⟨*FDIVCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteDivisorCategory()" [color=lightblue];
"FiniteDivisorCategory()" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

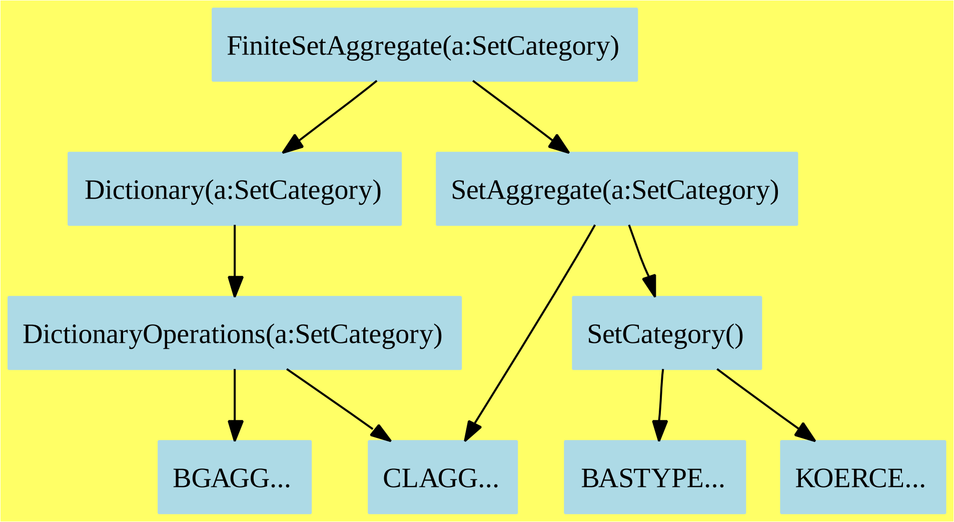
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

8.0.110 FiniteSetAggregate (FSAGG)


FiniteSetAggregate(a:SetCategory)
Dictionary(a:SetCategory)
SetAggregate(a:SetCategory)
DictionaryOperations(a:SetCategory)
SetCategory()
BGAGG...
CLAGG...
BASTYPE...
KOERCE...

⟨*FiniteSetAggregate.input*⟩≡
```
)set break resume
)sys rm -f FiniteSetAggregate.output
)spool FiniteSetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteSetAggregate
--R FiniteSetAggregate S: SetCategory  is a category constructor
--R Abbreviation for FiniteSetAggregate is FSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FSAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R bag : List S -> %                     brace : () -> %
--R brace : List S -> %                   coerce : % -> OutputForm
--R construct : List S -> %               copy : % -> %
--R dictionary : List S -> %              dictionary : () -> %
--R difference : (%,%) -> %               difference : (%,S) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R hash : % -> SingleInteger             insert! : (S,%) -> %
--R inspect : % -> S                      intersect : (%,%) -> %
```

```
--R latex : % -> String                 map : ((S -> S),%) -> %
--R max : % -> S if S has ORDSET        min : % -> S if S has ORDSET
--R sample : () -> %                    set : () -> %
--R set : List S -> %                   subset? : (%,%) -> Boolean
--R union : (%,%) -> %                  union : (%,S) -> %
--R union : (S,%) -> %                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R cardinality : % -> NonNegativeInteger
--R complement : % -> % if S has FINITE
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R random : () -> % if S has FINITE
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R symmetricDifference : (%,%) -> %
--R universe : () -> % if S has FINITE
--R
--E 1

)spool
```

```
)lisp (bye)
```

⟨*FiniteSetAggregate.help*⟩≡

```
====================================================================
FiniteSetAggregate examples
====================================================================

See Also:
o )show FiniteSetAggregate
```

See:

⇐ “Dictionary” (DIAGG) 7.0.99 on page 589
⇐ “SetAggregate” (SETAGG) 6.0.94 on page 547

Exports:

any?
bag
brace
cardinality
coerce
complement
construct
convert
copy
count
dictionary
difference
empty
empty?
eq?
eval
every?
extract!
find
hash
index
insert!
inspect
intersect
latex
less?
lookup
map
map!
max
member?
members
min
more?
parts
random
reduce
remove
remove!
removeDuplicates
sample
select
select!
set
size
size?
subset?
symmetricDifference
union
universe
#?
?<?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.
- **nil**

These are implemented by this category:

```
brace : List S -> %
cardinality : % -> NonNegativeInteger
coerce : % -> OutputForm
complement : % -> % if S has FINITE
construct : List S -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
difference : (%,%) -> %
index : PositiveInteger -> % if S has FINITE
intersect : (%,%) -> %
lookup : % -> PositiveInteger if S has FINITE
max : % -> S if S has ORDSET
min : % -> S if S has ORDSET
random : () -> % if S has FINITE
```

```
set : List S -> %
size : () -> NonNegativeInteger if S has FINITE
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
universe : () -> % if S has FINITE
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p589) Dictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
bag : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
member? : (S,%) -> Boolean
```

```
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT
    and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
```

These exports come from (p547) SetAggregate(S:SetCategory):

```
brace : () -> %
difference : (%,S) -> %
set : () -> %
union : (%,S) -> %
union : (S,%) -> %
```

⟨*category FSAGG FiniteSetAggregate*⟩≡

```
)abbrev category FSAGG FiniteSetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ A finite-set aggregate models the notion of a finite set, that is,
++ a collection of elements characterized by membership, but not
++ by order or multiplicity.
++ See \spadtype{Set} for an example.

FiniteSetAggregate(S:SetCategory): Category ==
  Join(Dictionary S, SetAggregate S) with
    finiteAggregate
    cardinality: % -> NonNegativeInteger
      ++ cardinality(u) returns the number of elements of u.
      ++ Note that \axiom{cardinality(u) = #u}.
    if S has Finite then
      Finite
      complement: % -> %
        ++ complement(u) returns the complement of the set u,
        ++ i.e. the set of all values not in u.
      universe: () -> %
        ++ universe()$D returns the universal set for finite set aggregate D.
    if S has OrderedSet then
      max: % -> S
        ++ max(u) returns the largest element of aggregate u.
      min: % -> S
        ++ min(u) returns the smallest element of aggregate u.

 add
   s < t           == #s < #t and s = intersect(s,t)
   s = t           == #s = #t and empty? difference(s,t)
   brace l           == construct l
   set         l           == construct l
   cardinality s   == #s
   construct l           == (s := set(); for x in l repeat insert_!(x,s); s)
   count(x:S, s:%) == (member?(x, s) => 1; 0)
   subset?(s, t)   == #s < #t and _and/[member?(x, t) for x in parts s]

   coerce(s:%):OutputForm ==
     brace [x::OutputForm for x in parts s]$List(OutputForm)

   intersect(s, t) ==
     i := {}
     for x in parts s | member?(x, t) repeat insert_!(x, i)
     i

   difference(s:%, t:%) ==
     m := copy s
     for x in parts t repeat remove_!(x, m)
```

```
      m

    symmetricDifference(s, t) ==
      d := copy s
      for x in parts t repeat
        if member?(x, s) then remove_!(x, d) else insert_!(x, d)
      d

    union(s:%, t:%) ==
       u := copy s
       for x in parts t repeat insert_!(x, u)
       u

    if S has Finite then
      universe()          == {index(i::PositiveInteger) for i in 1..size()$S}
      complement s == difference(universe(), s )
      size()  == 2 ** size()$S
      index i ==
        {index(j::PositiveInteger)$S for j in 1..size()$S | bit?(i-1,j-1)}
      random()  ==
        index((random()$Integer rem (size()$% + 1))::PositiveInteger)

      lookup s ==
        n:PositiveInteger := 1
        for x in parts s repeat _
          n := n + 2 ** ((lookup(x) - 1)::NonNegativeInteger)
        n

    if S has OrderedSet then
      max s ==
        empty?(l := parts s) => error "Empty set"
        reduce("max", l)

      min s ==
        empty?(l := parts s) => error "Empty set"
        reduce("min", l)
```

⟨*FSAGG.dotabb*⟩≡

```
  "FSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
  "FSAGG" -> "DIAGG"
  "FSAGG" -> "SETAGG"
```

⟨*FSAGG.dotfull*⟩≡

```
"FiniteSetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

⟨*FSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteSetAggregate(a:SetCategory)" [color=lightblue];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "CLAGG..."

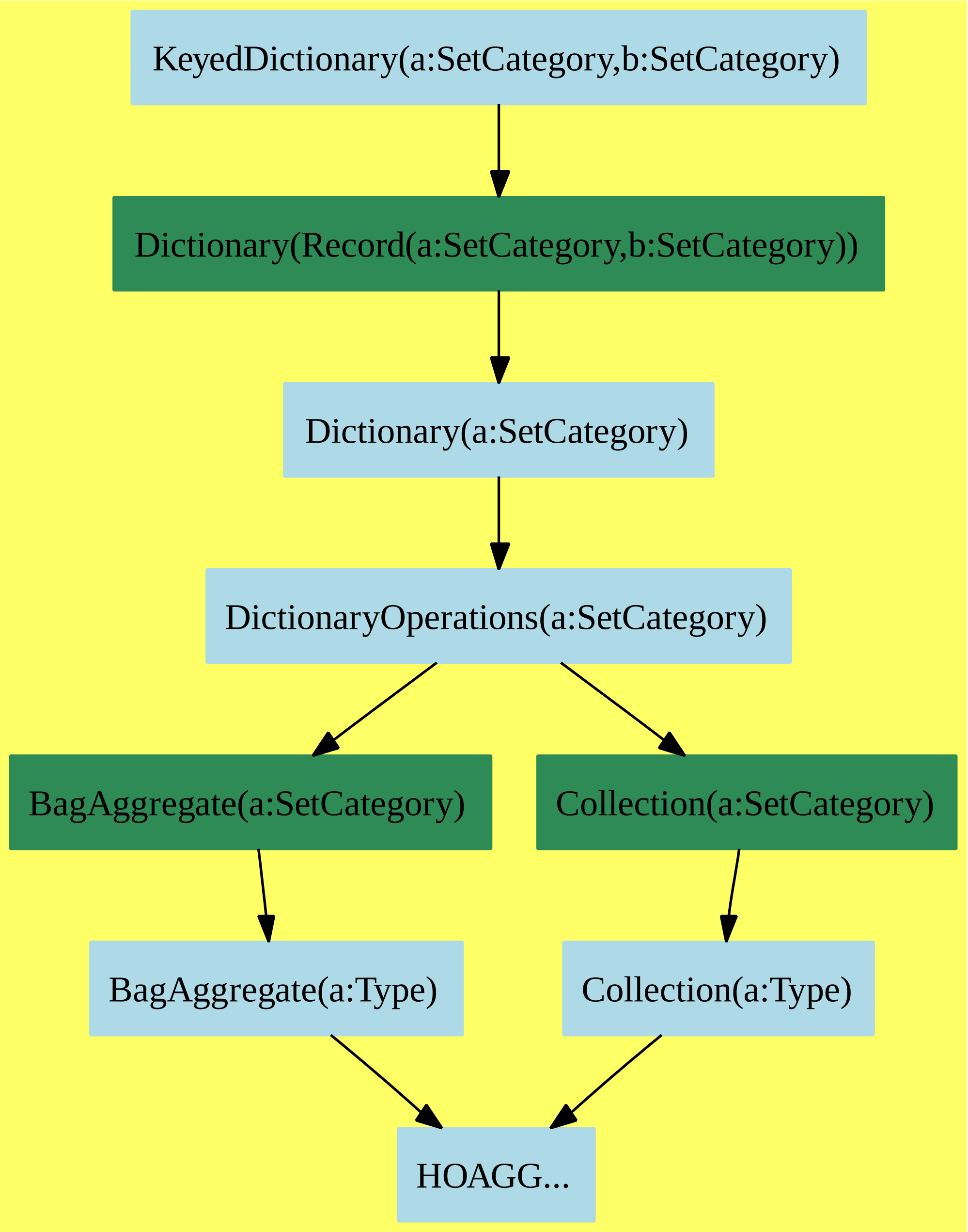
"SetCategory()" [color=lightblue];
"SetCategory()" -> "BASTYPE..."
"SetCategory()" -> "KOERCE..."

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"BASTYPE..." [color=lightblue];
"KOERCE..." [color=lightblue];
}
```

8.0.111 KeyedDictionary (KDAGG)


KeyedDictionary(a:SetCategory,b:SetCategory)
Dictionary(Record(a:SetCategory,b:SetCategory))
Dictionary(a:SetCategory)
DictionaryOperations(a:SetCategory)
BagAggregate(a:SetCategory)
Collection(a:SetCategory)
BagAggregate(a:Type)
Collection(a:Type)
HOAGG...

⟨*KeyedDictionary.input*⟩≡

```
)set break resume
```

```
)sys rm -f KeyedDictionary.output
)spool KeyedDictionary.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show KeyedDictionary
--R KeyedDictionary(Key: SetCategory,Entry: SetCategory)  is a category constructor
--R Abbreviation for KeyedDictionary is KDAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KDAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                key? : (Key,%) -> Boolean
--R keys : % -> List Key                  sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregat
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R extract! : % -> Record(key: Key,entry: Entry)
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ ha
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,entry: Entry)
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
```

```
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteA
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Ent
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finite
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ h
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteA
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finite
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*KeyedDictionary.help*⟩≡

```
====================================================================
KeyedDictionary examples
====================================================================

See Also:
o )show KeyedDictionary
```

See:

⇒ “TableAggregate” (TBAGG) 9.0.132 on page 884
⇐ “Dictionary” (DIAGG) 7.0.99 on page 589

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	key?	keys
latex	less?	map	map!	member?
members	more?	parts	reduce	remove
remove!	removeDuplicates	sample	search	select
select!	size?	#?	?=?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
remove! : (Key,%) -> Union(Entry,"failed")
search : (Key,%) -> Union(Entry,"failed")
```

These are implemented by this category:

```
key? : (Key,%) -> Boolean
member? : (Record(key: Key,entry: Entry),%) -> Boolean
    if Record(key: Key,entry: Entry) has SETCAT
    and $ has finiteAggregate
keys : % -> List Key
```

These exports come from (p589) Dictionary(R)
where R=Record(a:SetCategory,b:SetCategory))
and S=Record(key: Key,entry: Entry)

```
any? : ((S) -> Boolean),%) -> Boolean
    if $ has finiteAggregate
```

```
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
```

```
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
    if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category KDAGG KeyedDictionary*⟩≡

```
)abbrev category KDAGG KeyedDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A keyed dictionary is a dictionary of key-entry pairs for which there is
++ a unique entry for each key.

KeyedDictionary(Key:SetCategory, Entry:SetCategory): Category ==
  Dictionary Record(key:Key,entry:Entry) with
   key?: (Key, %) -> Boolean
     ++ key?(k,t) tests if k is a key in table t.
   keys: % -> List Key
     ++ keys(t) returns the list the keys in table t.
   -- to become keys: % -> Key* and keys: % -> Iterator(Entry,Entry)
   remove_!: (Key, %) -> Union(Entry,"failed")
     ++ remove!(k,t) searches the table t for the key k removing
     ++ (and return) the entry if there.
     ++ If t has no such key, \axiom{remove!(k,t)} returns "failed".
   search: (Key, %) -> Union(Entry,"failed")
     ++ search(k,t) searches the table t for the key k,
     ++ returning the entry stored in t for key k.
     ++ If t has no such key, \axiom{search(k,t)} returns "failed".
 add
   key?(k, t) == search(k, t) case Entry

   member?(p, t) ==
      r := search(p.key, t)
      r case Entry and r::Entry = p.entry

    if % has finiteAggregate then
      keys t == [x.key for x in parts t]
```

⟨*KDAGG.dotabb*⟩≡

```
"KDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KDAGG" -> "DIAGG"
```

⟨*KDAGG.dotfull*⟩≡

```
"KeyedDictionary(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"
```

⟨*KDAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

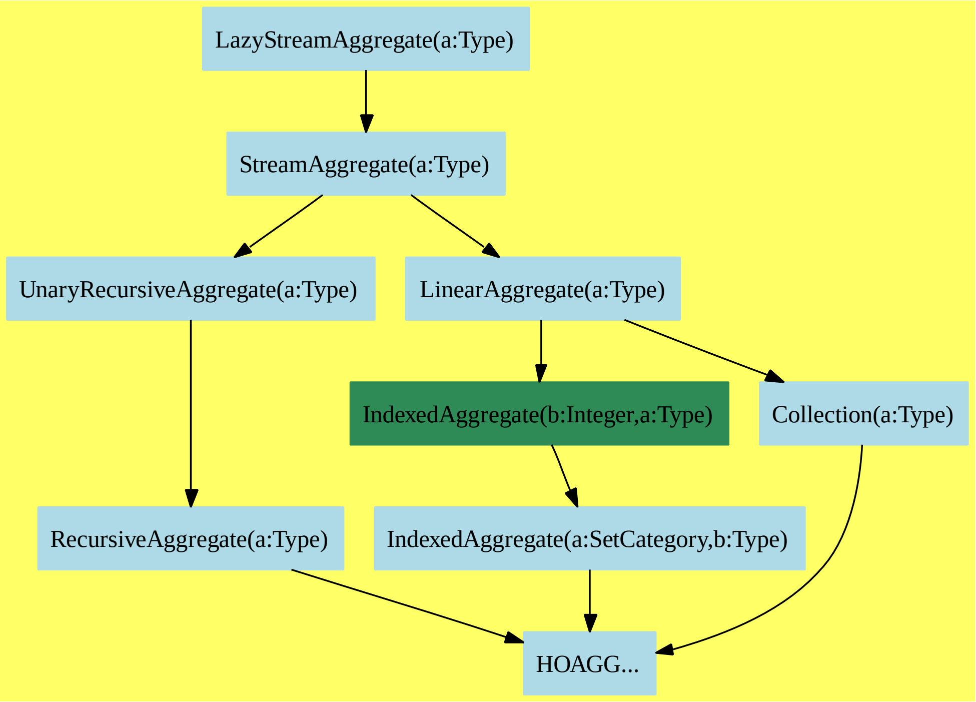
"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];
}
```

8.0.112 LazyStreamAggregate (LZSTAGG)


LazyStreamAggregate(a:Type)
StreamAggregate(a:Type)
UnaryRecursiveAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
Collection(a:Type)
RecursiveAggregate(a:Type)
IndexedAggregate(a:SetCategory,b:Type)
HOAGG...

⟨*LazyStreamAggregate.input*⟩≡

```
)set break resume
)sys rm -f LazyStreamAggregate.output
)spool LazyStreamAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LazyStreamAggregate
--R LazyStreamAggregate S: Type  is a category constructor
--R Abbreviation for LazyStreamAggregate is LZSTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LZSTAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               complete : % -> %
--R concat : (%,S) -> %                  concat : List % -> %
--R concat : (S,%) -> %                  concat : (%,%) -> %
--R construct : List S -> %              copy : % -> %
```

```
--R cycleEntry : % -> %                   cycleTail : % -> %
--R cyclic? : % -> Boolean                delete : (%,Integer) -> %
--R distance : (%,%) -> Integer           elt : (%,Integer,S) -> S
--R ?.? : (%,Integer) -> S                ?.last : (%,last) -> S
--R ?.rest : (%,rest) -> %                ?.first : (%,first) -> S
--R ?.value : (%,value) -> S              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                explicitEntries? : % -> Boolean
--R explicitlyEmpty? : % -> Boolean       explicitlyFinite? : % -> Boolean
--R extend : (%,Integer) -> %             first : % -> S
--R frst : % -> S                         index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (S,%,Integer) -> %
--R insert : (%,%,Integer) -> %           last : % -> S
--R lazy? : % -> Boolean                  lazyEvaluate : % -> %
--R leaf? : % -> Boolean                  leaves : % -> List S
--R map : (((S,S) -> S),%,%) -> %         map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %     nodes : % -> List %
--R possiblyInfinite? : % -> Boolean      qelt : (%,Integer) -> S
--R remove : ((S -> Boolean),%) -> %      rest : % -> %
--R rst : % -> %                          sample : () -> %
--R second : % -> S                       select : ((S -> Boolean),%) -> %
--R tail : % -> %                         third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
```

```
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R numberOfComputedEntries : % -> NonNegativeInteger
--R parts : % -> List S if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R rest : (%,NonNegativeInteger) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*LazyStreamAggregate.help*⟩≡

```
====================================================================
LazyStreamAggregate examples
====================================================================

See Also:
o )show LazyStreamAggregate
```

See:

⇐ "StreamAggregate" (STAGG) 7.0.107 on page 647

Exports:

any?
child?
children
coerce
complete
concat
concat!
construct
copy
convert
count
cycleEntry
cycleLength
cycleSplit!
cycleTail
cyclic?
delete
distance
elt
empty
empty?
entry?
entries
eq?
explicitEntries?
explicitlyEmpty?
explicitlyFinite?
extend
eval
every?
fill!
find
first
frst
hash
index?
indices
insert
last
latex
lazy?
lazyEvaluate
leaf?
leaves
less?
map
map!
maxIndex
member?
members
minIndex
more?
new
node?
nodes
numberOfComputedEntries
parts
possiblyInfinite?
qelt
qsetelt!
reduce
remove
removeDuplicates
rest
rst
sample
second
select
setchildren!
setelt
setfirst!
setlast!
setrest!
setvalue!
size?
split!
swap!
tail
third
value
#?
?=?
?.?
?.last
?.rest
?.first
?.value
?~=?

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
explicitEntries? : % -> Boolean
explicitlyEmpty? : % -> Boolean
frst : % -> S
lazy? : % -> Boolean
lazyEvaluate : % -> %
numberOfComputedEntries : % -> NonNegativeInteger
remove : ((S -> Boolean),%) -> %
rst : % -> %
select : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
```

```
    if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
complete : % -> %
construct : List S -> %
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
elt : (%,Integer,S) -> S
entries : % -> List S
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
extend : (%,Integer) -> %
first : (%,NonNegativeInteger) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
maxIndex : % -> Integer if Integer has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
node? : (%,%) -> Boolean if S has SETCAT
nodes : % -> List %
possiblyInfinite? : % -> Boolean
rest : % -> %
rest : (%,NonNegativeInteger) -> %
value : % -> S
size? : (%,NonNegativeInteger) -> Boolean
tail : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> S
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p647) StreamAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : (%,S) -> %
concat : (S,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
concat! : (%,S) -> % if $ has shallowlyMutable
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
cycleSplit! : % -> % if $ has shallowlyMutable
empty : () -> %
empty? : % -> Boolean
entry? : (S,%) -> Boolean
    if $ has finiteAggregate
    and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
new : (NonNegativeInteger,S) -> %
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
    if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
third : % -> S
?.value : (%,value) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category LZSTAGG LazyStreamAggregate*⟩≡

```
)abbrev category LZSTAGG LazyStreamAggregate
++ Category of streams with lazy evaluation
++ Author: Clifton J. Williamson
++ Date Created: 22 November 1989
++ Date Last Updated: 20 July 1990
++ Keywords: stream, infinite list, infinite sequence
++ Description:
++ LazyStreamAggregate is the category of streams with lazy
++ evaluation.  It is understood that the function 'empty?' will
++ cause lazy evaluation if necessary to determine if there are
++ entries.  Functions which call 'empty?', e.g. 'first' and 'rest',
++ will also cause lazy evaluation if necessary.

LazyStreamAggregate(S:Type): Category == StreamAggregate(S) with
  remove: (S -> Boolean,%) -> %
    ++ remove(f,st) returns a stream consisting of those elements of stream
    ++ st which do not satisfy the predicate f.
    ++ Note that \spad{remove(f,st) = [x for x in st | not f(x)]}.
    ++
    ++X m:=[i for i in 1..]
  ++X f(i:PositiveInteger):Boolean == even? i
  ++X remove(f,m)

select: (S -> Boolean,%) -> %
  ++ select(f,st) returns a stream consisting of those elements of stream
  ++ st satisfying the predicate f.
  ++ Note that \spad{select(f,st) = [x for x in st | f(x)]}.
  ++
  ++X m:=[i for i in 0..]
  ++X select(x+->prime? x,m)

explicitEntries?: % -> Boolean
  ++ explicitEntries?(s) returns true if the stream s has
  ++ explicitly computed entries, and false otherwise.
  ++
  ++X m:=[i for i in 0..]
  ++X explicitEntries? m

explicitlyEmpty?: % -> Boolean
  ++ explicitlyEmpty?(s) returns true if the stream is an
  ++ (explicitly) empty stream.
  ++ Note that this is a null test which will not cause lazy evaluation.
  ++
  ++X m:=[i for i in 0..]
  ++X explicitlyEmpty? m

lazy?: % -> Boolean
  ++ lazy?(s) returns true if the first node of the stream s
  ++ is a lazy evaluation mechanism which could produce an
  ++ additional entry to s.
  ++
  ++X m:=[i for i in 0..]
  ++X lazy? m

lazyEvaluate: % -> %
  ++ lazyEvaluate(s) causes one lazy evaluation of stream s.
  ++ Caution: the first node must be a lazy evaluation mechanism
  ++ (satisfies \spad{lazy?(s) = true}) as there is no error check.
  ++ Note that a call to this function may
  ++ or may not produce an explicit first entry
frst: % -> S
  ++ frst(s) returns the first element of stream s.
  ++ Caution: this function should only be called after a \spad{empty?}
  ++ test has been made since there no error check.
  ++
  ++X m:=[i for i in 0..]
  ++X frst m

 rst: % -> %
   ++ rst(s) returns a pointer to the next node of stream s.
   ++ Caution: this function should only be called after a \spad{empty?}
   ++ test has been made since there no error check.
   ++
   ++X m:=[i for i in 0..]
   ++X rst m

 numberOfComputedEntries: % -> NonNegativeInteger
   ++ numberOfComputedEntries(st) returns the number of explicitly
   ++ computed entries of stream st which exist immediately prior to the
   ++ time this function is called.
   ++
   ++X m:=[i for i in 0..]
   ++X numberOfComputedEntries m

 extend: (%,Integer) -> %
   ++ extend(st,n) causes entries to be computed, if necessary,
   ++ so that 'st' will have at least 'n' explicit entries or so
   ++ that all entries of 'st' will be computed if 'st' is finite
   ++ with length <= n.
   ++
   ++X m:=[i for i in 0..]
   ++X numberOfComputedEntries m
   ++X extend(m,20)
   ++X numberOfComputedEntries m

 complete: % -> %
   ++ complete(st) causes all entries of 'st' to be computed.
   ++ this function should only be called on streams which are
   ++ known to be finite.
   ++
   ++X m:=[i for i in 1..]
   ++X n:=filterUntil(i+->i>100,m)
   ++X numberOfComputedEntries n
   ++X complete n
   ++X numberOfComputedEntries n

add

 MIN ==> 1  -- minimal stream index

 I   ==> Integer
 NNI ==> NonNegativeInteger
```

```
    L   ==> List
    U   ==> UniversalSegment Integer

    indexx?            : (Integer,%) -> Boolean
    cycleElt           : % -> Union(%,"failed")
    computeCycleLength : % -> NNI
    computeCycleEntry  : (%,%) -> %

  --% SETCAT functions

    if S has SetCategory then

      x = y ==
        eq?(x,y) => true
        explicitlyFinite? x and explicitlyFinite? y =>
          entries x = entries y
        explicitEntries? x and explicitEntries? y =>
          frst x = frst y and EQ(rst x, rst y)$Lisp
        -- treat cyclic streams
        false

  --% HOAGG functions

    --null x == empty? x

    less?(x,n) ==
      n = 0    => false
      empty? x => true
      less?(rst x,(n-1) :: NNI)

    more?(x,n) ==
      empty? x => false
      n = 0    => true
      more?(rst x,(n-1) :: NNI)

    size?(x,n) ==
      empty? x => n = 0
      size?(rst x,(n-1) :: NNI)

    # x ==
      -- error if stream is not finite
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y  => return i
        lazy? y => error "#: infinite stream"
        y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "#: infinite stream"

--% CLAGG functions

  any?(f,x) ==
    -- error message only when x is a stream with lazy
    -- evaluation and f(s) = false for all stream elements
    -- 's' which have been computed when the function is
    -- called
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y  => return false
      lazy? y => error "any?: infinite stream"
      f frst y => return true
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => return false

  every?(f,x) ==
    -- error message only when x is a stream with lazy
    -- evaluation and f(s) = true for all stream elements
    -- 's' which have been computed when the function is
    -- called
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y => return true
      lazy? y => error "every?: infinite stream"
      not f frst y => return false
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => return true

-- following ops count and member? are only exported if $ has finiteAggregate

--  count(f:S -> Boolean,x:%) ==
--    -- error if stream is not finite
--    count : NNI := 0
--    y := x
--    for i in 0.. repeat
--      explicitlyEmpty? y  => return count
--      lazy? y => error "count: infinite stream"
--      if f frst y then count := count + 1
--      y := rst y
--      if odd? i then x := rst x
--      eq?(x,y) => error "count: infinite stream"
--  if S has SetCategory then

--      count(s:S,x:%) == count(#1 = s,x)
--        -- error if stream is not finite

--    member?(s,x) ==
--      -- error message only when x is a stream with lazy
--      -- evaluation and 's' is not among the stream elements
--      -- which have been computed when the function is called
--      y := x
--      for i in 0.. repeat
--        explicitlyEmpty? y  => return false
--        lazy? y => error "member?: infinite stream"
--        frst y = s => return true
--        y := rst y
--        if odd? i then x := rst x
--        eq?(x,y) => return false

  entries x ==
    -- returns a list of elements which have been computed
    -- error if infinite
    y := x
    l : L S := empty()
    for i in 0.. repeat
      explicitlyEmpty? y  => return reverse_! l
      lazy? y => error "infinite stream"
      l := concat(frst y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "infinite stream"

--% CNAGG functions

  construct l ==
    empty? l => empty()
    concat(first l, construct rest l)

  --entries x ==
    -- returns a list of the stream elements
    -- error if the stream is not finite
    --members x

--% ELTAGG functions
```

```
  elt(x:%,n:I) ==
    n < MIN or empty? x => error "elt: no such element"
    n = MIN => frst x
    elt(rst x,n - 1)

  elt(x:%,n:I,s:S) ==
    n < MIN or empty? x => s
    n = MIN => frst x
    elt(rst x,n - 1)

--% IXAGG functions

-- following assumes % has finiteAggregate and S has SetCategory
--  entry?(s,x) ==
--    -- error message only when x is a stream with lazy
--    -- evaluation and 's' is not among the stream elements
--    -- which have been computed when the function is called
--    member?(s,x)

  --entries x ==
    -- error if the stream is not finite
    --members x

  indexx?(n,x) ==
    empty? x => false
    n = MIN => true
    indexx?(n-1,rst x)

  index?(n,x) ==
    -- returns 'true' iff 'n' is the index of an entry which
    -- may or may not have been computed when the function is
    -- called
    -- additional entries are computed if necessary
    n < MIN => false
    indexx?(n,x)

  indices x ==
    -- error if stream is not finite
    y := x
    l : L I := empty()
    for i in MIN.. repeat
      explicitlyEmpty? y  => return reverse_! l
      lazy? y => error "indices: infinite stream"
      l := concat(i,l)
      y := rst y
      if odd? i then x := rst x
```

```
        eq?(x,y) => error "indices: infinite stream"

    maxIndex x ==
      -- error if stream is not finite
      empty? x =>
        error "maxIndex: no maximal index for empty stream"
      y := rst x
      for i in MIN.. repeat
        explicitlyEmpty? y  => return i
        lazy? y => error "maxIndex: infinite stream"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => error "maxIndex: infinite stream"

    minIndex x ==
      empty? x => error "minIndex: no minimal index for empty stream"
      MIN

  --% LNAGG functions

    delete(x:%,n:I) ==
    -- non-destructive
      not index?(n,x) => error "delete: index out of range"
      concat(first(x,(n - MIN) :: NNI), rest(x,(n - MIN + 1) :: NNI))

    delete(x:%,seg:U) ==
      low := lo seg
      hasHi seg =>
        high := hi seg
        high < low => copy x
        (not index?(low,x)) or (not index?(high,x)) =>
          error "delete: index out of range"
        concat(first(x,(low - MIN) :: NNI),rest(x,(high - MIN + 1) :: NNI))
      not index?(low,x) => error "delete: index out of range"
      first(x,(low - MIN) :: NNI)

    elt(x:%,seg:U) ==
      low := lo seg
      hasHi seg =>
        high := hi seg
        high < low => empty()
        (not index?(low,x)) or (not index?(high,x)) =>
          error "elt: index out of range"
        first(rest(x,(low - MIN) :: NNI),(high - low + 1) :: NNI)
      not index?(low,x) => error "elt: index out of range"
      rest(x,(low - MIN) :: NNI)
  insert(s:S,x:%,n:I) ==
    not index?(n,x) => error "insert: index out of range"
    nn := (n - MIN) :: NNI
    concat([first(x,nn), concat(s, empty()), rest(x,nn)])

  insert(y:%,x:%,n:I) ==
    not index?(n,x) => error "insert: index out of range"
    nn := (n - MIN) :: NNI
    concat([first(x,nn), y, rest(x,nn)])

--% RCAGG functions

  cycleElt x == cycleElt(x)$CyclicStreamTools(S,%)

  cyclic? x ==
    cycleElt(x) case "failed" => false
    true

  if S has SetCategory then
    child?(x,y) ==
      empty? y => error "child: no children"
      x = rst y

  children x ==
    empty? x => error "children: no children"
    [rst x]

  distance(x,z) ==
    y := x
    for i in 0.. repeat
      eq?(y,z) => return i
      (explicitlyEmpty? y) or (lazy? y) =>
        error "distance: 2nd arg not a descendent of the 1st"
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) =>
        error "distance: 2nd arg not a descendent of the 1st"

  if S has SetCategory then
    node?(z,x) ==
      -- error message only when x is a stream with lazy
      -- evaluation and 'y' is not a node of 'x'
      -- which has been computed when the function is called
      y := x
      for i in 0.. repeat
```

```
        z = y => return true
        explicitlyEmpty? y => return false
        lazy? y => error "node?: infinite stream"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => return false

  nodes x ==
    y := x
    l : L % := []
    for i in 0.. repeat
      explicitlyEmpty? y => return reverse_! l
      lazy? y => error "nodes: infinite stream"
      l := concat(y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "nodes: infinite stream"
    l -- @#$%^& compiler

  leaf? x == empty? rest x

  value x == first x

--% URAGG functions

  computeCycleLength cycElt ==
    computeCycleLength(cycElt)$CyclicStreamTools(S,%)

  computeCycleEntry(x,cycElt) ==
    computeCycleEntry(x,cycElt)$CyclicStreamTools(S,%)

  cycleEntry x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleEntry: non-cyclic stream"
    computeCycleEntry(x,cycElt::%)

  cycleLength x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleLength: non-cyclic stream"
    computeCycleLength(cycElt::%)

  cycleTail x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
    error "cycleTail: non-cyclic stream"
  y := x := computeCycleEntry(x,cycElt::%)
  z := rst x
  repeat
    eq?(x,z) => return y
    y := z ; z := rst z

elt(x,"first") == first x

first(x,n) ==
-- former name: take
  n = 0 or empty? x => empty()
  concat(frst x, first(rst x,(n-1) :: NNI))

rest x ==
  empty? x => error "Can't take the rest of an empty stream."
  rst x

elt(x,"rest") == rest x

rest(x,n) ==
-- former name: drop
  n = 0 or empty? x => x
  rest(rst x,(n-1) :: NNI)

last x ==
  -- error if stream is not finite
  empty? x => error "last: empty stream"
  y1 := x
  y2 := rst x
  for i in 0.. repeat
    explicitlyEmpty? y2 => return frst y1
    lazy? y2 => error "last: infinite stream"
    y1 := y2
    y2 := rst y2
    if odd? i then x := rst x
    eq?(x,y2) => error "last: infinite stream"

if % has finiteAggregate then -- # is only defined for finiteAggregates
  last(x,n) ==
    possiblyInfinite? x => error "last: infinite stream"
    m := # x
    m < n => error "last: index out of range"
    copy rest(x,(m-n)::NNI)

elt(x,"last") == last x
```

```
    tail x ==
      -- error if stream is not finite
      empty? x => error "tail: empty stream"
      y1 := x
      y2 := rst x
      for i in 0.. repeat
        explicitlyEmpty? y2 => return y1
        lazy? y2 => error "tail: infinite stream"
        y1 := y2
        y2 := rst y2
        if odd? i then x := rst x
        eq?(x,y2) => error "tail: infinite stream"

  --% STAGG functions

    possiblyInfinite? x ==
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y  => return false
        lazy? y => return true
        if odd? i then x := rst x
        y := rst y
        eq?(x,y) => return true

    explicitlyFinite? x == not possiblyInfinite? x

  --% LZSTAGG functions

    extend(x,n) ==
      y := x
      for i in 1..n while not empty? y repeat y := rst y
      x

    complete x ==
      y := x
      while not empty? y repeat y := rst y
      x
```

⟨*LZSTAGG.dotabb*⟩≡

```
  "LZSTAGG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
  "LZSTAGG" -> "STAGG"
```

⟨*LZSTAGG.dotfull*⟩≡
```
"LazyStreamAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
"LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"
```

⟨*LZSTAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LazyStreamAggregate(a:Type)" [color=lightblue];
"LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HOAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

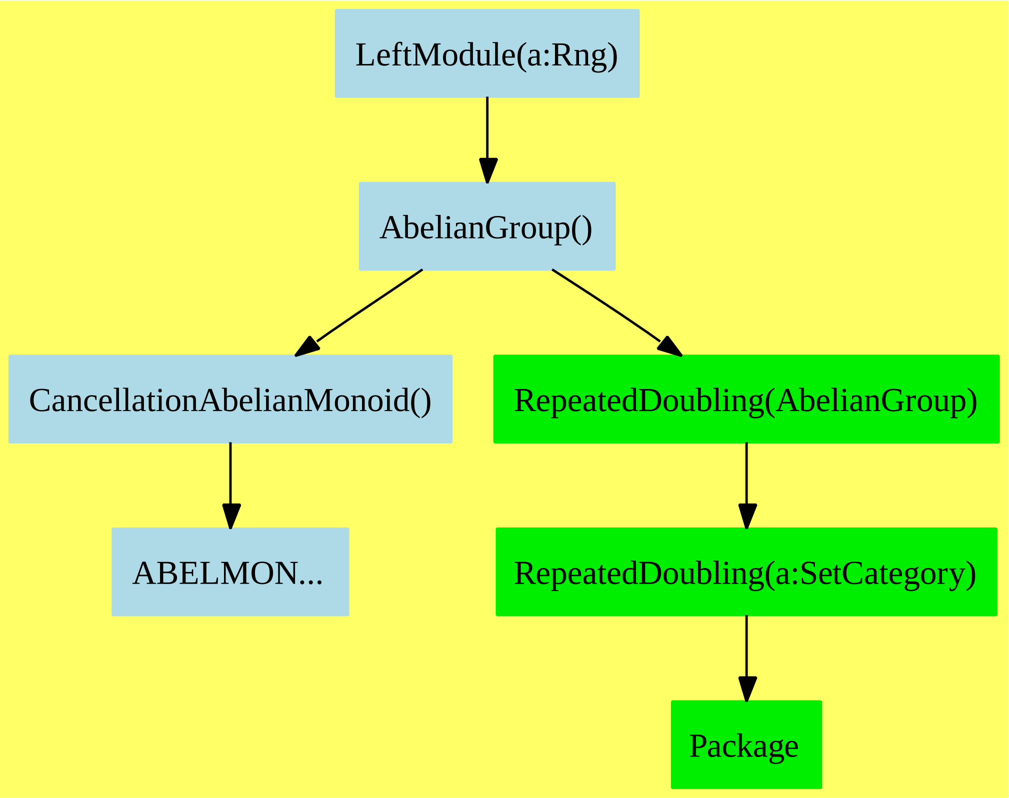
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];

}
```

8.0.113 LeftModule (LMODULE)


LeftModule(a:Rng)
AbelianGroup()
CancellationAbelianMonoid()
RepeatedDoubling(AbelianGroup)
ABELMON...
RepeatedDoubling(a:SetCategory)
Package

⟨*LeftModule.input*⟩≡

```
)set break resume
)sys rm -f LeftModule.output
)spool LeftModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftModule
--R LeftModule R: Rng  is a category constructor
--R Abbreviation for LeftModule is LMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*LeftModule.help*⟩≡

```
====================================================================
LeftModule examples
====================================================================

See Also:
o )show LeftModule
```

See:

⇒ “BiModule” (BMODULE) 9.0.122 on page 800
⇒ “LeftAlgebra” (LALG) 10.0.141 on page 956
⇒ “Ring” (RING) 9.0.129 on page 854
⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (R,%) -> %
```

These exports come from (p576) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

⟨*category LMODULE LeftModule*⟩≡

```
)abbrev category LMODULE LeftModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of left modules over an rng (ring not necessarily with unit).
++ This is an abelian group which supports left multiplation by elements of
++ the rng.
++
++ Axioms\br
++ \tab{5}\spad{ (a*b)*x = a*(b*x) }\br
++ \tab{5}\spad{ (a+b)*x = (a*x)+(b*x) }\br
++ \tab{5}\spad{ a*(x+y) = (a*x)+(a*y) }

LeftModule(R:Rng):Category == AbelianGroup with
   "*": (R,%) -> %
     ++ r*x returns the left multiplication of the module element x
     ++ by the ring element r.
```

⟨*LMODULE.dotabb*⟩≡

```
"LMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LMODULE" -> "ABELGRP"
```

⟨*LMODULE.dotfull*⟩≡

```
"LeftModule(a:Rng)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

⟨*LMODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

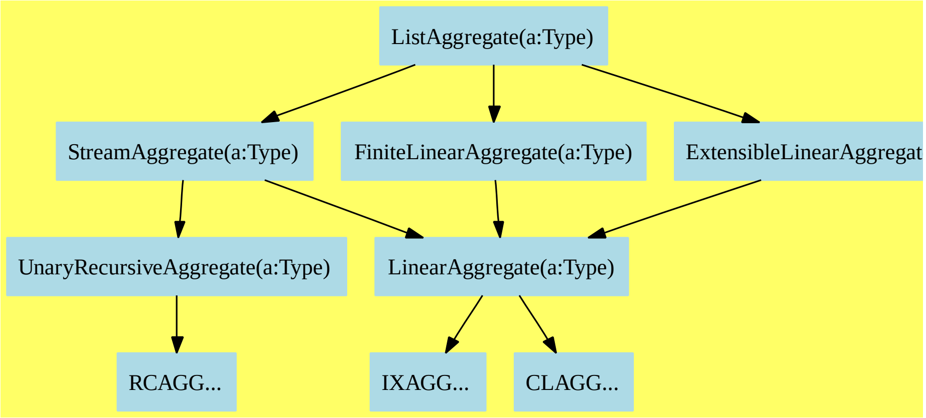
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
}
```

8.0.114 ListAggregate (LSAGG)


ListAggregate(a:Type)
StreamAggregate(a:Type)
FiniteLinearAggregate(a:Type)
ExtensibleLinearAggregat
UnaryRecursiveAggregate(a:Type)
LinearAggregate(a:Type)
RCAGG...
IXAGG...
CLAGG...

⟨*ListAggregate.input*⟩≡

```
)set break resume
)sys rm -f ListAggregate.output
)spool ListAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ListAggregate
--R ListAggregate S: Type  is a category constructor
--R Abbreviation for ListAggregate is LSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LSAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %              concat : (%,S) -> %
--R concat : List % -> %                concat : (S,%) -> %
--R concat : (%,%) -> %                 concat! : (%,S) -> %
--R concat! : (%,%) -> %                construct : List S -> %
--R copy : % -> %                       cycleEntry : % -> %
--R cycleTail : % -> %                  cyclic? : % -> Boolean
--R delete : (%,Integer) -> %           delete! : (%,Integer) -> %
--R distance : (%,%) -> Integer         elt : (%,Integer,S) -> S
--R ?.? : (%,Integer) -> S              ?.last : (%,last) -> S
--R ?.rest : (%,rest) -> %              ?.first : (%,first) -> S
--R ?.value : (%,value) -> S            empty : () -> %
--R empty? : % -> Boolean               entries : % -> List S
--R eq? : (%,%) -> Boolean              explicitlyFinite? : % -> Boolean
```

```
--R first : % -> S                       index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          insert : (S,%,Integer) -> %
--R insert : (%,%,Integer) -> %          insert! : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %         last : % -> S
--R leaf? : % -> Boolean                 leaves : % -> List S
--R list : S -> %                        map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %              new : (NonNegativeInteger,S) -> %
--R nodes : % -> List %                  possiblyInfinite? : % -> Boolean
--R qelt : (%,Integer) -> S              rest : % -> %
--R reverse : % -> %                     sample : () -> %
--R second : % -> S                      tail : % -> %
--R third : % -> S                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
```

```
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,%) -> Integer if S has SETCAT
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates! : % -> % if S has SETCAT
--R rest : (%,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((S,S) -> Boolean),%) -> %
--R sort : % -> % if S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
```

```
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*ListAggregate.help*⟩≡

```
====================================================================
ListAggregate examples
====================================================================

See Also:
o )show ListAggregate
```

See:

⇒ “AssociationListAggregate” (ALAGG) 10.0.134 on page 907
⇐ “ExtensibleLinearAggregate” (ELAGG) 7.0.101 on page 604
⇐ “FiniteLinearAggregate” (FLAGG) 7.0.102 on page 613

Exports:

any? concat copy cycleLength delete empty eq? fill! index? last less? max merge more? parts qsetelt! removeDuplicates reverse! select! setlast! sort swap! #? ?.first ?=?

children concat! copyInto! cycleSplit! delete! empty? eval find indices latex list maxIndex merge! new position reduce removeDuplicates! sample setchildren! setrest! sort! tail ?.? ?.value ?>?

child? construct count cycleTail distance entries every? first insert leaf? map member? min nodes possiblyInfinite? remove rest second setelt setvalue! sorted? third ?.last ?<? ?>=?

coerce convert cycleEntry cyclic? elt entry? explicitlyFinite? hash insert! leaves map! members minIndex node? qelt remove! reverse select setfirst! size? split! value ?.rest ?<=? ?~=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
copy : % -> %
```

```
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
find : ((S -> Boolean),%) -> Union(S,"failed")
insert! : (S,%,Integer) -> %
insert! : (%,%,Integer) -> %
list : S -> %
map : (((S,S) -> S),%,%) -> %
merge : (((S,S) -> Boolean),%,%) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
new : (NonNegativeInteger,S) -> %
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer if S has SETCAT
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
reverse! : % -> % if $ has shallowlyMutable
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p647) StreamAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (%,S) -> %
concat : List % -> %
concat : (S,%) -> %
concat : (%,%) -> %
concat! : (%,S) -> %
concat! : (%,%) -> %
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
```

```
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
distance : (%,%) -> Integer
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
possiblyInfinite? : % -> Boolean
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
remove : (S,%) -> %
         if S has SETCAT
         and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT
         and $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?.value : (%,value) -> S
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p613) FiniteLinearAggregate(S:Type)

```
max : (%,%) -> % if S has ORDSET
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : (((S,S) -> Boolean),%) -> %
sort : % -> % if S has ORDSET
sort! : % -> % if S has ORDSET and $ has shallowlyMutable
sorted? : % -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p604) ExtensibleLinearAggregate(S:Type):

```
merge! : (%,%) -> % if S has ORDSET
remove! : (S,%) -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

⟨*category LSAGG ListAggregate*⟩≡

```
)abbrev category LSAGG ListAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A list aggregate is a model for a linked list data structure.
++ A linked list is a versatile
++ data structure. Insertion and deletion are efficient and
++ searching is a linear operation.

ListAggregate(S:Type): Category == Join(StreamAggregate S,
   FiniteLinearAggregate S, ExtensibleLinearAggregate S) with
      list: S -> %
        ++ list(x) returns the list of one element x.
 add
   cycleMax ==> 1000

   mergeSort: ((S, S) -> Boolean, %, Integer) -> %

   sort_!(f, l)              == mergeSort(f, l, #l)
   list x                   == concat(x, empty())
   reduce(f, x)                   ==
    empty? x => _
      error "reducing over an empty list needs the 3 argument form"
    reduce(f, rest x, first x)
  merge(f, p, q)           == merge_!(f, copy p, copy q)

  select_!(f, x) ==
    while not empty? x and not f first x repeat x := rest x
    empty? x => x
    y := x
    z := rest y
    while not empty? z repeat
      if f first z then (y := z; z := rest z)
                   else (z := rest z; setrest_!(y, z))
    x

  merge_!(f, p, q) ==
    empty? p => q
    empty? q => p
    eq?(p, q) => error "cannot merge a list into itself"
    if f(first p, first q)
      then (r := t := p; p := rest p)
      else (r := t := q; q := rest q)
    while not empty? p and not empty? q repeat
      if f(first p, first q)
        then (setrest_!(t, p); t := p; p := rest p)
        else (setrest_!(t, q); t := q; q := rest q)
    setrest_!(t, if empty? p then q else p)
    r

  insert_!(s:S, x:%, i:Integer) ==
    i < (m := minIndex x) => error "index out of range"
    i = m => concat(s, x)
    y := rest(x, (i - 1 - m)::NonNegativeInteger)
    z := rest y
    setrest_!(y, concat(s, z))
    x

  insert_!(w:%, x:%, i:Integer) ==
    i < (m := minIndex x) => error "index out of range"
    i = m => concat_!(w, x)
    y := rest(x, (i - 1 - m)::NonNegativeInteger)
    z := rest y
    setrest_!(y, w)
    concat_!(y, z)
    x
```

```
remove_!(f:S -> Boolean, x:%) ==
  while not empty? x and f first x repeat x := rest x
  empty? x => x
  p := x
  q := rest x
  while not empty? q repeat
    if f first q then q := setrest_!(p, rest q)
                 else (p := q; q := rest q)
  x

delete_!(x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => rest x
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  setrest_!(y, rest(y, 2))
  x

delete_!(x:%, i:UniversalSegment(Integer)) ==
  (l := lo i) < (m := minIndex x) => error "index out of range"
  h := if hasHi i then hi i else maxIndex x
  h < l => x
  l = m => rest(x, (h + 1 - m)::NonNegativeInteger)
  t := rest(x, (l - 1 - m)::NonNegativeInteger)
  setrest_!(t, rest(t, (h - l + 2)::NonNegativeInteger))
  x

find(f, x) ==
  while not empty? x and not f first x repeat x := rest x
  empty? x => "failed"
  first x

position(f:S -> Boolean, x:%) ==
  for k in minIndex(x).. while not empty? x and not f first x repeat
    x := rest x
  empty? x => minIndex(x) - 1
  k

mergeSort(f, p, n) ==
  if n = 2 and f(first rest p, first p) then p := reverse_! p
  n < 3 => p
  l := (n quo 2)::NonNegativeInteger
  q := split_!(p, l)
  p := mergeSort(f, p, l)
  q := mergeSort(f, q, n - l)
  merge_!(f, p, q)
```

```
    sorted?(f, l) ==
      empty? l => true
      p := rest l
      while not empty? p repeat
        not f(first l, first p) => return false
        p := rest(l := p)
      true

    reduce(f, x, i) ==
      r := i
      while not empty? x repeat (r := f(r, first x); x := rest x)
      r

    if S has SetCategory then
       reduce(f, x, i,a) ==
         r := i
         while not empty? x and r ^= a repeat
           r := f(r, first x)
           x := rest x
         r

    new(n, s) ==
      l := empty()
      for k in 1..n repeat l := concat(s, l)
      l

    map(f, x, y) ==
      z := empty()
      while not empty? x and not empty? y repeat
        z := concat(f(first x, first y), z)
        x := rest x
        y := rest y
      reverse_! z

 -- map(f, x, y, d) ==
 --   z := empty()
 --   while not empty? x and not empty? y repeat
 --     z := concat(f(first x, first y), z)
 --     x := rest x
 --     y := rest y
 --   z := reverseInPlace z
 --   if not empty? x then
 --        z := concat_!(z, map(f(#1, d), x))
 --   if not empty? y then
 --        z := concat_!(z, map(f(d, #1), y))
 --   z
```

```
reverse_! x ==
  empty? x => x
  empty?(y := rest x) => x
  setrest_!(x, empty())
  while not empty? y repeat
    z := rest y
    setrest_!(y, x)
    x := y
    y := z
  x

copy x ==
  y := empty()
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := concat(first x, y)
    x := rest x
  reverse_! y

copyInto_!(y, x, s) ==
  s < (m := minIndex y) => error "index out of range"
  z := rest(y, (s - m)::NonNegativeInteger)
  while not empty? z and not empty? x repeat
    setfirst_!(z, first x)
    x := rest x
    z := rest z
  y

if S has SetCategory then
  position(w, x, s) ==
    s < (m := minIndex x) => error "index out of range"
    x := rest(x, (s - m)::NonNegativeInteger)
    for k in s.. while not empty? x and w ^= first x repeat
      x := rest x
    empty? x => minIndex x - 1
    k

  removeDuplicates_! l ==
    p := l
    while not empty? p repeat
      p := setrest_!(p, remove_!((x:S):Boolean +-> x = first p, rest p))
    l

if S has OrderedSet then
  x < y ==
          while not empty? x and not empty? y repeat
            first x ^= first y => return(first x < first y)
            x := rest x
            y := rest y
          empty? x => not empty? y
          false
```

⟨*LSAGG.dotabb*⟩≡

```
  "LSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
  "LSAGG" -> "FLAGG"
  "LSAGG" -> "ELAGG"
```

⟨*LSAGG.dotfull*⟩≡

```
  "ListAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
  "ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
  "ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
  "ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

  "ListAggregate(Record(a:SetCategory,b:SetCategory))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=LSAGG"];
  "ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
      "ListAggregate(a:Type)"
```

⟨*LSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

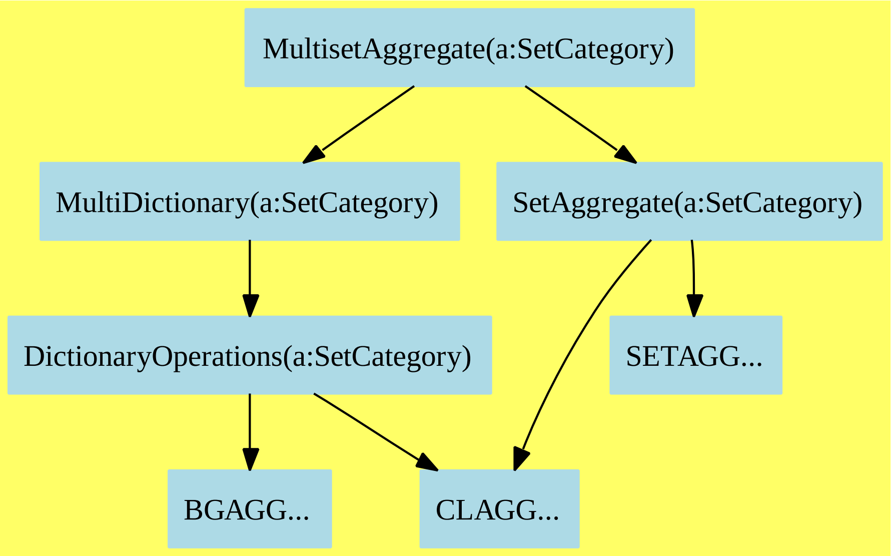
"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RCAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"RCAGG..." [color=lightblue];
}
```

8.0.115 MultisetAggregate (MSETAGG)


MultisetAggregate(a:SetCategory)
MultiDictionary(a:SetCategory)
SetAggregate(a:SetCategory)
DictionaryOperations(a:SetCategory)
SETAGG...
BGAGG...
CLAGG...

⟨*MultisetAggregate.input*⟩≡
```
)set break resume
)sys rm -f MultisetAggregate.output
)spool MultisetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MultisetAggregate
--R MultisetAggregate S: SetCategory  is a category constructor
--R Abbreviation for MultisetAggregate is MSETAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MSETAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R bag : List S -> %                     brace : () -> %
--R brace : List S -> %                   coerce : % -> OutputForm
--R construct : List S -> %               copy : % -> %
--R dictionary : List S -> %              dictionary : () -> %
--R difference : (%,%) -> %               difference : (%,S) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R hash : % -> SingleInteger             insert! : (S,%) -> %
```

```
--R inspect : % -> S                     intersect : (%,%) -> %
--R latex : % -> String                  map : ((S -> S),%) -> %
--R removeDuplicates! : % -> %           sample : () -> %
--R set : () -> %                        set : List S -> %
--R subset? : (%,%) -> Boolean            union : (%,%) -> %
--R union : (%,S) -> %                    union : (S,%) -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List Record(entry: S,count: NonNegativeInteger)
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R insert! : (S,%,NonNegativeInteger) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R symmetricDifference : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*MultisetAggregate.help*⟩≡

```
====================================================================
MultisetAggregate examples
====================================================================

See Also:
o )show MultisetAggregate
```

See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.0.128 on page 846
⇐ "MultiDictionary" (MDAGG) 7.0.104 on page 629
⇐ "SetAggregate" (SETAGG) 6.0.94 on page 547

Exports:

any? bag brace coerce
construct convert copy count
dictionary difference duplicates empty
empty? eq? eval every?
extract! find hash insert!
inspect intersect latex less?
map map! member? members
more? parts reduce remove
remove! removeDuplicates removeDuplicates! sample
select select! set size?
subset? symmetricDifference union #?
?~=? ?<? ?=?

Attributes exported:

- **partiallyOrderedSet** is true if a set with < which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These exports come from (p629) MultiDictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
```

```
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p547) SetAggregate(S:SetCategory):

```
brace : () -> %
brace : List S -> %
difference : (%,S) -> %
intersect : (%,%) -> %
sample : () -> %
set : () -> %
set : List S -> %
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

⟨category MSETAGG MultisetAggregate⟩≡

```
)abbrev category MSETAGG MultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A multi-set aggregate is a set which keeps track of the multiplicity
++ of its elements.

MultisetAggregate(S:SetCategory):
 Category == Join(MultiDictionary S, SetAggregate S)
```

⟨MSETAGG.dotabb⟩≡

```
"MSETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MSETAGG" -> "MDAGG"
"MSETAGG" -> "SETAGG"
```

⟨MSETAGG.dotfull⟩≡

```
"MultisetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

⟨*MSETAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

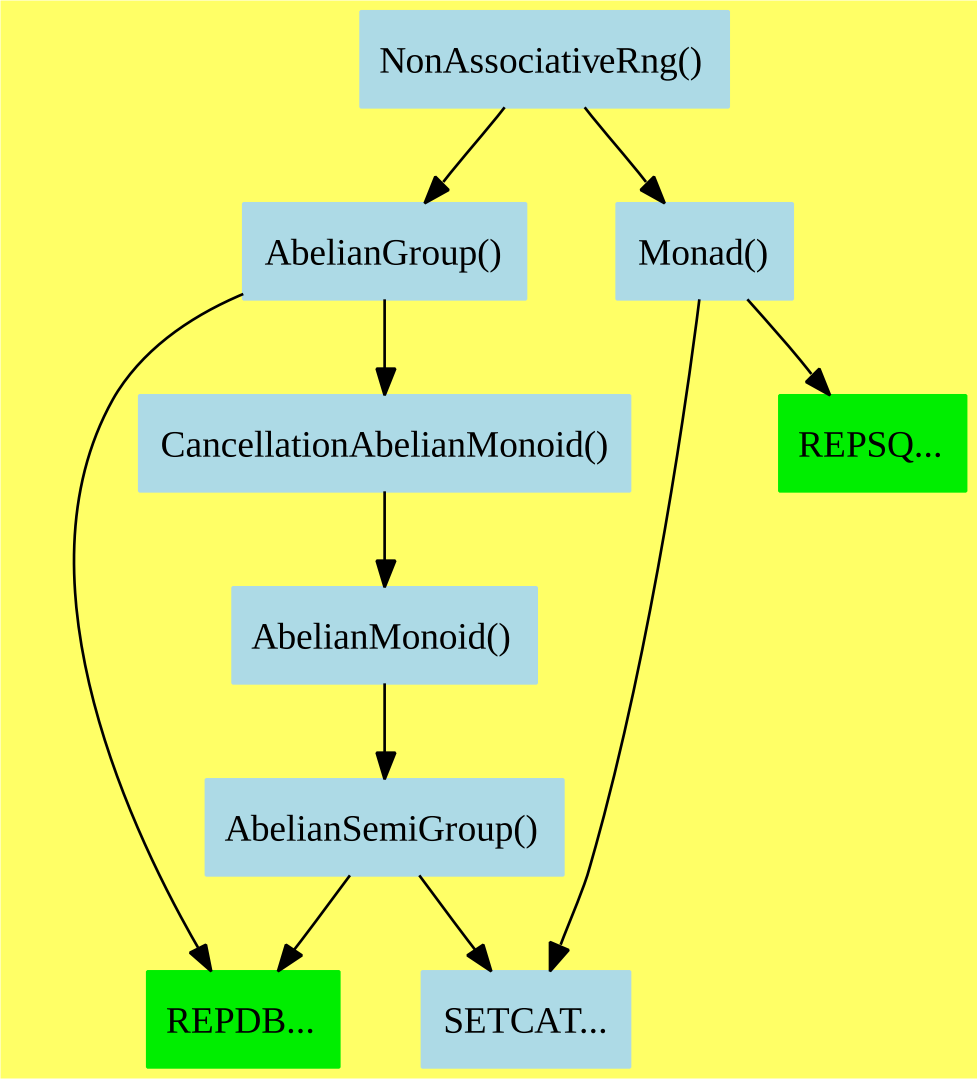
"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

8.0.116 NonAssociativeRng (NARNG)


NonAssociativeRng()
AbelianGroup()
Monad()
CancellationAbelianMonoid()
REPSQ...
AbelianMonoid()
AbelianSemiGroup()
REPDB...
SETCAT...

⟨*NonAssociativeRng.input*⟩≡

```
)set break resume
)sys rm -f NonAssociativeRng.output
)spool NonAssociativeRng.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show NonAssociativeRng
--R NonAssociativeRng  is a category constructor
--R Abbreviation for NonAssociativeRng is NARNG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NARNG
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %             coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*NonAssociativeRng.help*⟩≡

```
====================================================================
NonAssociativeRng examples
====================================================================

See Also:
o )show NonAssociativeRng
```

See:

⇒ “NonAssociativeAlgebra” (NAALG) 11.0.157 on page 1082
⇒ “NonAssociativeRing” (NASRING) 9.0.124 on page 816
⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576
⇐ “Monad” (MONAD) 4.0.52 on page 221

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	rightPower	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?~=?	

These are implemented by this category:

```
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
commutator : (%,%) -> %
```

These exports come from (p576) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p221) Monad():

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category NARNG NonAssociativeRng*⟩≡

```
)abbrev category NARNG NonAssociativeRng
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 03 July 1991
++ Basic Operations: +, *, -, **
++ Related Constructors: Rng, Ring, NonAssociativeRing
++ Also See:
++ AMS Classifications:
++ Keywords: not associative ring
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++ NonAssociativeRng is a basic ring-type structure, not necessarily
++ commutative or associative, and not necessarily with unit.\br
++ Axioms\br
++ \tab{5}x*(y+z) = x*y + x*z\br
++ \tab{5}(x+y)*z = x*z + y*z\br
++
++ Common Additional Axioms\br
++ \tab{5}noZeroDivisors\tab{5} ab = 0 => a=0 or b=0

NonAssociativeRng(): Category == Join(AbelianGroup,Monad)  with
    associator: (%,%,%) -> %
      ++ associator(a,b,c) returns \spad{(a*b)*c-a*(b*c)}.
    commutator: (%,%) -> %
      ++ commutator(a,b) returns \spad{a*b-b*a}.
    antiCommutator: (%,%) -> %
      ++ antiCommutator(a,b) returns \spad{a*b+b*a}.
  add
    associator(x,y,z) == (x*y)*z - x*(y*z)
    commutator(x,y) == x*y - y*x
    antiCommutator(x,y) == x*y + y*x
```

⟨*NARNG.dotabb*⟩≡

```
"NARNG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NARNG" -> "ABELGRP"
"NARNG" -> "MONAD"
```

⟨*NARNG.dotfull*⟩≡

```
"NonAssociativeRng()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"
```

⟨*NARNG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

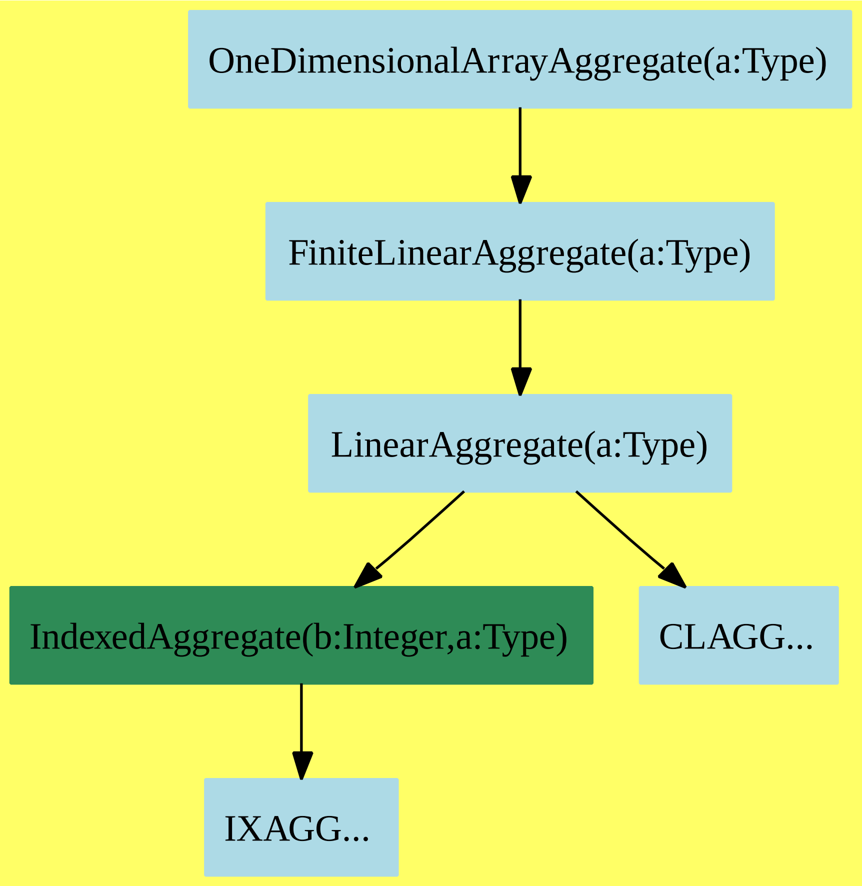
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

8.0.117 OneDimensionalArrayAggregate (A1AGG)


OneDimensionalArrayAggregate(a:Type)
FiniteLinearAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
CLAGG...
IXAGG...

⟨*OneDimensionalArrayAggregate.input*⟩≡

```
)set break resume
)sys rm -f OneDimensionalArrayAggregate.output
)spool OneDimensionalArrayAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OneDimensionalArrayAggregate
--R OneDimensionalArrayAggregate S: Type  is a category constructor
--R Abbreviation for OneDimensionalArrayAggregate is A1AGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for A1AGG
```

```
--R
--R------------------------------ Operations -------------------------------
--R concat : List % -> %                 concat : (%,%) -> %
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R construct : List S -> %             copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List S
--R eq? : (%,%) -> Boolean               index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %          map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %              new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S              reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
```

```
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*OneDimensionalArrayAggregate.help*⟩≡

```
====================================================================
OneDimensionalArrayAggregate examples
====================================================================

See Also:
o )show OneDimensionalArrayAggregate
```

See:

⇒ “BitAggregate” (BTAGG) 9.0.123 on page 806
⇒ “StringAggregate” (SRAGG) 9.0.131 on page 872
⇒ “VectorCategory” (VECTCAT) 9.0.133 on page 897
⇐ “FiniteLinearAggregate” (FLAGG) 7.0.102 on page 613

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	#?
?.?	?<?	?<=?	?~=?	?=?
?>?	?>=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : List % -> %
construct : List S -> %
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
```

```
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
parts : % -> List S if $ has finiteAggregate
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer
         if S has SETCAT
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
reverse! : % -> % if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p613) FiniteLinearAggregate(S:Type):

```
concat : (S,%) -> %
concat : (%,S) -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
```

```
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
max : (%,%) -> % if S has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,S) -> %
position : (S,%) -> Integer if S has SETCAT
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
remove : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
reverse : % -> %
sample : () -> %
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if S has ORDSET
sort : (((S,S) -> Boolean),%) -> %
sort! : % -> %
         if S has ORDSET and $ has shallowlyMutable
sorted? : % -> Boolean if S has ORDSET
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
```

⟨*category A1AGG OneDimensionalArrayAggregate*⟩≡

```
)abbrev category A1AGG OneDimensionalArrayAggregate
```

```
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ One-dimensional-array aggregates serves as models for one-dimensional
++ arrays. Categorically, these aggregates are finite linear aggregates
++ with the \spadatt{shallowlyMutable} property, that is, any component of
++ the array may be changed without affecting the
++ identity of the overall array.
++ Array data structures are typically represented by a fixed area in
++ storage and cannot efficiently grow or shrink on demand as can list
++ structures (see however \spadtype{FlexibleArray} for a data structure
++ which is a cross between a list and an array).
++ Iteration over, and access to, elements of arrays is extremely fast
++ (and often can be optimized to open-code).
++ Insertion and deletion however is generally slow since an entirely new
++ data structure must be created for the result.

OneDimensionalArrayAggregate(S:Type): Category ==
    FiniteLinearAggregate S with shallowlyMutable
  add
    parts x            == [qelt(x, i) for i in minIndex x .. maxIndex x]
    sort_!(f, a) == quickSort(f, a)$FiniteLinearAggregateSort(S, %)

    any?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return true
      false

    every?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        not(f qelt(a, i)) => return false
      true

    position(f:S -> Boolean, a:%) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return i
      minIndex(a) - 1

    find(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return qelt(a, i)
      "failed"

    count(f:S->Boolean, a:%) ==
      n:NonNegativeInteger := 0
      for i in minIndex a .. maxIndex a repeat
        if f(qelt(a, i)) then n := n+1
      n

    map_!(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        qsetelt_!(a, i, f qelt(a, i))
      a

    setelt(a:%, s:UniversalSegment(Integer), x:S) ==
      l := lo s; h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      for k in l..h repeat qsetelt_!(a, k, x)
      x

    reduce(f, a) ==
      empty? a => error "cannot reduce an empty aggregate"
      r := qelt(a, m := minIndex a)
      for k in m+1 .. maxIndex a repeat r := f(r, qelt(a, k))
      r

    reduce(f, a, identity) ==
      for k in minIndex a .. maxIndex a repeat
        identity := f(identity, qelt(a, k))
      identity

    if S has SetCategory then
       reduce(f, a, identity,absorber) ==
         for k in minIndex a .. maxIndex a while identity ^= absorber
                repeat identity := f(identity, qelt(a, k))
         identity

-- this is necessary since new has disappeared.
    stupidnew: (NonNegativeInteger, %, %) -> %
    stupidget: List % -> S
-- a and b are not both empty if n > 0
    stupidnew(n, a, b) ==
      zero? n => empty()
      new(n, (empty? a => qelt(b, minIndex b); qelt(a, minIndex a)))
-- at least one element of l must be non-empty
```

```
    stupidget l ==
      for a in l repeat
        not empty? a => return first a
      error "Should not happen"

    map(f, a, b) ==
      m := max(minIndex a, minIndex b)
      n := min(maxIndex a, maxIndex b)
      l := max(0, n - m + 1)::NonNegativeInteger
      c := stupidnew(l, a, b)
      for i in minIndex(c).. for j in m..n repeat
        qsetelt_!(c, i, f(qelt(a, j), qelt(b, j)))
      c

--  map(f, a, b, x) ==
--    m := min(minIndex a, minIndex b)
--    n := max(maxIndex a, maxIndex b)
--    l := (n - m + 1)::NonNegativeInteger
--    c := new l
--    for i in minIndex(c).. for j in m..n repeat
--        qsetelt_!(c, i, f(a(j, x), b(j, x)))
--    c

    merge(f, a, b) ==
      r := stupidnew(#a + #b, a, b)
      i := minIndex a
      m := maxIndex a
      j := minIndex b
      n := maxIndex b
      for k in minIndex(r).. while i <= m and j <= n repeat
        if f(qelt(a, i), qelt(b, j)) then
          qsetelt_!(r, k, qelt(a, i))
          i := i+1
        else
          qsetelt_!(r, k, qelt(b, j))
          j := j+1
      for k in k.. for i in i..m repeat qsetelt_!(r, k, elt(a, i))
      for k in k.. for j in j..n repeat qsetelt_!(r, k, elt(b, j))
      r

    elt(a:%, s:UniversalSegment(Integer)) ==
      l := lo s
      h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      r := stupidnew(max(0, h - l + 1)::NonNegativeInteger, a, a)
      for k in minIndex r.. for i in l..h repeat
```

```
        qsetelt_!(r, k, qelt(a, i))
      r

    insert(a:%, b:%, i:Integer) ==
      m := minIndex b
      n := maxIndex b
      i < m or i > n => error "index out of range"
      y := stupidnew(#a + #b, a, b)
      for k in minIndex y.. for j in m..i-1 repeat
        qsetelt_!(y, k, qelt(b, j))
      for k in k.. for j in minIndex a .. maxIndex a repeat
        qsetelt_!(y, k, qelt(a, j))
      for k in k.. for j in i..n repeat qsetelt_!(y, k, qelt(b, j))
      y

    copy x ==
      y := stupidnew(#x, x, x)
      for i in minIndex x .. maxIndex x for j in minIndex y .. repeat
        qsetelt_!(y, j, qelt(x, i))
      y

    copyInto_!(y, x, s) ==
      s < minIndex y or s + #x > maxIndex y + 1 =>
                                          error "index out of range"
      for i in minIndex x .. maxIndex x for j in s.. repeat
        qsetelt_!(y, j, qelt(x, i))
      y

    construct l ==
--    a := new(#l)
      empty? l => empty()
      a := new(#l, first l)
      for i in minIndex(a).. for x in l repeat qsetelt_!(a, i, x)
      a

    delete(a:%, s:UniversalSegment(Integer)) ==
      l := lo s; h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      h < l => copy a
      r := stupidnew((#a - h + l - 1)::NonNegativeInteger, a, a)
      for k in minIndex(r).. for i in minIndex a..l-1 repeat
        qsetelt_!(r, k, qelt(a, i))
      for k in k.. for i in h+1 .. maxIndex a repeat
        qsetelt_!(r, k, qelt(a, i))
      r
```

```
delete(x:%, i:Integer) ==
  i < minIndex x or i > maxIndex x => error "index out of range"
  y := stupidnew((#x - 1)::NonNegativeInteger, x, x)
  for i in minIndex(y).. for j in minIndex x..i-1 repeat
    qsetelt_!(y, i, qelt(x, j))
  for i in i .. for j in i+1 .. maxIndex x repeat
    qsetelt_!(y, i, qelt(x, j))
  y

reverse_! x ==
  m := minIndex x
  n := maxIndex x
  for i in 0..((n-m) quo 2) repeat swap_!(x, m+i, n-i)
  x

concat l ==
  empty? l => empty()
  n := _+/[#a for a in l]
  i := minIndex(r := new(n, stupidget l))
  for a in l repeat
    copyInto_!(r, a, i)
    i := i + #a
  r

sorted?(f, a) ==
  for i in minIndex(a)..maxIndex(a)-1 repeat
    not f(qelt(a, i), qelt(a, i + 1)) => return false
  true

concat(x:%, y:%) ==
  z := stupidnew(#x + #y, x, y)
  copyInto_!(z, x, i := minIndex z)
  copyInto_!(z, y, i + #x)
  z

if S has SetCategory then
  x = y ==
    #x ^= #y => false
    for i in minIndex x .. maxIndex x repeat
      not(qelt(x, i) = qelt(y, i)) => return false
    true

  coerce(r:%):OutputForm ==
    bracket commaSeparate
          [qelt(r, k)::OutputForm for k in minIndex r .. maxIndex r]
    position(x:S, t:%, s:Integer) ==
      n := maxIndex t
      s < minIndex t or s > n => error "index out of range"
      for k in s..n repeat
        qelt(t, k) = x => return k
      minIndex(t) - 1

  if S has OrderedSet then
    a < b ==
      for i in minIndex a .. maxIndex a
        for j in minIndex b .. maxIndex b repeat
          qelt(a, i) ^= qelt(b, j) => return a.i < b.j
      #a < #b
```

⟨*A1AGG.dotabb*⟩≡

```
"A1AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"A1AGG" -> "FLAGG"
```

⟨*A1AGG.dotfull*⟩≡

```
"OneDimensionalArrayAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"OneDimensionalArrayAggregate(Character)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"
```

⟨*A1AGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

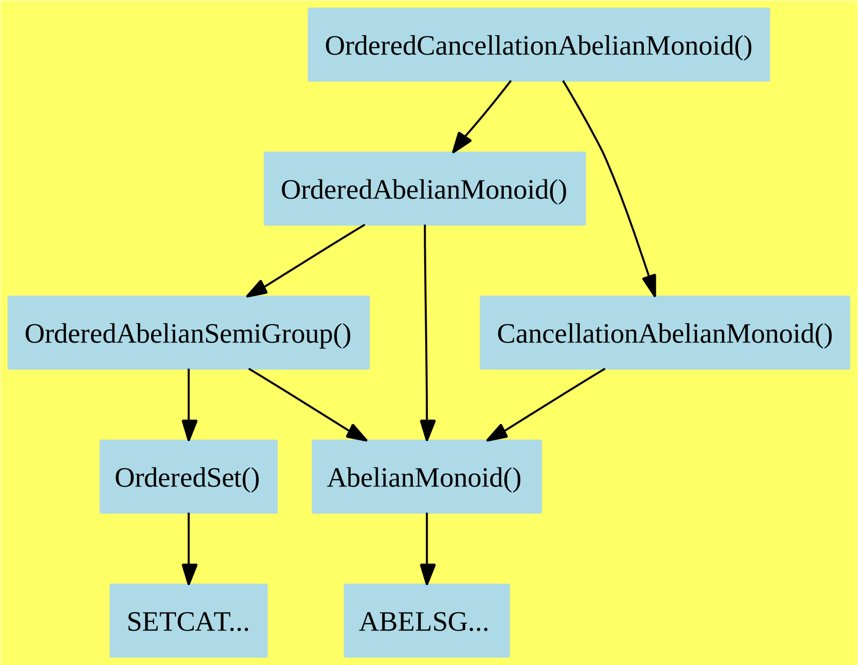
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

8.0.118 OrderedCancellationAbelianMonoid (OCAMON)


OrderedCancellationAbelianMonoid()
OrderedAbelianMonoid()
OrderedAbelianSemiGroup()
CancellationAbelianMonoid()
OrderedSet()
AbelianMonoid()
SETCAT...
ABELSG...

⟨*OrderedCancellationAbelianMonoid.input*⟩≡

```
)set break resume
)sys rm -f OrderedCancellationAbelianMonoid.output
)spool OrderedCancellationAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedCancellationAbelianMonoid
--R OrderedCancellationAbelianMonoid  is a category constructor
--R Abbreviation for OrderedCancellationAbelianMonoid is OCAMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OCAMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
```

```
--R latex : % -> String                    max : (%,%) -> %
--R min : (%,%) -> %                       sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedCancellationAbelianMonoid.help*⟩≡

```
====================================================================
OrderedCancellationAbelianMonoid examples
====================================================================

See Also:
o )show OrderedCancellationAbelianMonoid
```

See:

⇒ “OrderedAbelianGroup” (OAGROUP) 9.0.126 on page 836
⇒ “OrderedAbelianMonoidSup” (OAMONS) 9.0.127 on page 841
⇐ “CancellationAbelianMonoid” (CABMON) 6.0.83 on page 423
⇐ “OrderedAbelianMonoid” (OAMON) 7.0.105 on page 636

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	zero?	?˜=?
?*?	?+?	?<?	?<=?	?=?
?>?	?>=?			

These exports come from (p636) OrderedAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

These exports come from (p423) CancellationAbelianMonoid():

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

⟨*category OCAMON OrderedCancellationAbelianMonoid*⟩≡

```
)abbrev category OCAMON OrderedCancellationAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ Ordered sets which are also abelian cancellation monoids,
++ such that the addition preserves the ordering.

OrderedCancellationAbelianMonoid(): Category ==
        Join(OrderedAbelianMonoid, CancellationAbelianMonoid)
```

⟨*OCAMON.dotabb*⟩≡

```
"OCAMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OCAMON" -> "OAMON"
"OCAMON" -> "CABMON"
```

⟨*OCAMON.dotfull*⟩≡

```
"OrderedCancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"
```

⟨*OCAMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

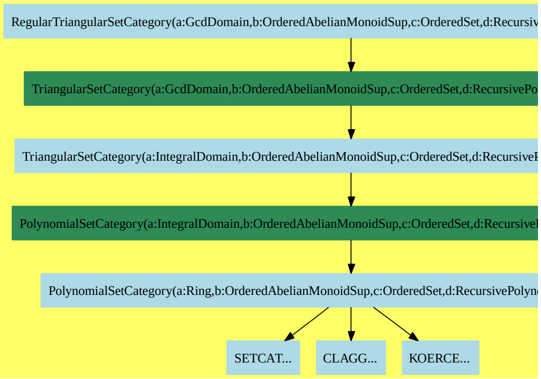
"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

8.0.119 RegularTriangularSetCategory (RSETCAT)


RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursiv
TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveI
PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursivel
PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolyn
SETCAT...
CLAGG...
KOERCE...

⟨*RegularTriangularSetCategory.input*⟩≡
```
)set break resume
)sys rm -f RegularTriangularSetCategory.output
)spool RegularTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RegularTriangularSetCategory
--R RegularTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: Order
--R Abbreviation for RegularTriangularSetCategory is RSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List V      augment : (List P,%) -> List %
--R augment : (P,List %) -> List %        augment : (P,%) -> List %
--R coerce : % -> List P                  coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List P -> %               copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List P,%) -> List %         extend : (P,List %) -> List %
--R extend : (P,%) -> List %              extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
```

```
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List P                internalAugment : (P,%) -> %
--R intersect : (P,List %) -> List %      intersect : (List P,%) -> List %
--R intersect : (P,%) -> List %           invertible? : (P,%) -> Boolean
--R invertibleSet : (P,%) -> List %       last : % -> Union(P,"failed")
--R latex : % -> String                   mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List V           map : ((P -> P),%) -> %
--R mvar : % -> V                         normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean        purelyAlgebraic? : % -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List P -> %
--R sample : () -> %                      stronglyReduce : (P,%) -> P
--R stronglyReduced? : % -> Boolean       trivialIdeal? : % -> Boolean
--R variables : % -> List V               zeroSetSplit : List P -> List %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List P,List %) -> List %
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List P,List %) -> List %
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R internalAugment : (List P,%) -> %
--R intersect : (List P,List %) -> List %
--R invertible? : (P,%) -> List Record(val: Boolean,tower: %)
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
--R lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R purelyAlgebraic? : (P,%) -> Boolean
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List Record(val: P,tower: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List P,Boolean) -> List %
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List
--R
--E 1

)spool
)lisp (bye)
```

⟨*RegularTriangularSetCategory.help*⟩≡

```
====================================================================
RegularTriangularSetCategory examples
====================================================================

See Also:
o )show RegularTriangularSetCategory
```

See:

⇒ “NormalizedTriangularSetCategory” (NTSCAT) 9.0.125 on page 823
⇒ “SquareFreeRegularTriangularSetCategory” (SFRTCAT) 9.0.130 on page 860

⇐ “TriangularSetCategory” (TSETCAT) 7.0.108 on page 657

Exports:

algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
convert
copy
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
less?
latex
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduceByQuasiMonic
reduced?
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of

elements.

- **nil**

These are directly exported but not implemented:

```
augment : (P,%) -> List %
extend : (P,%) -> List %
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,%) -> List %
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertible? : (P,%) -> Boolean
invertibleSet : (P,%) -> List %
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
zeroSetSplit : (List P,Boolean) -> List %
```

These are implemented by this category:

```
algebraicCoefficients? : (P,%) -> Boolean
augment : (P,List %) -> List %
augment : (List P,%) -> List %
augment : (List P,List %) -> List %
extend : (P,List %) -> List %
extend : (List P,%) -> List %
extend : (List P,List %) -> List %
intersect : (List P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,List %) -> List %
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
```

These exports come from (p657) TriangularSetCategory(R,E,V,P) where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
```

```
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
retractIfCan : List P -> Union(%,"failed")
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
select : (%,V) -> Union(P,"failed")
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplitIntoTriangularSystems :
   List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category RSETCAT RegularTriangularSetCategory*⟩≡

```
)abbrev category RSETCAT RegularTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1998
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References :
++  [1] M. KALKBRENER "Three contributions to elimination theory"
++      Phd Thesis, University of Linz, Austria, 1991.
++  [2] M. KALKBRENER "Algorithmic properties of polynomial rings"
++      Journal of Symbol. Comp. 1998
++  [3] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++  [4] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ The category of regular triangular sets, introduced under
++ the name regular chains in [1] (and other papers).
++ In [3] it is proved that regular triangular sets and towers of simple
++ extensions of a field are equivalent notions.
++ In the following definitions, all polynomials and ideals
++ are taken from the polynomial ring \spad{k[x1,...,xn]} where \spad{k}
++ is the fraction field of \spad{R}.
++ The triangular set \spad{[t1,...,tm]} is regular
++ iff for every \spad{i} the initial of \spad{ti+1} is invertible
++ in the tower of simple extensions associated with \spad{[t1,...,ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given ideal \spad{I}
++ iff the radical of \spad{I} is equal to the intersection
++ of the radical ideals generated by the saturated ideals
++ of the \spad{[T1,...,Ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given triangular set \spad{T}
++ iff it is a split of Kalkbrener of the saturated ideal of \spad{T}.
++ Let \spad{K} be an algebraic closure of \spad{k}.
++ Assume that \spad{V} is finite with cardinality
++ \spad{n} and let \spad{A} be the affine space \spad{K^n}.
++ For a regular triangular set \spad{T} let denote by \spad{W(T)} the
++ set of regular zeros of \spad{T}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given subset \spad{S} of \spad{A}
++ iff the union of the \spad{W(Ti)} contains \spad{S} and
++ is contained in the closure of \spad{S} (w.r.t. Zariski topology).
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given triangular set \spad{T}
++ if it is a split of Lazard of \spad{W(T)}.
++ Note that if \spad{[T1,...,Ts]} is a split of Lazard of
++ \spad{T} then it is also a split of Kalkbrener of \spad{T}.
++ The converse is false.
++ This category provides operations related to both kinds of
++ splits, the former being related to ideals decomposition whereas
++ the latter deals with varieties decomposition.
++ See the example illustrating the RegularTriangularSet constructor for more
++ explanations about decompositions by means of regular triangular sets.

RegularTriangularSetCategory(R:GcdDomain, E:OrderedAbelianMonoidSup,_
  V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
         Category ==
   TriangularSetCategory(R,E,V,P) with
    purelyAlgebraic?: (P,$) -> Boolean
      ++ \spad{purelyAlgebraic?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} is algebraic w.r.t. \spad{ts}.
    purelyTranscendental? : (P,$) -> Boolean
      ++ \spad{purelyTranscendental?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} is not algebraic w.r.t. \spad{ts}
    algebraicCoefficients?  : (P,$) -> Boolean
      ++ \spad{algebraicCoefficients?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} which is not the main one of \spad{p}
      ++ is algebraic w.r.t. \spad{ts}.
    purelyAlgebraic?: $ -> Boolean
      ++ \spad{purelyAlgebraic?(ts)} returns true iff for every algebraic
      ++ variable \spad{v} of \spad{ts} we have
      ++ \spad{algebraicCoefficients?(t_v,ts_v_-)} where \spad{ts_v}
      ++ is select from TriangularSetCategory(ts,v) and
      ++ \spad{ts_v_-} is
      ++ collectUnder from TriangularSetCategory(ts,v).
    purelyAlgebraicLeadingMonomial?: (P, $) -> Boolean
      ++ \spad{purelyAlgebraicLeadingMonomial?(p,ts)} returns true iff
      ++ the main variable of any non-constant iterarted initial
      ++ of \spad{p} is algebraic w.r.t. \spad{ts}.
    invertibleElseSplit? : (P,$) -> Union(Boolean,List $)
      ++ \spad{invertibleElseSplit?(p,ts)} returns \spad{true} (resp.
      ++ \spad{false}) if \spad{p} is invertible in the tower
      ++ associated with \spad{ts} or returns a split of Kalkbrener
      ++ of \spad{ts}.
    invertible? : (P,$) -> List Record(val : Boolean, tower : $)
      ++ \spad{invertible?(p,ts)} returns \spad{lbwt} where \spad{lbwt.i}
      ++ is the result of \spad{invertibleElseSplit?(p,lbwt.i.tower)} and
      ++ the list of the \spad{(lqrwt.i).tower} is a split of Kalkbrener of
      ++ \spad{ts}.
    invertible?: (P,$) -> Boolean
  ++ \spad{invertible?(p,ts)} returns true iff \spad{p} is invertible
  ++ in the tower associated with \spad{ts}.
invertibleSet: (P,$) -> List $
  ++ \spad{invertibleSet(p,ts)} returns a split of Kalkbrener of the
  ++ quotient ideal of the ideal \axiom{I}  by \spad{p} where \spad{I} is
  ++ the radical of saturated of \spad{ts}.
lastSubResultantElseSplit: (P, P, $) -> Union(P,List $)
  ++ \spad{lastSubResultantElseSplit(p1,p2,ts)} returns either
  ++ \spad{g} a quasi-monic gcd of \spad{p1} and \spad{p2} w.r.t.
  ++ the \spad{ts} or a split of Kalkbrener of \spad{ts}.
  ++ This assumes that \spad{p1} and \spad{p2} have the same maim
  ++ variable and that this variable is greater that any variable
  ++ occurring in \spad{ts}.
lastSubResultant: (P, P, $) -> List Record(val : P, tower : $)
  ++ \spad{lastSubResultant(p1,p2,ts)} returns \spad{lpwt} such that
  ++ \spad{lpwt.i.val} is a quasi-monic gcd of \spad{p1} and \spad{p2}
  ++ w.r.t. \spad{lpwt.i.tower}, for every \spad{i}, and such
  ++ that the list of the \spad{lpwt.i.tower} is a split of Kalkbrener of
  ++ \spad{ts}. Moreover, if \spad{p1} and \spad{p2} do not
  ++ have a non-trivial gcd w.r.t. \spad{lpwt.i.tower} then
  ++ \spad{lpwt.i.val} is the resultant of these polynomials w.r.t.
  ++ \spad{lpwt.i.tower}. This assumes that \spad{p1} and \spad{p2} have
  ++ the same main variable and that this variable is greater that any
  ++ variable occurring in \spad{ts}.
squareFreePart: (P,$) -> List Record(val : P, tower : $)
  ++ \spad{squareFreePart(p,ts)} returns \spad{lpwt} such that
  ++ \spad{lpwt.i.val} is a square-free polynomial
  ++ w.r.t. \spad{lpwt.i.tower}, this polynomial being associated with
  ++ \spad{p} modulo \spad{lpwt.i.tower}, for every \spad{i}. Moreover,
  ++ the list of the \spad{lpwt.i.tower} is a split
  ++ of Kalkbrener of \spad{ts}.
  ++ WARNING: This assumes that \spad{p} is a non-constant polynomial
  ++ such that if \spad{p} is added to \spad{ts}, then the resulting set
  ++ is a regular triangular set.
intersect: (P,$) -> List $
  ++ \spad{intersect(p,ts)} returns the same as
  ++ \spad{intersect([p],ts)}
intersect: (List P, $) -> List $
  ++ \spad{intersect(lp,ts)} returns \spad{lts} a split of Lazard
  ++ of the intersection of the affine variety associated
  ++ with \spad{lp} and the regular zero set of \spad{ts}.
intersect: (List P, List $) -> List $
  ++ \spad{intersect(lp,lts)} returns the same as
  ++ \spad{concat([intersect(lp,ts) for ts in lts])|}
intersect: (P, List $) -> List $
  ++ \spad{intersect(p,lts)} returns the same as
  ++ \spad{intersect([p],lts)}
augment: (P,$) -> List $
  ++ \spad{augment(p,ts)} assumes that \spad{p} is a non-constant
  ++ polynomial whose main variable is greater than any variable
  ++ of \spad{ts}. This operation assumes also that if \spad{p} is
  ++ added to \spad{ts} the resulting set, say \spad{ts+p}, is a
  ++ regular triangular set. Then it returns a split of Kalkbrener
  ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
  ++ instance \spad{ts+p} is required to be square-free.
augment: (P,List $) -> List $
  ++ \spad{augment(p,lts)} returns the same as
  ++ \spad{concat([augment(p,ts) for ts in lts])}
augment: (List P,$) -> List $
  ++ \spad{augment(lp,ts)} returns \spad{ts} if \spad{empty? lp},
  ++ \spad{augment(p,ts)} if \spad{lp = [p]}, otherwise
  ++ \spad{augment(first lp, augment(rest lp, ts))}
augment: (List P,List $) -> List $
  ++ \spad{augment(lp,lts)} returns the same as
  ++ \spad{concat([augment(lp,ts) for ts in lts])}
internalAugment: (P, $) -> $
  ++ \spad{internalAugment(p,ts)} assumes that \spad{augment(p,ts)}
  ++ returns a singleton and returns it.
internalAugment: (List P, $) -> $
  ++ \spad{internalAugment(lp,ts)} returns \spad{ts} if \spad{lp}
  ++ is empty otherwise returns
  ++ \spad{internalAugment(rest lp, internalAugment(first lp, ts))}
extend: (P,$) -> List $
  ++ \spad{extend(p,ts)} assumes that \spad{p} is a non-constant
  ++ polynomial whose main variable is greater than any variable
  ++ of \spad{ts}. Then it returns a split of Kalkbrener
  ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
  ++ instance \spad{ts+p} is not a regular triangular set.
extend: (P, List $)  -> List $
  ++ \spad{extend(p,lts)} returns the same as
  ++ \spad{concat([extend(p,ts) for ts in lts])|}
extend: (List P,$) -> List $
  ++ \spad{extend(lp,ts)} returns \spad{ts} if \spad{empty? lp}
  ++ \spad{extend(p,ts)} if \spad{lp = [p]} else
  ++ \spad{extend(first lp, extend(rest lp, ts))}
extend: (List P,List $) -> List $
  ++ \spad{extend(lp,lts)} returns the same as
  ++ \spad{concat([extend(lp,ts) for ts in lts])|}
zeroSetSplit: (List P, Boolean) -> List $
  ++ \spad{zeroSetSplit(lp,clos?)} returns \spad{lts} a split of
  ++ Kalkbrener of the radical ideal associated with \spad{lp}.
  ++ If \spad{clos?} is false, it is also a decomposition of the
++ variety associated with \spad{lp} into the regular zero set of the
++ \spad{ts} in \spad{lts} (or, in other words, a split of Lazard of
++ this variety). See the example illustrating the
++ \spadtype{RegularTriangularSet} constructor for more explanations
++ about decompositions by means of regular triangular sets.
```

add

```
    NNI ==> NonNegativeInteger
    INT ==> Integer
    LP ==> List P
    PWT ==> Record(val : P, tower : $)
    LpWT ==> Record(val : (List P), tower : $)
    Split ==> List $
    pack ==> PolynomialSetUtilitiesPackage(R,E,V,P)

    purelyAlgebraic?(p: P, ts: $): Boolean ==
      ground? p => true
      not algebraic?(mvar(p),ts) => false
      algebraicCoefficients?(p,ts)

    purelyTranscendental?(p:P,ts:$): Boolean  ==
      empty? ts => true
      lv : List V := variables(p)$P
      while (not empty? lv) and (not algebraic?(first(lv),ts)) repeat _
        lv := rest lv
      empty? lv

    purelyAlgebraicLeadingMonomial?(p: P, ts: $): Boolean  ==
      ground? p => true
      algebraic?(mvar(p),ts) and purelyAlgebraicLeadingMonomial?(init(p), ts)

    algebraicCoefficients?(p:P,ts:$): Boolean  ==
      ground? p => true
      (not ground? init(p)) and not (algebraic?(mvar(init(p)),ts)) => false
      algebraicCoefficients?(init(p),ts) =>
        ground? tail(p) => true
        mvar(tail(p)) = mvar(p) =>
          algebraicCoefficients?(tail(p),ts)
        algebraic?(mvar(tail(p)),ts) =>
          algebraicCoefficients?(tail(p),ts)
        false
      false

    if V has Finite
    then
```

```
 purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    size()$V = #ts => true
    lp: LP := sort(infRittWu?,members(ts))
    i: NonNegativeInteger := size()$V
    for p in lp repeat
      v: V := mvar(p)
      (i = (lookup(v)$V)::NNI) =>
        i := subtractIfCan(i,1)::NNI
      univariate?(p)$pack =>
        i := subtractIfCan(i,1)::NNI
      not algebraicCoefficients?(p,collectUnder(ts,v)) =>
        return false
      i := subtractIfCan(i,1)::NNI
    true

else

  purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    v: V := mvar(ts)
    p: P := select(ts,v)::P
    ts := collectUnder(ts,v)
    empty? ts => univariate?(p)$pack
    not purelyAlgebraic?(ts) => false
    algebraicCoefficients?(p,ts)

augment(p:P,lts:List $) ==
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(p,ts),toSave)
  toSave

augment(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    toSave := augment(p,toSave)
  toSave

augment(lp:LP,lts:List $) ==
  empty? lp => lts
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(lp,ts),toSave)
  toSave

extend(p:P,lts:List $) ==
  toSave : Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(extend(p,ts),toSave)
  toSave

extend(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    toSave := extend(p,toSave)
  toSave

extend(lp:LP,lts:List $) ==
  empty? lp => lts
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(extend(lp,ts),toSave)
  toSave

intersect(lp:LP,lts:List $): List $  ==
  -- A VERY GENERAL default algorithm
  (empty? lp) or (empty? lts) => lts
  lp := [primitivePart(p) for p in lp]
  lp := removeDuplicates lp
  lp := remove(zero?,lp)
  any?(ground?,lp) => []
  toSee: List LpWT := [[lp,ts]$LpWT for ts in lts]
  toSave: List $ := []
  lp: LP
  p: P
```

```
         ts: $
         lus: List $
         while (not empty? toSee) repeat
           lpwt := first toSee; toSee := rest toSee
           lp := lpwt.val; ts := lpwt.tower
           empty? lp => toSave := cons(ts, toSave)
           p := first lp;  lp := rest lp
           lus := intersect(p,ts)
           toSee := concat([[lp,us]$LpWT for us in lus], toSee)
         toSave

       intersect(lp: LP,ts: $): List $  ==
         intersect(lp,[ts])

       intersect(p: P,lts: List $): List $  ==
         intersect([p],lts)
```

⟨*RSETCAT.dotabb*⟩≡

```
  "RSETCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
  "RSETCAT" -> "TSETCAT"
```

⟨*RSETCAT.dotfull*⟩≡

```
  "RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
  "RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
    ->
  "TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
```

⟨*RSETCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolyno

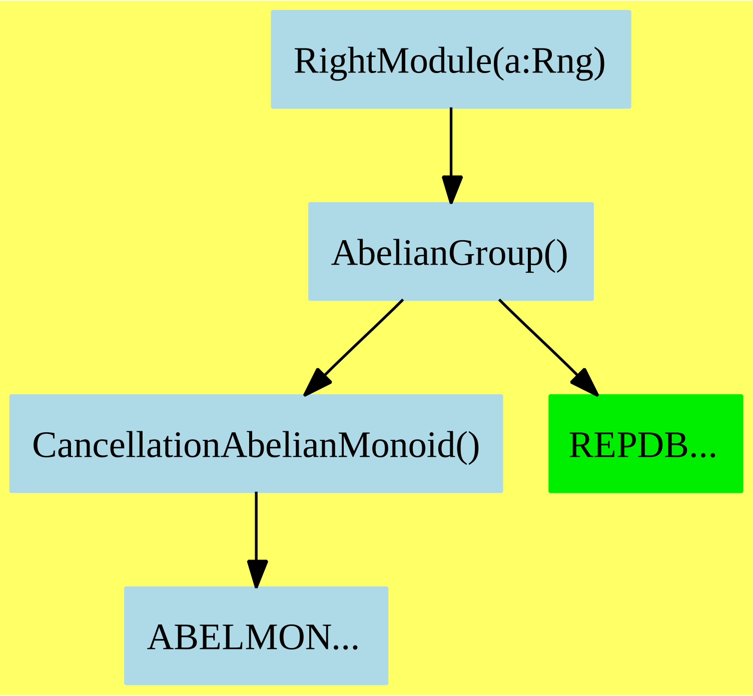
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolyno
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolyno
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolyno

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialC
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialC
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialC
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialC
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

8.0.120 RightModule (RMODULE)


RightModule(a:Rng)
AbelianGroup()
CancellationAbelianMonoid()
REPDB...
ABELMON...

⟨*RightModule.input*⟩≡

```
)set break resume
)sys rm -f RightModule.output
)spool RightModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RightModule
--R RightModule R: Rng  is a category constructor
--R Abbreviation for RightModule is RMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
```

```
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*RightModule.help*⟩≡

```
====================================================================
RightModule examples
====================================================================

See Also:
o )show RightModule
```

See:

⇒ "BiModule" (BMODULE) 9.0.122 on page 800
⇐ "AbelianGroup" (ABELGRP) 7.0.97 on page 576

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (%,R) -> %
```

These exports come from (p576) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

⟨*category RMODULE RightModule*⟩≡

```
)abbrev category RMODULE RightModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of right modules over an rng (ring not necessarily
++ with unit). This is an abelian group which supports right
++ multiplication by elements of the rng.
++
++ Axioms\br
++ \tab{5}\spad{ x*(a*b) = (x*a)*b }\br
++ \tab{5}\spad{ x*(a+b) = (x*a)+(x*b) }\br
++ \tab{5}\spad{ (x+y)*x = (x*a)+(y*a) }

RightModule(R:Rng):Category == AbelianGroup with
   "*": (%,R) -> %
     ++ x*r returns the right multiplication of the module element x
     ++ by the ring element r.
```

⟨*RMODULE.dotabb*⟩≡

```
"RMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RMODULE" -> "ABELGRP"
```

⟨*RMODULE.dotfull*⟩≡

```
"RightModule(a:Rng)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Rng)" -> "AbelianGroup()"

"RightModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"
```

⟨*RMODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

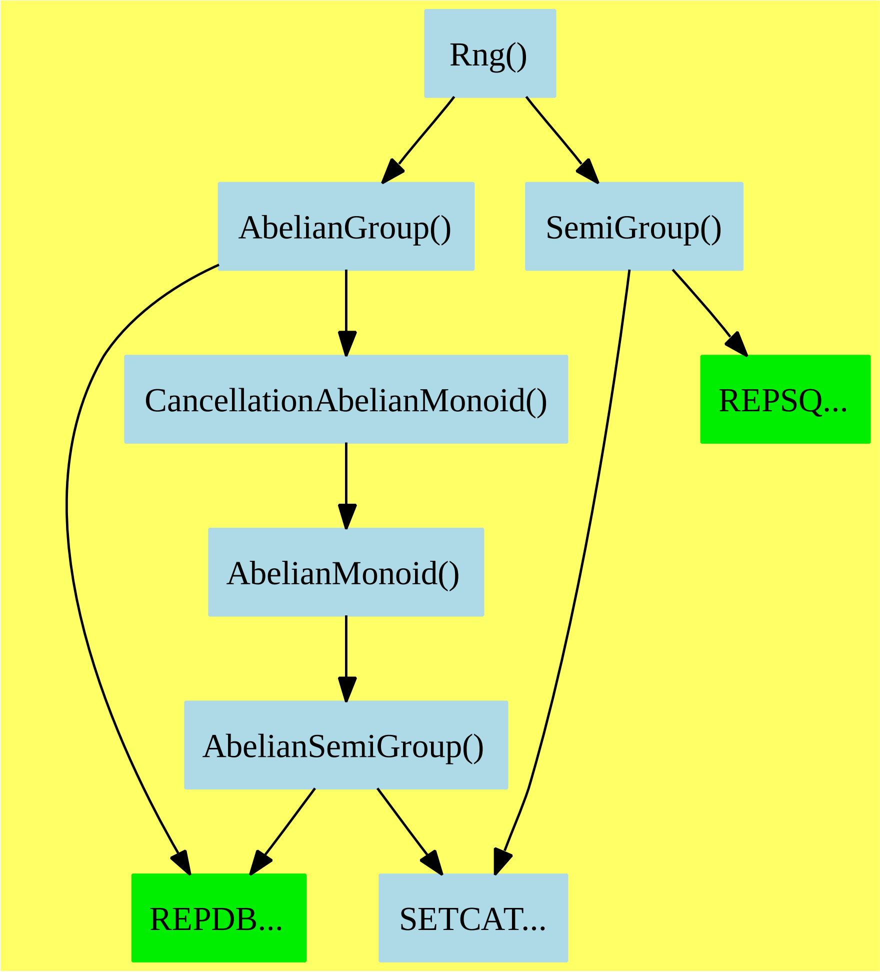
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

8.0.121 Rng (RNG)


Rng()
AbelianGroup()
SemiGroup()
CancellationAbelianMonoid()
REPSQ...
AbelianMonoid()
AbelianSemiGroup()
REPDB...
SETCAT...

Rng is a Ring that does not necessarily have a unit.

⟨*Rng.input*⟩≡

```
)set break resume
)sys rm -f Rng.output
)spool Rng.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Rng
```

```
--R Rng  is a category constructor
--R Abbreviation for Rng is RNG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RNG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*Rng.help*⟩≡

```
====================================================================
Rng examples
====================================================================

See Also:
o )show Rng
```

See:

⇒ “Ring” (RING) 9.0.129 on page 854
⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576
⇐ “SemiGroup” (SGROUP) 4.0.61 on page 269

Exports:

0	coerce	hash	latex	sample
zero?	subtractIfCan	?*?	?**?	?+?
?-?	-?	?=?	?~=?	?^?

These exports come from (p576) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p269) SemiGroup():

```
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category RNG Rng*⟩≡

```
)abbrev category RNG Rng
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of associative rings, not necessarily commutative, and not
++ necessarily with a 1. This is a combination of an abelian group
++ and a semigroup, with multiplication distributing over addition.
++
++ Axioms\br
++ \tab{5}\spad{ x*(y+z) = x*y + x*z}\br
++ \tab{5}\spad{ (x+y)*z = x*z + y*z }
++
++ Conditional attributes\br
++ \tab{5}noZeroDivisors\tab{5}\spad{  ab = 0 => a=0 or b=0}

Rng(): Category == Join(AbelianGroup,SemiGroup)
```

⟨*RNG.dotabb*⟩≡

```
"RNG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"RNG" -> "ABELGRP"
"RNG" -> "SGROUP"
```

⟨*RNG.dotfull*⟩≡

```
"Rng()" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"
```

⟨*RNG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

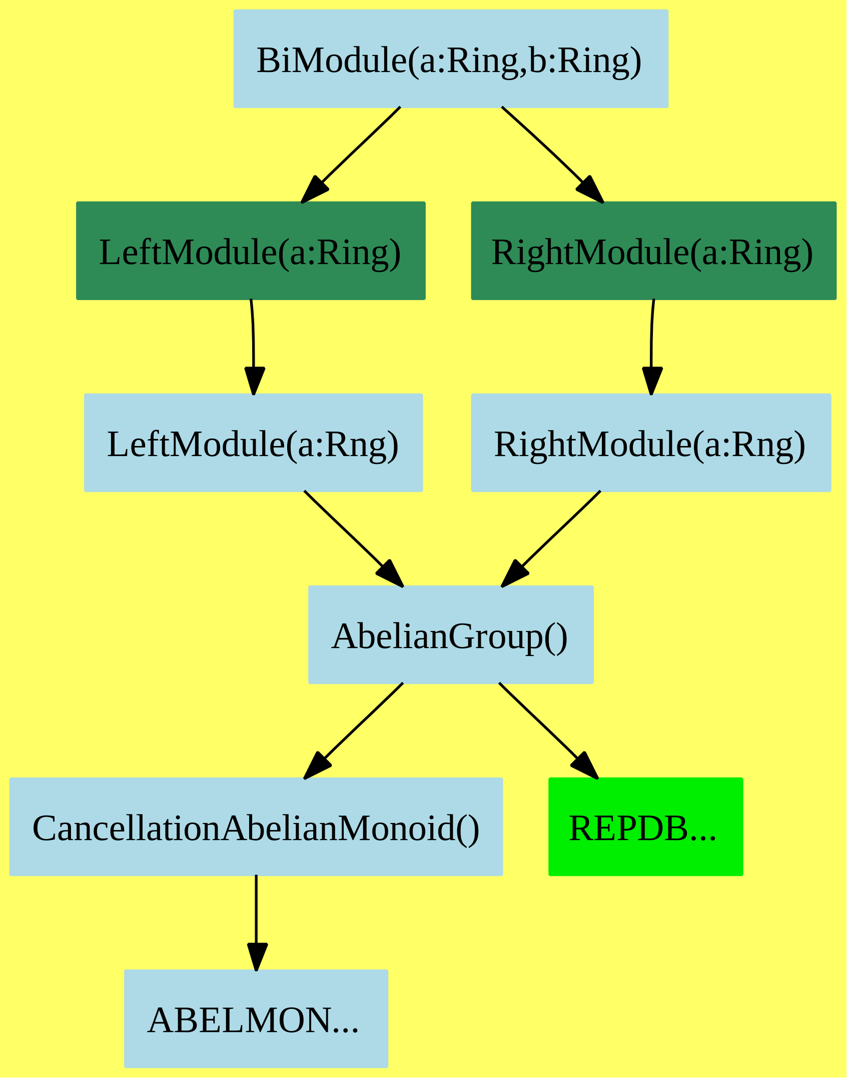
"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

Chapter 9

Category Layer 8

9.0.122 BiModule (BMODULE)


BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
AbelianGroup()
CancellationAbelianMonoid()
REPDB...
ABELMON...

⟨*BiModule.input*⟩≡
```
)set break resume
)sys rm -f BiModule.output
)spool BiModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BiModule
--R BiModule(R: Ring,S: Ring)  is a category constructor
--R Abbreviation for BiModule is BMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                       ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                 ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 0 : () -> %                            coerce : % -> OutputForm
--R hash : % -> SingleInteger              latex : % -> String
--R sample : () -> %                       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*BiModule.help*⟩≡
```
====================================================================
BiModule examples
====================================================================

See Also:
o )show BiModule
```

See:

⇒ “AbelianMonoidRing” (AMR) 13.0.170 on page 1213
⇒ “CommutativeRing” (COMRING) 10.0.137 on page 932
⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “EntireRing” (ENTIRER) 10.0.139 on page 944
⇒ “FreeModuleCat” (FMCAT) 10.0.140 on page 950
⇒ “Module” (MODULE) 10.0.143 on page 966
⇒ “MonogenicLinearOperator” (MLO) 12.0.165 on page 1160
⇒ “RectangularMatrixCategory” (RMATCAT) 10.0.147 on page 994
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.168 on page 1192
⇒ “UnivariateSkewPolynomialCategory” (OREPCAT) 10.0.150 on page 1024
⇒ “XAlgebra” (XALG) 10.0.151 on page 1038
⇐ “LeftModule” (LMODULE) 8.0.113 on page 722
⇐ “RightModule” (RMODULE) 8.0.120 on page 788

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p722) LeftModule(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p788) RightModule(S:Ring):

```
?*? : (%,S) -> %
```

⟨*category BMODULE BiModule*⟩≡

```
)abbrev category BMODULE BiModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A \spadtype{BiModule} is both a left and right module with respect
++ to potentially different rings.
++
++ Axiom\br
++ \tab{5}\spad{ r*(x*s) = (r*x)*s }

BiModule(R:Ring,S:Ring):Category ==
  Join(LeftModule(R),RightModule(S)) with
     leftUnitary
       ++ \spad{1 * x = x}
     rightUnitary
       ++ \spad{x * 1 = x}
```

⟨*BMODULE.dotabb*⟩≡

```
"BMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BMODULE" -> "LMODULE"
"BMODULE" -> "RMODULE"
```

⟨BMODULE.dotfull⟩≡

```
"BiModule(a:Ring,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"BiModule(a:CommutativeRing,b:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:OrderedAbelianMonoid)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:Ring,b:OrderedAbelianMonoid)" -> "BiModule(a:Ring,b:Ring)"
```

⟨*BMODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

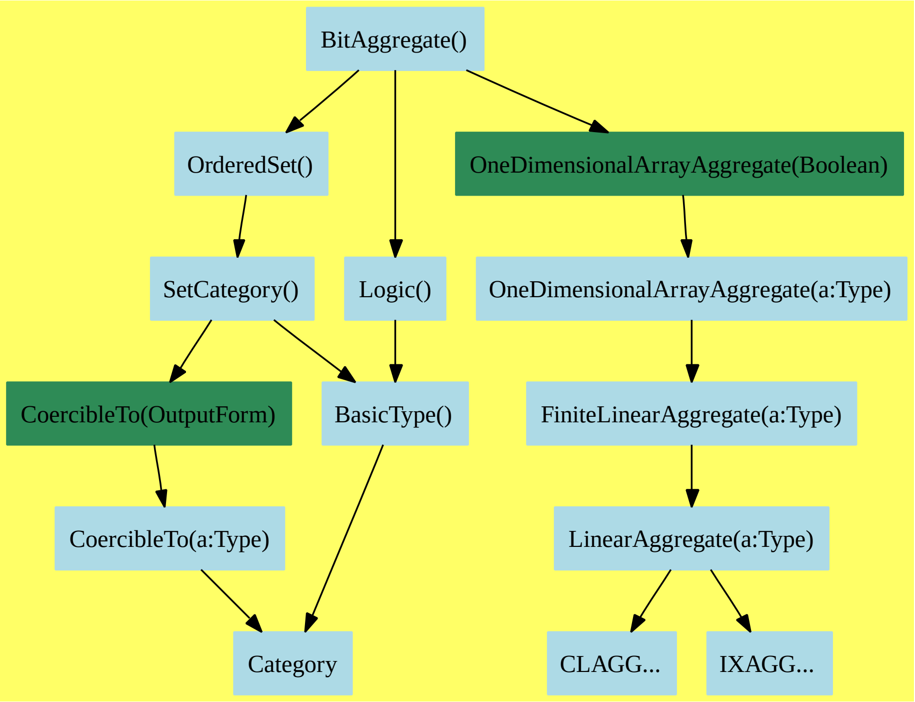
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

9.0.123 BitAggregate (BTAGG)


BitAggregate()
OrderedSet()
OneDimensionalArrayAggregate(Boolean)
SetCategory()
Logic()
OneDimensionalArrayAggregate(a:Type)
CoercibleTo(OutputForm)
BasicType()
FiniteLinearAggregate(a:Type)
CoercibleTo(a:Type)
LinearAggregate(a:Type)
Category
CLAGG...
IXAGG...

⟨*BitAggregate.input*⟩≡

```
)set break resume
)sys rm -f BitAggregate.output
)spool BitAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BitAggregate
--R BitAggregate  is a category constructor
--R Abbreviation for BitAggregate is BTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BTAGG
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                     ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                     ^? : % -> %
```

```
--R ?and? : (%,%) -> %                    coerce : % -> OutputForm
--R concat : (%,Boolean) -> %             concat : (Boolean,%) -> %
--R concat : (%,%) -> %                   concat : List % -> %
--R construct : List Boolean -> %         copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> Boolean
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Boolean           eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      nand : (%,%) -> %
--R nor : (%,%) -> %                      not? : % -> %
--R ?or? : (%,%) -> %                     qelt : (%,Integer) -> Boolean
--R reverse : % -> %                      sample : () -> %
--R xor : (%,%) -> %                      ~? : % -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if Boolean has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Boolean -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Boolean,%) -> NonNegativeInteger if Boolean has SETCAT and $ has finiteAggregat
--R delete : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Boolean) -> Boolean
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT
--R eval : (%,List Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETC
--R eval : (%,Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,Boolean,Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,List Boolean,List Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has
--R every? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Boolean) -> % if $ has shallowlyMutable
--R find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
--R first : % -> Boolean if Integer has ORDSET
--R insert : (Boolean,%,Integer) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Boolean -> Boolean),%) -> %
--R map : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R map! : ((Boolean -> Boolean),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Boolean,%) -> Boolean if Boolean has SETCAT and $ has finiteAggregate
--R members : % -> List Boolean if $ has finiteAggregate
--R merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if Boolean has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
```

```
--R new : (NonNegativeInteger,Boolean) -> %
--R parts : % -> List Boolean if $ has finiteAggregate
--R position : ((Boolean -> Boolean),%) -> Integer
--R position : (Boolean,%) -> Integer if Boolean has SETCAT
--R position : (Boolean,%,Integer) -> Integer if Boolean has SETCAT
--R qsetelt! : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean if Boo
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean if $ has finit
--R reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean if $ has finiteAggrega
--R remove : (Boolean,%) -> % if Boolean has SETCAT and $ has finiteAggregate
--R remove : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Boolean has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,Boolean) -> Boolean if $ has shallowlyMu
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((Boolean,Boolean) -> Boolean),%) -> %
--R sort : % -> % if Boolean has ORDSET
--R sort! : (((Boolean,Boolean) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if Boolean has ORDSET and $ has shallowlyMutable
--R sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if Boolean has ORDSET
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

⟨*BitAggregate.help*⟩≡

```
====================================================================
BitAggregate examples
====================================================================

See Also:
o )show BitAggregate
```

See:

⇐ “Logic” (LOGIC) 3.0.29 on page 110
⇐ “OneDimensionalArrayAggregate” (A1AGG) 8.0.117 on page 754
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	nand	new	nor	not?
parts	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	xor	#?	?/\?
?<?	?<=?	?=?	?>?	?>=?
?\/?	ˆ?	?and?	?.?	?or?
˜?	?˜=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
?and? : (%,%) -> %
?or? : (%,%) -> %
xor : (%,%) -> %
```

These are implemented by this category:

```
not? : % -> %
^? : % -> %
~? : % -> %
?/\? : (%,%) -> %
?\/? : (%,%) -> %
```

```
nand : (%,%) -> %
nor : (%,%) -> %
```

These exports come from (p242) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

TPDHERE: Note that none of the exports of Logic are needed. Perhaps this can be eliminated.

These exports come from (p754) OneDimensionalArrayAggregate(Boolean):

```
any? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
concat : (%,%) -> %
concat : List % -> %
concat : (%,Boolean) -> %
concat : (Boolean,%) -> %
construct : List Boolean -> %
convert : % -> InputForm
    if Boolean has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Boolean,%) -> NonNegativeInteger
    if Boolean has SETCAT and $ has finiteAggregate
count : ((Boolean -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Boolean) -> Boolean
empty : () -> %
empty? : % -> Boolean
entry? : (Boolean,%) -> Boolean
    if $ has finiteAggregate and Boolean has SETCAT
entries : % -> List Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Equation Boolean) -> %
```

```
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Boolean,Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,List Boolean,List Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
every? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Boolean) -> %
    if $ has shallowlyMutable
find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
first : % -> Boolean if Integer has ORDSET
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Boolean,%,Integer) -> %
insert : (%,%,Integer) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((Boolean -> Boolean),%) -> %
map : (((Boolean,Boolean) -> Boolean),%,%) -> %
map! : ((Boolean -> Boolean),%) -> %
    if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (Boolean,%) -> Boolean
    if Boolean has SETCAT and $ has finiteAggregate
members : % -> List Boolean if $ has finiteAggregate
merge : (%,%) -> % if Boolean has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Boolean) -> %
merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
parts : % -> List Boolean if $ has finiteAggregate
position : ((Boolean -> Boolean),%) -> Integer
position : (Boolean,%,Integer) -> Integer
    if Boolean has SETCAT
position : (Boolean,%) -> Integer
    if Boolean has SETCAT
qelt : (%,Integer) -> Boolean
qsetelt! : (%,Integer,Boolean) -> Boolean
    if $ has shallowlyMutable
reverse : % -> %
reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean
    if $ has finiteAggregate
reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean
    if $ has finiteAggregate
reduce :
 (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean
    if Boolean has SETCAT and $ has finiteAggregate
remove : (Boolean,%) -> %
    if Boolean has SETCAT and $ has finiteAggregate
remove : ((Boolean -> Boolean),%) -> %
    if $ has finiteAggregate
```

```
removeDuplicates : % -> %
    if Boolean has SETCAT and $ has finiteAggregate
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
setelt : (%,UniversalSegment Integer,Boolean) -> Boolean
    if $ has shallowlyMutable
select : ((Boolean -> Boolean),%) -> %
    if $ has finiteAggregate
setelt : (%,Integer,Boolean) -> Boolean
    if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : (((Boolean,Boolean) -> Boolean),%) -> %
sort : % -> % if Boolean has ORDSET
sort! : % -> %
    if Boolean has ORDSET and $ has shallowlyMutable
sort! : (((Boolean,Boolean) -> Boolean),%) -> %
    if $ has shallowlyMutable
sorted? : % -> Boolean if Boolean has ORDSET
sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
?.? : (%,UniversalSegment Integer) -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> Boolean
```

⟨*category BTAGG BitAggregate*⟩≡

```
)abbrev category BTAGG BitAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The bit aggregate category models aggregates representing large
++ quantities of Boolean data.

BitAggregate(): Category ==
  Join(OrderedSet, Logic, OneDimensionalArrayAggregate Boolean) with
    "not": % -> %
      ++ not(b) returns the logical not of bit aggregate
      ++ \axiom{b}.
    "^"  : % -> %
      ++ ^ b returns the logical not of bit aggregate
      ++ \axiom{b}.
    nand : (%, %) -> %
```

```
      ++ nand(a,b) returns the logical nand of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    nor         : (%, %) -> %
      ++ nor(a,b) returns the logical nor of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    _and : (%, %) -> %
      ++ a and b returns the logical and of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    _or         : (%, %) -> %
      ++ a or b returns the logical or of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    xor         : (%, %) -> %
      ++ xor(a,b) returns the logical exclusive-or of bit aggregates
      ++ \axiom{a} and \axiom{b}.

 add
   not v      == map(_not, v)
   _^ v             == map(_not, v)
   _~(v)      == map(_~, v)
   _/_\(v, u) == map(_/_\, v, u)
   _\_/(v, u) == map(_\_/, v, u)
   nand(v, u) == map(nand, v, u)
   nor(v, u)  == map(nor, v, u)
```

⟨*BTAGG.dotabb*⟩≡

```
"BTAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BTAGG" -> "ORDSET"
"BTAGG" -> "LOGIC"
"BTAGG" -> "A1AGG"
```

⟨*BTAGG.dotfull*⟩≡

```
"BitAggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"
```

⟨*BTAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BitAggregate()" [color=lightblue];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

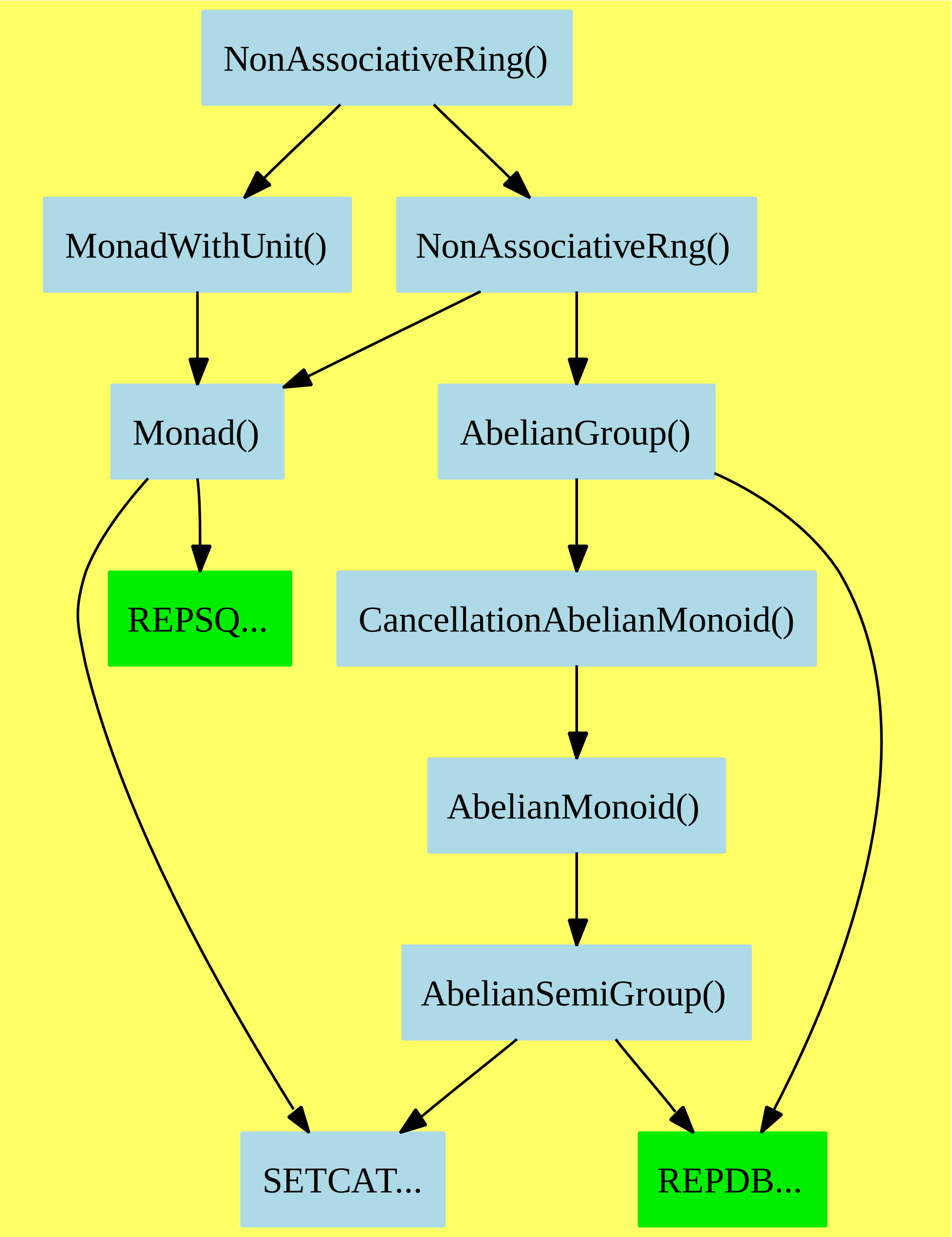
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Logic()" [color=lightblue];
"Logic()" -> "BasicType()"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
"Category" [color=lightblue];

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

9.0.124 NonAssociativeRing (NASRING)


NonAssociativeRing()
MonadWithUnit()
NonAssociativeRng()
Monad()
AbelianGroup()
REPSQ...
CancellationAbelianMonoid()
AbelianMonoid()
AbelianSemiGroup()
SETCAT...
REPDB...

⟨*NonAssociativeRing.input*⟩≡

```
)set break resume
```

```
)sys rm -f NonAssociativeRing.output
)spool NonAssociativeRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonAssociativeRing
--R NonAssociativeRing  is a category constructor
--R Abbreviation for NonAssociativeRing is NASRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NASRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R antiCommutator : (%,%) -> %           associator : (%,%,%) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R leftPower : (%,NonNegativeInteger) -> %
--R leftPower : (%,PositiveInteger) -> %
--R leftRecip : % -> Union(%,"failed")
--R rightPower : (%,NonNegativeInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*NonAssociativeRing.help*⟩≡

```
====================================================================
NonAssociativeRing examples
====================================================================

See Also:
o )show NonAssociativeRing
```

See:

⇐ “MonadWithUnit” (MONADWU) 5.0.75 on page 365
⇐ “NonAssociativeRng” (NARNG) 8.0.116 on page 749

Exports:

0	1	antiCommutator	associator	characteristic
coerce	commutator	hash	latex	leftPower
leftRecip	one?	recip	rightPower	rightRecip
sample	subtractIfCan	zero?	?*?	?~=?
?**?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p749) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p365) MonadWithUnit():

```
1 : () -> %
```

```
leftPower : (%,NonNegativeInteger) -> %
leftRecip : % -> Union(%,"failed")
one? : % -> Boolean
recip : % -> Union(%,"failed")
rightPower : (%,NonNegativeInteger) -> %
rightRecip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
```

⟨*category NASRING NonAssociativeRing*⟩≡

```
)abbrev category NASRING NonAssociativeRing
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: +, *, -, **
++ Related Constructors: NonAssociativeRng, Rng, Ring
++ Also See:
++ AMS Classifications:
++ Keywords: non-associative ring with unit
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++ A NonAssociativeRing is a non associative rng which has a unit,
++ the multiplication is not necessarily commutative or associative.

NonAssociativeRing(): Category == Join(NonAssociativeRng,MonadWithUnit) with
      characteristic: -> NonNegativeInteger
        ++ characteristic() returns the characteristic of the ring.
        --we can not make this a constant, since some domains are mutable
      coerce: Integer -> %
        ++ coerce(n) coerces the integer n to an element of the ring.
   add
      n:Integer
      coerce(n) == n * 1$%
```

⟨*NASRING.dotabb*⟩≡

```
"NASRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NASRING" -> "MONADWU"
"NASRING" -> "NARNG"
```

⟨*NASRING.dotfull*⟩≡
```
"NonAssociativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NonAssociativeRing()" -> "NonAssociativeRng()"
"NonAssociativeRing()" -> "MonadWithUnit()"
```

⟨*NASRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NonAssociativeRing()" -> "NonAssociativeRng()"
"NonAssociativeRing()" -> "MonadWithUnit()"

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

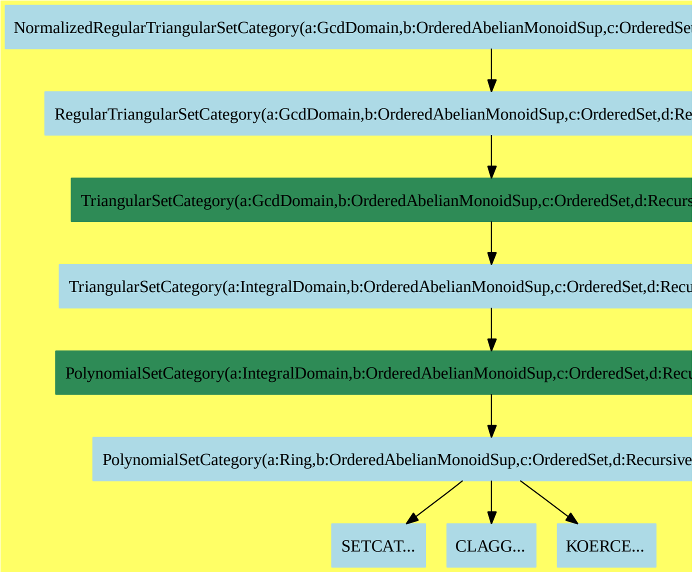
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

9.0.125 NormalizedTriangularSetCategory (NTSCAT)


NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Re
TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs
TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recu
PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recu
PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
SETCAT...
CLAGG...
KOERCE...

⟨*NormalizedTriangularSetCategory.input*⟩≡

```
)set break resume
)sys rm -f NormalizedTriangularSetCategory.output
)spool NormalizedTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NormalizedTriangularSetCategory
--R NormalizedTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P
--R Abbreviation for NormalizedTriangularSetCategory is NTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List V      augment : (List P,%) -> List %
--R augment : (P,List %) -> List %        augment : (P,%) -> List %
--R coerce : % -> List P                  coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List P -> %               copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
```

```
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List P,%) -> List %         extend : (P,List %) -> List %
--R extend : (P,%) -> List %              extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List P                internalAugment : (P,%) -> %
--R intersect : (P,List %) -> List %      intersect : (List P,%) -> List %
--R intersect : (P,%) -> List %           invertible? : (P,%) -> Boolean
--R invertibleSet : (P,%) -> List %       last : % -> Union(P,"failed")
--R latex : % -> String                   mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List V           map : ((P -> P),%) -> %
--R mvar : % -> V                         normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean        purelyAlgebraic? : % -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List P -> %
--R sample : () -> %                      stronglyReduce : (P,%) -> P
--R stronglyReduced? : % -> Boolean       trivialIdeal? : % -> Boolean
--R variables : % -> List V               zeroSetSplit : List P -> List %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List P,List %) -> List %
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"f
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List P,List %) -> List %
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R internalAugment : (List P,%) -> %
--R intersect : (List P,List %) -> List %
--R invertible? : (P,%) -> List Record(val: Boolean,tower: %)
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
--R lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R purelyAlgebraic? : (P,%) -> Boolean
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List Record(val: P,tower: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List P,Boolean) -> List %
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List P)
--R
--E 1

)spool
)lisp (bye)
```

⟨*NormalizedTriangularSetCategory.help*⟩≡

```
====================================================================
NormalizedTriangularSetCategory examples
====================================================================

See Also:
o )show NormalizedTriangularSetCategory
```

See:

⇒ "SquareFreeNormalizedTriangularSetCategory" (SNTSCAT) 10.0.148 on page 1004

⇐ "RegularTriangularSetCategory" (RSETCAT) 8.0.119 on page 772

Exports:

algebraic?
algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
copy
convert
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduceByQuasiMonic
reduced?
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p772) RegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
copy : % -> %
convert : % -> InputForm if P has KONVERT INFORM
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
```

```
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category NTSCAT NormalizedTriangularSetCategory*⟩≡

```
)abbrev category NTSCAT NormalizedTriangularSetCategory
```

```
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References :
++  [1] D. LAZARD "A new method for solving algebraic systems of
++      positive dimension" Discr. App. Math. 33:147-160,1991
++  [2] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++  [3] M. MORENO MAZA and R. RIOBOO "Computations of gcd over
++      algebraic towers of simple extensions" In proceedings of AAECC11
++      Paris, 1995.
++  [4] M. MORENO MAZA "Calculs de pgcd au-dessus des tours
++      d'extensions simples et resolution des systemes d'equations
++      algebriques" These, Universite P.etM. Curie, Paris, 1997.
++ Description:
++ The category of normalized triangular sets. A triangular
++ set \spad{ts} is said normalized if for every algebraic
++ variable \spad{v} of \spad{ts} the polynomial \spad{select(ts,v)}
++ is normalized w.r.t. every polynomial in \spad{collectUnder(ts,v)}.
++ A polynomial \spad{p} is said normalized w.r.t. a non-constant
++ polynomial \spad{q} if \spad{p} is constant or \spad{degree(p,mdeg(q)) = 0}
++ and \spad{init(p)} is normalized w.r.t. \spad{q}. One of the important
++ features of normalized triangular sets is that they are regular sets.

NormalizedTriangularSetCategory(R:GcdDomain,E:OrderedAbelianMonoidSup,_
 V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
         Category ==  RegularTriangularSetCategory(R,E,V,P)
```

⟨*NTSCAT.dotabb*⟩≡

```
"NTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NTSCAT" -> "RSETCAT"
```

⟨NTSCAT.dotfull⟩≡

```
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
```

⟨*NTSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 [color=lightblue];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

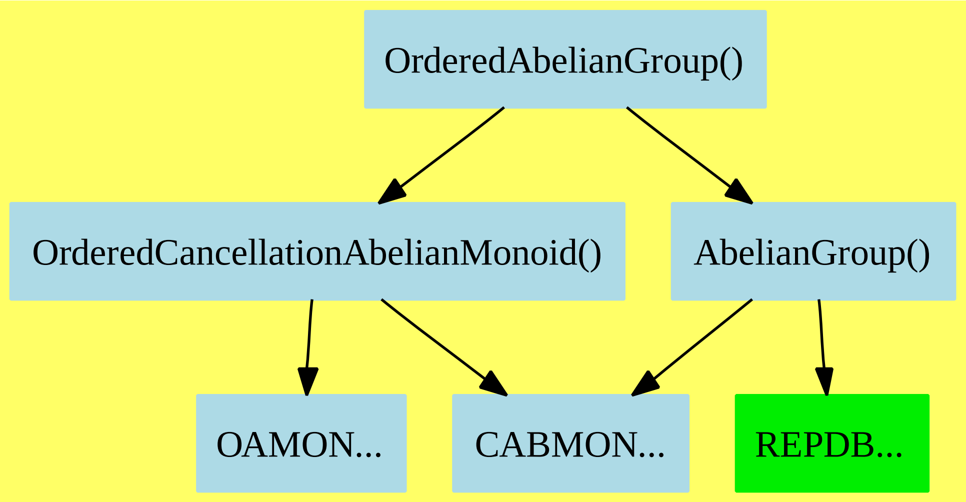
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedS

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.0.126 OrderedAbelianGroup (OAGROUP)


OrderedAbelianGroup()
OrderedCancellationAbelianMonoid()
AbelianGroup()
OAMON...
CABMON...
REPDB...

⟨*OrderedAbelianGroup.input*⟩≡

```
)set break resume
)sys rm -f OrderedAbelianGroup.output
)spool OrderedAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianGroup
--R OrderedAbelianGroup  is a category constructor
--R Abbreviation for OrderedAbelianGroup is OAGROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAGROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      -? : % -> %
--R ?-? : (%,%) -> %                      ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
```

```
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedAbelianGroup.help*⟩≡

```
====================================================================
OrderedAbelianGroup examples
====================================================================

See Also:
o )show OrderedAbelianGroup
```

See:

⇒ “OrderedRing” (ORDRING) 10.0.144 on page 971
⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576
⇐ “OrderedCancellationAbelianMonoid” (OCAMON) 8.0.118 on page 767

Exports:

0 coerce hash latex max
min sample subtractIfCan zero? ?~=?
?*? ?+? -? ?-? ?<?
?<=? ?=? ?>? ?>=?

These exports come from (p767) OrderedCancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

These exports come from (p576) AbelianGroup():

```
-? : % -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
```

⟨*category OAGROUP OrderedAbelianGroup*⟩≡

```
)abbrev category OAGROUP OrderedAbelianGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian groups, such that the
++ addition preserves the ordering.

OrderedAbelianGroup(): Category ==
        Join(OrderedCancellationAbelianMonoid, AbelianGroup)
```

⟨*OAGROUP.dotabb*⟩≡

```
"OAGROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OAGROUP" -> "OCAMON"
"OAGROUP" -> "ABELGRP"
```

⟨*OAGROUP.dotfull*⟩≡

```
"OrderedAbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"
```

⟨*OAGROUP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

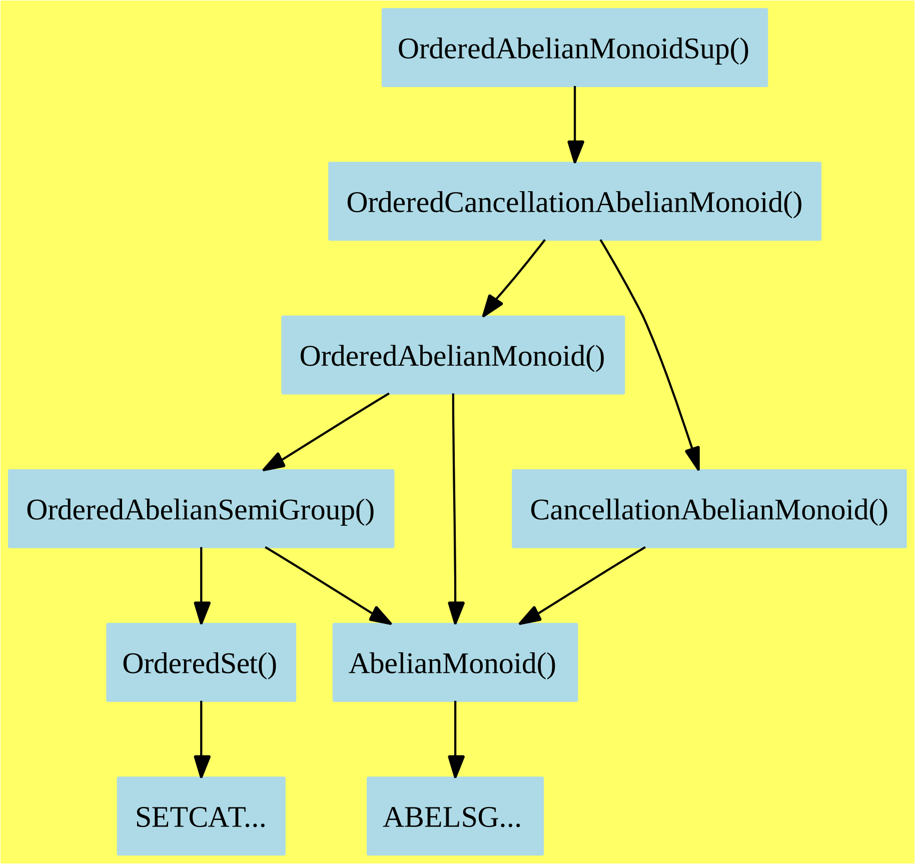
"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OAMON..."
"OrderedCancellationAbelianMonoid()" -> "CABMON..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"OAMON..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

9.0.127 OrderedAbelianMonoidSup (OAMONS)


OrderedAbelianMonoidSup()
OrderedCancellationAbelianMonoid()
OrderedAbelianMonoid()
OrderedAbelianSemiGroup()
CancellationAbelianMonoid()
OrderedSet()
AbelianMonoid()
SETCAT...
ABELSG...

⟨*OrderedAbelianMonoidSup.input*⟩≡

```
)set break resume
)sys rm -f OrderedAbelianMonoidSup.output
)spool OrderedAbelianMonoidSup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianMonoidSup
--R OrderedAbelianMonoidSup  is a category constructor
--R Abbreviation for OrderedAbelianMonoidSup is OAMONS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAMONS
--R
```

```
--R------------------------------ Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  max : (%,%) -> %
--R min : (%,%) -> %                     sample : () -> %
--R sup : (%,%) -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedAbelianMonoidSup.help*⟩≡

```
====================================================================
OrderedAbelianMonoidSup examples
====================================================================

See Also:
o )show OrderedAbelianMonoidSup
```

See:

⇐ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇐ “OrderedCancellationAbelianMonoid” (OCAMON) 8.0.118 on page 767

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	sup	zero?
?~=?	?*?	?<=?	?+?	?<?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
sup : (%,%) -> %
```

These exports come from (p767) OrderedCancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category OAMONS OrderedAbelianMonoidSup*⟩≡

```
)abbrev category OAMONS OrderedAbelianMonoidSup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ This domain is an OrderedAbelianMonoid with a \spadfun{sup}
++ operation added. The purpose of the \spadfun{sup} operator
++ in this domain is to act as a supremum with respect to the
++ partial order imposed by \spadop{-}, rather than with respect to
++ the total \spad{>} order (since that is "max").
++
++ Axioms\br
++ \tab{5}\spad{sup(a,b)-a \~~= "failed"}\br
++ \tab{5}\spad{sup(a,b)-b \~~= "failed"}\br
++ \tab{5}\spad{x-a \~~= "failed" and x-b \~~= "failed" => x >= sup(a,b)}\br

OrderedAbelianMonoidSup(): Category == OrderedCancellationAbelianMonoid with
  sup: (%,%) -> %
    ++ sup(x,y) returns the least element from which both
    ++ x and y can be subtracted.
```

⟨*OAMONS.dotabb*⟩≡

```
"OAMONS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OAMONS" -> "OCAMON"
```

⟨*OAMONS.dotfull*⟩≡

```
"OrderedAbelianMonoidSup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"
```

⟨*OAMONS.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoidSup()" [color=lightblue];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

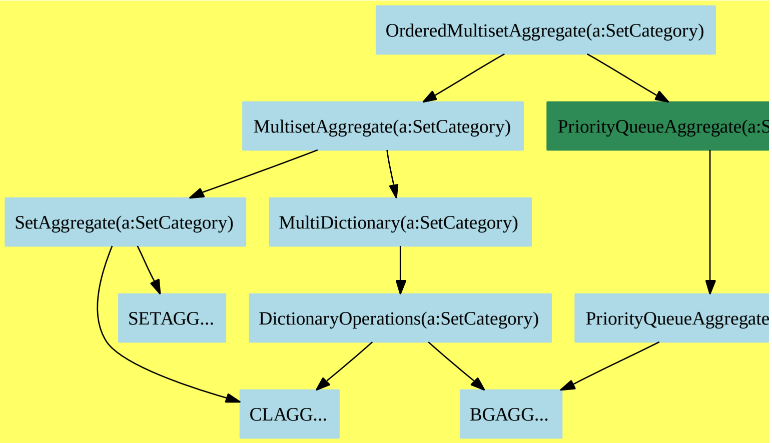
"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

9.0.128 OrderedMultisetAggregate (OMSAGG)


OrderedMultisetAggregate(a:SetCategory)
MultisetAggregate(a:SetCategory)
PriorityQueueAggregate(a:S
SetAggregate(a:SetCategory)
MultiDictionary(a:SetCategory)
SETAGG...
DictionaryOperations(a:SetCategory)
PriorityQueueAggregate
CLAGG...
BGAGG...

⟨*OrderedMultisetAggregate.input*⟩≡

```
)set break resume
)sys rm -f OrderedMultisetAggregate.output
)spool OrderedMultisetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedMultisetAggregate
--R OrderedMultisetAggregate S: OrderedSet  is a category constructor
--R Abbreviation for OrderedMultisetAggregate is OMSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OMSAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R bag : List S -> %                     brace : () -> %
--R brace : List S -> %                   coerce : % -> OutputForm
--R construct : List S -> %               copy : % -> %
--R dictionary : List S -> %              dictionary : () -> %
--R difference : (%,%) -> %               difference : (%,S) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R hash : % -> SingleInteger             insert! : (S,%) -> %
--R inspect : % -> S                      intersect : (%,%) -> %
--R latex : % -> String                   map : ((S -> S),%) -> %
--R max : % -> S                          merge : (%,%) -> %
```

```
--R merge! : (%,%) -> %                   min : % -> S
--R removeDuplicates! : % -> %           sample : () -> %
--R set : () -> %                        set : List S -> %
--R subset? : (%,%) -> Boolean           union : (%,%) -> %
--R union : (%,S) -> %                   union : (S,%) -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List Record(entry: S,count: NonNegativeInteger)
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R insert! : (S,%,NonNegativeInteger) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R symmetricDifference : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedMultisetAggregate.help*⟩≡

```
====================================================================
OrderedMultisetAggregate examples
====================================================================

See Also:
o )show OrderedMultisetAggregate
```

See:

⇐ “MultisetAggregate” (MSETAGG) 8.0.115 on page 742
⇐ “PriorityQueueAggregate” (PRQAGG) 6.0.92 on page 534

Exports:

any?
bag
brace
coerce
construct
convert
copy
count
dictionary
difference
empty
empty?
eq?
duplicates
eval
every?
extract!
find
hash
insert!
inspect
intersect
latex
less?
map
map!
max
member?
members
merge
merge!
min
more?
parts
reduce
remove
remove!
removeDuplicates
removeDuplicates!
sample
select
select!
set
size?
subset?
symmetricDifference
union
#?
?<?
?=?
?~?

Attributes exported:

- **partiallyOrderedSet** is true if a set with < which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
min : % -> S
```

These exports come from (p742) MultisetAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
brace : () -> %
brace : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
```

```
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
difference : (%,S) -> %
difference : (%,%) -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
intersect : (%,%) -> %
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
```

```
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> %
    if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
sample : () -> %
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
set : () -> %
set : List S -> %
size? : (%,NonNegativeInteger) -> Boolean
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p534) PriorityQueueAggregate(S:OrderedSet):

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

⟨*category OMSAGG OrderedMultisetAggregate*⟩≡

```
)abbrev category OMSAGG OrderedMultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An ordered-multiset aggregate is a multiset built over an ordered set S
++ so that the relative sizes of its entries can be assessed.
++ These aggregates serve as models for priority queues.

OrderedMultisetAggregate(S:OrderedSet): Category ==
   Join(MultisetAggregate S,PriorityQueueAggregate S) with
   -- max: % -> S                      ++ smallest entry in the set
   -- duplicates: % -> List Record(entry:S,count:NonNegativeInteger)
```

```
          ++ to become an in order iterator
     -- parts: % -> List S             ++ in order iterator
        min: % -> S
          ++ min(u) returns the smallest entry in the multiset aggregate u.
```

⟨*OMSAGG.dotabb*⟩≡

```
  "OMSAGG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
  "OMSAGG" -> "MSETAGG"
  "OMSAGG" -> "PRQAGG"
```

⟨*OMSAGG.dotfull*⟩≡

```
  "OrderedMultisetAggregate(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
  "OrderedMultisetAggregate(a:SetCategory)" ->
      "MultisetAggregate(a:SetCategory)"
  "OrderedMultisetAggregate(a:SetCategory)" ->
     "PriorityQueueAggregate(a:SetCategory)"
```

⟨*OMSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedMultisetAggregate(a:SetCategory)" [color=lightblue];
"OrderedMultisetAggregate(a:SetCategory)" ->
    "MultisetAggregate(a:SetCategory)"
"OrderedMultisetAggregate(a:SetCategory)" ->
   "PriorityQueueAggregate(a:SetCategory)"

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

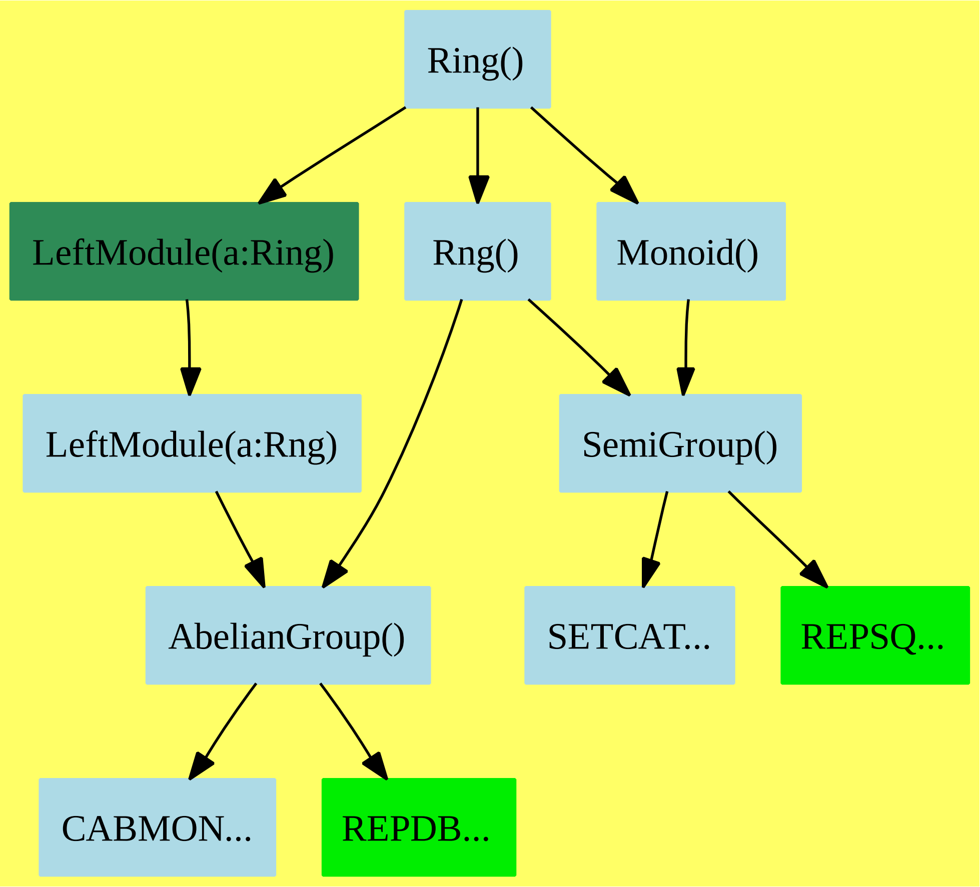
"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"PriorityQueueAggregate(a:SetCategory)" [color=seagreen];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BGAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

9.0.129 Ring (RING)


Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*Ring.input*⟩≡

```
)set break resume
)sys rm -f Ring.output
)spool Ring.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Ring
--R Ring  is a category constructor
--R Abbreviation for Ring is RING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RING
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %         coerce : Integer -> %
--R coerce : % -> OutputForm               hash : % -> SingleInteger
--R latex : % -> String                    one? : % -> Boolean
--R recip : % -> Union(%,"failed")         sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*Ring.help*⟩≡

```
====================================================================
Ring examples
====================================================================

See Also:
o )show Ring
```

See:

⇒ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇒ “AbelianMonoidRing” (AMR) 13.0.170 on page 1213
⇒ “CharacteristicNonZero” (CHARNZ) 10.0.135 on page 922
⇒ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇒ “CommutativeRing” (COMRING) 10.0.137 on page 932
⇒ “DifferentialExtension” (DIFEXT) 11.0.153 on page 1053
⇒ “DifferentialRing” (DIFRING) 10.0.138 on page 938
⇒ “EntireRing” (ENTIRER) 10.0.139 on page 944
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “LeftAlgebra” (LALG) 10.0.141 on page 956
⇒ “LinearlyExplicitRingOver” (LINEXP) 10.0.142 on page 961
⇒ “MonogenicLinearOperator” (MLO) 12.0.165 on page 1160
⇒ “OrderedRing” (ORDRING) 10.0.144 on page 971
⇒ “PartialDifferentialRing” (PDRING) 10.0.145 on page 977
⇒ “UnivariateSkewPolynomialCategory” (OREPCAT) 10.0.150 on page 1024
⇒ “XAlgebra” (XALG) 10.0.151 on page 1038
⇒ “XFreeAlgebra” (XFALG) 11.0.159 on page 1094
⇐ “LeftModule” (LMODULE) 8.0.113 on page 722
⇐ “Monoid” (MONOID) 5.0.76 on page 371
⇐ “Rng” (RNG) 8.0.121 on page 793

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p793) Rng():

```
0 : () -> %
```

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p371) Monoid():

```
1 : () -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category RING Ring*⟩≡

```
)abbrev category RING Ring
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of rings with unity, always associative, but
++ not necessarily commutative.

Ring(): Category == Join(Rng,Monoid,LeftModule(%)) with
      characteristic: () -> NonNegativeInteger
        ++ characteristic() returns the characteristic of the ring
        ++ this is the smallest positive integer n such that
        ++ \spad{n*x=0} for all x in the ring, or zero if no such n
        ++ exists.
        --We can not make this a constant, since some domains are mutable
```

```
       coerce: Integer -> %
         ++ coerce(i) converts the integer i to a member of the given domain.
--     recip: % -> Union(%,"failed") -- inherited from Monoid
       unitsKnown
         ++ recip truly yields
         ++ reciprocal or "failed" if not a unit.
         ++ Note that \spad{recip(0) = "failed"}.
    add
       n:Integer
       coerce(n) == n * 1$%
```

⟨*RING.dotabb*⟩≡

```
"RING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"RING" -> "RNG"
"RING" -> "MONOID"
"RING" -> "LMODULE"
```

⟨*RING.dotfull*⟩≡

```
"Ring()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"
```

⟨*RING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

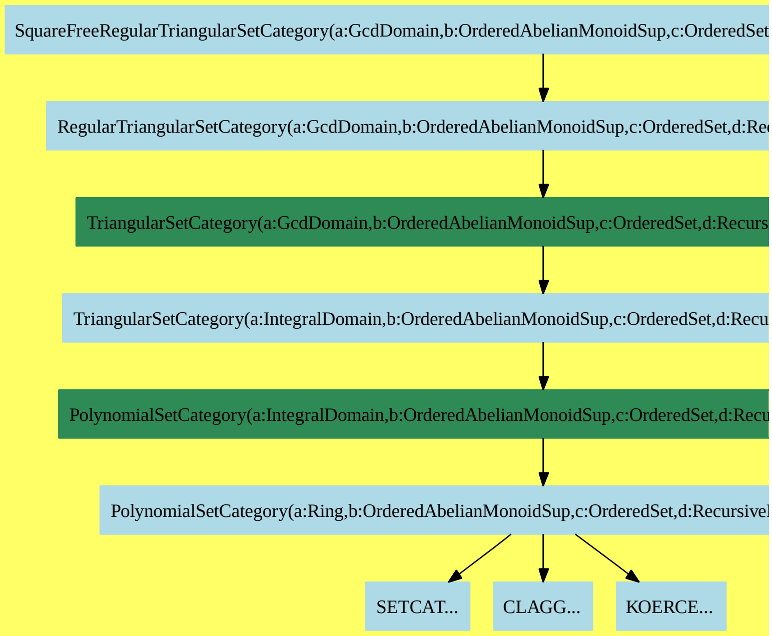
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

9.0.130 SquareFreeRegularTriangularSetCategory (SFRTCAT)


SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet
RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Re
TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs
TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recu
PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recu
PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
SETCAT...
CLAGG...
KOERCE...

⟨*SquareFreeRegularTriangularSetCategory.input*⟩≡

```
)set break resume
)sys rm -f SquareFreeRegularTriangularSetCategory.output
)spool SquareFreeRegularTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SquareFreeRegularTriangularSetCategory
--R SquareFreeRegularTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSu
--R Abbreviation for SquareFreeRegularTriangularSetCategory is SFRTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SFRTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List V      augment : (List P,%) -> List %
--R augment : (P,List %) -> List %        augment : (P,%) -> List %
--R coerce : % -> List P                  coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
```

```
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List P -> %               copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List P,%) -> List %         extend : (P,List %) -> List %
--R extend : (P,%) -> List %              extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List P                internalAugment : (P,%) -> %
--R intersect : (P,List %) -> List %      intersect : (List P,%) -> List %
--R intersect : (P,%) -> List %           invertible? : (P,%) -> Boolean
--R invertibleSet : (P,%) -> List %       last : % -> Union(P,"failed")
--R latex : % -> String                   mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List V           map : ((P -> P),%) -> %
--R mvar : % -> V                         normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean        purelyAlgebraic? : % -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List P -> %
--R sample : () -> %                      stronglyReduce : (P,%) -> P
--R stronglyReduced? : % -> Boolean       trivialIdeal? : % -> Boolean
--R variables : % -> List V               zeroSetSplit : List P -> List %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List P,List %) -> List %
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List P,List %) -> List %
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R internalAugment : (List P,%) -> %
--R intersect : (List P,List %) -> List %
--R invertible? : (P,%) -> List Record(val: Boolean,tower: %)
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
--R lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R purelyAlgebraic? : (P,%) -> Boolean
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List Record(val: P,tower: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List P,Boolean) -> List %
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List
--R
--E 1

)spool
)lisp (bye)
```

⟨*SquareFreeRegularTriangularSetCategory.help*⟩≡

```
====================================================================
SquareFreeRegularTriangularSetCategory examples
====================================================================

See Also:
o )show SquareFreeRegularTriangularSetCategory
```

See:

⇒ “SquareFreeNormalizedTriangularSetCategory” (SNTSCAT) 10.0.148 on page 1004

⇐ “RegularTriangularSetCategory” (RSETCAT) 8.0.119 on page 772

Exports:

algebraic?
algebraicVariables
augment
basicSet
coHeight
collectQuasiMonic
collectUpper
copy
count
empty
eq?
every?
extendIfCan
first
headReduce
headRemainder
initiallyReduce
initials
intersect
invertibleElseSplit?
last
lastSubResultantElseSplit
less?
mainVariables
map!
members
mvar
parts
purelyAlgebraicLeadingMonomial?
quasiComponent
reduceByQuasiMonic
remainder
removeDuplicates
rest
retractIfCan
rewriteIdealWithRemainder
roughBase?
roughSubIdeal?
sample
size?
squareFreePart
stronglyReduced?
trivialIdeal?
zeroSetSplit
#?
?˜=?

algebraicCoefficients?
any?
autoReduced?
coerce
collect
collectUnder
construct
convert
degree
empty?
eval
extend
find
hash
headReduced?
infRittWu?
initiallyReduced?
internalAugment
invertible?
invertibleSet
lastSubResultant
latex
mainVariable?
map
member?
more?
normalized?
purelyAlgebraic?
purelyTranscendental?
reduce
reduced?
remove
removeZero
retract
rewriteIdealWithHeadRemainder
rewriteSetWithReduction
roughEqualIdeals?
roughUnitIdeal?
select
sort
stronglyReduce
triangular?
variables
zeroSetSplitIntoTriangularSystems
?=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p772) RegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
```

```
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SFRTCAT SquareFreeRegularTriangularSetCategory*⟩≡

```
)abbrev category SFRTCAT SquareFreeRegularTriangularSetCategory
```

```
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1996
++ Date Last Updated: 09/10/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References :
++  [1] D. LAZARD "A new method for solving algebraic systems of
++      positive dimension" Discr. App. Math. 33:147-160,1991
++  [2] M. KALKBRENER "Algorithmic properties of polynomial rings"
++      Habilitation Thesis, ETZH, Zurich, 1995.
++  [3] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ The category of square-free regular triangular sets.
++ A regular triangular set \spad{ts} is square-free if
++ the gcd of any polynomial \spad{p} in \spad{ts} and
++ differentiate(p,mvar(p)) w.r.t. collectUnder(ts,mvar(p))
++ has degree zero w.r.t. \spad{mvar(p)}. Thus any square-free regular
++ set defines a tower of square-free simple extensions.

SquareFreeRegularTriangularSetCategory(R:GcdDomain,E:OrderedAbelianMonoidSup,_
  V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
         Category ==
   RegularTriangularSetCategory(R,E,V,P)
```

⟨*SFRTCAT.dotabb*⟩≡

```
"SFRTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SFRTCAT" -> "RSETCAT"
```

⟨*SFRTCAT.dotfull*⟩≡

```
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
```

⟨*SFRTCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 [color=lightblue];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

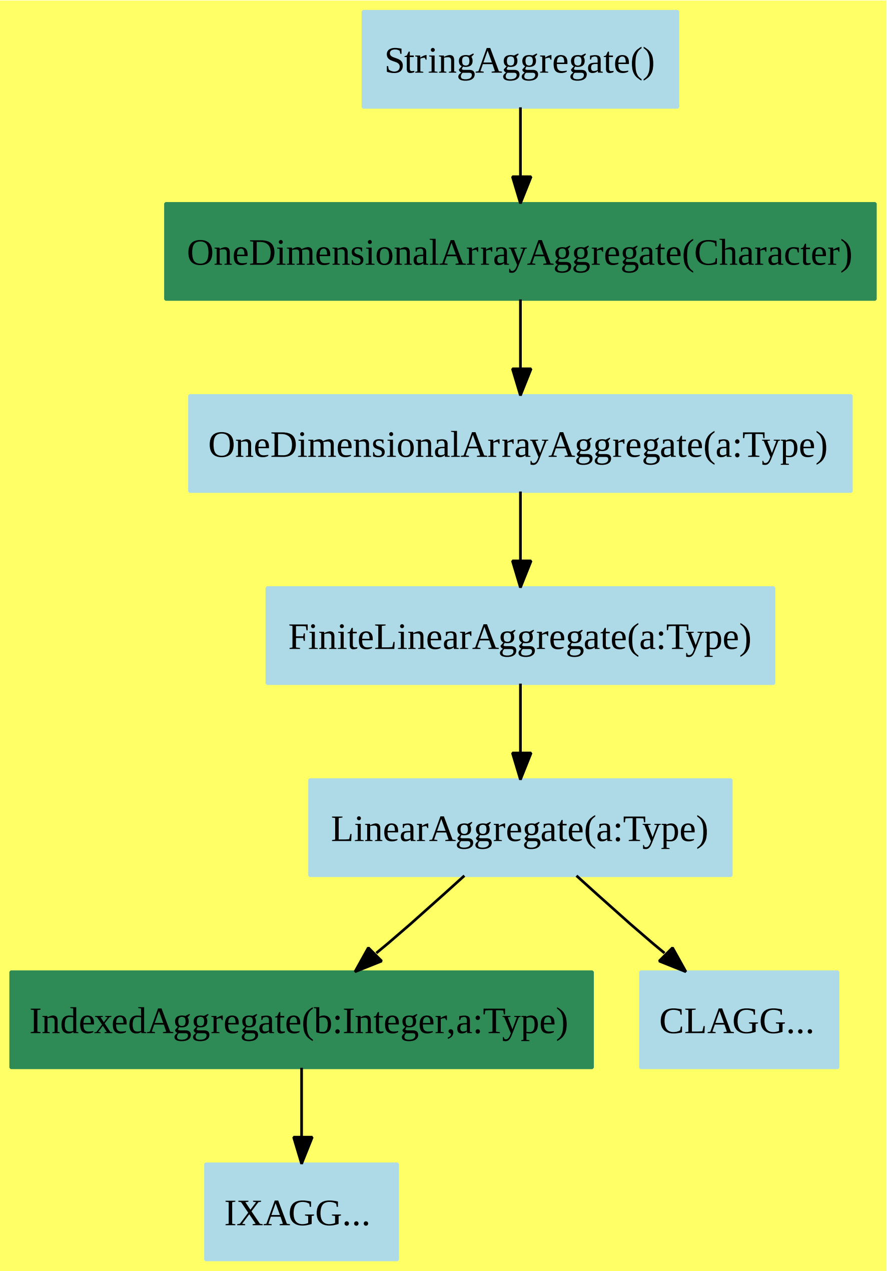
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedS

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.0.131 StringAggregate (SRAGG)


StringAggregate()
OneDimensionalArrayAggregate(Character)
OneDimensionalArrayAggregate(a:Type)
FiniteLinearAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
CLAGG...
IXAGG...

⟨*StringAggregate.input*⟩≡

```
)set break resume
```

```
)sys rm -f StringAggregate.output
)spool StringAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StringAggregate
--R StringAggregate  is a category constructor
--R Abbreviation for StringAggregate is SRAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SRAGG
--R
--R------------------------------- Operations --------------------------------
--R coerce : Character -> %               concat : List % -> %
--R concat : (%,%) -> %                   concat : (Character,%) -> %
--R concat : (%,Character) -> %           construct : List Character -> %
--R copy : % -> %                         delete : (%,Integer) -> %
--R ?.? : (%,%) -> %                      ?.? : (%,Integer) -> Character
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Character         eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           leftTrim : (%,Character) -> %
--R lowerCase : % -> %                    lowerCase! : % -> %
--R prefix? : (%,%) -> Boolean            qelt : (%,Integer) -> Character
--R reverse : % -> %                      rightTrim : (%,Character) -> %
--R sample : () -> %                      split : (%,Character) -> List %
--R suffix? : (%,%) -> Boolean            trim : (%,CharacterClass) -> %
--R trim : (%,Character) -> %             upperCase : % -> %
--R upperCase! : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Character has ORDSET
--R ?<=? : (%,%) -> Boolean if Character has ORDSET
--R ?=? : (%,%) -> Boolean if Character has SETCAT
--R ?>? : (%,%) -> Boolean if Character has ORDSET
--R ?>=? : (%,%) -> Boolean if Character has ORDSET
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Character has SETCAT
--R convert : % -> InputForm if Character has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if Character has SETCAT and $ has finiteAggr
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
```

```
--R eval : (%,List Character,List Character) -> % if Character has EVALAB CHAR an
--R eval : (%,Character,Character) -> % if Character has EVALAB CHAR and Characte
--R eval : (%,Equation Character) -> % if Character has EVALAB CHAR and Character
--R eval : (%,List Equation Character) -> % if Character has EVALAB CHAR and Char
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R hash : % -> SingleInteger if Character has SETCAT
--R insert : (Character,%,Integer) -> %
--R latex : % -> String if Character has SETCAT
--R leftTrim : (%,CharacterClass) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (%,%,Character) -> NonNegativeInteger
--R match? : (%,%,Character) -> Boolean
--R max : (%,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Character,%) -> Boolean if Character has SETCAT and $ has finiteAg
--R members : % -> List Character if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List Character if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finit
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Char
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if Character has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if Character has SETCAT and $ has finiteAggregate
--R replace : (%,UniversalSegment Integer,%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Character) -> Character if $ has shallow
```

```
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if Character has ORDSET and $ has shallowlyMutable
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List %
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Character has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*StringAggregate.help*⟩≡

```
====================================================================
StringAggregate examples
====================================================================

See Also:
o )show StringAggregate
```

See:

⇒ “StringCategory” (STRICAT) 10.0.149 on page 1014
⇐ “OneDimensionalArrayAggregate” (A1AGG) 8.0.117 on page 754

Exports:

any?	coerce	concat	construct	copy
convert	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eval
every?	eq?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	prefix?
qelt	qsetelt!	reduce	remove	removeDuplicates
replace	reverse	reverse!	rightTrim	sample
setelt	size?	sort	sort!	sorted?
split	substring?	suffix?	swap!	trim
upperCase	upperCase!	#?	?~=?	?.?
?<?	?<=?	?=?	?>?	?>=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
lowerCase! : % -> %
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
position : (CharacterClass,%,Integer) -> Integer
position : (%,%,Integer) -> Integer
replace : (%,UniversalSegment Integer,%) -> %
rightTrim : (%,Character) -> %
rightTrim : (%,CharacterClass) -> %
split : (%,Character) -> List %
split : (%,CharacterClass) -> List %
```

```
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
upperCase! : % -> %
```

These are implemented by this category:

```
coerce : Character -> %
lowerCase : % -> %
prefix? : (%,%) -> Boolean
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
?.? : (%,%) -> %
```

These exports come from (p754) OneDimensionalArrayAggregate(Character):

```
any? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Character has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
convert : % -> InputForm
         if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
         if Character has SETCAT
         and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entries : % -> List Character
entry? : (Character,%) -> Boolean
         if $ has finiteAggregate
         and Character has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,Character,Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
```

```
eval : (%,Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,List Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
every? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,Character) -> %
         if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
         if Integer has ORDSET
hash : % -> SingleInteger
         if Character has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Character,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if Character has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Character -> Character),%) -> %
map : (((Character,Character) -> Character),%,%) -> %
map! : ((Character -> Character),%) -> %
         if $ has shallowlyMutable
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
         if Character has SETCAT
         and $ has finiteAggregate
members : % -> List Character
         if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
         if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
         if Character has SETCAT
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character,Character)
      -> Character
         if Character has SETCAT and $ has finiteAggregate
remove : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (Character,%) -> %
         if Character has SETCAT
         and $ has finiteAggregate
removeDuplicates : % -> %
         if Character has SETCAT
         and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,UniversalSegment Integer,Character) -> Character
         if $ has shallowlyMutable
sort! : (((Character,Character) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((Character,Character) -> Boolean),%) -> Boolean
setelt : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if Character has ORDSET
sort : (((Character,Character) -> Boolean),%) -> %
sort! : % -> %
         if Character has ORDSET
         and $ has shallowlyMutable
sorted? : % -> Boolean if Character has ORDSET
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean if Character has SETCAT
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> Character
?<? : (%,%) -> Boolean if Character has ORDSET
?<=? : (%,%) -> Boolean if Character has ORDSET
?=? : (%,%) -> Boolean if Character has SETCAT
?>? : (%,%) -> Boolean if Character has ORDSET
?>=? : (%,%) -> Boolean if Character has ORDSET
```

⟨*category SRAGG StringAggregate*⟩≡

```
)abbrev category SRAGG StringAggregate
++ Author: Stephen Watt and Michael Monagan.
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A string aggregate is a category for strings, that is,
++ one dimensional arrays of characters.

StringAggregate: Category == OneDimensionalArrayAggregate Character with
    lowerCase           : % -> %
      ++ lowerCase(s) returns the string with all characters in lower case.
    lowerCase_!: % -> %
      ++ lowerCase!(s) destructively replaces the alphabetic characters
      ++ in s by lower case.
    upperCase           : % -> %
      ++ upperCase(s) returns the string with all characters in upper case.
    upperCase_!: % -> %
      ++ upperCase!(s) destructively replaces the alphabetic characters
      ++ in s by upper case characters.
    prefix?            : (%, %) -> Boolean
      ++ prefix?(s,t) tests if the string s is the initial substring of t.
      ++ Note that \axiom{prefix?(s,t) ==
      ++   reduce(and,[s.i = t.i for i in 0..maxIndex s])}.
    suffix?            : (%, %) -> Boolean
      ++ suffix?(s,t) tests if the string s is the final substring of t.
      ++ Note that \axiom{suffix?(s,t) ==
      ++  reduce(and,[s.i = t.(n - m + i) for i in 0..maxIndex s])}
      ++ where m and n denote the maxIndex of s and t respectively.
    substring?: (%, %, Integer) -> Boolean
      ++ substring?(s,t,i) tests if s is a substring of t beginning at
      ++ index i.
      ++ Note that \axiom{substring?(s,t,0) = prefix?(s,t)}.
    match: (%, %, Character) -> NonNegativeInteger
      ++ match(p,s,wc) tests if pattern \axiom{p} matches subject \axiom{s}
      ++ where \axiom{wc} is a wild card character. If no match occurs,
      ++ the index \axiom{0} is returned; otheriwse, the value returned
      ++ is the first index of the first character in the subject matching
      ++ the subject (excluding that matched by an initial wild-card).
      ++ For example, \axiom{match("*to*","yorktown","*")} returns \axiom{5}
      ++ indicating a successful match starting at index \axiom{5} of
      ++ \axiom{"yorktown"}.
    match?: (%, %, Character) -> Boolean
      ++ match?(s,t,c) tests if s matches t except perhaps for
```

```
  ++ multiple and consecutive occurrences of character c.
  ++ Typically c is the blank character.
replace           : (%, UniversalSegment(Integer), %) -> %
  ++ replace(s,i..j,t) replaces the substring \axiom{s(i..j)}
  ++ of s by string t.
position           : (%, %, Integer) -> Integer
  ++ position(s,t,i) returns the position j of the substring s in
  ++ string t, where \axiom{j >= i} is required.
position           : (CharacterClass, %, Integer) -> Integer
  ++ position(cc,t,i) returns the position \axiom{j >= i} in t of
  ++ the first character belonging to cc.
coerce            : Character -> %
  ++ coerce(c) returns c as a string s with the character c.
split: (%, Character) -> List %
  ++ split(s,c) returns a list of substrings delimited by character c.
split: (%, CharacterClass) -> List %
  ++ split(s,cc) returns a list of substrings delimited by
  ++ characters in cc.
trim: (%, Character) -> %
  ++ trim(s,c) returns s with all characters c deleted from right
  ++ and left ends.
  ++ For example, \axiom{trim(" abc ", char " ")} returns \axiom{"abc"}.
trim: (%, CharacterClass) -> %
  ++ trim(s,cc) returns s with all characters in cc deleted from right
  ++ and left ends.
  ++ For example, \axiom{trim("(abc)", charClass "()")}
  ++ returns \axiom{"abc"}.
leftTrim: (%, Character) -> %
  ++ leftTrim(s,c) returns s with all leading characters c deleted.
  ++ For example, \axiom{leftTrim("  abc  ", char " ")}
  ++ returns \axiom{"abc  "}.
leftTrim: (%, CharacterClass) -> %
  ++ leftTrim(s,cc) returns s with all leading characters in cc deleted.
  ++ For example, \axiom{leftTrim("(abc)", charClass "()")}
  ++ returns \axiom{"abc)"}.
rightTrim: (%, Character) -> %
  ++ rightTrim(s,c) returns s with all trailing occurrences of c deleted.
  ++ For example, \axiom{rightTrim("  abc  ", char " ")}
  ++ returns \axiom{"  abc"}.
rightTrim: (%, CharacterClass) -> %
  ++ rightTrim(s,cc) returns s with all trailing occurences of
  ++ characters in cc deleted.
  ++ For example, \axiom{rightTrim("(abc)", charClass "()")}
  ++ returns \axiom{"(abc"}.
elt: (%, %) -> %
  ++ elt(s,t) returns the concatenation of s and t. It is provided to
```

```
        ++ allow juxtaposition of strings to work as concatenation.
        ++ For example, \axiom{"smoo" "shed"} returns \axiom{"smooshed"}.
   add
     trim(s: %, c:  Character)          == leftTrim(rightTrim(s, c),        c)
     trim(s: %, cc: CharacterClass) == leftTrim(rightTrim(s, cc), cc)
     lowerCase s                 == lowerCase_! copy s
     upperCase s                 == upperCase_! copy s
     prefix?(s, t)         == substring?(s, t, minIndex t)
     coerce(c:Character):% == new(1, c)
     elt(s:%, t:%): %         == concat(s,t)$%
```

⟨*SRAGG.dotabb*⟩≡

```
  "SRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
  "SRAGG" -> "A1AGG"
```

⟨*SRAGG.dotfull*⟩≡

```
  "StringAggregate()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
  "StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"
```

⟨*SRAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"

"OneDimensionalArrayAggregate(Character)" [color=seagreen];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

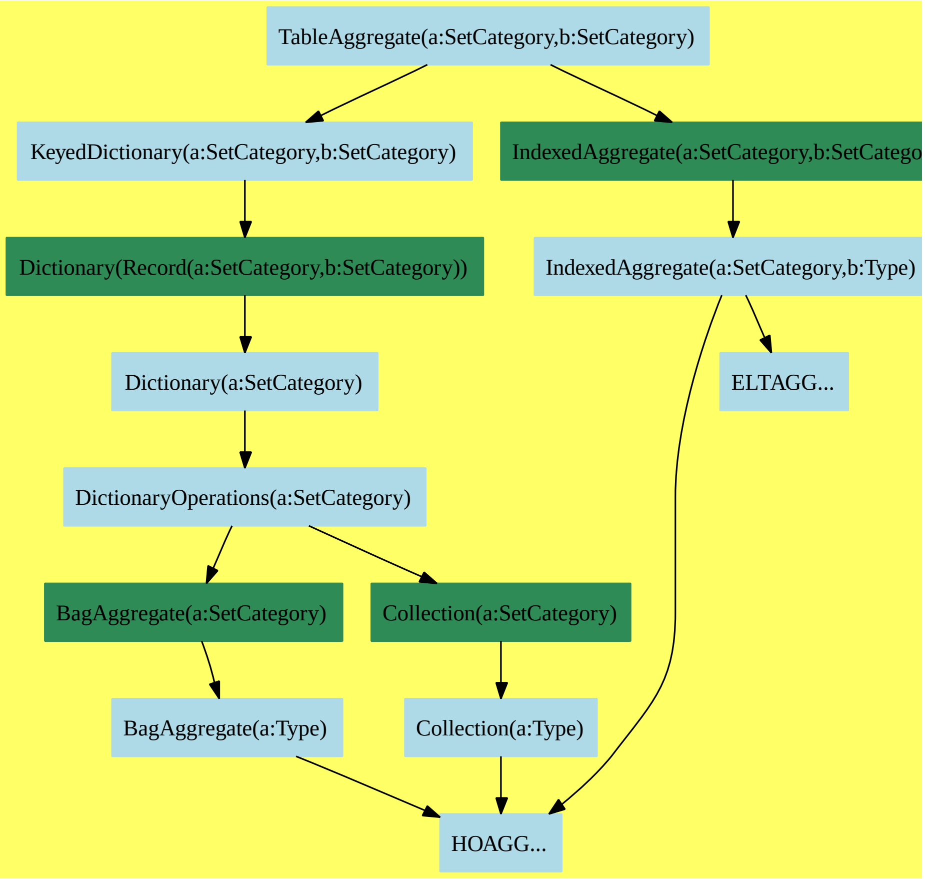
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

9.0.132 TableAggregate (TBAGG)


TableAggregate(a:SetCategory,b:SetCategory)
KeyedDictionary(a:SetCategory,b:SetCategory)
IndexedAggregate(a:SetCategory,b:SetCatego
Dictionary(Record(a:SetCategory,b:SetCategory))
IndexedAggregate(a:SetCategory,b:Type)
Dictionary(a:SetCategory)
ELTAGG...
DictionaryOperations(a:SetCategory)
BagAggregate(a:SetCategory)
Collection(a:SetCategory)
BagAggregate(a:Type)
Collection(a:Type)
HOAGG...

⟨*TableAggregate.input*⟩≡

```
)set break resume
)sys rm -f TableAggregate.output
)spool TableAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TableAggregate
--R TableAggregate(Key: SetCategory,Entry: SetCategory)  is a category constructo
--R Abbreviation for TableAggregate is TBAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TBAGG
--R
```

```
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R elt : (%,Key,Entry) -> Entry          ?.? : (%,Key) -> Entry
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Entry             eq? : (%,%) -> Boolean
--R index? : (Key,%) -> Boolean           indices : % -> List Key
--R key? : (Key,%) -> Boolean             keys : % -> List Key
--R map : ((Entry -> Entry),%) -> %       qelt : (%,Key) -> Entry
--R sample : () -> %                      setelt : (%,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregat
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteAggregate
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCA
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
```

```
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,ent
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteA
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Ent
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finite
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ h
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteA
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finite
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List Record(key: Key,entry: Entry) -> %
--R ?~=? : (%,%) -> Boolean if Entry has SETCAT or Record(key: Key,entry: Entry) 
--R
--E 1

)spool
)lisp (bye)
```

⟨*TableAggregate.help*⟩≡

```
====================================================================
TableAggregate examples
====================================================================

See Also:
o )show TableAggregate
```

See:

⇒ “AssociationListAggregate” (ALAGG) 10.0.134 on page 907
⇐ “IndexedAggregate” (IXAGG) 5.0.74 on page 356
⇐ “KeyedDictionary” (KDAGG) 8.0.111 on page 694

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?.?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
setelt : (%,Key,Entry) -> Entry
```

These are implemented by this category:

```
any? : ((Entry -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Key,Entry) -> Entry
entries : % -> List Entry
```

```
every? : ((Entry -> Boolean),%) -> Boolean
        if $ has finiteAggregate
extract! : % -> Record(key: Key,entry: Entry)
find :
  ((Record(key: Key,entry: Entry) -> Boolean),%)
     -> Union(Record(key: Key,entry: Entry),"failed")
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (Record(key: Key,entry: Entry),%) -> %
inspect : % -> Record(key: Key,entry: Entry)
map : (((Entry,Entry) -> Entry),%,%) -> %
map : ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! :
  ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%)
       -> % if $ has shallowlyMutable
parts : % -> List Entry if $ has finiteAggregate
parts : % -> List Record(key: Key,entry: Entry)
         if $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
table : () -> %
table : List Record(key: Key,entry: Entry) -> %
?.? : (%,Key) -> Entry
?=? : (%,%) -> Boolean
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
```

These exports come from (p694) KeyedDictionary(Key,Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm if RecKE has KONVERT INFORM
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List RecKE,List RecKE) -> %
```

```
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
key? : (Key,%) -> Boolean
keys : % -> List Key
hash : % -> SingleInteger
         if Entry has SETCAT
         or RecKE has SETCAT
latex : % -> String
         if Entry has SETCAT
         or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (RecKE,%) -> Boolean
    if RecKE has SETCAT
     and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
reduce :
 (((RecKE,RecKE) -> RecKE),%) -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE) -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE,RecKE)
   -> RecKE
         if RecKE has SETCAT
         and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (RecKE,%) -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
remove! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
sample : () -> %
```

```
search : (Key,%) -> Union(Entry,"failed")
select : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
         if Entry has SETCAT
         or RecKE has SETCAT
```

These exports come from (p356) IndexedAggregate(Key,Entry))
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eval : (%,List Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Entry,Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,List Entry,List Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
fill! : (%,Entry) -> % if $ has shallowlyMutable
first : % -> Entry if Key has ORDSET
map : ((Entry -> Entry),%) -> %
maxIndex : % -> Key if Key has ORDSET
member? : (Entry,%) -> Boolean
         if Entry has SETCAT
         and $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
minIndex : % -> Key if Key has ORDSET
qelt : (%,Key) -> Entry
qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
```

⟨*category TBAGG TableAggregate*⟩≡

```
)abbrev category TBAGG TableAggregate
++ Author: Michael Monagan, Stephen Watt;
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A table aggregate is a model of a table, i.e. a discrete many-to-one
++ mapping from keys to entries.

TableAggregate(Key:SetCategory, Entry:SetCategory): Category ==
  Join(KeyedDictionary(Key,Entry),IndexedAggregate(Key,Entry)) with
   setelt: (%,Key,Entry) -> Entry           -- setelt_! later
     ++ setelt(t,k,e) (also written \axiom{t.k := e}) is equivalent
     ++ to \axiom{(insert([k,e],t); e)}.
   table: () -> %
     ++ table()$T creates an empty table of type T.
     ++
     ++E Data:=Record(age:Integer,gender:String)
     ++E a1:AssociationList(String,Data):=table()
     ++E a1."tim":=[55,"male"]$Data
   table: List Record(key:Key,entry:Entry) -> %
     ++ table([x,y,...,z]) creates a table consisting of entries
     ++ \axiom{x,y,...,z}.
   -- to become table: Record(key:Key,entry:Entry)* -> %
   map: ((Entry, Entry) -> Entry, %, %) -> %
     ++ map(fn,t1,t2) creates a new table t from given tables t1 and t2 with
     ++ elements fn(x,y) where x and y are corresponding elements from t1
     ++ and t2 respectively.
 add
   table()               == empty()
   table l               == dictionary l
-- empty()               == dictionary()

   insert_!(p, t)      == (t(p.key) := p.entry; t)
   indices t               == keys t

   coerce(t:%):OutputForm ==
     prefix("table"::OutputForm,
                    [k::OutputForm = (t.k)::OutputForm for k in keys t])

   elt(t, k) ==
      (r := search(k, t)) case Entry => r::Entry
      error "key not in table"
   elt(t, k, e) ==
      (r := search(k, t)) case Entry => r::Entry
      e

   map_!(f, t) ==
      for k in keys t repeat t.k := f t.k
      t

   map(f:(Entry, Entry) -> Entry, s:%, t:%) ==
      z := table()
      for k in keys s | key?(k, t) repeat z.k := f(s.k, t.k)
      z

-- map(f, s, t, x) ==
--    z := table()
--    for k in keys s repeat z.k := f(s.k, t(k, x))
--    for k in keys t | not key?(k, s) repeat z.k := f(t.k, x)
--    z

   if % has finiteAggregate then
     parts(t:%):List Record(key:Key,entry:Entry) ==
         [[k, t.k] for k in keys t]
     parts(t:%):List Entry   == [t.k for k in keys t]
     entries(t:%):List Entry == parts(t)

     s:% = t:% ==
       eq?(s,t) => true
       #s ^= #t => false
       for k in keys s repeat
         (e := search(k, t)) _
           case "failed" or (e::Entry) ^= s.k => return false
       true

     map(f: Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%==
       z := table()
       for k in keys t repeat
         ke: Record(key:Key,entry:Entry) := f [k, t.k]
         z ke.key := ke.entry
       z
     map_!(f:Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%_
      ==
       lke: List Record(key:Key,entry:Entry) := nil()
       for k in keys t repeat
         lke := cons(f [k, remove_!(k,t)::Entry], lke)
       for ke in lke repeat
```

```
        t ke.key := ke.entry
      t

    inspect(t: %): Record(key:Key,entry:Entry) ==
      ks := keys t
      empty? ks => error "Cannot extract from an empty aggregate"
      [first ks, t first ks]

    find(f: Record(key:Key,entry:Entry)->Boolean, t:%):_
        Union(Record(key:Key,entry:Entry), "failed") ==
      for ke in parts(t)@List(Record(key:Key,entry:Entry)) _
         repeat if f ke then return ke
      "failed"

    index?(k: Key, t: %): Boolean ==
      search(k,t) case Entry

    remove_!(x:Record(key:Key,entry:Entry), t:%) ==
      if member?(x, t) then remove_!(x.key, t)
      t
    extract_!(t: %): Record(key:Key,entry:Entry) ==
      k: Record(key:Key,entry:Entry) := inspect t
      remove_!(k.key, t)
      k

    any?(f: Entry->Boolean, t: %): Boolean ==
      for k in keys t | f t k repeat return true
      false
    every?(f: Entry->Boolean, t: %): Boolean ==
      for k in keys t | not f t k repeat return false
      true
    count(f: Entry->Boolean, t: %): NonNegativeInteger ==
      tally: NonNegativeInteger := 0
      for k in keys t | f t k repeat tally := tally + 1
      tally
```

⟨*TBAGG.dotabb*⟩≡

```
"TBAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TBAGG" -> "KDAGG"
"TBAGG" -> "IXAGG"
```

⟨*TBAGG.dotfull*⟩≡

```
"TableAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"
```

⟨*TBAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."
"IndexedAggregate(a:SetCategory,b:Type)" -> "ELTAGG..."

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

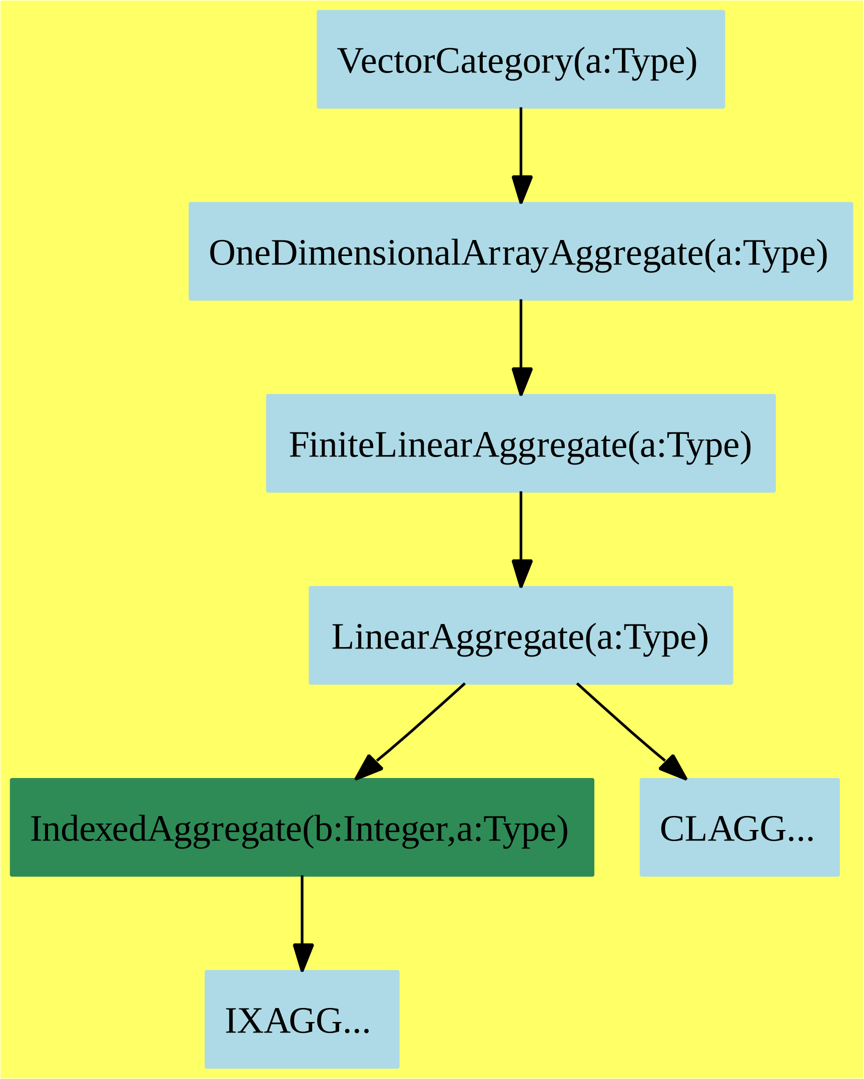
"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."
"ELTAGG..." [color=lightblue];
"HOAGG..." [color=lightblue];
}
```

9.0.133 VectorCategory (VECTCAT)


VectorCategory(a:Type)
OneDimensionalArrayAggregate(a:Type)
FiniteLinearAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
CLAGG...
IXAGG...

⟨*VectorCategory.input*⟩≡

```
)set break resume
)sys rm -f VectorCategory.output
)spool VectorCategory.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show VectorCategory
--R VectorCategory R: Type  is a category constructor
--R Abbreviation for VectorCategory is VECTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for VECTCAT
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP          concat : List % -> %
--R concat : (%,%) -> %                   concat : (R,%) -> %
--R concat : (%,R) -> %                   construct : List R -> %
--R copy : % -> %                         delete : (%,Integer) -> %
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List R                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           insert : (R,%,Integer) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R new : (NonNegativeInteger,R) -> %     qelt : (%,Integer) -> R
--R reverse : % -> %                      sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,R) -> % if R has MONOID
--R ?*? : (R,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,%) -> % if R has RING
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> R if R has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
```

```
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RING and R has RADCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RING and R has RADCAT
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,%) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R sort! : % -> % if R has ORDSET and $ has shallowlyMutable
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
```

```
--E 1

)spool
)lisp (bye)
```

⟨*VectorCategory.help*⟩≡

```
====================================================================
VectorCategory examples
====================================================================

See Also:
o )show VectorCategory
```

See:

⇒ “PointCategory” (PTCAT) 10.0.146 on page 985
⇐ “OneDimensionalArrayAggregate” (A1AGG) 8.0.117 on page 754

Exports:

any?
coerce
concat
construct
convert
copy
copyInto!
count
cross
delete
dot
entry?
elt
empty
empty?
entries
eq?
eval
every?
fill!
find
first
hash
index?
indices
insert
latex
length
less?
magnitude
map
map!
max
maxIndex
member?
members
merge
min
minIndex
more?
new
outerProduct
parts
position
qelt
qsetelt!
reduce
remove
removeDuplicates
reverse
reverse!
sample
select
setelt
size?
sort
sort!
sorted?
swap!
zero
#?
?*?
?+?
?-?
?<?
?<=?
?=?
?>?
?>=?
-?
?.?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
cross : (%,%) -> % if R has RING
dot : (%,%) -> R if R has RING
length : % -> R if R has RING and R has RADCAT
magnitude : % -> R if R has RING and R has RADCAT
outerProduct : (%,%) -> Matrix R if R has RING
zero : NonNegativeInteger -> % if R has ABELMON
?*? : (Integer,%) -> % if R has ABELGRP
?*? : (%,R) -> % if R has MONOID
?*? : (R,%) -> % if R has MONOID
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p754) OneDimensionalArrayAggregate(R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : (R,%) -> %
concat : (%,R) -> %
concat : List % -> %
concat : (%,%) -> %
construct : List R -> %
convert : % -> InputForm if R has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
parts : % -> List R if $ has finiteAggregate
position : (R,%) -> Integer if R has SETCAT
position : ((R -> Boolean),%) -> Integer
position : (R,%,Integer) -> Integer if R has SETCAT
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((R -> Boolean),%) -> % if $ has finiteAggregate
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if R has ORDSET
sort : (((R,R) -> Boolean),%) -> %
sort! : % -> % if R has ORDSET and $ has shallowlyMutable
sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
sorted? : % -> Boolean if R has ORDSET
sorted? : (((R,R) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if R has SETCAT
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?~=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category VECTCAT VectorCategory*⟩≡

```
)abbrev category VECTCAT VectorCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: DirectProductCategory, Vector, IndexedVector
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{VectorCategory} represents the type of vector like objects,
++ i.e. finite sequences indexed by some finite segment of the
++ integers. The operations available on vectors depend on the structure
++ of the underlying components. Many operations from the component domain
++ are defined for vectors componentwise. It can by assumed that extraction or
++ updating components can be done in constant time.

VectorCategory(R:Type): Category == OneDimensionalArrayAggregate R with
    if R has AbelianSemiGroup then
      _+ : (%, %) -> %
        ++ x + y returns the component-wise sum of the vectors x and y.
        ++ Error: if x and y are not of the same length.
    if R has AbelianMonoid then
      zero: NonNegativeInteger -> %
        ++ zero(n) creates a zero vector of length n.
    if R has AbelianGroup then
      _- : % -> %
        ++ -x negates all components of the vector x.
      _- : (%, %) -> %
        ++ x - y returns the component-wise difference of the vectors x and y.
        ++ Error: if x and y are not of the same length.
      _* : (Integer, %) -> %
        ++ n * y multiplies each component of the vector y by the integer n.
    if R has Monoid then
      _* : (R, %) -> %
        ++ r * y multiplies the element r times each component of the vector y.
      _* : (%, R) -> %
        ++ y * r multiplies each component of the vector y by the element r.
    if R has Ring then
      dot: (%, %) -> R
        ++ dot(x,y) computes the inner product of the two vectors x and y.
        ++ Error: if x and y are not of the same length.
      outerProduct: (%, %) -> Matrix R
        ++ outerProduct(u,v) constructs the matrix whose (i,j)'th element is
        ++ u(i)*v(j).
      cross: (%, %) -> %
        ++ vectorProduct(u,v) constructs the cross product of u and v.
        ++ Error: if u and v are not of length 3.
    if R has RadicalCategory and R has Ring then
      length: % -> R
        ++ length(v) computes the sqrt(dot(v,v)), i.e. the magnitude
      magnitude: % -> R
        ++ magnitude(v) computes the sqrt(dot(v,v)), i.e. the length
 add
    if R has AbelianSemiGroup then
      u + v ==
        (n := #u) ^= #v => error "Vectors must be of the same length"
        map(_+ , u, v)
    if R has AbelianMonoid then
      zero n == new(n, 0)

    if R has AbelianGroup then
      - u              == map(x +-> -x, u)

      n:Integer * u:% == map(x +-> n * x, u)

      u - v            == u + (-v)

    if R has Monoid then
      u:% * r:R        == map(x +-> x * r, u)

      r:R * u:%        == map(x +-> r * x, u)

    if R has Ring then
      dot(u, v) ==
        #u ^= #v => error "Vectors must be of the same length"
        _+/[qelt(u, i) * qelt(v, i) for i in minIndex u .. maxIndex u]

      outerProduct(u, v) ==
        matrix [[qelt(u, i) * qelt(v,j) for i in minIndex u .. maxIndex u] _
                for j in minIndex v .. maxIndex v]

      cross(u, v) ==
        #u ^= 3 or #v ^= 3 => error "Vectors must be of length 3"
        construct [qelt(u, 2)*qelt(v, 3) - qelt(u, 3)*qelt(v, 2) , _
                   qelt(u, 3)*qelt(v, 1) - qelt(u, 1)*qelt(v, 3) , _
                   qelt(u, 1)*qelt(v, 2) - qelt(u, 2)*qelt(v, 1) ]

    if R has RadicalCategory and R has Ring then
      length p ==
         sqrt(dot(p,p))

      magnitude p ==
         sqrt(dot(p,p))
```

⟨*VECTCAT.dotabb*⟩≡

```
"VECTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VECTCAT" -> "A1AGG"
```

⟨*VECTCAT.dotfull*⟩≡

```
"VectorCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"VectorCategory(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"
```

⟨*VECTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

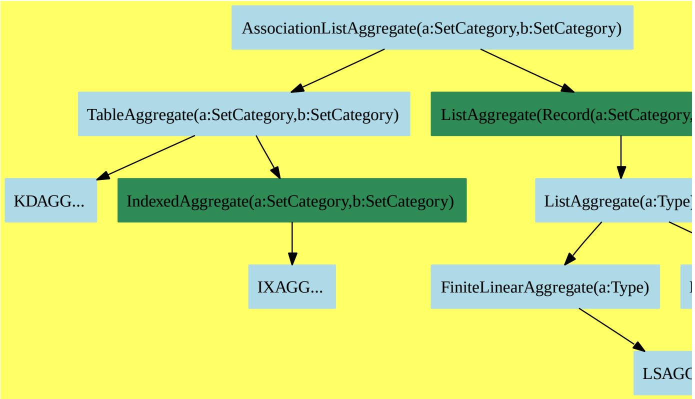
"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

Chapter 10

Category Layer 9

10.0.134 AssociationListAggregate (ALAGG)


AssociationListAggregate(a:SetCategory,b:SetCategory)
TableAggregate(a:SetCategory,b:SetCategory)
ListAggregate(Record(a:SetCategory,
KDAGG...
IndexedAggregate(a:SetCategory,b:SetCategory)
ListAggregate(a:Type)
IXAGG...
FiniteLinearAggregate(a:Type)
LSAGG

⟨*AssociationListAggregate.input*⟩≡

```
)set break resume
)sys rm -f AssociationListAggregate.output
)spool AssociationListAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociationListAggregate
--R AssociationListAggregate(Key: SetCategory,Entry: SetCategory)  is a category constructo
--R Abbreviation for AssociationListAggregate is ALAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ALAGG
```

```
--R
--R------------------------------- Operations -------------------------------
--R children : % -> List %                concat : (%,%) -> %
--R concat : List % -> %                  concat! : (%,%) -> %
--R copy : % -> %                         cycleEntry : % -> %
--R cycleTail : % -> %                    cyclic? : % -> Boolean
--R delete : (%,Integer) -> %             delete! : (%,Integer) -> %
--R dictionary : () -> %                  distance : (%,%) -> Integer
--R ?.rest : (%,rest) -> %                elt : (%,Key,Entry) -> Entry
--R ?.? : (%,Key) -> Entry                empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List Entry
--R eq? : (%,%) -> Boolean                explicitlyFinite? : % -> Boolean
--R index? : (Integer,%) -> Boolean       index? : (Key,%) -> Boolean
--R indices : % -> List Integer           indices : % -> List Key
--R insert : (%,%,Integer) -> %           insert! : (%,%,Integer) -> %
--R key? : (Key,%) -> Boolean             keys : % -> List Key
--R leaf? : % -> Boolean                  map : ((Entry -> Entry),%) -> %
--R nodes : % -> List %                   possiblyInfinite? : % -> Boolean
--R qelt : (%,Key) -> Entry               rest : % -> %
--R reverse : % -> %                      sample : () -> %
--R setelt : (%,Key,Entry) -> Entry       table : () -> %
--R tail : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?<=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry h
--R ?>? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?>=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has fin
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has fin
--R assoc : (Key,%) -> Union(Record(key: Key,entry: Entry),"failed")
--R bag : List Record(key: Key,entry: Entry) -> %
--R child? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry
--R concat : (Record(key: Key,entry: Entry),%) -> %
--R concat : (%,Record(key: Key,entry: Entry)) -> %
--R concat! : (%,Record(key: Key,entry: Entry)) -> %
--R construct : List Record(key: Key,entry: Entry) -> %
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteA
```

```
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R ?.value : (%,value) -> Record(key: Key,entry: Entry)
--R ?.first : (%,first) -> Record(key: Key,entry: Entry)
--R ?.last : (%,last) -> Record(key: Key,entry: Entry)
--R ?.? : (%,UniversalSegment Integer) -> %
--R ?.? : (%,Integer) -> Record(key: Key,entry: Entry)
--R elt : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)
--R entries : % -> List Record(key: Key,entry: Entry)
--R entry? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Recor
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Record(key: Key,entry: Entry)) -> % if $ has shallowlyMutable
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R first : % -> Record(key: Key,entry: Entry)
--R first : (%,NonNegativeInteger) -> %
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCA
--R insert : (Record(key: Key,entry: Entry),%,Integer) -> %
--R insert! : (Record(key: Key,entry: Entry),%,Integer) -> %
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R last : % -> Record(key: Key,entry: Entry)
--R last : (%,NonNegativeInteger) -> %
```

```
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has S
--R leaves : % -> List Record(key: Key,entry: Entry)
--R less? : (%,NonNegativeInteger) -> Boolean
--R list : Record(key: Key,entry: Entry) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) ->
--R map : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Reco
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) ->
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -
--R max : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,ent
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,ent
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R merge : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R merge : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Bo
--R merge! : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R merge! : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> B
--R min : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Record(key: Key,entry: Entry)) -> %
--R node? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R position : (Record(key: Key,entry: Entry),%,Integer) -> Integer if Record(key
--R position : (Record(key: Key,entry: Entry),%) -> Integer if Record(key: Key,en
--R position : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Integer
--R qelt : (%,Integer) -> Record(key: Key,entry: Entry)
--R qsetelt! : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> R
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteA
```

```
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SET
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SET
--R remove! : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SH
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> %
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finiteAg
--R removeDuplicates! : % -> % if Record(key: Key,entry: Entry) has SETCAT
--R rest : (%,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R search : (Key,%) -> Union(Entry,"failed")
--R second : % -> Record(key: Key,entry: Entry)
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> %
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ h
--R setelt : (%,first,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ h
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,last,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ ha
--R setelt : (%,UniversalSegment Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,e
--R setelt : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if S
--R setfirst! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has
--R setlast! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has s
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Record(key: Key,entry: Entry) has ORDSET
--R sort : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%) ->
--R sort! : % -> % if Record(key: Key,entry: Entry) has ORDSET and $ has shallowlyMutable
--R sort! : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%) -
--R sorted? : % -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R sorted? : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%)
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List Record(key: Key,entry: Entry) -> %
--R third : % -> Record(key: Key,entry: Entry)
--R value : % -> Record(key: Key,entry: Entry)
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R
--E 1
)spool
)lisp (bye)
```

⟨*AssociationListAggregate.help*⟩≡

```
====================================================================
AssociationListAggregate examples
====================================================================

See Also:
o )show AssociationListAggregate
```

See:

⇐ “ListAggregate” (LSAGG) 8.0.114 on page 727
⇐ “TableAggregate” (TBAGG) 9.0.132 on page 884

Exports:

any?
assoc
bag
children
child?
coerce
concat
concat!
construct
convert
copy
copyInto!
count
cycleEntry
cycleLength
cycleSplit!
cycleTail
cyclic?
delete
delete!
dictionary
distance
elt
empty
empty?
entries
entry?
eq?
eval
every?
explicitlyFinite?
extract!
fill!
find
first
hash
index?
indices
insert
insert!
inspect
key?
keys
last
latex
leaf?
leaves
less?
list
map
map!
max
maxIndex
member?
members
merge
merge!
min
minIndex
more?
new
nodes
node?
parts
position
possiblyInfinite?
qelt
qsetelt!
reduce
remove
remove!
removeDuplicates
removeDuplicates!
rest
reverse
reverse!
sample
search
second
select
select!
setchildren!
setelt
setfirst!
setlast!
setrest!
setvalue!
size?
sort
sort!
sorted?
split!
swap!
table
tail
third
value
#?
?<?
?<=?
?=?
?>?
?>=?
?~=?
?.rest
?.value
?.first
?.last
?.?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
assoc : (Key,%) -> Union(Record(key: Key,entry: Entry),"failed")
```

These exports come from (p884) TableAggregate(Key, Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE = Record(key: Key,entry: Entry)

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm
         if RecKE has KONVERT INFORM
         or RecKE has KONVERT INFORM
copy : % -> %
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
elt : (%,Key,Entry) -> Entry
elt : (%,Integer,RecKE) -> RecKE
empty : () -> %
empty? : % -> Boolean
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
```

```
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Entry,Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,List Entry,List Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
every? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
extract! : % -> RecKE
fill! : (%,Entry) -> % if $ has shallowlyMutable
find : ((RecKE -> Boolean),%) -> Union(RecKE,"failed")
first : % -> Entry if Key has ORDSET
hash : % -> SingleInteger
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (RecKE,%) -> %
inspect : % -> RecKE
key? : (Key,%) -> Boolean
keys : % -> List Key
```

```
latex : % -> String
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Entry -> Entry),%) -> %
map : ((RecKE -> RecKE),%) -> %
map : (((Entry,Entry) -> Entry),%,%) -> %
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
maxIndex : % -> Key if Key has ORDSET
member? : (RecKE,%) -> Boolean
         if RecKE has SETCAT
         and $ has finiteAggregate
member? : (Entry,%) -> Boolean
         if Entry has SETCAT
         and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
minIndex : % -> Key if Key has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
parts : % -> List RecKE if $ has finiteAggregate
qelt : (%,Key) -> Entry
qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
reduce :
 (((RecKE,RecKE) -> RecKE),%)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE,RecKE)
   -> RecKE
         if RecKE has SETCAT
         and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> % if $ has finiteAggregate
remove : (RecKE,%) -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
remove! : (RecKE,%) -> % if RecKE has SETCAT
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
```

```
          if RecKE has SETCAT
          and $ has finiteAggregate
          or RecKE has SETCAT
          and $ has finiteAggregate
sample : () -> %
search : (Key,%) -> Union(Entry,"failed")
select : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
setelt : (%,Key,Entry) -> Entry
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
table : () -> %
table : List RecKE -> %
?~=? : (%,%) -> Boolean
          if RecKE has SETCAT
          or Entry has SETCAT
          or RecKE has SETCAT
?.? : (%,Key) -> Entry
```

These exports come from (p727) ListAggregate(a)
where a is Record(key:Key,entry:Entry)
and RecKE=Record(key: Key,entry: Entry)

```
children : % -> List %
child? : (%,%) -> Boolean if RecKE has SETCAT
coerce : % -> OutputForm
          if RecKE has SETCAT
          or Entry has SETCAT
          or RecKE has SETCAT
concat : (%,%) -> %
concat : List % -> %
concat : (RecKE,%) -> %
concat : (%,RecKE) -> %
concat! : (%,%) -> %
concat! : (%,RecKE) -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete! : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
entries : % -> List RecKE
entry? : (RecKE,%) -> Boolean
```

```
         if $ has finiteAggregate
         and RecKE has SETCAT
explicitlyFinite? : % -> Boolean
fill! : (%,RecKE) -> % if $ has shallowlyMutable
first : % -> RecKE
first : (%,NonNegativeInteger) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (RecKE,%,Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (RecKE,%,Integer) -> %
last : % -> RecKE
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List RecKE
list : RecKE -> %
map : (((RecKE,RecKE) -> RecKE),%,%) -> %
max : (%,%) -> % if RecKE has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
merge : (%,%) -> % if RecKE has ORDSET
merge : (((RecKE,RecKE) -> Boolean),%,%) -> %
merge! : (%,%) -> % if RecKE has ORDSET
merge! : (((RecKE,RecKE) -> Boolean),%,%) -> %
min : (%,%) -> % if RecKE has ORDSET
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,RecKE) -> %
nodes : % -> List %
node? : (%,%) -> Boolean if RecKE has SETCAT
position : (RecKE,%,Integer) -> Integer
         if RecKE has SETCAT
position : (RecKE,%) -> Integer
         if RecKE has SETCAT
position : ((RecKE -> Boolean),%) -> Integer
possiblyInfinite? : % -> Boolean
qelt : (%,Integer) -> RecKE
qsetelt! : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
remove! : ((RecKE -> Boolean),%) -> %
remove! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
removeDuplicates! : % -> % if RecKE has SETCAT
rest : % -> %
rest : (%,NonNegativeInteger) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
second : % -> RecKE
select! : ((RecKE -> Boolean),%) -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,RecKE) -> RecKE
```

```
         if $ has shallowlyMutable
setelt : (%,first,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,last,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setfirst! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setlast! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
sort : % -> % if RecKE has ORDSET
sort : (((RecKE,RecKE) -> Boolean),%) -> %
sort! : % -> %
         if RecKE has ORDSET
         and $ has shallowlyMutable
sort! : (((RecKE,RecKE) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : % -> Boolean if RecKE has ORDSET
sorted? : (((RecKE,RecKE) -> Boolean),%) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
tail : % -> %
third : % -> RecKE
value : % -> RecKE
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if RecKE has ORDSET
?<=? : (%,%) -> Boolean if RecKE has ORDSET
?=? : (%,%) -> Boolean
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
?>? : (%,%) -> Boolean if RecKE has ORDSET
?>=? : (%,%) -> Boolean if RecKE has ORDSET
?.value : (%,value) -> RecKE
?.first : (%,first) -> RecKE
?.last : (%,last) -> RecKE
?.rest : (%,rest) -> %
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> RecKE
```

⟨*category ALAGG AssociationListAggregate*⟩≡

```
)abbrev category ALAGG AssociationListAggregate
```

```
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An association list is a list of key entry pairs which may be viewed
++ as a table.        It is a poor mans version of a table:
++ searching for a key is a linear operation.

AssociationListAggregate(Key:SetCategory,Entry:SetCategory): Category ==
   Join(TableAggregate(Key, Entry), _
         ListAggregate Record(key:Key,entry:Entry)) with
      assoc: (Key, %) -> Union(Record(key:Key,entry:Entry), "failed")
        ++ assoc(k,u) returns the element x in association list u stored
        ++ with key k, or "failed" if u has no key k.
```

⟨ALAGG.dotabb⟩≡

```
"ALAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"ALAGG" -> "TBAGG"
"ALAGG" -> "LSAGG"
```

⟨ALAGG.dotfull⟩≡

```
"AssociationListAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"
```

⟨*ALAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AssociationListAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" -> "KDAGG..."
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" -> "IXAGG..."

"ListAggregate(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
    "ListAggregate(a:Type)"

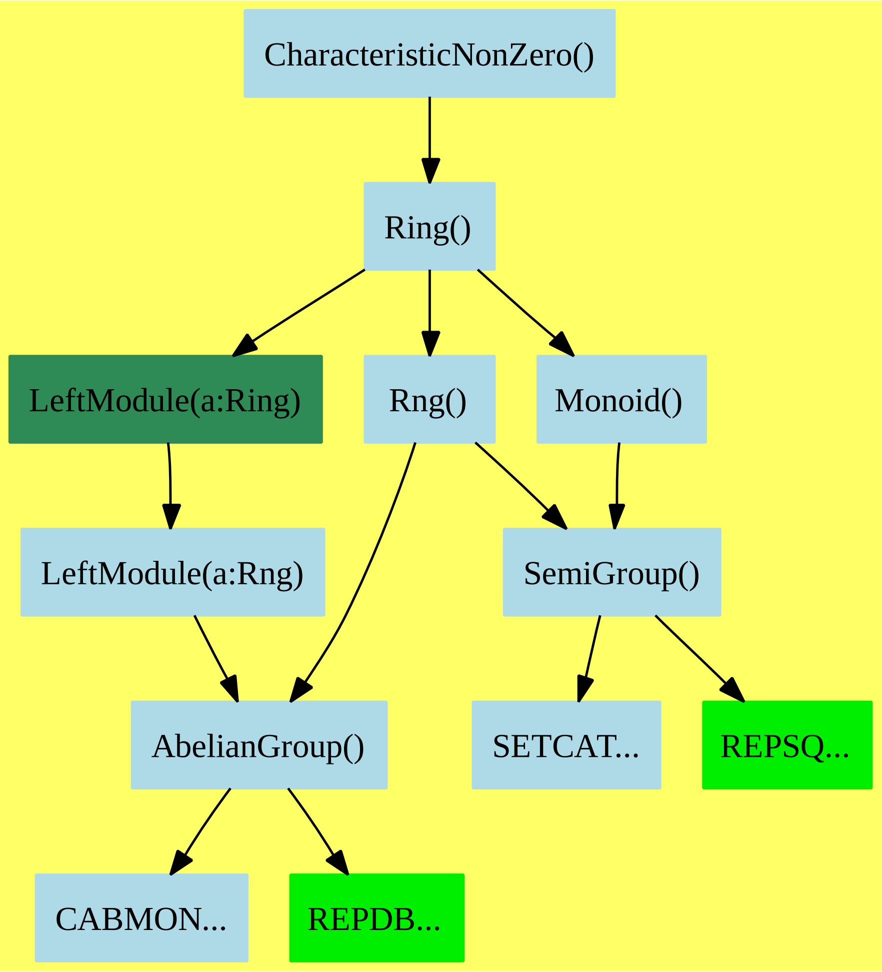
"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LSAGG..."

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LSAGG..."

"KDAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"LSAGG..." [color=lightblue];
}
```

10.0.135 CharacteristicNonZero (CHARNZ)


CharacteristicNonZero()
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*CharacteristicNonZero.input*⟩≡

```
)set break resume
)sys rm -f CharacteristicNonZero.output
)spool CharacteristicNonZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CharacteristicNonZero
--R CharacteristicNonZero  is a category constructor
--R Abbreviation for CharacteristicNonZero is CHARNZ
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CHARNZ
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coerce : Integer -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  one? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*CharacteristicNonZero.help*⟩≡

```
====================================================================
CharacteristicNonZero examples
====================================================================

See Also:
o )show CharacteristicNonZero
```

See:

⇒ "FieldOfPrimeCharacteristic" (FPC) 17.0.193 on page 1457
⇒ "FiniteRankAlgebra" (FINRALG) 17.0.194 on page 1465
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "QuotientFieldCategory" (QFCAT) 17.0.198 on page 1516
⇐ "Ring" (RING) 9.0.129 on page 854

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
charthRoot	subtractIfCan	?~=?	?^?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
charthRoot : % -> Union(%,"failed")
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
```

```
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category CHARNZ CharacteristicNonZero*⟩≡

```
)abbrev category CHARNZ CharacteristicNonZero
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Rings of Characteristic Non Zero

CharacteristicNonZero():Category == Ring with
    charthRoot: % -> Union(%,"failed")
       ++ charthRoot(x) returns the pth root of x
       ++ where p is the characteristic of the ring.
```

⟨*CHARNZ.dotabb*⟩≡

```
"CHARNZ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CHARNZ" -> "RING"
```

⟨*CHARNZ.dotfull*⟩≡

```
"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "Ring()"
```

⟨*CHARNZ.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

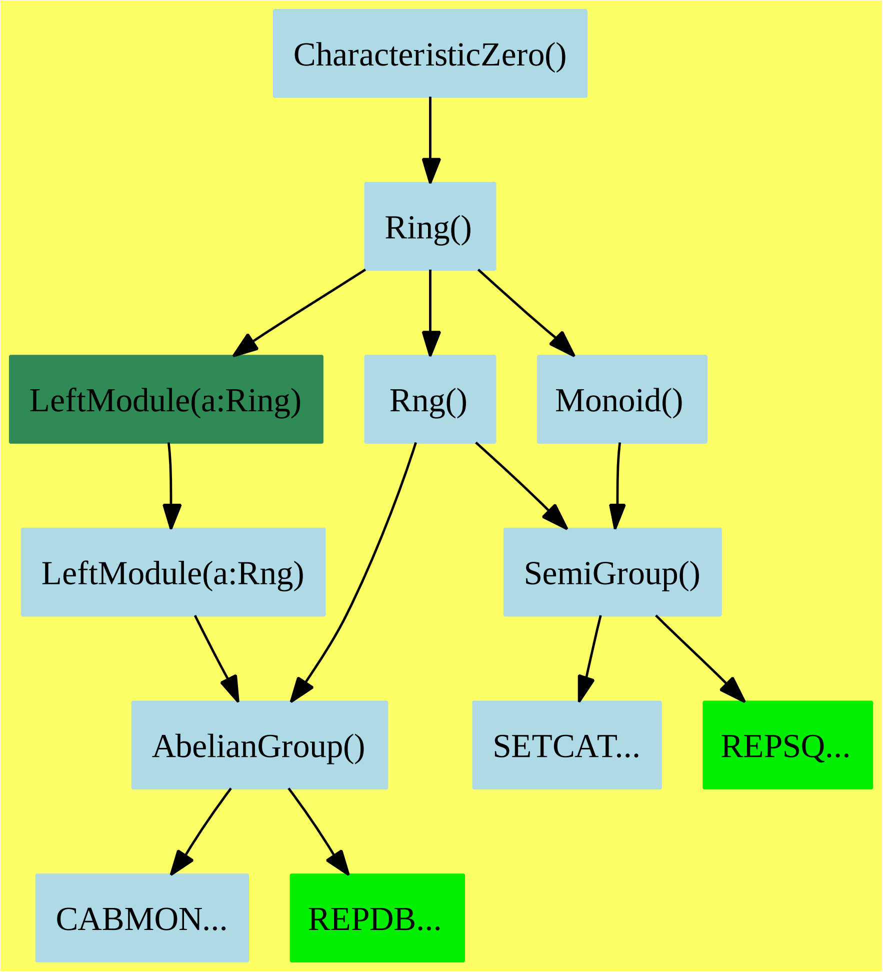
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.136 CharacteristicZero (CHARZ)


CharacteristicZero()
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*CharacteristicZero.input*⟩≡

```
)set break resume
)sys rm -f CharacteristicZero.output
)spool CharacteristicZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CharacteristicZero
--R CharacteristicZero  is a category constructor
--R Abbreviation for CharacteristicZero is CHARZ
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CHARZ
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*CharacteristicZero.help*⟩≡

```
====================================================================
CharacteristicZero examples
====================================================================

See Also:
o )show CharacteristicZero
```

See:

⇒ “FiniteRankAlgebra” (FINRALG) 17.0.194 on page 1465
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “PAdicIntegerCategory” (PADICCT) 16.0.188 on page 1377
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
```

```
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category CHARZ CharacteristicZero*⟩≡

```
)abbrev category CHARZ CharacteristicZero
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Rings of Characteristic Zero.

CharacteristicZero():Category == Ring
```

⟨*CHARZ.dotabb*⟩≡

```
"CHARZ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
"CHARZ" -> "RING"
```

⟨*CHARZ.dotfull*⟩≡

```
"CharacteristicZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
"CharacteristicZero()" -> "Ring()"
```

⟨*CHARZ.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

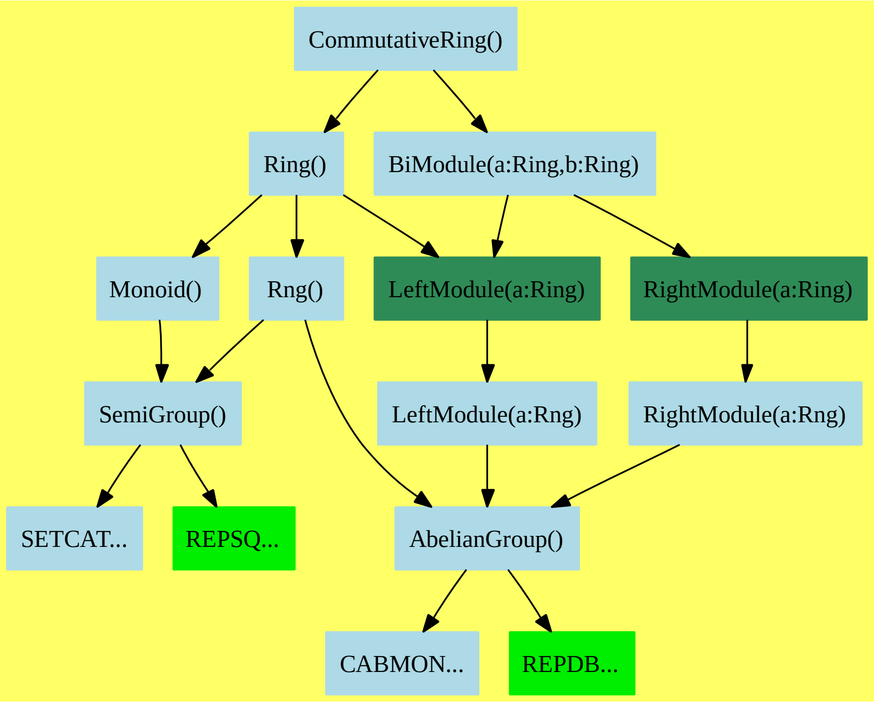
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.137 CommutativeRing (COMRING)


CommutativeRing()
Ring()
BiModule(a:Ring,b:Ring)
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
AbelianGroup()
CABMON...
REPDB...

⟨*CommutativeRing.input*⟩≡

```
)set break resume
)sys rm -f CommutativeRing.output
)spool CommutativeRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CommutativeRing
--R CommutativeRing  is a category constructor
--R Abbreviation for CommutativeRing is COMRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coerce : Integer -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  one? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*CommutativeRing.help*⟩≡

```
====================================================================
CommutativeRing examples
====================================================================

See Also:
o )show CommutativeRing
```

Commutative Rings are a subset of IntegralDomains.
⇒ “IntegralDomain” (INTDOM) 12.0.164 on page 1153.

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.216 on page 1779
⇒ “IntegralDomain” (INTDOM) 12.0.164 on page 1153
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes exported:

- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category COMRING CommutativeRing*⟩≡

```
)abbrev category COMRING CommutativeRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of commutative rings with unity, i.e. rings where
++ \spadop{*} is commutative, and which have a multiplicative identity
++ element.

--CommutativeRing():Category == Join(Ring,BiModule(%:Ring,%:Ring)) with
CommutativeRing():Category == Join(Ring,BiModule(%,%)) with
    commutative("*")  ++ multiplication is commutative.
```

⟨*COMRING.dotabb*⟩≡

```
"COMRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
"COMRING" -> "RING"
"COMRING" -> "BMODULE"
```

⟨*COMRING.dotfull*⟩≡

```
"CommutativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
"CommutativeRing()" -> "Ring()"
"CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"
```

⟨*COMRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "Ring()"
"CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

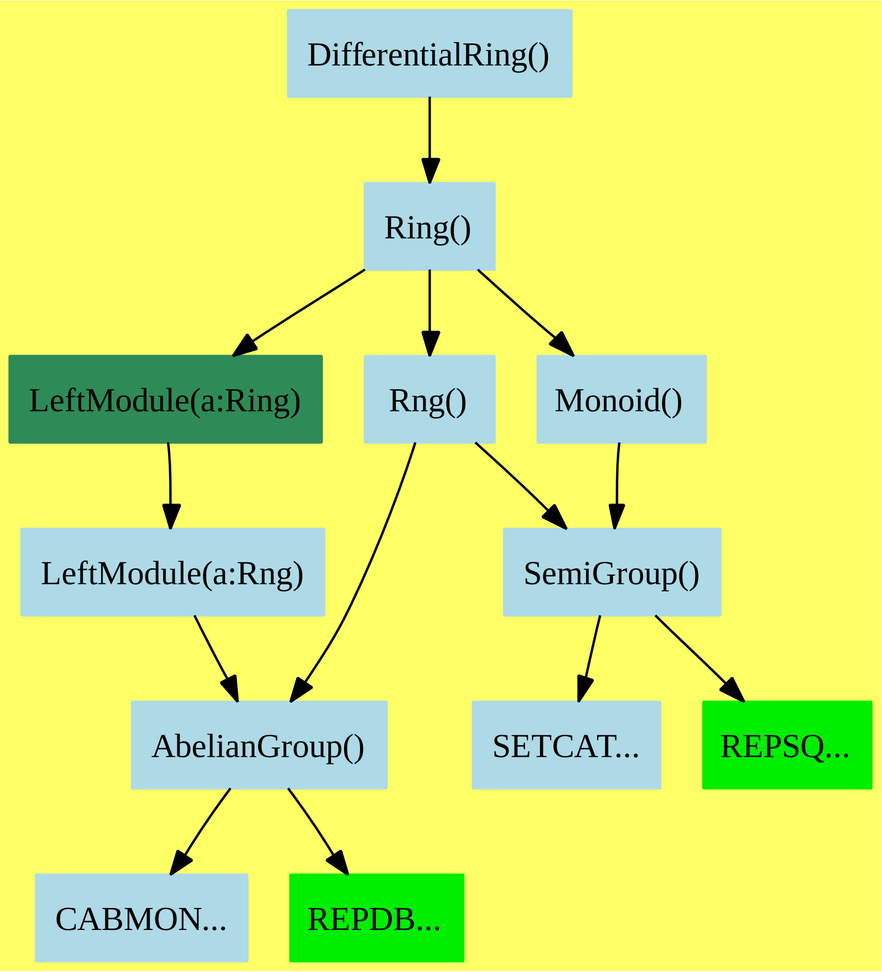
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."
"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.138 DifferentialRing (DIFRING)


DifferentialRing()
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*DifferentialRing.input*⟩≡

```
)set break resume
)sys rm -f DifferentialRing.output
)spool DifferentialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialRing
--R DifferentialRing  is a category constructor
--R Abbreviation for DifferentialRing is DIFRING
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIFRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,NonNegativeInteger) -> %       D : % -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              differentiate : % -> %
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*DifferentialRing.help*⟩≡

```
====================================================================
DifferentialRing examples
====================================================================

See Also:
o )show DifferentialRing
```

See:

⇒ "IntegerNumberSystem" (INS) 16.0.186 on page 1355
⇒ "FiniteFieldCategory" (FFIELDC) 18.0.207 on page 1671
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.204 on page 1620
⇐ "Ring" (RING) 9.0.129 on page 854

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?~=?	?**?
?^?	?*?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : % -> %
```

These are implemented by this category:

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

```
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category DIFRING DifferentialRing*⟩≡

```
)abbrev category DIFRING DifferentialRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An ordinary differential ring, that is, a ring with an operation
++ \spadfun{differentiate}.
++
++ Axioms\br
++ \tab{5}\spad{differentiate(x+y) = differentiate(x)+differentiate(y)}\br
++ \tab{5}\spad{differentiate(x*y) = x*differentiate(y) + differentiate(x)*y}

DifferentialRing(): Category == Ring with
    differentiate: % -> %
         ++ differentiate(x) returns the derivative of x.
         ++ This function is a simple differential operator
         ++ where no variable needs to be specified.
    D: % -> %
         ++ D(x) returns the derivative of x.
         ++ This function is a simple differential operator
         ++ where no variable needs to be specified.
    differentiate: (%, NonNegativeInteger) -> %
         ++ differentiate(x, n) returns the n-th derivative of x.
    D: (%, NonNegativeInteger) -> %
         ++ D(x, n) returns the n-th derivative of x.
  add
    D r == differentiate r
    differentiate(r, n) ==
      for i in 1..n repeat r := differentiate r
```

```
        r
      D(r,n) == differentiate(r,n)
```

⟨*DIFRING.dotabb*⟩≡

```
"DIFRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
"DIFRING" -> "RING"
```

⟨*DIFRING.dotfull*⟩≡

```
"DifferentialRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
"DifferentialRing()" -> "Ring()"
```

⟨*DIFRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

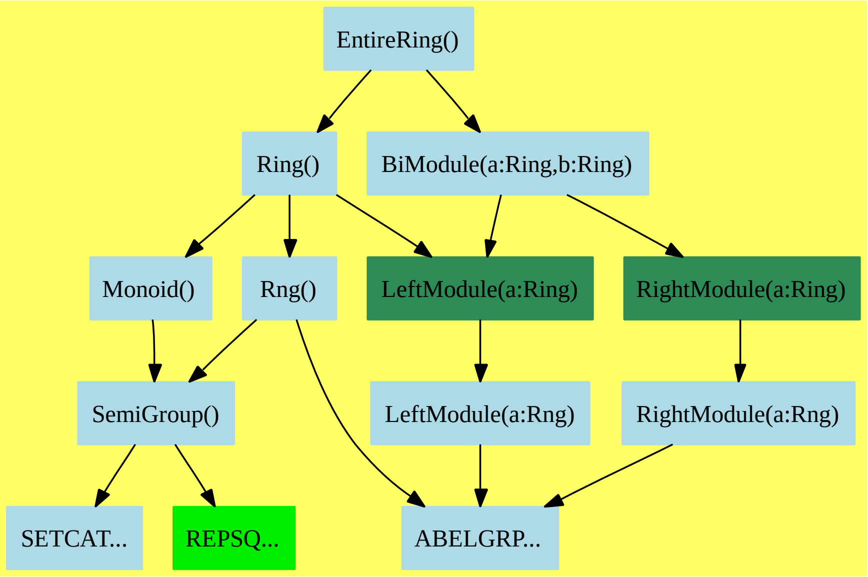
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.139 EntireRing (ENTIRER)


EntireRing()
Ring()
BiModule(a:Ring,b:Ring)
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
ABELGRP...

⟨*EntireRing.input*⟩≡

```
  )set break resume
  )sys rm -f EntireRing.output
  )spool EntireRing.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show EntireRing
  --R EntireRing  is a category constructor
  --R Abbreviation for EntireRing is ENTIRER
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for ENTIRER
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
  --R 1 : () -> %                           0 : () -> %
  --R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
  --R coerce : % -> OutputForm              hash : % -> SingleInteger
  --R latex : % -> String                   one? : % -> Boolean
```

```
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*EntireRing.help*⟩≡

```
====================================================================
EntireRing examples
====================================================================

See Also:
o )show EntireRing
```

See:

⇒ “DivisionRing” (DIVRING) 12.0.161 on page 1116
⇒ “IntegralDomain” (INTDOM) 12.0.164 on page 1153
⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category ENTIRER EntireRing*⟩≡

```
)abbrev category ENTIRER EntireRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Entire Rings (non-commutative Integral Domains), i.e. a ring
++ not necessarily commutative which has no zero divisors.
++
++ Axioms\br
++ \tab{5}\spad{ab=0 => a=0 or b=0}  -- known as noZeroDivisors\br
++ \tab{5}\spad{not(1=0)}

--EntireRing():Category == Join(Ring,BiModule(%:Ring,%:Ring)) with
EntireRing():Category == Join(Ring,BiModule(%,%)) with
      noZeroDivisors  ++ if a product is zero then one of the factors
                      ++ must be zero.
```

⟨*ENTIRER.dotabb*⟩≡

```
"ENTIRER"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
"ENTIRER" -> "RING"
"ENTIRER" -> "BMODULE"
```

⟨*ENTIRER.dotfull*⟩≡

```
"EntireRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
"EntireRing()" -> "Ring()"
"EntireRing()" -> "BiModule(a:Ring,b:Ring)"
```

⟨*ENTIRER.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "Ring()"
"EntireRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

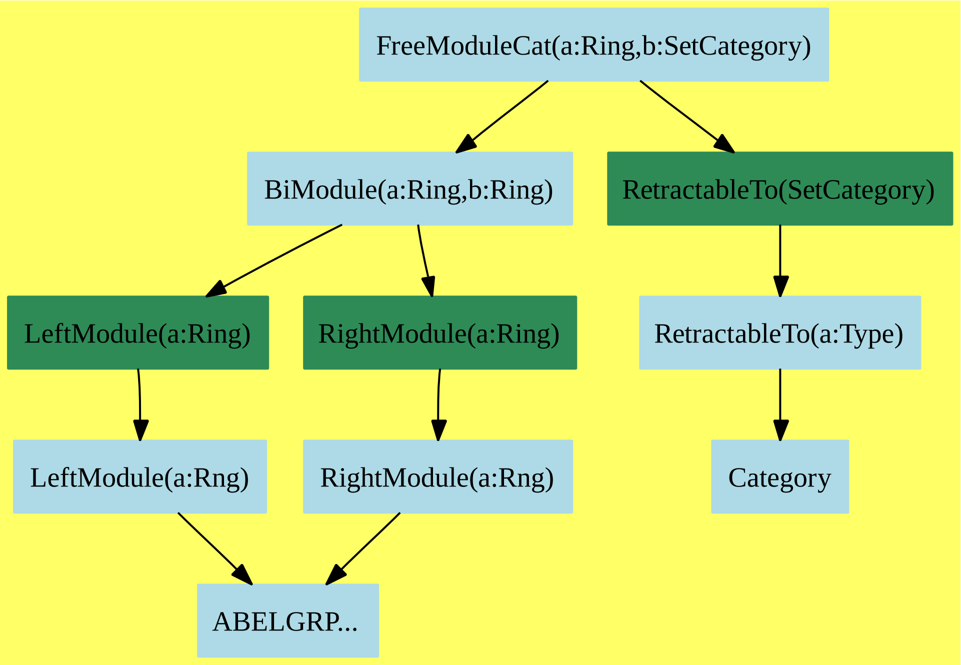
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.0.140 FreeModuleCat (FMCAT)


FreeModuleCat(a:Ring,b:SetCategory)
BiModule(a:Ring,b:Ring)
RetractableTo(SetCategory)
LeftModule(a:Ring)
RightModule(a:Ring)
RetractableTo(a:Type)
LeftModule(a:Rng)
RightModule(a:Rng)
Category
ABELGRP...

⟨*FreeModuleCat.input*⟩≡

```
)set break resume
)sys rm -f FreeModuleCat.output
)spool FreeModuleCat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeModuleCat
--R FreeModuleCat(R: Ring,Basis: SetCategory)  is a category constructor
--R Abbreviation for FreeModuleCat is FMCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,Basis) -> %                  ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
```

```
--R coefficient : (%,Basis) -> R          coefficients : % -> List R
--R coerce : Basis -> %                   coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> R           leadingMonomial : % -> Basis
--R map : ((R -> R),%) -> %               monom : (Basis,R) -> %
--R monomial? : % -> Boolean              monomials : % -> List %
--R reductum : % -> %                     retract : % -> Basis
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R leadingTerm : % -> Record(k: Basis,c: R)
--R listOfTerms : % -> List Record(k: Basis,c: R)
--R numberOfMonomials : % -> NonNegativeInteger
--R retractIfCan : % -> Union(Basis,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FreeModuleCat.help*⟩≡

```
====================================================================
FreeModuleCat examples
====================================================================

See Also:
o )show FreeModuleCat
```

See:

⇐ "BiModule" (BMODULE) 9.0.122 on page 800
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0
hash
leadingTerm
monomial?
retract
zero?
?-?
coefficient
latex
listOfTerms
monomials
retractIfCan
?˜=?
-?
coefficients
leadingCoefficient
map
numberOfMonomials
sample
?*?
?=?
coerce
leadingMonomial
monom
reductum
subtractIfCan
?+?

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Basis) -> R
coefficients : % -> List R
leadingCoefficient : % -> R
leadingMonomial : % -> Basis
leadingTerm : % -> Record(k: Basis,c: R)
listOfTerms : % -> List Record(k: Basis,c: R)
map : ((R -> R),%) -> %
monom : (Basis,R) -> %
monomial? : % -> Boolean
monomials : % -> List %
numberOfMonomials : % -> NonNegativeInteger
reductum : % -> %
?*? : (R,Basis) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
```

```
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,R) -> %
```

These exports come from (p62) RetractableTo(Basis:SetCategory):

```
coerce : Basis -> %
retract : % -> Basis
retractIfCan : % -> Union(Basis,"failed")
```

⟨*category FMCAT FreeModuleCat*⟩≡

```
)abbrev category FMCAT FreeModuleCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A domain of this category
++ implements formal linear combinations
++ of elements from a domain \spad{Basis} with coefficients
++ in a domain \spad{R}. The domain \spad{Basis} needs only
++ to belong to the category \spadtype{SetCategory} and \spad{R}
++ to the category \spadtype{Ring}. Thus the coefficient ring
++ may be non-commutative.
++ See the \spadtype{XDistributedPolynomial} constructor
++ for examples of domains built with the \spadtype{FreeModuleCat}
++ category constructor.

FreeModuleCat(R, Basis):Category == Exports where
   R: Ring
   Basis: SetCategory
   TERM ==> Record(k: Basis, c: R)

   Exports == Join(BiModule(R,R), RetractableTo Basis) with
     "*"                : (R, Basis) -> %
       ++ \spad{r*b} returns the product of \spad{r} by \spad{b}.
     coefficient        : (%, Basis) -> R
```

```
  ++ \spad{coefficient(x,b)} returns the coefficient
  ++ of \spad{b} in \spad{x}.
map                 : (R -> R, %) -> %
  ++ \spad{map(fn,u)} maps function \spad{fn} onto the coefficients
  ++  of the non-zero monomials of \spad{u}.
monom               : (Basis, R) -> %
  ++ \spad{monom(b,r)} returns the element with the single monomial
  ++  \spad{b} and coefficient \spad{r}.
monomial?           : % -> Boolean
  ++ \spad{monomial?(x)} returns true if \spad{x} contains a single
  ++ monomial.
listOfTerms         : % -> List TERM
  ++ \spad{listOfTerms(x)} returns a list \spad{lt} of terms with type
  ++ \spad{Record(k: Basis, c: R)} such that \spad{x} equals
  ++ \spad{reduce(+, map(x +-> monom(x.k, x.c), lt))}.
coefficients        : % -> List R
  ++ \spad{coefficients(x)} returns the list of coefficients of \spad{x}
monomials           : % -> List %
  ++ \spad{monomials(x)} returns the list of \spad{r_i*b_i}
  ++ whose sum is \spad{x}.
numberOfMonomials  : % -> NonNegativeInteger
  ++ \spad{numberOfMonomials(x)} returns the number of monomials
  ++ of \spad{x}.
leadingMonomial     : % -> Basis
  ++ \spad{leadingMonomial(x)} returns the first element from
  ++ \spad{Basis} which appears in \spad{listOfTerms(x)}.
leadingCoefficient : % -> R
  ++ \spad{leadingCoefficient(x)} returns the first coefficient
  ++ which appears in \spad{listOfTerms(x)}.
leadingTerm         : % -> TERM
  ++ \spad{leadingTerm(x)} returns the first term which
  ++ appears in \spad{listOfTerms(x)}.
reductum            : % -> %
  ++ \spad{reductum(x)} returns \spad{x} minus its leading term.

 -- attributs
if R has CommutativeRing then Module(R)
```

⟨*FMCAT.dotabb*⟩≡

```
  "FMCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
  "FMCAT" -> "BMODULE"
  "FMCAT" -> "RETRACT"
```

⟨*FMCAT.dotfull*⟩≡

```
"FreeModuleCat(a:Ring,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"
```

⟨*FMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeModuleCat(a:Ring,b:SetCategory)" [color=lightblue];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

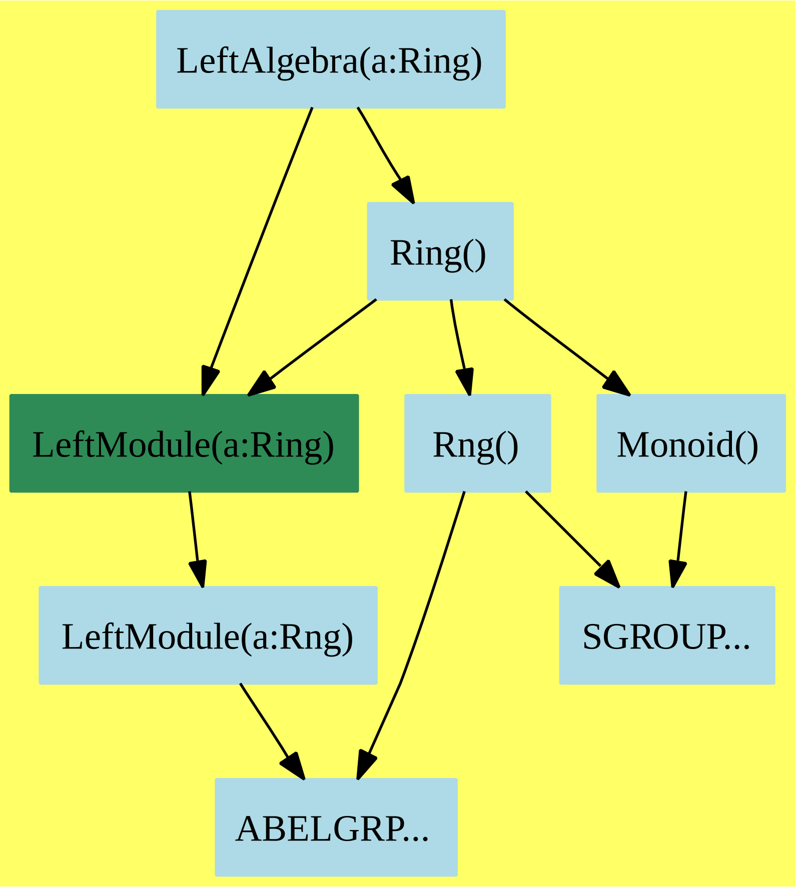
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.0.141 LeftAlgebra (LALG)


LeftAlgebra(a:Ring)
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SGROUP...
ABELGRP...

⟨*LeftAlgebra.input*⟩≡

```
)set break resume
)sys rm -f LeftAlgebra.output
)spool LeftAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftAlgebra
--R LeftAlgebra R: Ring  is a category constructor
--R Abbreviation for LeftAlgebra is LALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LALG
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  one? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*LeftAlgebra.help*⟩≡

```
====================================================================
LeftAlgebra examples
====================================================================

See Also:
o )show LeftAlgebra
```

See:

⇐ “LeftModule” (LMODULE) 8.0.113 on page 722
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

0	1	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p722) LeftModule(R:Type):

```
?*? : (R,%) -> %
```

⟨*category LALG LeftAlgebra*⟩≡

```
)abbrev category LALG LeftAlgebra
++ Author: Larry A. Lambe
++ Date  : 03/01/89; revised 03/17/89; revised 12/02/90.
++ Description:
++ The category of all left algebras over an arbitrary ring.

LeftAlgebra(R:Ring): Category == Join(Ring, LeftModule R) with
      coerce: R -> %
        ++ coerce(r) returns r * 1 where 1 is the identity of the
        ++ left algebra.
    add
      coerce(x:R):% == x * 1$%
```

⟨*LALG.dotabb*⟩≡

```
"LALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LALG" -> "LMODULE"
"LALG" -> "RING"
```

⟨*LALG.dotfull*⟩≡

```
"LeftAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"
```

⟨*LALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LeftAlgebra(a:Ring)" [color=lightblue];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

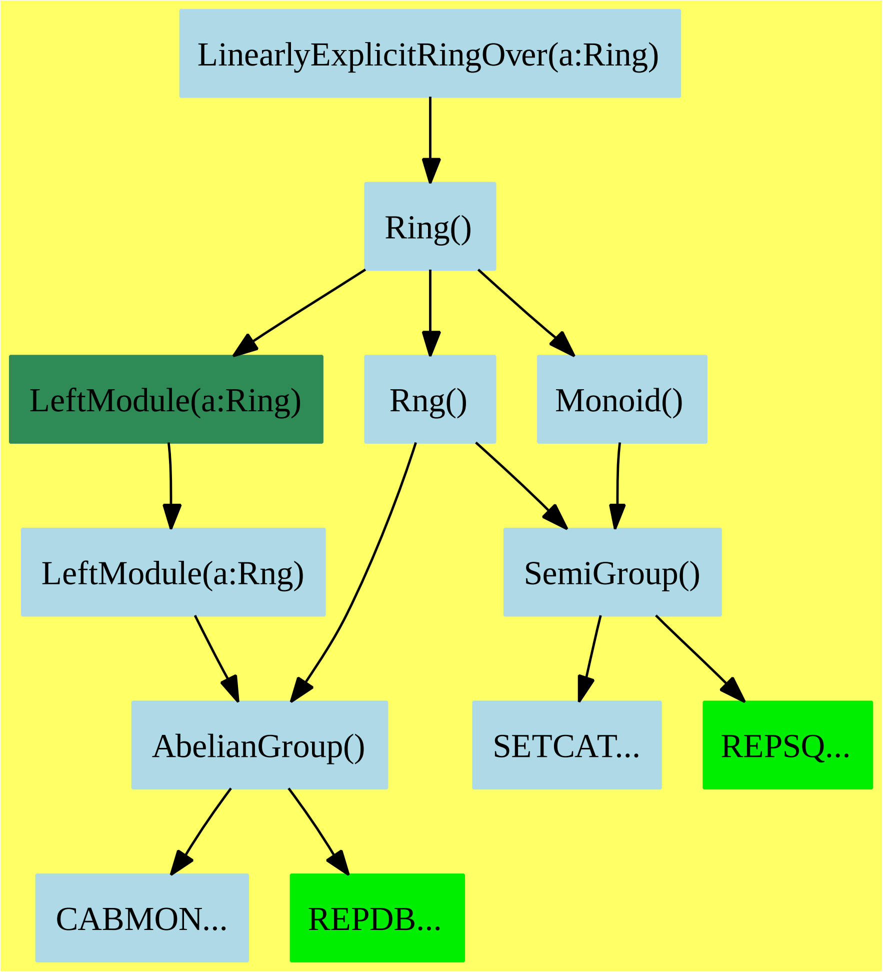
"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SGROUP..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.0.142 LinearlyExplicitRingOver (LINEXP)


LinearlyExplicitRingOver(a:Ring)
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*LinearlyExplicitRingOver.input*⟩≡

```
)set break resume
)sys rm -f LinearlyExplicitRingOver.output
)spool LinearlyExplicitRingOver.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LinearlyExplicitRingOver
--R LinearlyExplicitRingOver R: Ring  is a category constructor
--R Abbreviation for LinearlyExplicitRingOver is LINEXP
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LINEXP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : Matrix % -> Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*LinearlyExplicitRingOver.help*⟩≡

```
====================================================================
LinearlyExplicitRingOver examples
====================================================================

See Also:
o )show LinearlyExplicitRingOver
```

See:

⇒ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.154 on page 1060
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	reducedSystem	subtractIfCan
sample	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return

These are directly exported but not implemented:

```
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category LINEXP LinearlyExplicitRingOver*⟩≡

```
)abbrev category LINEXP LinearlyExplicitRingOver
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An extension ring with an explicit linear dependence test.

LinearlyExplicitRingOver(R:Ring): Category == Ring with
  reducedSystem: Matrix % -> Matrix R
    ++ reducedSystem(A) returns a matrix B such that \spad{A x = 0}
    ++ and \spad{B x = 0} have the same solutions in R.
  reducedSystem: (Matrix %,Vector %) -> Record(mat:Matrix R,vec:Vector R)
    ++ reducedSystem(A, v) returns a matrix B and a vector w such that
    ++ \spad{A x = v} and \spad{B x = w} have the same solutions in R.
```

⟨*LINEXP.dotabb*⟩≡

```
"LINEXP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LINEXP" -> "RING"
```

⟨*LINEXP.dotfull*⟩≡

```
"LinearlyExplicitRingOver(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"LinearlyExplicitRingOver(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(Integer)" -> "LinearlyExplicitRingOver(a:Ring)"

"LinearlyExplicitRingOver(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(Fraction(Integer))" ->
    "LinearlyExplicitRingOver(a:Ring)"
```

⟨*LINEXP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

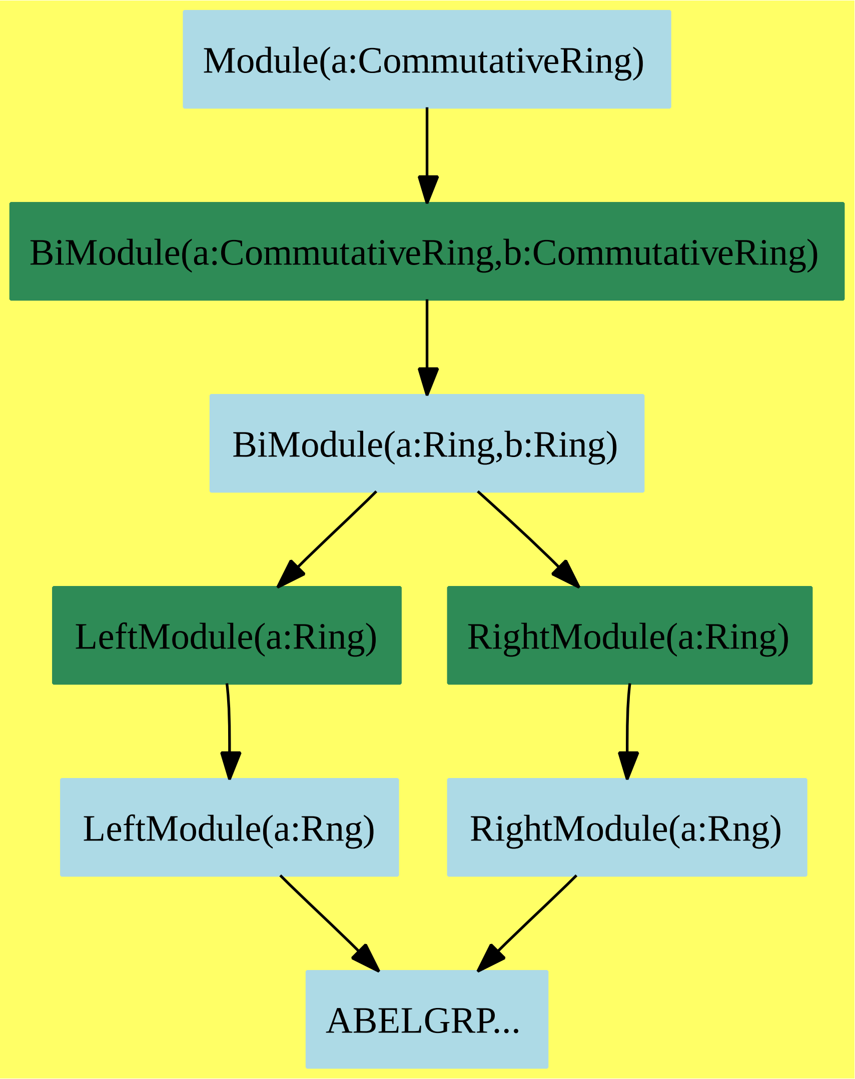
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.143 Module (MODULE)


Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*Module.input*⟩≡
```
)set break resume
)sys rm -f Module.output
)spool Module.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show Module
--R Module R: CommutativeRing  is a category constructor
--R Abbreviation for Module is MODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                        ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                  ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                        ?-? : (%,%) -> %
--R -? : % -> %                             ?=? : (%,%) -> Boolean
--R 0 : () -> %                             coerce : % -> OutputForm
--R hash : % -> SingleInteger               latex : % -> String
--R sample : () -> %                        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*Module.help*⟩≡

```
====================================================================
Module examples
====================================================================

See Also:
o )show Module
```

See:

⇒ "Algebra" (ALGEBRA) 11.0.152 on page 1045
⇒ "LieAlgebra" (LIECAT) 11.0.155 on page 1067
⇒ "NonAssociativeAlgebra" (NAALG) 11.0.157 on page 1082
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.147 on page 994
⇒ "VectorSpace" (VSPACE) 11.0.158 on page 1089
⇐ "BiModule" (BMODULE) 9.0.122 on page 800

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
?*? : (%,R) -> %
```

These exports come from (p800) BiModule(a:Ring,b:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category MODULE Module*⟩≡

```
)abbrev category MODULE Module
++ Author:
++ Date Created:
++ Date Last Updated:
```

```
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of modules over a commutative ring.
++
++ Axioms\br
++ \tab{5}\spad{1*x = x}\br
++ \tab{5}\spad{(a*b)*x = a*(b*x)}\br
++ \tab{5}\spad{(a+b)*x = (a*x)+(b*x)}\br
++ \tab{5}\spad{a*(x+y) = (a*x)+(a*y)}

Module(R:CommutativeRing): Category == BiModule(R,R)
  add
    if not(R is %) then x:%*r:R == r*x
```

⟨*MODULE.dotabb*⟩≡

```
"MODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"MODULE" -> "BMODULE"
```

⟨*MODULE.dotfull*⟩≡

```
"Module(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"Module(Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(Field)" -> "Module(a:CommutativeRing)"
```

⟨*MODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

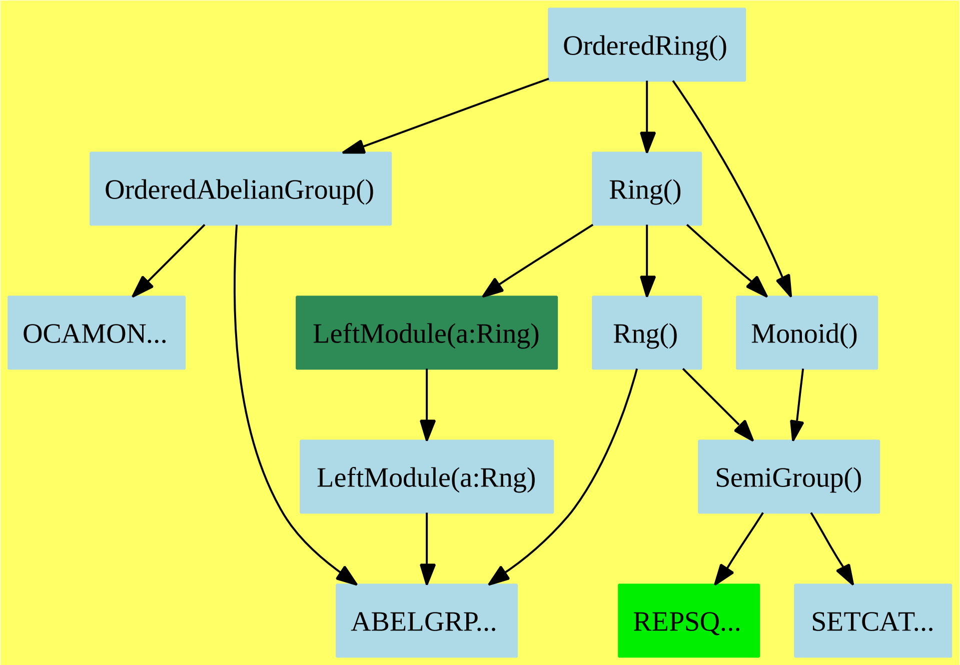
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.0.144 OrderedRing (ORDRING)


OrderedRing()
OrderedAbelianGroup()
Ring()
OCAMON...
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
ABELGRP...
REPSQ...
SETCAT...

⟨*OrderedRing.input*⟩≡

```
)set break resume
)sys rm -f OrderedRing.output
)spool OrderedRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedRing
--R OrderedRing  is a category constructor
--R Abbreviation for OrderedRing is ORDRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      -? : % -> %
--R ?-? : (%,%) -> %                      ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
```

```
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       abs : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R negative? : % -> Boolean             one? : % -> Boolean
--R positive? : % -> Boolean             recip : % -> Union(%,"failed")
--R sample : () -> %                     sign : % -> Integer
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedRing.help*⟩≡

```
====================================================================
OrderedRing examples
====================================================================

See Also:
o )show OrderedRing
```

See:

⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “OrderedIntegralDomain” (OINTDOM) 13.0.174 on page 1253
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇐ “Monoid” (MONOID) 5.0.76 on page 371
⇐ “OrderedAbelianGroup” (OAGROUP) 9.0.126 on page 836
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1	0	abs	characteristic	coerce
hash	latex	max	min	negative?
one?	positive?	recip	sample	sign
subtractIfCan	zero?	?^?	?~=?	?*?
?**?	?+?	-?	?-?	?<?
?<=?	?=?	?>?	?>=?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are implemented by this category:

```
abs : % -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
```

These exports come from (p836) OrderedAbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?+? : (%,%) -> %
-? : % -> %
?-? : (%,%) -> %
```

These exports come from (p854) Ring():

```
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category ORDRING OrderedRing*⟩≡

```
)abbrev category ORDRING OrderedRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also rings, that is, domains where the ring
++ operations are compatible with the ordering.
++
++ Axiom\br
++ \tab{5}\spad{0<a and b<c => ab< ac}

OrderedRing(): Category == Join(OrderedAbelianGroup,Ring,Monoid) with
     positive?: % -> Boolean
       ++ positive?(x) tests whether x is strictly greater than 0.
     negative?: % -> Boolean
       ++ negative?(x) tests whether x is strictly less than 0.
     sign     : % -> Integer
       ++ sign(x) is 1 if x is positive, -1 if x is negative,
       ++ 0 if x equals 0.
     abs      : % -> %
```

```
       ++ abs(x) returns the absolute value of x.
 add
    positive? x == x>0
    negative? x == x<0
    sign x ==
      positive? x => 1
      negative? x => -1
      zero? x => 0
      error "x satisfies neither positive?, negative? or zero?"
    abs x ==
      positive? x => x
      negative? x => -x
      zero? x => 0
      error "x satisfies neither positive?, negative? or zero?"
```

⟨*ORDRING.dotabb*⟩≡

```
"ORDRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"ORDRING" -> "OAGROUP"
"ORDRING" -> "RING"
"ORDRING" -> "MONOID"
```

⟨*ORDRING.dotfull*⟩≡

```
"OrderedRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"
```

⟨*ORDRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OCAMON..."
"OrderedAbelianGroup()" -> "ABELGRP..."

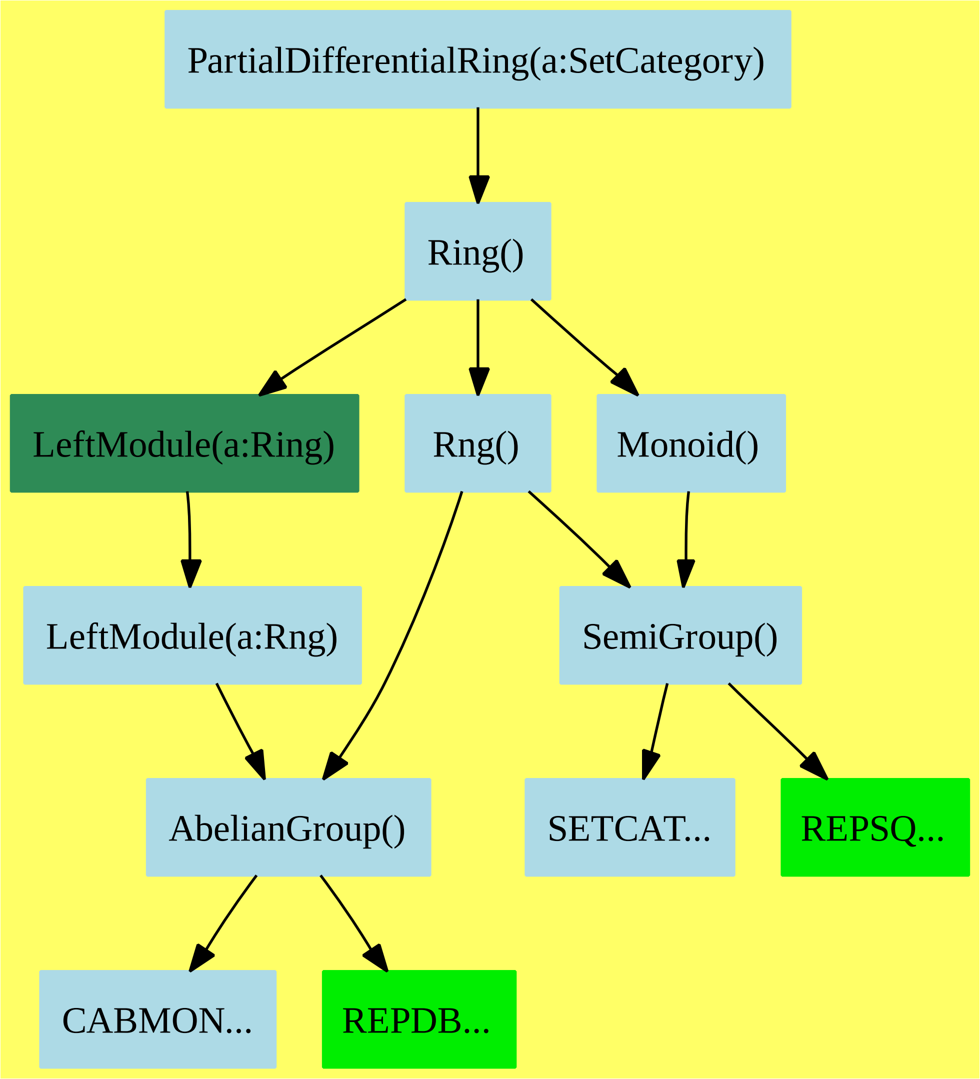
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"OCAMON..." [color=lightblue];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.0.145 PartialDifferentialRing (PDRING)


PartialDifferentialRing(a:SetCategory)
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*PartialDifferentialRing.input*⟩≡

```
)set break resume
)sys rm -f PartialDifferentialRing.output
)spool PartialDifferentialRing.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show PartialDifferentialRing
--R PartialDifferentialRing S: SetCategory  is a category constructor
--R Abbreviation for PartialDifferentialRing is PDRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PDRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,List S) -> %                   D : (%,S) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              differentiate : (%,List S) -> %
--R differentiate : (%,S) -> %            hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,List S,List NonNegativeInteger) -> %
--R D : (%,S,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,List S,List NonNegativeInteger) -> %
--R differentiate : (%,S,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*PartialDifferentialRing.help*⟩≡

```
====================================================================
PartialDifferentialRing examples
====================================================================

See Also:
o )show PartialDifferentialRing
```

See:

⇒ "DifferentialExtension" (DIFEXT) 11.0.153 on page 1053
⇒ "FunctionSpace" (FS) 17.0.195 on page 1472
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.0.182 on page 1316
⇒ "PolynomialCategory" (POLYCAT) 16.0.189 on page 1385
⇐ "Ring" (RING) 9.0.129 on page 854

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?^?	?*?
?~=?	?**?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,S) -> %
```

These are implemented by this category:

```
D : (%,S) -> %
D : (%,List S) -> %
D : (%,S,NonNegativeInteger) -> %
D : (%,List S,List NonNegativeInteger) -> %
differentiate : (%,List S) -> %
differentiate : (%,S,NonNegativeInteger) -> %
differentiate : (%,List S,List NonNegativeInteger) -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category PDRING PartialDifferentialRing*⟩≡

```
)abbrev category PDRING PartialDifferentialRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A partial differential ring with differentiations indexed by a
++ parameter type S.
++
++ Axioms\br
++ \tab{5}\spad{differentiate(x+y,e)=differentiate(x,e)+differentiate(y,e)}\br
++ \tab{5}\spad{differentiate(x*y,e)=x*differentiate(y,e)+differentiate(x,e)*y}

PartialDifferentialRing(S:SetCategory): Category == Ring with
    differentiate: (%, S) -> %
        ++ differentiate(x,v) computes the partial derivative of x
        ++ with respect to v.
    differentiate: (%, List S) -> %
        ++ differentiate(x,[s1,...sn]) computes successive partial
        ++ derivatives,
        ++ i.e. \spad{differentiate(...differentiate(x, s1)..., sn)}.
    differentiate: (%, S, NonNegativeInteger) -> %
        ++ differentiate(x, s, n) computes multiple partial derivatives, i.e.
        ++ n-th derivative of x with respect to s.
    differentiate: (%, List S, List NonNegativeInteger) -> %
```

```
        ++ differentiate(x, [s1,...,sn], [n1,...,nn]) computes
        ++ multiple partial derivatives, i.e.
    D: (%, S) -> %
        ++ D(x,v) computes the partial derivative of x
        ++ with respect to v.
    D: (%, List S) -> %
        ++ D(x,[s1,...sn]) computes successive partial derivatives,
        ++ i.e. \spad{D(...D(x, s1)..., sn)}.
    D: (%, S, NonNegativeInteger) -> %
        ++ D(x, s, n) computes multiple partial derivatives, i.e.
        ++ n-th derivative of x with respect to s.
    D: (%, List S, List NonNegativeInteger) -> %
        ++ D(x, [s1,...,sn], [n1,...,nn]) computes
        ++ multiple partial derivatives, i.e.
        ++ \spad{D(...D(x, s1, n1)..., sn, nn)}.
  add
    differentiate(r:%, l:List S) ==
      for s in l repeat r := differentiate(r, s)
      r

    differentiate(r:%, s:S, n:NonNegativeInteger) ==
      for i in 1..n repeat r := differentiate(r, s)
      r

    differentiate(r:%, ls:List S, ln:List NonNegativeInteger) ==
      for s in ls for n in ln repeat r := differentiate(r, s, n)
      r

    D(r:%, v:S) == differentiate(r,v)
    D(r:%, lv:List S) == differentiate(r,lv)
    D(r:%, v:S, n:NonNegativeInteger) == differentiate(r,v,n)
    D(r:%, lv:List S, ln:List NonNegativeInteger) == differentiate(r, lv, ln)
```

⟨*PDRING.dotabb*⟩≡

```
  "PDRING"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
  "PDRING" -> "RING"
```

⟨*PDRING.dotfull*⟩≡

```
"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"PartialDifferentialRing(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:OrderedSet)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"
```

⟨*PDRING.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

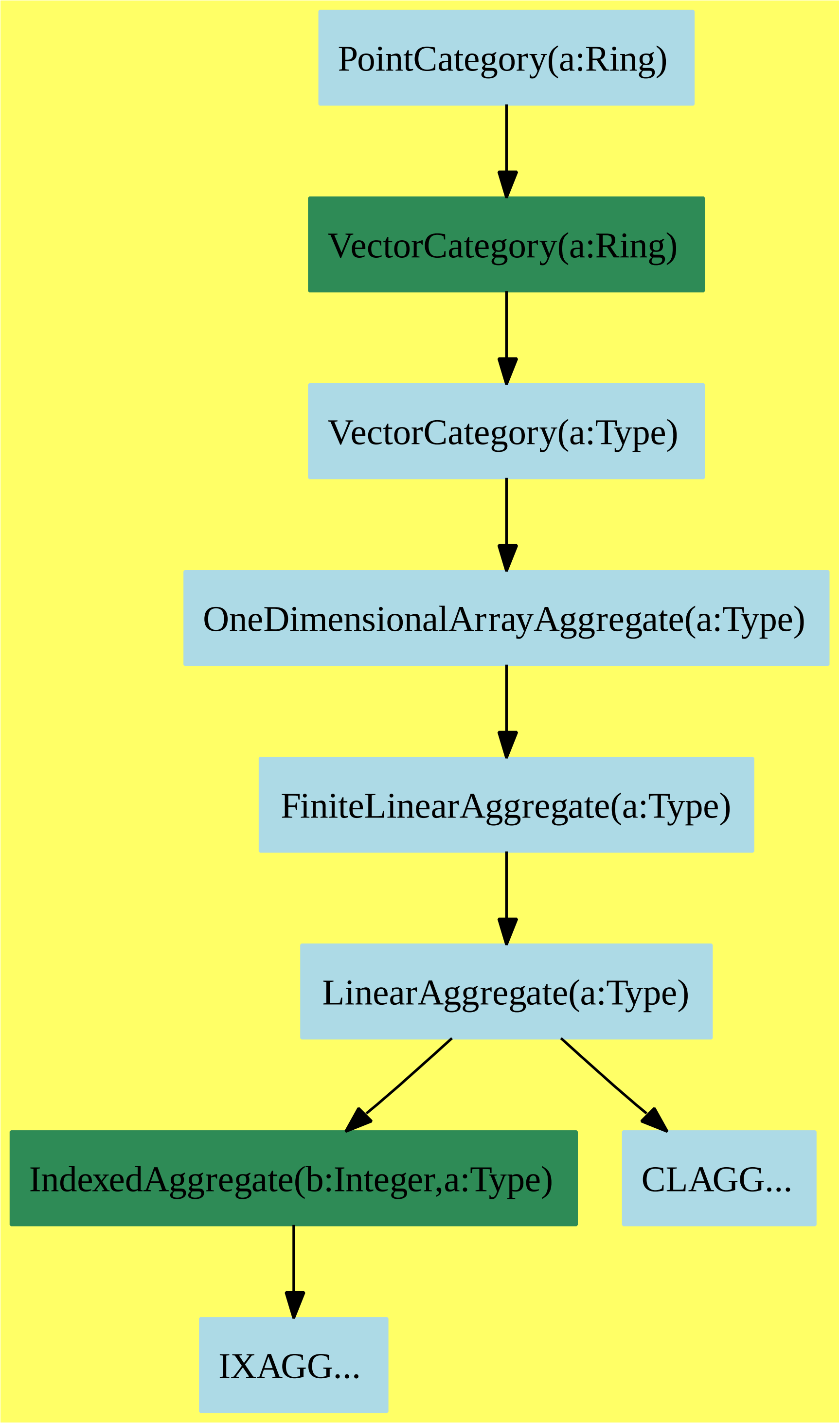
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.146 PointCategory (PTCAT)


PointCategory(a:Ring)
VectorCategory(a:Ring)
VectorCategory(a:Type)
OneDimensionalArrayAggregate(a:Type)
FiniteLinearAggregate(a:Type)
LinearAggregate(a:Type)
IndexedAggregate(b:Integer,a:Type)
CLAGG...
IXAGG...

⟨*PointCategory.input*⟩≡
```
)set break resume
)sys rm -f PointCategory.output
)spool PointCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PointCategory
--R PointCategory R: Ring  is a category constructor
--R Abbreviation for PointCategory is PTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PTCAT
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP          concat : List % -> %
--R concat : (%,%) -> %                   concat : (R,%) -> %
--R concat : (%,R) -> %                   construct : List R -> %
--R convert : List R -> %                 copy : % -> %
--R cross : (%,%) -> %                    delete : (%,Integer) -> %
--R dimension : % -> PositiveInteger      ?.? : (%,Integer) -> R
--R elt : (%,Integer,R) -> R              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List R
--R eq? : (%,%) -> Boolean                extend : (%,List R) -> %
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           insert : (R,%,Integer) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R new : (NonNegativeInteger,R) -> %     point : List R -> %
--R qelt : (%,Integer) -> R               reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,R) -> % if R has MONOID
--R ?*? : (R,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
```

```
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> R if R has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RING and R has RADCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RING and R has RADCAT
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,%) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
```

```
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R sort! : % -> % if R has ORDSET and $ has shallowlyMutable
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*PointCategory.help*⟩≡

```
====================================================================
PointCategory examples
====================================================================

See Also:
o )show PointCategory
```

See:

⇐ “VectorCategory” (VECTCAT) 9.0.133 on page 897

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	cross	count	delete
dimension	dot	elt	empty	empty?
entry?	entries	eq?	eval	every?
extend	fill!	find	first	hash
index?	indices	insert	latex	length
less?	magnitude	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	new	outerProduct	parts
point	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	zero	#?	?.?
?~=?	-?	?*?	?+?	?-?
?<?	?<=?	?=?	?>?	?>=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
convert : List R -> %
cross : (%,%) -> %
dimension : % -> PositiveInteger
extend : (%,List R) -> %
point : List R -> %
```

These exports come from (p897) VectorCategory(R:Ring):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (R,%) -> %
concat : (%,R) -> %
construct : List R -> %
convert : % -> InputForm if R has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
dot : (%,%) -> R if R has RING
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
entries : % -> List R
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
length : % -> R if R has RING and R has RADCAT
less? : (%,NonNegativeInteger) -> Boolean
magnitude : % -> R if R has RING and R has RADCAT
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
outerProduct : (%,%) -> Matrix R if R has RING
parts : % -> List R if $ has finiteAggregate
position : (R,%,Integer) -> Integer if R has SETCAT
position : (R,%) -> Integer if R has SETCAT
position : ((R -> Boolean),%) -> Integer
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((R -> Boolean),%) -> % if $ has finiteAggregate
setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if R has ORDSET
sort : (((R,R) -> Boolean),%) -> %
sort! : % -> % if R has ORDSET and $ has shallowlyMutable
sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
sorted? : % -> Boolean if R has ORDSET
sorted? : (((R,R) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
zero : NonNegativeInteger -> % if R has ABELMON
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?.? : (%,UniversalSegment Integer) -> %
?~=? : (%,%) -> Boolean if R has SETCAT
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?=? : (%,%) -> Boolean if R has SETCAT
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?*? : (Integer,%) -> % if R has ABELGRP
?*? : (%,R) -> % if R has MONOID
?*? : (R,%) -> % if R has MONOID
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
?+? : (%,%) -> % if R has ABELSG
```

⟨*category PTCAT PointCategory*⟩≡

```
)abbrev category PTCAT PointCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Operations: point, elt, setelt, copy, dimension, minIndex, maxIndex,
++ convert
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ PointCategory is the category of points in space which
++ may be plotted via the graphics facilities.  Functions are provided for
++ defining points and handling elements of points.

PointCategory(R:Ring) : Category == VectorCategory(R) with
  point: List R -> %
    ++ point(l) returns a point category defined by a list l of elements from
    ++ the domain R.
  dimension: % -> PositiveInteger
    ++ dimension(s) returns the dimension of the point category s.
  convert: List R -> %
    ++ convert(l) takes a list of elements, l, from the domain Ring and
    ++ returns the form of point category.
  cross: (%,%) -> %
      ++ cross(p,q) computes the cross product of the two points \spad{p}
      ++ and \spad{q}. Error if the p and q are not 3 dimensional
  extend : (%,List R) -> %
        ++ extend(x,l,r) \undocumented
```

⟨*PTCAT.dotabb*⟩≡

```
"PTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PTCAT" -> "VECTCAT"
```

⟨*PTCAT.dotfull*⟩≡

```
"PointCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"
```

⟨*PTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PointCategory(a:Ring)" [color=lightblue];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"

"VectorCategory(a:Ring)" [color=seagreen];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

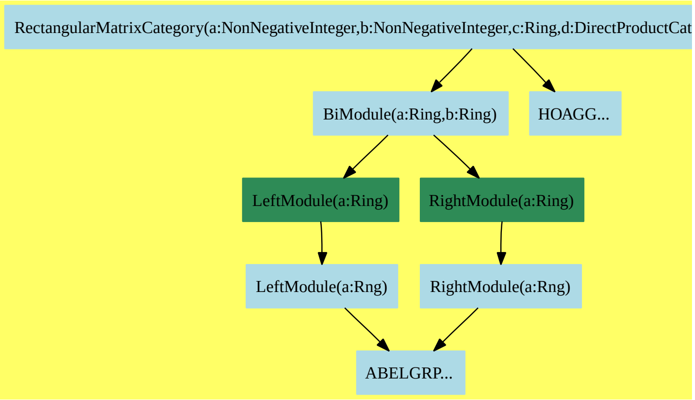
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

10.0.147 RectangularMatrixCategory (RMATCAT)


RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCat
BiModule(a:Ring,b:Ring)
HOAGG...
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

RectangularMatrixCategory does not automatically inherit MatrixCategory. Note that domains in DirectProductCategory(n,R), which are expected as parameters of RectangularMatrixCategory do not satisfy FiniteLinearAggregate(R) as required in MatrixCategory.

The RectangularMatrix domain is matrices of fixed dimension.

⟨*RectangularMatrixCategory.input*⟩≡
```
)set break resume
)sys rm -f RectangularMatrixCategory.output
)spool RectangularMatrixCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RectangularMatrixCategory
--R RectangularMatrixCategory(m: NonNegativeInteger,n: NonNegativeInteger,R: Ring
--R Abbreviation for RectangularMatrixCategory is RMATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RMATCAT
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antisymmetric? : % -> Boolean
--R coerce : % -> OutputForm              column : (%,Integer) -> Col
--R copy : % -> %                         diagonal? : % -> Boolean
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   listOfLists : % -> List List R
--R map : (((R,R) -> R),%,%) -> %        map : ((R -> R),%) -> %
--R matrix : List List R -> %             maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       qelt : (%,Integer,Integer) -> R
--R row : (%,Integer) -> Row              sample : () -> %
--R square? : % -> Boolean                symmetric? : % -> Boolean
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (NonNegativeInteger,%) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List Col if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate
--R rank : % -> NonNegativeInteger if R has INTDOM
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*RectangularMatrixCategory.help*⟩≡

```
====================================================================
RectangularMatrixCategory examples
====================================================================

See Also:
o )show RectangularMatrixCategory
```

See:

⇒ “SquareMatrixCategory” (SMATCAT) 12.0.168 on page 1192
⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “HomogeneousAggregate” (HOAGG) 4.0.49 on page 203
⇐ “MatrixCategory” (MATCAT) 6.0.88 on page 458

Exports:

0	antisymmetric?	any?	coerce	column
copy	count	diagonal?	elt	empty
empty?	eq?	eval	every?	exquo
hash	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
nrows	nullSpace	nullity	parts	qelt
rank	row	rowEchelon	sample	size?
square?	subtractIfCan	symmetric?	zero?	#?
?*?	?/?	?+?	?-?	-?
?=?	?~=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
column : (%,Integer) -> Col
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
listOfLists : % -> List List R
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
matrix : List List R -> %
minColIndex : % -> Integer
minRowIndex : % -> Integer
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
qelt : (%,Integer,Integer) -> R
```

```
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
diagonal? : % -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
square? : % -> Boolean
symmetric? : % -> Boolean
```

These exports come from (p800) BiModule(a:Ring,b:Ring)

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p203) HomogeneousAggregate(Ring)”

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
less? : (%,NonNegativeInteger) -> Boolean
```

```
map! : ((R -> R),%) -> % if $ has shallowlyMutable
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

⟨*category RMATCAT RectangularMatrixCategory*⟩≡

```
)abbrev category RMATCAT RectangularMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: RectangularMatrix(m,n,R)
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{RectangularMatrixCategory} is a category of matrices of fixed
++ dimensions. The dimensions of the matrix will be parameters of the
++ domain. Domains in this category will be R-modules and will be non-mutable.

RectangularMatrixCategory(m,n,R,Row,Col): Category == Definition where
  m,n : NonNegativeInteger
  R   : Ring
  Row : DirectProductCategory(n,R)
  Col : DirectProductCategory(m,R)

  Definition ==> Join(BiModule(R,R),HomogeneousAggregate(R)) with

    finiteAggregate
      ++ matrices are finite

    if R has CommutativeRing then Module(R)

--% Matrix creation

    matrix: List List R -> %
      ++ \spad{matrix(l)} converts the list of lists l to a matrix, where the
      ++ list of lists is viewed as a list of the rows of the matrix.

--% Predicates

    square?  : % -> Boolean
```

```
        ++ \spad{square?(m)} returns true if m is a square matrix (i.e. if m
        ++ has the same number of rows as columns) and false otherwise.
      diagonal?: % -> Boolean
        ++ \spad{diagonal?(m)} returns true if the matrix m is square and
        ++ diagonal (i.e. all entries of m not on the diagonal are zero) and
        ++ false otherwise.
      symmetric?: % -> Boolean
        ++ \spad{symmetric?(m)} returns true if the matrix m is square and
        ++ symmetric (i.e. \spad{m[i,j] = m[j,i]} for all \spad{i} and j) and
        ++ false otherwise.
      antisymmetric?: % -> Boolean
        ++ \spad{antisymmetric?(m)} returns true if the matrix m is square and
        ++ antisymmetric (i.e. \spad{m[i,j] = -m[j,i]} for all \spad{i} and j)
        ++ and false otherwise.

  --% Size inquiries

      minRowIndex : % -> Integer
        ++ \spad{minRowIndex(m)} returns the index of the 'first' row of the
        ++ matrix m.
      maxRowIndex : % -> Integer
        ++ \spad{maxRowIndex(m)} returns the index of the 'last' row of the
        ++ matrix m.
      minColIndex : % -> Integer
        ++ \spad{minColIndex(m)} returns the index of the 'first' column of the
        ++ matrix m.
      maxColIndex : % -> Integer
        ++ \spad{maxColIndex(m)} returns the index of the 'last' column of the
        ++ matrix m.
      nrows : % -> NonNegativeInteger
        ++ \spad{nrows(m)} returns the number of rows in the matrix m.
      ncols : % -> NonNegativeInteger
        ++ \spad{ncols(m)} returns the number of columns in the matrix m.

  --% Part extractions

      listOfLists: % -> List List R
        ++ \spad{listOfLists(m)} returns the rows of the matrix m as a list
        ++ of lists.
      elt: (%,Integer,Integer) -> R
        ++ \spad{elt(m,i,j)} returns the element in the \spad{i}th row and
        ++ \spad{j}th column of the matrix m.
        ++ Error: if indices are outside the proper
        ++ ranges.
      qelt: (%,Integer,Integer) -> R
        ++ \spad{qelt(m,i,j)} returns the element in the \spad{i}th row and
      ++ \spad{j}th column of
      ++ the matrix m. Note that there is NO error check to determine
      ++ if indices are in the proper ranges.
    elt: (%,Integer,Integer,R) -> R
      ++ \spad{elt(m,i,j,r)} returns the element in the \spad{i}th row and
      ++ \spad{j}th column of the matrix m, if m has an \spad{i}th row and a
      ++ \spad{j}th column, and returns r otherwise.
    row: (%,Integer) -> Row
      ++ \spad{row(m,i)} returns the \spad{i}th row of the matrix m.
      ++ Error: if the index is outside the proper range.
    column: (%,Integer) -> Col
      ++ \spad{column(m,j)} returns the \spad{j}th column of the matrix m.
      ++ Error: if the index outside the proper range.

--% Map and Zip

    map: (R -> R,%) -> %
      ++ \spad{map(f,a)} returns b, where \spad{b(i,j) = a(i,j)} for all i, j.
    map:((R,R) -> R,%,%) -> %
      ++ \spad{map(f,a,b)} returns c, where c is such that
      ++ \spad{c(i,j) = f(a(i,j),b(i,j))} for all \spad{i}, j.

--% Arithmetic

    if R has IntegralDomain then
      "exquo": (%,R) -> Union(%,"failed")
        ++ \spad{exquo(m,r)} computes the exact quotient of the elements
        ++ of m by r, returning \axiom{"failed"} if this is not possible.
    if R has Field then
      "/": (%,R) -> %
        ++ \spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.

--% Linear algebra

    if R has EuclideanDomain then
      rowEchelon: % -> %
        ++ \spad{rowEchelon(m)} returns the row echelon form of the matrix m.
    if R has IntegralDomain then
      rank: % -> NonNegativeInteger
        ++ \spad{rank(m)} returns the rank of the matrix m.
      nullity: % -> NonNegativeInteger
        ++ \spad{nullity(m)} returns the nullity of the matrix m. This is
        ++ the dimension of the null space of the matrix m.
      nullSpace: % -> List Col
        ++ \spad{nullSpace(m)}+ returns a basis for the null space of
        ++ the matrix m.
```

```
  add
    nrows x == m

    ncols x == n

    square? x == m = n

    diagonal? x ==
      not square? x => false
      for i in minRowIndex x .. maxRowIndex x repeat
        for j in minColIndex x .. maxColIndex x
          | (j - minColIndex x) ^= (i - minRowIndex x) repeat
            not zero? qelt(x, i, j) => return false
      true

    symmetric? x ==
      m ^= n => false
      mr := minRowIndex x; mc := minColIndex x
      for i in 0..(n - 1) repeat
        for j in (i + 1)..(n - 1) repeat
          qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
      true

    antisymmetric? x ==
      m ^= n => false
      mr := minRowIndex x; mc := minColIndex x
      for i in 0..(n - 1) repeat
        for j in i..(n - 1) repeat
          qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
            return false
      true
```

⟨*RMATCAT.dotabb*⟩≡

```
 "RMATCAT"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
 "RMATCAT" -> "BMODULE"
 "RMATCAT" -> "HOAGG"
```

⟨*RMATCAT.dotfull*⟩≡
```
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "HomogeneousAggregate(Ring)"
```

⟨*RMATCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "HOAGG..."

"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

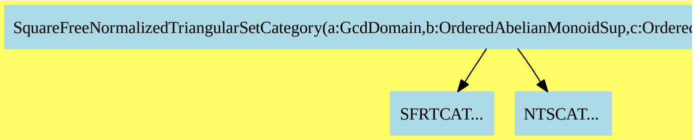
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

}
```

10.0.148 SquareFreeNormalizedTriangularSetCategory (SNTSCAT)


SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:Ordere
SFRTCAT...
NTSCAT...

⟨*SquareFreeNormalizedTriangularSetCategory.input*⟩≡

```
  )set break resume
  )sys rm -f SquareFreeNormalizedTriangularSetCategory.output
  )spool SquareFreeNormalizedTriangularSetCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show SquareFreeNormalizedTriangularSetCategory
  --R SquareFreeNormalizedTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoi
  --R Abbreviation for SquareFreeNormalizedTriangularSetCategory is SNTSCAT
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for SNTSCAT
  --R
  --R------------------------------- Operations --------------------------------
  --R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
  --R algebraicVariables : % -> List V      augment : (List P,%) -> List %
  --R augment : (P,List %) -> List %        augment : (P,%) -> List %
  --R coerce : % -> List P                  coerce : % -> OutputForm
  --R collect : (%,V) -> %                  collectQuasiMonic : % -> %
  --R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
  --R construct : List P -> %               copy : % -> %
  --R degree : % -> NonNegativeInteger      empty : () -> %
  --R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
  --R extend : (List P,%) -> List %         extend : (P,List %) -> List %
  --R extend : (P,%) -> List %              extend : (%,P) -> %
  --R first : % -> Union(P,"failed")        hash : % -> SingleInteger
  --R headReduce : (P,%) -> P               headReduced? : % -> Boolean
  --R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
  --R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
  --R initials : % -> List P                internalAugment : (P,%) -> %
  --R intersect : (P,List %) -> List %      intersect : (List P,%) -> List %
  --R intersect : (P,%) -> List %           invertible? : (P,%) -> Boolean
  --R invertibleSet : (P,%) -> List %       last : % -> Union(P,"failed")
  --R latex : % -> String                   mainVariable? : (V,%) -> Boolean
  --R mainVariables : % -> List V           map : ((P -> P),%) -> %
  --R mvar : % -> V                         normalized? : % -> Boolean
```

```
--R normalized? : (P,%) -> Boolean        purelyAlgebraic? : % -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List P -> %
--R sample : () -> %                      stronglyReduce : (P,%) -> P
--R stronglyReduced? : % -> Boolean       trivialIdeal? : % -> Boolean
--R variables : % -> List V               zeroSetSplit : List P -> List %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List P,List %) -> List %
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List P,List %) -> List %
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R internalAugment : (List P,%) -> %
--R intersect : (List P,List %) -> List %
--R invertible? : (P,%) -> List Record(val: Boolean,tower: %)
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
--R lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R purelyAlgebraic? : (P,%) -> Boolean
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List Record(val: P,tower: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List P,Boolean) -> List %
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List
--R
--E 1

)spool
)lisp (bye)
```

⟨SquareFreeNormalizedTriangularSetCategory.help⟩≡

```
====================================================================
SquareFreeNormalizedTriangularSetCategory examples
====================================================================

See Also:
o )show SquareFreeNormalizedTriangularSetCategory
```

See:

⇐ “NormalizedTriangularSetCategory” (NTSCAT) 9.0.125 on page 823
⇐ “SquareFreeRegularTriangularSetCategory” (SFRTCAT) 9.0.130 on page 860

Exports:

algebraic?
algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
convert
copy
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduced?
reduceByQuasiMonic
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from
(p860) SquareFreeRegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
```

```
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (List P,List %) -> List %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
```

```
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
reduceByQuasiMonic : (P,%) -> P
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
   List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SNTSCAT SquareFreeNormalizedTriangularSetCategory*⟩≡

```
)abbrev category SNTSCAT SquareFreeNormalizedTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/16/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References :
++ [1] D. LAZARD "A new method for solving algebraic systems of
++ positive dimension" Discr. App. Math. 33:147-160,1991
++ Description:
++ The category of square-free and normalized triangular sets.
++ Thus, up to the primitivity axiom of [1], these sets are Lazard
++ triangular sets.

SquareFreeNormalizedTriangularSetCategory(R:GcdDomain,_
                                          E:OrderedAbelianMonoidSup,_
                                          V:OrderedSet,_
                                         P:RecursivePolynomialCategory(R,E,V)):
         Category ==
   Join(SquareFreeRegularTriangularSetCategory(R,E,V,P), _
        NormalizedTriangularSetCategory(R,E,V,P))
```

⟨*SNTSCAT.dotabb*⟩≡

```
"SNTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SNTSCAT" -> "NTSCAT"
"SNTSCAT" -> "SFRTCAT"
```

⟨*SNTSCAT.dotfull*⟩≡

```
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 ->
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 ->
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
```

⟨*SNTSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

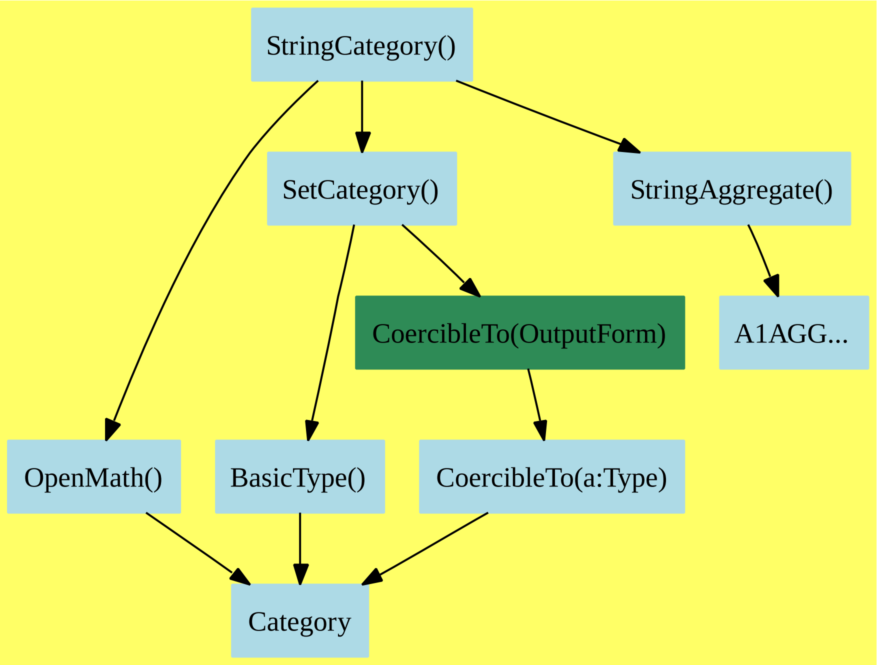
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 [color=lightblue];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 -> "SFRTCAT..."

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 -> "NTSCAT..."

"SFRTCAT..."  [color=lightblue];
"NTSCAT..."  [color=lightblue];

}
```

10.0.149 StringCategory (STRICAT)


StringCategory()
SetCategory()
StringAggregate()
CoercibleTo(OutputForm)
A1AGG...
OpenMath()
BasicType()
CoercibleTo(a:Type)
Category

⟨*StringCategory.input*⟩≡

```
)set break resume
)sys rm -f StringCategory.output
)spool StringCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StringCategory
--R StringCategory  is a category constructor
--R Abbreviation for StringCategory is STRICAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STRICAT
--R
--R------------------------------ Operations -------------------------------
--R ?=? : (%,%) -> Boolean              OMwrite : (%,Boolean) -> String
--R OMwrite : % -> String               coerce : % -> OutputForm
--R coerce : Character -> %             concat : List % -> %
--R concat : (%,%) -> %                 concat : (Character,%) -> %
--R concat : (%,Character) -> %         construct : List Character -> %
```

```
--R copy : % -> %                         delete : (%,Integer) -> %
--R ?.? : (%,%) -> %                      ?.? : (%,Integer) -> Character
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Character         eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R latex : % -> String                   leftTrim : (%,Character) -> %
--R lowerCase : % -> %                    lowerCase! : % -> %
--R prefix? : (%,%) -> Boolean            qelt : (%,Integer) -> Character
--R reverse : % -> %                      rightTrim : (%,Character) -> %
--R sample : () -> %                      split : (%,Character) -> List %
--R string : Integer -> %                 suffix? : (%,%) -> Boolean
--R trim : (%,CharacterClass) -> %        trim : (%,Character) -> %
--R upperCase : % -> %                    upperCase! : % -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Character has ORDSET
--R ?<=? : (%,%) -> Boolean if Character has ORDSET
--R ?>? : (%,%) -> Boolean if Character has ORDSET
--R ?>=? : (%,%) -> Boolean if Character has ORDSET
--R OMwrite : (OpenMathDevice,%,Boolean) -> Void
--R OMwrite : (OpenMathDevice,%) -> Void
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if Character has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if Character has SETCAT and $ has finiteAggr
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R eval : (%,List Character,List Character) -> % if Character has EVALAB CHAR and Character
--R eval : (%,Character,Character) -> % if Character has EVALAB CHAR and Character has SETCA
--R eval : (%,Equation Character) -> % if Character has EVALAB CHAR and Character has SETCAT
--R eval : (%,List Equation Character) -> % if Character has EVALAB CHAR and Character has S
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R insert : (Character,%,Integer) -> %
--R leftTrim : (%,CharacterClass) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (%,%,Character) -> NonNegativeInteger
```

```
--R match? : (%,%,Character) -> Boolean
--R max : (%,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Character,%) -> Boolean if Character has SETCAT and $ has finiteAg
--R members : % -> List Character if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List Character if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finit
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Char
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if Character has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if Character has SETCAT and $ has finiteAggregate
--R replace : (%,UniversalSegment Integer,%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Character) -> Character if $ has shallow
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if Character has ORDSET and $ has shallowlyMutable
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List %
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

⟨*StringCategory.help*⟩≡

```
====================================================================
StringCategory examples
====================================================================

See Also:
o )show StringCategory
```

See:

⇐ “OpenMath” (OM) 2.0.13 on page 43
⇐ “SetCategory” (SETCAT) 3.0.34 on page 130
⇐ “StringAggregate” (SRAGG) 9.0.131 on page 872

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	OMwrite	parts	position
prefix?	qelt	qsetelt!	reduce	remove
removeDuplicates	replace	reverse	reverse!	rightTrim
sample	select	setelt	size?	sort
sort!	sorted?	split	string	substring?
suffix?	swap!	trim	upperCase	upperCase!
#?	?<?	?<=?	?>?	?>=?
?=?	?.?	?~=?		

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
string : Integer -> %
```

These exports come from (p872) StringAggregate():

```
any? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
coerce : Character -> %
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
convert : % -> InputForm
    if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
    if Character has SETCAT
    and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entry? : (Character,%) -> Boolean
    if $ has finiteAggregate
    and Character has SETCAT
entries : % -> List Character
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Character,Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,List Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
every? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Character) -> %
    if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
    if Integer has ORDSET
hash : % -> SingleInteger
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (Character,%,Integer) -> %
latex : % -> String
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
```

```
less? : (%,NonNegativeInteger) -> Boolean
lowerCase : % -> %
lowerCase! : % -> %
map : (((Character,Character) -> Character),%,%) -> %
map : ((Character -> Character),%) -> %
map! : ((Character -> Character),%) -> %
    if $ has shallowlyMutable
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
    if Character has SETCAT
    and $ has finiteAggregate
members : % -> List Character
    if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
    if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
    if Character has SETCAT
position : (CharacterClass,%,Integer) -> Integer
position : (%,%,Integer) -> Integer
prefix? : (%,%) -> Boolean
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%)
   -> Character
    if $ has finiteAggregate
reduce : (((Character,Character) -> Character),%,Character)
   -> Character
    if $ has finiteAggregate
reduce :
 (((Character,Character) -> Character),%,Character,Character)
   -> Character
    if Character has SETCAT
    and $ has finiteAggregate
remove : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (Character,%) -> %
    if Character has SETCAT
    and $ has finiteAggregate
```

```
removeDuplicates : % -> %
    if Character has SETCAT
    and $ has finiteAggregate
replace : (%,UniversalSegment Integer,%) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
rightTrim : (%,CharacterClass) -> %
rightTrim : (%,Character) -> %
sample : () -> %
select : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
setelt :
  (%,UniversalSegment Integer,Character) -> Character
    if $ has shallowlyMutable
setelt : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if Character has ORDSET
sort : (((Character,Character) -> Boolean),%) -> %
sort! : % -> %
    if Character has ORDSET
    and $ has shallowlyMutable
sort! : (((Character,Character) -> Boolean),%) -> %
    if $ has shallowlyMutable
sorted? : (((Character,Character) -> Boolean),%) -> Boolean
sorted? : % -> Boolean if Character has ORDSET
split : (%,CharacterClass) -> List %
split : (%,Character) -> List %
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
swap! : (%,Integer,Integer) -> Void
    if $ has shallowlyMutable
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
upperCase! : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if Character has ORDSET
?<=? : (%,%) -> Boolean if Character has ORDSET
?>? : (%,%) -> Boolean if Character has ORDSET
?>=? : (%,%) -> Boolean if Character has ORDSET
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> Character
?.? : (%,%) -> %
```

These exports come from (p130) SetCategory():

These exports come from (p43) OpenMath():

```
OMwrite : (%,Boolean) -> String
OMwrite : % -> String
OMwrite : (OpenMathDevice,%,Boolean) -> Void
OMwrite : (OpenMathDevice,%) -> Void
```

⟨*category STRICAT StringCategory*⟩≡

```
)abbrev category STRICAT StringCategory
-- Note that StringCategory is built into the old compiler
-- redundant SetCategory added to help A# compiler
++ Description:
++ A category for string-like objects

StringCategory():Category == _
     Join(StringAggregate(), SetCategory, OpenMath) with
  string: Integer -> %
    ++ string(i) returns the decimal representation of i in a string
```

⟨*STRICAT.dotabb*⟩≡

```
"STRICAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"STRICAT" -> "OM"
"STRICAT" -> "SETCAT"
"STRICAT" -> "SRAGG"
```

⟨*STRICAT.dotfull*⟩≡

```
"StringCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"
```

⟨*STRICAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StringCategory()" [color=lightblue];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"

"OpenMath()" [color=lightblue];
"OpenMath()" -> "Category"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

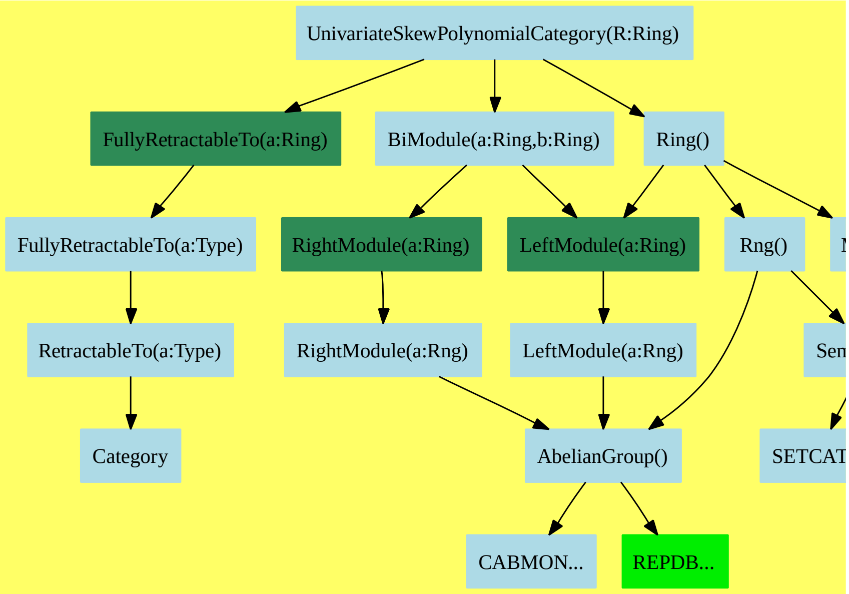
"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "A1AGG..."

"A1AGG..." [color=lightblue];

"Category" [color=lightblue];

}
```

10.0.150 UnivariateSkewPolynomialCategory (OREPCAT)


UnivariateSkewPolynomialCategory(R:Ring)
FullyRetractableTo(a:Ring)
BiModule(a:Ring,b:Ring)
Ring()
FullyRetractableTo(a:Type)
RightModule(a:Ring)
LeftModule(a:Ring)
Rng()
N
RetractableTo(a:Type)
RightModule(a:Rng)
LeftModule(a:Rng)
Sem
Category
AbelianGroup()
SETCAT
CABMON...
REPDB...

⟨*UnivariateSkewPolynomialCategory.input*⟩≡

```
  )set break resume
  )sys rm -f UnivariateSkewPolynomialCategory.output
  )spool UnivariateSkewPolynomialCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show UnivariateSkewPolynomialCategory
  --R UnivariateSkewPolynomialCategory R: Ring  is a category constructor
  --R Abbreviation for UnivariateSkewPolynomialCategory is OREPCAT
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for OREPCAT
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
  --R 1 : () -> %                           0 : () -> %
  --R ?^? : (%,PositiveInteger) -> %        apply : (%,R,R) -> R
```

```
--R coefficients : % -> List R           coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger     hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> R
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                    retract : % -> R
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (%,%) -> % if R has FIELD
--R leftLcm : (%,%) -> % if R has FIELD
--R leftQuotient : (%,%) -> % if R has FIELD
--R leftRemainder : (%,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R rightGcd : (%,%) -> % if R has FIELD
--R rightLcm : (%,%) -> % if R has FIELD
--R rightQuotient : (%,%) -> % if R has FIELD
--R rightRemainder : (%,%) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariateSkewPolynomialCategory.help*⟩≡

```
====================================================================
UnivariateSkewPolynomialCategory examples
====================================================================

See Also:
o )show UnivariateSkewPolynomialCategory
```

See:

⇒ “LinearOrdinaryDifferentialOperatorCategory” (LODOCAT) 11.0.156 on page 1072

⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

0	1	apply	characteristic
coefficient	coefficients	coerce	content
degree	exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient	leftExtendedGcd
leftGcd	leftLcm	leftQuotient	leftRemainder
minimumDegree	monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip	reductum
retract	retractIfCan	rightDivide	rightExactQuotient
rightExtendedGcd	rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (%,R,R) -> R
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
```

These are implemented by this category:

```
coefficients : % -> List R
coerce : R -> %
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
leftGcd : (%,%) -> % if R has FIELD
leftLcm : (%,%) -> % if R has FIELD
leftQuotient : (%,%) -> % if R has FIELD
leftRemainder : (%,%) -> % if R has FIELD
primitivePart : % -> % if R has GCDDOM
retractIfCan : % -> Union(R,"failed")
rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
rightGcd : (%,%) -> % if R has FIELD
rightLcm : (%,%) -> % if R has FIELD
rightQuotient : (%,%) -> % if R has FIELD
rightRemainder : (%,%) -> % if R has FIELD
?*? : (R,%) -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
?*? : (%,R) -> %
```

These exports come from (p100) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> R
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

⟨*category OREPCAT UnivariateSkewPolynomialCategory*⟩≡

```
)abbrev category OREPCAT UnivariateSkewPolynomialCategory
++ Author: Manuel Bronstein, Jean Della Dora, Stephen M. Watt
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++ This is the category of univariate skew polynomials over an Ore
++ coefficient ring.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.
++ This category is an evolution of the types
++ MonogenicLinearOperator, OppositeMonogenicLinearOperator, and
++ NonCommutativeOperatorDivision

UnivariateSkewPolynomialCategory(R:Ring):
  Category == Join(Ring, BiModule(R, R), FullyRetractableTo R) with
      degree: $ -> NonNegativeInteger
          ++ degree(l) is \spad{n} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      minimumDegree: $ -> NonNegativeInteger
          ++ minimumDegree(l) is the smallest \spad{k} such that
          ++ \spad{a(k) ^= 0} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      leadingCoefficient: $ -> R
          ++ leadingCoefficient(l) is \spad{a(n)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      reductum: $ -> $
          ++ reductum(l) is \spad{l - monomial(a(n),n)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      coefficient: ($, NonNegativeInteger) -> R
          ++ coefficient(l,k) is \spad{a(k)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      monomial: (R, NonNegativeInteger) -> $
          ++ monomial(c,k) produces c times the k-th power of
          ++ the generating operator, \spad{monomial(1,1)}.
      coefficients: % -> List R
          ++ coefficients(l) returns the list of all the nonzero
      ++ coefficients of l.
  apply: (%, R, R) -> R
    ++ apply(p, c, m) returns \spad{p(m)} where the action is
    ++ given by \spad{x m = c sigma(m) + delta(m)}.
  if R has CommutativeRing then Algebra R
  if R has IntegralDomain then
    "exquo": (%, R) -> Union(%, "failed")
      ++ exquo(l, a) returns the exact quotient of l by a,
      ++ returning \axiom{"failed"} if this is not possible.
    monicLeftDivide:   (%, %) -> Record(quotient: %, remainder: %)
      ++ monicLeftDivide(a,b) returns the pair \spad{[q,r]} such that
      ++ \spad{a = b*q + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ \spad{b} must be monic.
      ++ This process is called ``left division''.
    monicRightDivide:   (%, %) -> Record(quotient: %, remainder: %)
      ++ monicRightDivide(a,b) returns the pair \spad{[q,r]} such that
      ++ \spad{a = q*b + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ \spad{b} must be monic.
      ++ This process is called ``right division''.
  if R has GcdDomain then
    content: % -> R
      ++ content(l) returns the gcd of all the coefficients of l.
    primitivePart: % -> %
      ++ primitivePart(l) returns l0 such that \spad{l = a * l0}
      ++ for some a in R, and \spad{content(l0) = 1}.
  if R has Field then
    leftDivide:   (%, %) -> Record(quotient: %, remainder: %)
      ++ leftDivide(a,b) returns the pair \spad{[q,r]} such that
      ++ \spad{a = b*q + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ This process is called ``left division''.
    leftQuotient:  (%, %) -> %
      ++ leftQuotient(a,b) computes the pair \spad{[q,r]} such that
      ++ \spad{a = b*q + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ The value \spad{q} is returned.
    leftRemainder:  (%, %) -> %
      ++ leftRemainder(a,b) computes the pair \spad{[q,r]} such that
      ++ \spad{a = b*q + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ The value \spad{r} is returned.
    leftExactQuotient:(%, %) -> Union(%, "failed")
      ++ leftExactQuotient(a,b) computes the value \spad{q}, if it exists,
      ++  such that \spad{a = b*q}.
```

```
leftGcd:   (%, %) -> %
  ++ leftGcd(a,b) computes the value \spad{g} of highest degree
  ++ such that
  ++    \spad{a = g*aa}
  ++    \spad{b = g*bb}
  ++ for some values \spad{aa} and \spad{bb}.
  ++ The value \spad{g} is computed using left-division.
leftExtendedGcd:   (%, %) -> Record(coef1:%, coef2:%, generator:%)
  ++ leftExtendedGcd(a,b) returns \spad{[c,d]} such that
  ++ \spad{g = a * c + b * d = leftGcd(a, b)}.
rightLcm:   (%, %) -> %
  ++ rightLcm(a,b) computes the value \spad{m} of lowest degree
  ++ such that \spad{m = a*aa = b*bb} for some values
  ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
  ++ computed using left-division.
rightDivide:   (%, %) -> Record(quotient: %, remainder: %)
  ++ rightDivide(a,b) returns the pair \spad{[q,r]} such that
  ++ \spad{a = q*b + r} and the degree of \spad{r} is
  ++ less than the degree of \spad{b}.
  ++ This process is called ``right division''.
rightQuotient:  (%, %) -> %
  ++ rightQuotient(a,b) computes the pair \spad{[q,r]} such that
  ++ \spad{a = q*b + r} and the degree of \spad{r} is
  ++ less than the degree of \spad{b}.
  ++ The value \spad{q} is returned.
rightRemainder:  (%, %) -> %
  ++ rightRemainder(a,b) computes the pair \spad{[q,r]} such that
  ++ \spad{a = q*b + r} and the degree of \spad{r} is
  ++ less than the degree of \spad{b}.
  ++ The value \spad{r} is returned.
rightExactQuotient:(%, %) -> Union(%, "failed")
  ++ rightExactQuotient(a,b) computes the value \spad{q}, if it exists
  ++ such that \spad{a = q*b}.
rightGcd:   (%, %) -> %
  ++ rightGcd(a,b) computes the value \spad{g} of highest degree
  ++ such that
  ++    \spad{a = aa*g}
  ++    \spad{b = bb*g}
  ++ for some values \spad{aa} and \spad{bb}.
  ++ The value \spad{g} is computed using right-division.
rightExtendedGcd:   (%, %) -> Record(coef1:%, coef2:%, generator:%)
  ++ rightExtendedGcd(a,b) returns \spad{[c,d]} such that
  ++ \spad{g = c * a + d * b = rightGcd(a, b)}.
leftLcm:   (%, %) -> %
  ++ leftLcm(a,b) computes the value \spad{m} of lowest degree
  ++ such that \spad{m = aa*a = bb*b} for some values
```

```
        ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
        ++ computed using right-division.

 add
    coerce(x:R):% == monomial(x, 0)

    coefficients l ==
      ans:List(R) := empty()
      while l ^= 0 repeat
        ans := concat(leadingCoefficient l, ans)
        l   := reductum l
      ans

    a:R * y:% ==
      z:% := 0
      while y ^= 0 repeat
        z := z + monomial(a * leadingCoefficient y, degree y)
        y := reductum y
      z

    retractIfCan(x:%):Union(R, "failed") ==
      zero? x or zero? degree x => leadingCoefficient x
      "failed"

    if R has IntegralDomain then
      l exquo a ==
        ans:% := 0
        while l ^= 0 repeat
          (u := (leadingCoefficient(l) exquo a)) case "failed" =>
             return "failed"
          ans := ans + monomial(u::R, degree l)
          l   := reductum l
        ans

    if R has GcdDomain then
      content l       == gcd coefficients l

      primitivePart l == (l exquo content l)::%

    if R has Field then
      leftEEA:  (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
      rightEEA: (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
      ncgcd:    (%, %, (%, %) -> %) -> %
      nclcm:  (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) -> %
      exactQuotient: Record(quotient:%, remainder:%) -> Union(%, "failed")
      extended: (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) ->
```

```
                                Record(coef1:%, coef2:%, generator:%)

leftQuotient(a, b)         == leftDivide(a,b).quotient

leftRemainder(a, b)        == leftDivide(a,b).remainder

leftExtendedGcd(a, b)      == extended(a, b, leftEEA)

rightLcm(a, b)             == nclcm(a, b, leftEEA)

rightQuotient(a, b)        == rightDivide(a,b).quotient

rightRemainder(a, b)       == rightDivide(a,b).remainder

rightExtendedGcd(a, b)     == extended(a, b, rightEEA)

leftLcm(a, b)              == nclcm(a, b, rightEEA)

leftExactQuotient(a, b)    == exactQuotient leftDivide(a, b)

rightExactQuotient(a, b)   == exactQuotient rightDivide(a, b)

rightGcd(a, b)             == ncgcd(a, b, rightRemainder)

leftGcd(a, b)              == ncgcd(a, b, leftRemainder)

exactQuotient qr  == (zero?(qr.remainder) => qr.quotient; "failed")

-- returns [g = leftGcd(a, b), c, d, l = rightLcm(a, b)]
-- such that g := a c + b d
leftEEA(a, b) ==
  a0 := a
  u0:% := v:% := 1
  v0:% := u:% := 0
  while b ^= 0 repeat
    qr     := leftDivide(a, b)
    (a, b) := (b, qr.remainder)
    (u0, u):= (u, u0 - u * qr.quotient)
    (v0, v):= (v, v0 - v * qr.quotient)
  [a, u0, v0, a0 * u]

ncgcd(a, b, ncrem) ==
  zero? a => b
  zero? b => a
  degree a < degree b => ncgcd(b, a, ncrem)
  while b ^= 0 repeat (a, b) := (b, ncrem(a, b))
```

```
            a

          extended(a, b, eea) ==
            zero? a => [0, 1, b]
            zero? b => [1, 0, a]
            degree a < degree b =>
              rec := eea(b, a)
              [rec.coef2, rec.coef1, rec.gcd]
            rec := eea(a, b)
            [rec.coef1, rec.coef2, rec.gcd]

          nclcm(a, b, eea) ==
            zero? a or zero? b => 0
            degree a < degree b => nclcm(b, a, eea)
            rec := eea(a, b)
            rec.lcm

          -- returns [g = rightGcd(a, b), c, d, l = leftLcm(a, b)]
          -- such that g := a c + b d
          rightEEA(a, b) ==
            a0 := a
            u0:% := v:% := 1
            v0:% := u:% := 0
            while b ^= 0 repeat
              qr     := rightDivide(a, b)
              (a, b) := (b, qr.remainder)
              (u0, u):= (u, u0 - qr.quotient * u)
              (v0, v):= (v, v0 - qr.quotient * v)
            [a, u0, v0, u * a0]
```

⟨*OREPCAT.dotabb*⟩≡

```
"OREPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"OREPCAT" -> "BMODULE"
"OREPCAT" -> "FRETRCT"
"OREPCAT" -> "RING"
```

⟨*OREPCAT.dotfull*⟩≡
```
"UnivariateSkewPolynomialCategory(R:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"
```

⟨*OREPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

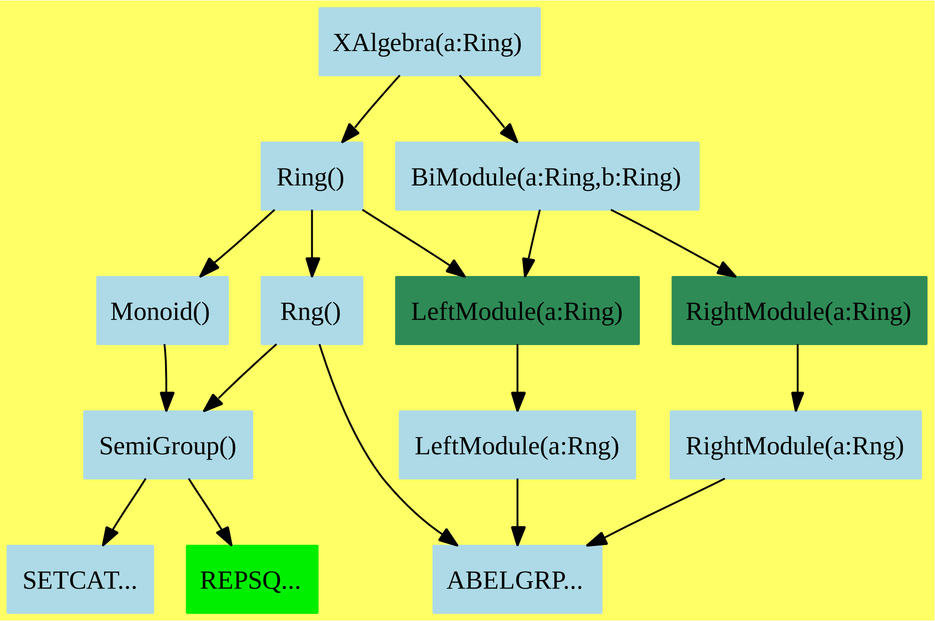
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.151 XAlgebra (XALG)


XAlgebra(a:Ring)
Ring()
BiModule(a:Ring,b:Ring)
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
ABELGRP...

⟨*XAlgebra.input*⟩≡

```
)set break resume
)sys rm -f XAlgebra.output
)spool XAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XAlgebra
--R XAlgebra R: Ring  is a category constructor
--R Abbreviation for XAlgebra is XALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coerce : R -> %
```

```
--R coerce : Integer -> %                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*XAlgebra.help*⟩≡

```
====================================================================
XAlgebra examples
====================================================================

See Also:
o )show XAlgebra
```

See:

⇒ "XFreeAlgebra" (XFALG) 11.0.159 on page 1094
⇐ "BiModule" (BMODULE) 9.0.122 on page 800
⇐ "Ring" (RING) 9.0.129 on page 854

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coerce : R -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
```

```
 -? : % -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?**? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
 ?*? : (R,%) -> %
 ?*? : (%,R) -> %
```

⟨*category XALG XAlgebra*⟩≡

```
)abbrev category XALG XAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This is the category of algebras over non-commutative rings.
++ It is used by constructors of non-commutative algebras such as
++ XPolynomialRing and XFreeAlgebra

XAlgebra(R: Ring): Category ==
  Join(Ring, BiModule(R,R)) with
    coerce: R -> %
      ++ \spad{coerce(r)} equals  \spad{r*1}.
    if R has CommutativeRing then Algebra(R)
```

⟨*XALG.dotabb*⟩≡

```
"XALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XALG" -> "BMODULE"
"XALG" -> "RING"
```

⟨*XALG.dotfull*⟩≡

```
"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
```

⟨*XALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

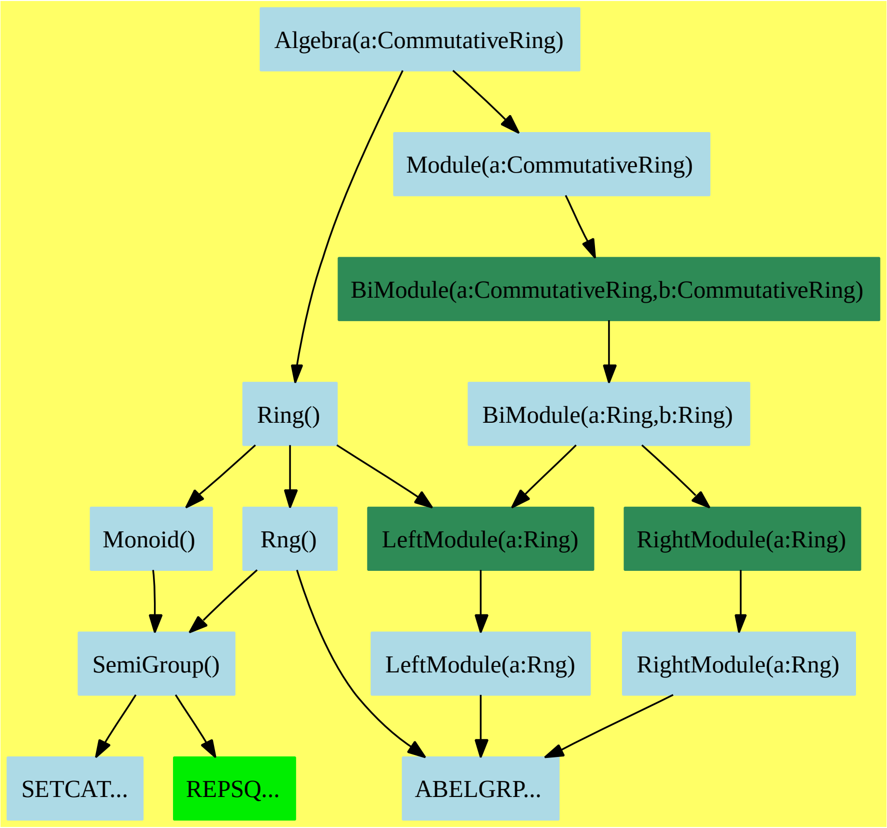
"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 11

Category Layer 10

11.0.152 Algebra (ALGEBRA)


Algebra(a:CommutativeRing)
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
Ring()
BiModule(a:Ring,b:Ring)
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
ABELGRP...

⟨*Algebra.input*⟩≡

```
)set break resume
)sys rm -f Algebra.output
)spool Algebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Algebra
--R Algebra R: CommutativeRing  is a category constructor
--R Abbreviation for Algebra is ALGEBRA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ALGEBRA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*Algebra.help*⟩≡
```
====================================================================
Algebra examples
====================================================================

See Also:
o )show Algebra
```

See:

⇒ “DivisionRing” (DIVRING) 12.0.161 on page 1116
⇒ “FiniteRankAlgebra” (FINRALG) 17.0.194 on page 1465
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “IntegralDomain” (INTDOM) 12.0.164 on page 1153
⇒ “MonogenicLinearOperator” (MLO) 12.0.165 on page 1160
⇒ “OctonionCategory” (OC) 12.0.166 on page 1167
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “QuaternionCategory” (QUATCAT) 12.0.167 on page 1180
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇐ “Module” (MODULE) 10.0.143 on page 966
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p966) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

⟨*category ALGEBRA Algebra*⟩≡

```
)abbrev category ALGEBRA Algebra
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of associative algebras (modules which are themselves rings).
++
++ Axioms\br
++ \tab{5}\spad{(b+c)::% = (b::%) + (c::%)}\br
++ \tab{5}\spad{(b*c)::% = (b::%) * (c::%)}\br
++ \tab{5}\spad{(1::R)::% = 1::%}\br
++ \tab{5}\spad{b*x = (b::%)*x}\br
++ \tab{5}\spad{r*(a*b) = (r*a)*b = a*(r*b)}

Algebra(R:CommutativeRing): Category ==
  Join(Ring, Module R) with
      coerce: R -> %
          ++ coerce(r) maps the ring element r to a member of the algebra.
 add
  coerce(x:R):% == x * 1$%
```

⟨*ALGEBRA.dotabb*⟩≡

```
"ALGEBRA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"ALGEBRA" -> "RING"
"ALGEBRA" -> "MODULE"
```

⟨*ALGEBRA.dotfull*⟩≡

```
"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Algebra(a:Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:Field)" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Integer)" -> "Algebra(a:CommutativeRing)"

"Algebra(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(IntegralDomain)" -> "Algebra(a:CommutativeRing)"
```

⟨*ALGEBRA.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

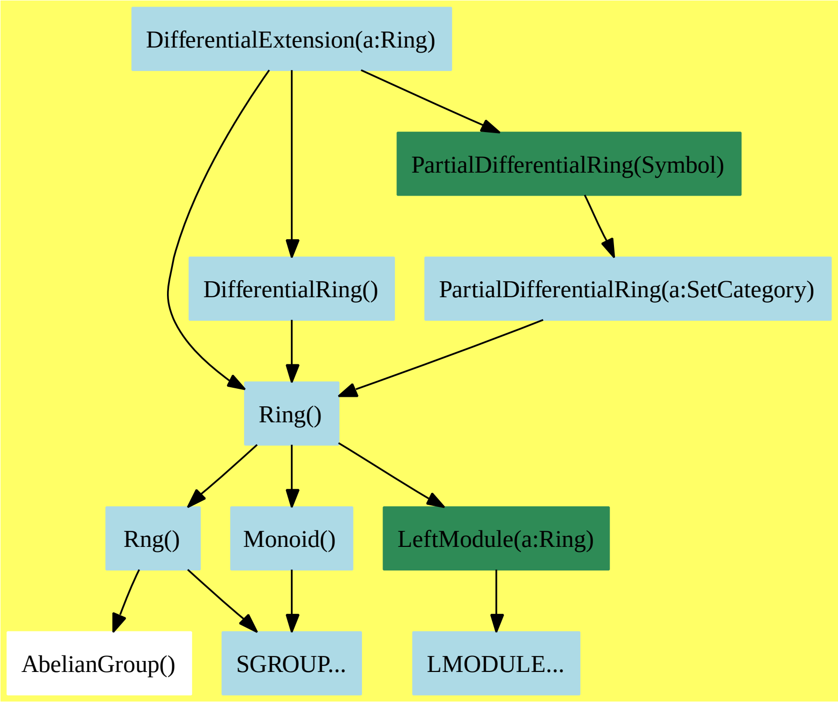
"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

11.0.153 DifferentialExtension (DIFEXT)


DifferentialExtension(a:Ring)
PartialDifferentialRing(Symbol)
DifferentialRing()
PartialDifferentialRing(a:SetCategory)
Ring()
Rng()
Monoid()
LeftModule(a:Ring)
AbelianGroup()
SGROUP...
LMODULE...

⟨*DifferentialExtension.input*⟩≡

```
)set break resume
)sys rm -f DifferentialExtension.output
)spool DifferentialExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialExtension
--R DifferentialExtension R: Ring  is a category constructor
--R Abbreviation for DifferentialExtension is DIFEXT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIFEXT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R D : % -> % if R has DIFRING           D : (%,(R -> R)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,(R -> R)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*DifferentialExtension.help*⟩≡

```
====================================================================
DifferentialExtension examples
====================================================================

See Also:
o )show DifferentialExtension
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.216 on page 1779
⇒ “DifferentialPolynomialCategory” (DPOLCAT) 17.0.192 on page 1439
⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “QuaternionCategory” (QUATCAT) 12.0.167 on page 1180
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.168 on page 1192
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇐ “PartialDifferentialRing” (PDRING) 10.0.145 on page 977
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

1
0
characteristic
coerce
D
differentiate
hash
latex
one?
recip
sample
subtractIfCan
zero?
?*?
?~=?
?**?
?+?
?-?
-?
?=?
?^?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,(R -> R)) -> %
```

These are implemented by this category:

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
```

```
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p938) DifferentialRing():

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
```

These exports come from (p977) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

⟨*category DIFEXT DifferentialExtension*⟩≡

```
)abbrev category DIFEXT DifferentialExtension
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ Differential extensions of a ring R.
++ Given a differentiation on R, extend it to a differentiation on %.

DifferentialExtension(R:Ring): Category == Ring with
    differentiate: (%, R -> R) -> %
       ++ differentiate(x, deriv) differentiates x extending
       ++ the derivation deriv on R.
    differentiate: (%, R -> R, NonNegativeInteger) -> %
       ++ differentiate(x, deriv, n) differentiate x n times
       ++ using a derivation which extends deriv on R.
    D: (%, R -> R) -> %
       ++ D(x, deriv) differentiates x extending
       ++ the derivation deriv on R.
    D: (%, R -> R, NonNegativeInteger) -> %
       ++ D(x, deriv, n) differentiate x n times
       ++ using a derivation which extends deriv on R.
    if R has DifferentialRing then DifferentialRing
    if R has PartialDifferentialRing(Symbol) then
             PartialDifferentialRing(Symbol)
  add
    differentiate(x:%, derivation: R -> R, n:NonNegativeInteger):% ==
      for i in 1..n repeat x := differentiate(x, derivation)
      x
    D(x:%, derivation: R -> R) == differentiate(x, derivation)
    D(x:%, derivation: R -> R, n:NonNegativeInteger) ==
            differentiate(x, derivation, n)

    if R has DifferentialRing then
      differentiate x == differentiate(x, differentiate$R)

    if R has PartialDifferentialRing Symbol then
      differentiate(x:%, v:Symbol):% ==
        differentiate(x, s +-> differentiate(s, v)$R)
```

⟨*DIFEXT.dotabb*⟩≡

```
"DIFEXT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DIFEXT" -> "RING"
"DIFEXT" -> "DIFRING"
"DIFEXT" -> "PDRING"
```

⟨*DIFEXT.dotfull*⟩≡
```
"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"DifferentialExtension(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(IntegralDomain)" ->
  "DifferentialExtension(a:Ring)"

"DifferentialExtension(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(CommutativeRing)" ->
  "DifferentialExtension(a:Ring)"
```

⟨*DIFEXT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

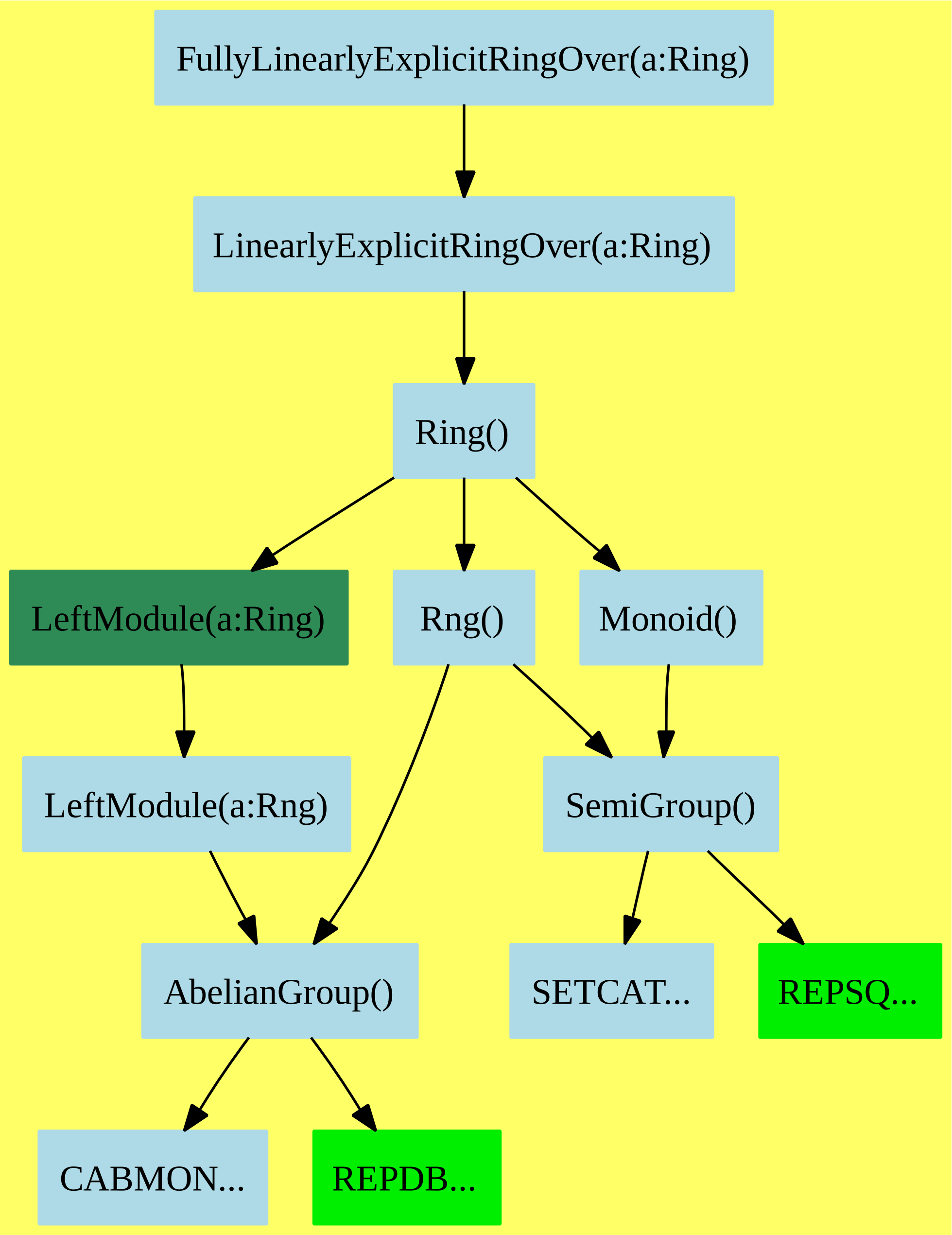
"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
}
```

11.0.154 FullyLinearlyExplicitRingOver (FLINEXP)


FullyLinearlyExplicitRingOver(a:Ring)
LinearlyExplicitRingOver(a:Ring)
Ring()
LeftModule(a:Ring)
Rng()
Monoid()
LeftModule(a:Rng)
SemiGroup()
AbelianGroup()
SETCAT...
REPSQ...
CABMON...
REPDB...

⟨*FullyLinearlyExplicitRingOver.input*⟩≡

```
)set break resume
```

```
)sys rm -f FullyLinearlyExplicitRingOver.output
)spool FullyLinearlyExplicitRingOver.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyLinearlyExplicitRingOver
--R FullyLinearlyExplicitRingOver R: Ring  is a category constructor
--R Abbreviation for FullyLinearlyExplicitRingOver is FLINEXP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLINEXP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : Matrix % -> Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*FullyLinearlyExplicitRingOver.help*⟩≡

```
====================================================================
FullyLinearlyExplicitRingOver examples
====================================================================

See Also:
o )show FullyLinearlyExplicitRingOver
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.216 on page 1779
⇒ “DirectProductCategory” (DIRPCAT) 12.0.160 on page 1103
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇒ “QuaternionCategory” (QUATCAT) 12.0.167 on page 1180
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.168 on page 1192
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.0.142 on page 961

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	reducedSystem	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are implemented by this category:

```
reducedSystem : Matrix % ->
    Matrix Integer if R has LINEXP INT
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
         if R has LINEXP INT
```

These exports come from (p961) LinearlyExplicitRingOver(a:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
sample : () -> %
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category FLINEXP FullyLinearlyExplicitRingOver*⟩≡

```
)abbrev category FLINEXP FullyLinearlyExplicitRingOver
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ S is \spadtype{FullyLinearlyExplicitRingOver R} means that S is a
++ \spadtype{LinearlyExplicitRingOver R} and, in addition, if R is a
++ \spadtype{LinearlyExplicitRingOver Integer}, then so is S

FullyLinearlyExplicitRingOver(R:Ring):Category ==
  LinearlyExplicitRingOver R with
    if (R has LinearlyExplicitRingOver Integer) then
            LinearlyExplicitRingOver Integer
 add
  if not(R is Integer) then
    if (R has LinearlyExplicitRingOver Integer) then
      reducedSystem(m:Matrix %):Matrix(Integer) ==
        reducedSystem(reducedSystem(m)@Matrix(R))

      reducedSystem(m:Matrix %, v:Vector %):
        Record(mat:Matrix(Integer), vec:Vector(Integer)) ==
          rec := reducedSystem(m, v)@Record(mat:Matrix R, vec:Vector R)
          reducedSystem(rec.mat, rec.vec)
```

⟨*FLINEXP.dotabb*⟩≡

```
"FLINEXP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FLINEXP" -> "LINEXP"
```

⟨*FLINEXP.dotfull*⟩≡

```
"FullyLinearlyExplicitRingOver(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:CommutativeRing)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(IntegralDomain)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"
```

⟨*FLINEXP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyLinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

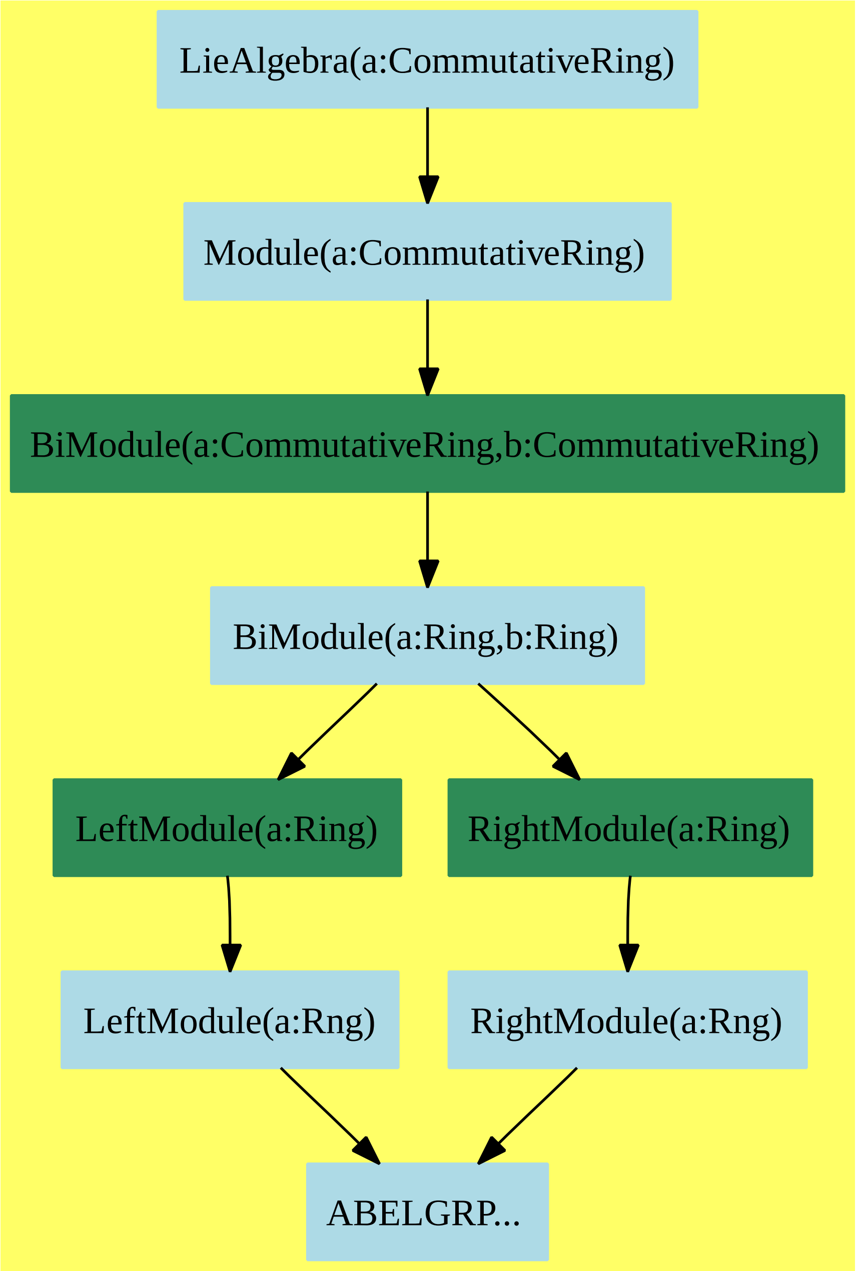
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.0.155 LieAlgebra (LIECAT)


LieAlgebra(a:CommutativeRing)
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*LieAlgebra.input*⟩≡

```
)set break resume
```

```
)sys rm -f LieAlgebra.output
)spool LieAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LieAlgebra
--R LieAlgebra R: CommutativeRing  is a category constructor
--R Abbreviation for LieAlgebra is LIECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LIECAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R construct : (%,%) -> %                hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*LieAlgebra.help*⟩≡

```
====================================================================
LieAlgebra examples
====================================================================

See Also:
o )show LieAlgebra
```

See:

⇒ "FreeLieAlgebra" (FLALG) 12.0.163 on page 1146
⇐ "Module" (MODULE) 10.0.143 on page 966

Exports:

0	coerce	construct	hash	latex
sample	subtractIfCan	zero?	?˜=?	?/?
?*?	?+?	?-?	-?	?=?

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
construct : (%,%) -> %
```

These are implemented by this category:

```
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p966) Module(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category LIECAT LieAlgebra*⟩≡

```
)abbrev category LIECAT LieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Keywords:
++ References:
++ Description:
++ The category of Lie Algebras.
++ It is used by the domains of non-commutative algebra,
++ LiePolynomial and XPBWPolynomial.

LieAlgebra(R: CommutativeRing): Category ==  Module(R) with
    construct:  ($,$) -> $
      ++ \axiom{construct(x,y)} returns the Lie bracket of \axiom{x}
      ++ and \axiom{y}.
    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
    if R has Field then
       "/"   :  ($,R) -> $
         ++ \axiom{x/r} returns the division of \axiom{x} by \axiom{r}.
  add
    if R has Field then x / r == inv(r)$R * x
```

⟨*LIECAT.dotabb*⟩≡

```
"LIECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LIECAT" -> "MODULE"
```

⟨*LIECAT.dotfull*⟩≡

```
"LieAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

⟨*LIECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

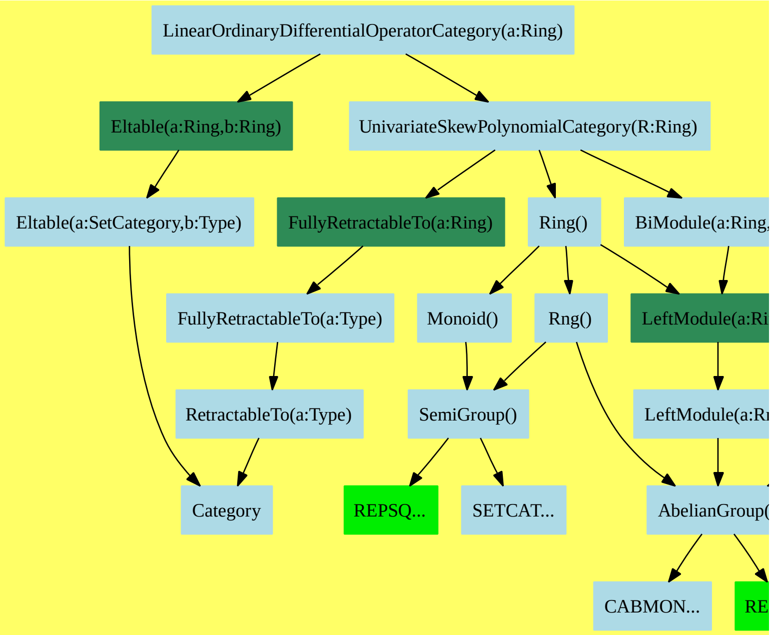
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.0.156 LinearOrdinaryDifferentialOperatorCategory (LODOCAT)


LinearOrdinaryDifferentialOperatorCategory(a:Ring)
Eltable(a:Ring,b:Ring)
UnivariateSkewPolynomialCategory(R:Ring)
Eltable(a:SetCategory,b:Type)
FullyRetractableTo(a:Ring)
Ring()
BiModule(a:Ring,
FullyRetractableTo(a:Type)
Monoid()
Rng()
LeftModule(a:Ri
RetractableTo(a:Type)
SemiGroup()
LeftModule(a:Ri
Category
REPSQ...
SETCAT...
AbelianGroup(
CABMON...
RE

⟨*LinearOrdinaryDifferentialOperatorCategory.input*⟩≡

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperatorCategory.output
)spool LinearOrdinaryDifferentialOperatorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LinearOrdinaryDifferentialOperatorCategory
--R LinearOrdinaryDifferentialOperatorCategory A: Ring  is a category constructor
--R Abbreviation for LinearOrdinaryDifferentialOperatorCategory is LODOCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LODOCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (A,%) -> %                      ?*? : (%,A) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : () -> %                           1 : () -> %
```

```
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R adjoint : % -> %                      apply : (%,A,A) -> A
--R coefficients : % -> List A            coerce : A -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      ?.? : (%,A) -> A
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> A           one? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> A                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> A
--R coerce : Fraction Integer -> % if A has RETRACT FRAC INT
--R content : % -> A if A has GCDDOM
--R directSum : (%,%) -> % if A has FIELD
--R exquo : (%,A) -> Union(%,"failed") if A has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if A has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
--R leftGcd : (%,%) -> % if A has FIELD
--R leftLcm : (%,%) -> % if A has FIELD
--R leftQuotient : (%,%) -> % if A has FIELD
--R leftRemainder : (%,%) -> % if A has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if A has INTDOM
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if A has INTDOM
--R monomial : (A,NonNegativeInteger) -> %
--R primitivePart : % -> % if A has GCDDOM
--R retract : % -> Fraction Integer if A has RETRACT FRAC INT
--R retract : % -> Integer if A has RETRACT INT
--R retractIfCan : % -> Union(A,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if A has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if A has RETRACT INT
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if A has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
--R rightGcd : (%,%) -> % if A has FIELD
--R rightLcm : (%,%) -> % if A has FIELD
--R rightQuotient : (%,%) -> % if A has FIELD
--R rightRemainder : (%,%) -> % if A has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R symmetricPower : (%,NonNegativeInteger) -> % if A has FIELD
--R symmetricProduct : (%,%) -> % if A has FIELD
```

```
--R symmetricSquare : % -> % if A has FIELD
--R
--E 1

)spool
)lisp (bye)
```

⟨*LinearOrdinaryDifferentialOperatorCategory.help*⟩≡

```
====================================================================
LinearOrdinaryDifferentialOperatorCategory examples
====================================================================

See Also:
o )show LinearOrdinaryDifferentialOperatorCategory
```

See:

⇐ “Eltable” (ELTAB) 2.0.10 on page 32
⇐ “UnivariateSkewPolynomialCategory” (OREPCAT) 10.0.150 on page 1024

Exports:

0
1
adjoint
apply
characteristic
coefficient
coefficients
coerce
content
D
degree
directSum
exquo
hash
latex
leadingCoefficient
leftDivide
leftExactQuotient
leftExtendedGcd
leftGcd
leftLcm
leftQuotient
leftRemainder
minimumDegree
monicLeftDivide
monicRightDivide
monomial
one?
primitivePart
recip
reductum
retract
retractIfCan
rightDivide
rightExactQuotient
rightExtendedGcd
rightGcd
rightLcm
rightQuotient
rightRemainder
sample
subtractIfCan
symmetricPower
symmetricProduct
symmetricSquare
zero?
?^?
?.?
?~=?
?*?
?**?
?+?
?-?
-?
?=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
directSum : (%,%) -> % if A has FIELD
symmetricPower : (%,NonNegativeInteger) -> % if A has FIELD
symmetricProduct : (%,%) -> % if A has FIELD
```

These are implemented by this category:

```
adjoint : % -> %
D : () -> %
symmetricSquare : % -> % if A has FIELD
```

These exports come from (p1024) UnivariateSkewPolynomialCategory(A:Ring):

```
0 : () -> %
1 : () -> %
apply : (%,A,A) -> A
characteristic : () -> NonNegativeInteger
coefficient : (%,NonNegativeInteger) -> A
coefficients : % -> List A
coerce : % -> OutputForm
coerce : Integer -> %
coerce : A -> %
coerce : Fraction Integer -> % if A has RETRACT FRAC INT
content : % -> A if A has GCDDOM
degree : % -> NonNegativeInteger
exquo : (%,A) -> Union(%,"failed") if A has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> A
leftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
leftExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %)
   if A has FIELD
leftGcd : (%,%) -> % if A has FIELD
leftLcm : (%,%) -> % if A has FIELD
leftQuotient : (%,%) -> % if A has FIELD
leftRemainder : (%,%) -> % if A has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monomial : (A,NonNegativeInteger) -> %
one? : % -> Boolean
primitivePart : % -> % if A has GCDDOM
recip : % -> Union(%,"failed")
reductum : % -> %
retract : % -> A
retract : % -> Fraction Integer if A has RETRACT FRAC INT
retract : % -> Integer if A has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
   if A has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
   if A has RETRACT INT
retractIfCan : % -> Union(A,"failed")
rightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
rightExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
rightExtendedGcd :
  (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
```

```
rightGcd : (%,%) -> % if A has FIELD
rightLcm : (%,%) -> % if A has FIELD
rightQuotient : (%,%) -> % if A has FIELD
rightRemainder : (%,%) -> % if A has FIELD
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?*? : (%,A) -> %
?*? : (A,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p32) Eltable(A:Ring,A:Ring):

```
?.? : (%,A) -> A
```

⟨*category LODOCAT LinearOrdinaryDifferentialOperatorCategory*⟩≡

```
)abbrev category LODOCAT LinearOrdinaryDifferentialOperatorCategory
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 15 April 1994
++ Keywords: differential operator
++ Description:
++ LinearOrdinaryDifferentialOperatorCategory is the category
++ of differential operators with coefficients in a ring A with a given
++ derivation.
++
++ Multiplication of operators corresponds to functional composition:\br
++ (L1 * L2).(f) = L1 L2 f

LinearOrdinaryDifferentialOperatorCategory(A:Ring): Category ==
  Join(UnivariateSkewPolynomialCategory A, Eltable(A, A)) with
        D: () -> %
            ++ D() provides the operator corresponding to a derivation
            ++ in the ring \spad{A}.
        adjoint: % -> %
            ++ adjoint(a) returns the adjoint operator of a.
        if A has Field then
          symmetricProduct: (%, %) -> %
            ++ symmetricProduct(a,b) computes an operator \spad{c} of
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the products of a solution of \spad{a} by
            ++ a solution of \spad{b}.
          symmetricPower  : (%, NonNegativeInteger) -> %
            ++ symmetricPower(a,n) computes an operator \spad{c} of
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the products of \spad{n} solutions
            ++ of \spad{a}.
          symmetricSquare : % -> %
            ++ symmetricSquare(a) computes \spad{symmetricProduct(a,a)}
            ++ using a more efficient method.
          directSum: (%, %) -> %
            ++ directSum(a,b) computes an operator \spad{c} of
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the sums of a solution of \spad{a} by
            ++ a solution of \spad{b}.
   add
        m1monom: NonNegativeInteger -> %

        D() == monomial(1, 1)

        m1monom n ==
          a:A := (odd? n => -1; 1)
          monomial(a, n)

        adjoint a ==
          ans:% := 0
          while a ^= 0 repeat
            ans := ans + m1monom(degree a) * leadingCoefficient(a)::%
            a   := reductum a
          ans

        if A has Field then symmetricSquare l == symmetricPower(l, 2)
```

⟨*LODOCAT.dotabb*⟩≡

```
  "LODOCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
  "LODOCAT" -> "ELTAB"
  "LODOCAT" -> "OREPCAT"
```

⟨*LODOCAT.dotfull*⟩≡

```
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "UnivariateSkewPolynomialCategory(R:Ring)"
```

⟨*LODOCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearOrdinaryDifferentialOperatorCategory(a:Ring)" [color=lightblue];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "UnivariateSkewPolynomialCategory(R:Ring)"

"Eltable(a:Ring,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:Ring,b:Ring)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

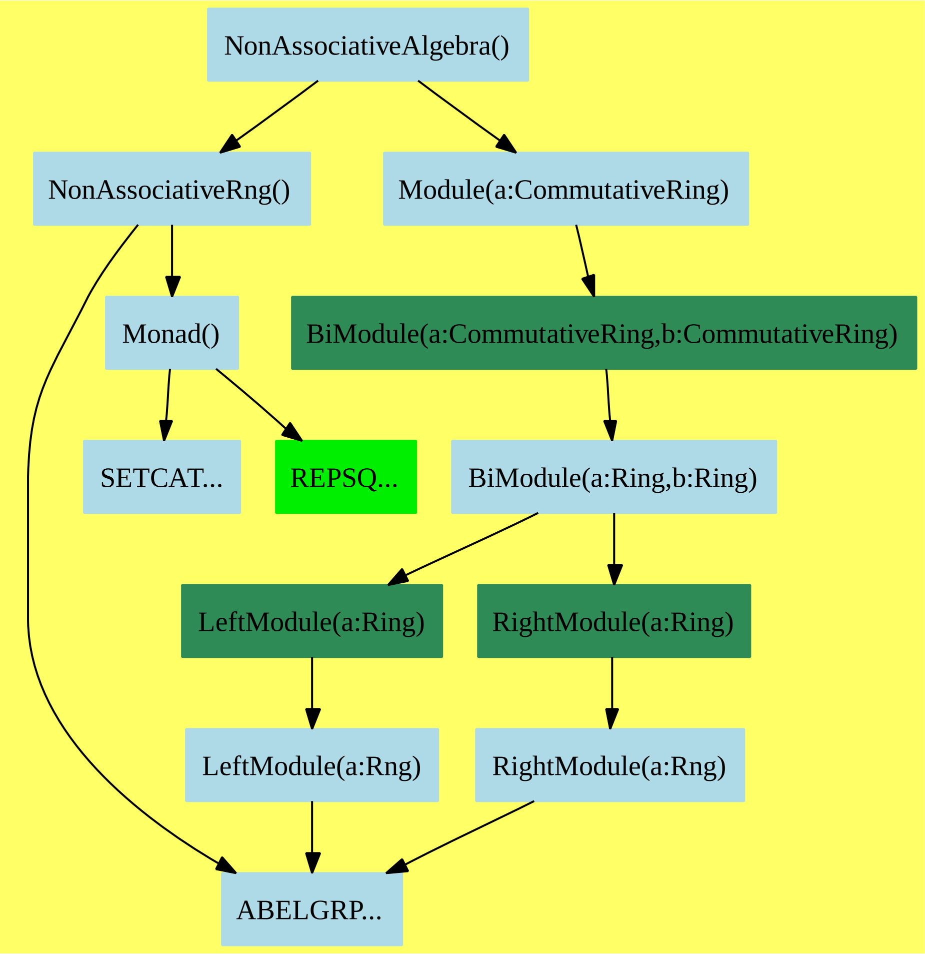
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.0.157 NonAssociativeAlgebra (NAALG)


NonAssociativeAlgebra()
NonAssociativeRng()
Module(a:CommutativeRing)
Monad()
BiModule(a:CommutativeRing,b:CommutativeRing)
SETCAT...
REPSQ...
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*NonAssociativeAlgebra.input*⟩≡

```
)set break resume
)sys rm -f NonAssociativeAlgebra.output
)spool NonAssociativeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonAssociativeAlgebra
--R NonAssociativeAlgebra R: CommutativeRing  is a category constructor
--R Abbreviation for NonAssociativeAlgebra is NAALG
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NAALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %             coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R leftPower : (%,PositiveInteger) -> %
--R plenaryPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*NonAssociativeAlgebra.help*⟩≡

```
====================================================================
NonAssociativeAlgebra examples
====================================================================

See Also:
o )show NonAssociativeAlgebra
```

See:

⇒ "FiniteRankNonAssociativeAlgebra" (FINAALG) 12.0.162 on page 1122
⇐ "Module" (MODULE) 10.0.143 on page 966
⇐ "NonAssociativeRng" (NARNG) 8.0.116 on page 749

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	plenaryPower	rightPower
sample	subtractIfCan	zero?	?~=?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
plenaryPower : (%,PositiveInteger) -> %
```

These exports come from (p749) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p966) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

⟨*category NAALG NonAssociativeAlgebra*⟩≡

```
)abbrev category NAALG NonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: +, -, *, **
++ Related Constructors: Algebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra
++ Reference:
++ R.D. Schafer: An Introduction to Nonassociative Algebras
++ Academic Press, New York, 1966
++ Description:
++ NonAssociativeAlgebra is the category of non associative algebras
++ (modules which are themselves non associative rngs).\br
++
++ Axioms\br
++ \tab{5}r*(a*b) = (r*a)*b = a*(r*b)

NonAssociativeAlgebra(R:CommutativeRing): Category == _
  Join(NonAssociativeRng, Module R) with
    plenaryPower : (%,PositiveInteger) -> %
      ++ plenaryPower(a,n) is recursively defined to be
      ++ \spad{plenaryPower(a,n-1)*plenaryPower(a,n-1)} for \spad{n>1}
      ++ and \spad{a} for \spad{n=1}.
  add
    plenaryPower(a,n) ==
--      one? n => a
      ( n = 1 ) => a
      n1 : PositiveInteger := (n-1)::NonNegativeInteger::PositiveInteger
      plenaryPower(a,n1) * plenaryPower(a,n1)
```

⟨*NAALG.dotabb*⟩≡

```
"NAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NAALG" -> "NARNG"
"NAALG" -> "MODULE"
```

⟨*NAALG.dotfull*⟩≡

```
"NonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

⟨*NAALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

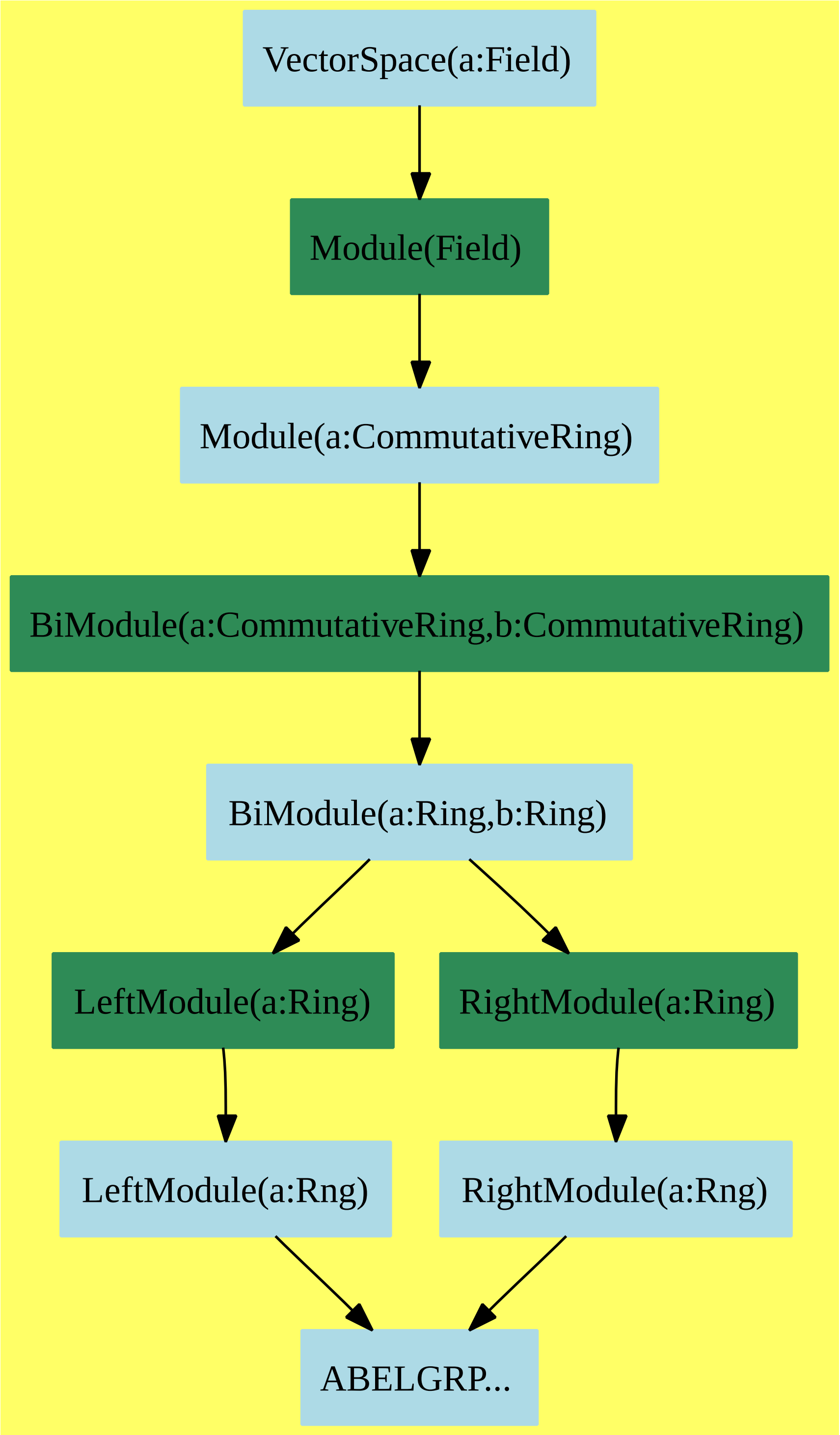
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

11.0.158 VectorSpace (VSPACE)


VectorSpace(a:Field)
Module(Field)
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*VectorSpace.input*⟩≡
```
)set break resume
)sys rm -f VectorSpace.output
)spool VectorSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show VectorSpace
--R VectorSpace S: Field  is a category constructor
--R Abbreviation for VectorSpace is VSPACE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for VSPACE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,S) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              dimension : () -> CardinalNumber
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*VectorSpace.help*⟩≡
```
====================================================================
VectorSpace examples
====================================================================

See Also:
o )show VectorSpace
```

See:

⇒ "ExtensionField" (XF) 18.0.206 on page 1662
⇐ "Module" (MODULE) 10.0.143 on page 966

Exports:

0	coerce	dimension	hash	latex
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?/?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
dimension : () -> CardinalNumber
```

These are implemented by this category:

```
?/? : (%,S) -> %
```

These exports come from (p966) Module():

```
?*? : (%,S) -> %
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (S,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category VSPACE VectorSpace*⟩≡

```
)abbrev category VSPACE VectorSpace
++ Author:
++ Date Created:
```

```
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Vector Spaces (not necessarily finite dimensional) over a field.

VectorSpace(S:Field): Category ==  Module(S) with
    "/"      : (%, S) -> %
      ++ x/y divides the vector x by the scalar y.
    dimension: () -> CardinalNumber
      ++ dimension() returns the dimensionality of the vector space.
  add
    (v:% / s:S):% == inv(s) * v
```

⟨*VSPACE.dotabb*⟩≡

```
"VSPACE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
"VSPACE" -> "MODULE"
```

⟨*VSPACE.dotfull*⟩≡

```
"VectorSpace(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
"VectorSpace(a:Field)" -> "Module(Field)"
```

⟨*VSPACE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "Module(Field)"

"Module(Field)" [color=seagreen];
"Module(Field)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

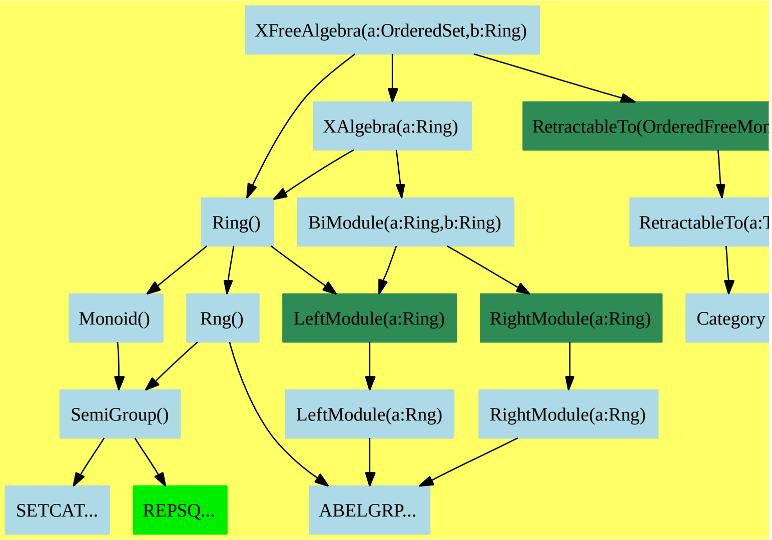
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.0.159 XFreeAlgebra (XFALG)


XFreeAlgebra(a:OrderedSet,b:Ring)
XAlgebra(a:Ring)
RetractableTo(OrderedFreeMon
Ring()
BiModule(a:Ring,b:Ring)
RetractableTo(a:T
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
Category
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
ABELGRP...

⟨*XFreeAlgebra.input*⟩≡

```
)set break resume
)sys rm -f XFreeAlgebra.output
)spool XFreeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XFreeAlgebra
--R XFreeAlgebra(vl: OrderedSet,R: Ring)  is a category constructor
--R Abbreviation for XFreeAlgebra is XFALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XFALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (vl,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coef : (%,%) -> R                     coerce : vl -> %
--R coerce : R -> %                       coerce : Integer -> %
```

```
--R coerce : % -> OutputForm              constant : % -> R
--R constant? : % -> Boolean              hash : % -> SingleInteger
--R latex : % -> String                   lquo : (%,%) -> %
--R lquo : (%,vl) -> %                    map : ((R -> R),%) -> %
--R mirror : % -> %                       monomial? : % -> Boolean
--R one? : % -> Boolean                   quasiRegular : % -> %
--R quasiRegular? : % -> Boolean          recip : % -> Union(%,"failed")
--R rquo : (%,%) -> %                     rquo : (%,vl) -> %
--R sample : () -> %                      varList : % -> List vl
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid vl) -> R
--R coerce : OrderedFreeMonoid vl -> %
--R lquo : (%,OrderedFreeMonoid vl) -> %
--R mindeg : % -> OrderedFreeMonoid vl
--R mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R monom : (OrderedFreeMonoid vl,R) -> %
--R retract : % -> OrderedFreeMonoid vl
--R retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
--R rquo : (%,OrderedFreeMonoid vl) -> %
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R sh : (%,%) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*XFreeAlgebra.help*⟩≡

```
====================================================================
XFreeAlgebra examples
====================================================================

See Also:
o )show XFreeAlgebra
```

See:

⇒ "XPolynomialsCat" (XPOLYC) 12.0.169 on page 1205
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62
⇐ "Ring" (RING) 9.0.129 on page 854
⇐ "XAlgebra" (XALG) 10.0.151 on page 1038

Exports:

0	1	characteristic	coef	coerce
constant	constant?	hash	latex	lquo
map	mindeg	mindegTerm	mirror	monom
monomial?	one?	quasiRegular	quasiRegular?	recip
retract	retractIfCan	rquo	sample	sh
subtractIfCan	varList	zero?	?*?	?**?
?+?	?-?	-?	?=?	?^?
?~=?				

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
lquo : (%,vl) -> %
lquo : (%,%) -> %
lquo : (%,OrderedFreeMonoid vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
```

```
monomial? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
varList : % -> List vl
?*? : (vl,%) -> %
?*? : (%,R) -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p1038) XAlgebra(a:Ring):

```
coerce : R -> %
?*? : (R,%) -> %
```

These exports come from (p62) RetractableTo(WORD)
where WORD:OrderedFreeMonoid(OrderedSet))

```
coerce : OrderedFreeMonoid vl -> %
```

```
retract : % -> OrderedFreeMonoid vl
retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
```

⟨category XFALG XFreeAlgebra⟩≡

```
)abbrev category XFALG XFreeAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category specifies opeations for  polynomials
++ and formal series with non-commutative variables.

XFreeAlgebra(vl:OrderedSet,R:Ring):Category == Catdef where
   WORD   ==> OrderedFreeMonoid(vl)          -- monoide libre
   NNI    ==> NonNegativeInteger
   I      ==> Integer
   TERM   ==> Record(k: WORD, c: R)

   Catdef == Join(Ring, XAlgebra(R), RetractableTo WORD)
     with
       "*": (vl,%) -> %
         ++ \spad{v * x} returns the product of a variable \spad{x}
         ++ by \spad{x}.
       "*": (%, R) -> %
         ++ \spad{x * r} returns the product of \spad{x} by \spad{r}.
         ++ Usefull if \spad{R} is a non-commutative Ring.
       mindeg: % -> WORD
         ++ \spad{mindeg(x)} returns the little word which appears
         ++ in \spad{x}. Error if \spad{x=0}.
       mindegTerm: % -> TERM
         ++ \spad{mindegTerm(x)} returns the term whose word is
         ++ \spad{mindeg(x)}.
       coef  : (%,WORD) -> R
         ++ \spad{coef(x,w)} returns the coefficient of the word \spad{w}
         ++ in \spad{x}.
       coef  : (%,%) -> R
         ++ \spad{coef(x,y)} returns scalar product of \spad{x} by \spad{y},
         ++ the set of words being regarded as an orthogonal basis.
       lquo  : (%,vl) -> %
```

```
  ++ \spad{lquo(x,v)} returns the left simplification of \spad{x}
  ++ by the variable \spad{v}.
lquo  : (%,WORD) -> %
  ++ \spad{lquo(x,w)} returns the left simplification of \spad{x}
  ++ by the word \spad{w}.
lquo  : (%,%) -> %
  ++ \spad{lquo(x,y)} returns the left simplification of \spad{x}
  ++ by \spad{y}.
rquo  : (%,vl) -> %
  ++ \spad{rquo(x,v)} returns the right simplification of \spad{x}
  ++ by the variable \spad{v}.
rquo  : (%,WORD) -> %
  ++ \spad{rquo(x,w)} returns the right simplification of \spad{x}
  ++ by \spad{w}.
rquo  : (%,%) -> %
  ++ \spad{rquo(x,y)} returns the right simplification of \spad{x}
  ++ by \spad{y}.
monom : (WORD , R) -> %
  ++ \spad{monom(w,r)} returns the product of the word \spad{w}
  ++ by the coefficient \spad{r}.
monomial? : % -> Boolean
  ++ \spad{monomial?(x)} returns true if \spad{x} is a monomial
mirror: % -> %
  ++ \spad{mirror(x)} returns \spad{Sum(r_i mirror(w_i))} if
  ++ \spad{x} writes \spad{Sum(r_i w_i)}.
coerce : vl -> %
  ++ \spad{coerce(v)} returns \spad{v}.
constant?:% -> Boolean
  ++ \spad{constant?(x)} returns true if \spad{x} is constant.
constant: % -> R
  ++ \spad{constant(x)} returns the constant term of \spad{x}.
quasiRegular? : % -> Boolean
  ++ \spad{quasiRegular?(x)} return true if \spad{constant(x)} is zero
quasiRegular : % -> %
  ++ \spad{quasiRegular(x)} return \spad{x} minus its constant term.
if R has CommutativeRing then
   sh :(%,%) -> %
      ++ \spad{sh(x,y)} returns the shuffle-product of \spad{x}
      ++ by \spad{y}.
      ++ This multiplication is associative and commutative.
   sh :(%,NNI) -> %
      ++ \spad{sh(x,n)} returns the shuffle power of \spad{x} to
      ++ the \spad{n}.
map   : (R -> R, %) -> %
  ++ \spad{map(fn,x)} returns \spad{Sum(fn(r_i) w_i)} if \spad{x}
  ++ writes \spad{Sum(r_i w_i)}.
```

```
        varList: % -> List vl
          ++ \spad{varList(x)} returns the list of variables which
          ++ appear in \spad{x}.

      -- Attributs
        if R has noZeroDivisors then noZeroDivisors
```

⟨*XFALG.dotabb*⟩≡

```
"XFALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFALG" -> "RETRACT"
"XFALG" -> "RING"
"XFALG" -> "XALG"
```

⟨*XFALG.dotfull*⟩≡

```
"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"
```

⟨XFALG.dotpic⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

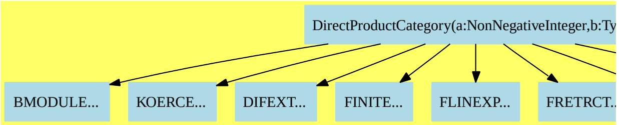
"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 12

Category Layer 11

12.0.160 DirectProductCategory (DIRPCAT)


DirectProductCategory(a:NonNegativeInteger,b:Ty
BMODULE...
KOERCE...
DIFEXT...
FINITE...
FLINEXP...
FRETRCT..

⟨*DirectProductCategory.input*⟩≡

```
)set break resume
)sys rm -f DirectProductCategory.output
)spool DirectProductCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductCategory
--R DirectProductCategory(dim: NonNegativeInteger,R: Type)  is a category constructor
--R Abbreviation for DirectProductCategory is DIRPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIRPCAT
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has RING             1 : () -> % if R has MONOID
--R 0 : () -> % if R has CABMON           coerce : % -> Vector R
--R copy : % -> %                         directProduct : Vector R -> %
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List R                 eq? : (%,%) -> Boolean
```

```
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R map : ((R -> R),%) -> %               qelt : (%,Integer) -> R
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if R has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if R has CABMON
--R ?*? : (R,%) -> % if R has RING
--R ?*? : (%,R) -> % if R has RING
--R ?*? : (%,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has RING
--R ?**? : (%,PositiveInteger) -> % if R has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if R has MONOID
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has RING
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?<=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?>=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R D : (%,(R -> R)) -> % if R has RING
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if and(has(R,PartialDifferen
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing S
--R D : (%,List Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,R
--R D : (%,Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
--R D : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Ring))
--R D : % -> % if and(has(R,DifferentialRing),has(R,Ring))
--R ?^? : (%,PositiveInteger) -> % if R has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if R has MONOID
--R abs : % -> % if R has ORDRING
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if R has RING
--R coerce : R -> % if R has SETCAT
--R coerce : Fraction Integer -> % if and(has(R,RetractableTo Fraction Integer),h
--R coerce : Integer -> % if and(has(R,RetractableTo Integer),has(R,SetCategory))
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(R -> R)) -> % if R has RING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if and(has(R,Par
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDiffere
--R differentiate : (%,List Symbol) -> % if and(has(R,PartialDifferentialRing Sym
--R differentiate : (%,Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),ha
--R differentiate : % -> % if and(has(R,DifferentialRing),has(R,Ring))
--R dimension : () -> CardinalNumber if R has FIELD
--R dot : (%,%) -> R if R has RING
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R index : PositiveInteger -> % if R has FINITE
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if R has FINITE
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has ORDRING or R has OAMONS
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R min : (%,%) -> % if R has ORDRING or R has OAMONS
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has MONOID
--R parts : % -> List R if $ has finiteAggregate
--R positive? : % -> Boolean if R has ORDRING
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R random : () -> % if R has FINITE
--R recip : % -> Union(%,"failed") if R has MONOID
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if R has RIN
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if and(has(R,LinearlyExplicitRingOver Intege
--R retract : % -> R if R has SETCAT
--R retract : % -> Fraction Integer if and(has(R,RetractableTo Fraction Integer),has(R,SetCa
--R retract : % -> Integer if and(has(R,RetractableTo Integer),has(R,SetCategory))
--R retractIfCan : % -> Union(R,"failed") if R has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if and(has(R,RetractableTo Fractio
--R retractIfCan : % -> Union(Integer,"failed") if and(has(R,RetractableTo Integer),has(R,Se
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R sign : % -> Integer if R has ORDRING
--R size : () -> NonNegativeInteger if R has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
--R sup : (%,%) -> % if R has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if R has RING
--R zero? : % -> Boolean if R has CABMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

⟨*DirectProductCategory.help*⟩≡

```
====================================================================
DirectProductCategory examples
====================================================================

See Also:
o )show DirectProductCategory
```

See:

⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “CoercibleTo” (KOERCE) 2.0.6 on page 19
⇐ “DifferentialExtension” (DIFEXT) 11.0.153 on page 1053
⇐ “IndexedAggregate” (IXAGG) 5.0.74 on page 356
⇐ “Finite” (FINITE) 4.0.46 on page 188
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.154 on page 1060
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100
⇐ “OrderedRing” (ORDRING) 10.0.144 on page 971
⇐ “OrderedAbelianMonoidSup” (OAMONS) 9.0.127 on page 841

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entry?	entries	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
?˜=?	-?	?.?	#?	?*?
?**?	?+?	?-?	?/? ?<?	
?<=?	?=?	?>?	?>=?	?ˆ?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- if #2 has commutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation ” $*$ ” : $(D, D) - > D$ which is commutative.
- if #2 has unitsKnown then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- if #2 has Ring then rightUnitary where **leftUnitary** is true if $1 * x = x$ for all x.
- if #2 has Ring then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.

- **nil**

These are directly exported but not implemented:

```
directProduct : Vector R -> %
dot : (%,%) -> R if R has RING
unitVector : PositiveInteger -> % if R has RING
?*? : (R,%) -> % if R has MONOID
?*? : (%,R) -> % if R has MONOID
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger if R has RING
coerce : Integer -> %
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
  or R has RING
differentiate : (%,(R -> R)) -> % if R has RING
dimension : () -> CardinalNumber if R has FIELD
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem :
   (Matrix %,Vector %) ->
      Record(mat: Matrix R,vec: Vector R)
         if R has RING
size : () -> NonNegativeInteger if R has FINITE
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p356) IndexedAggregate(a:SetCategory,R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
copy : % -> %
count : (R,%) -> NonNegativeInteger
  if R has SETCAT
  and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean
  if $ has finiteAggregate
  and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,R,R) -> %
```

```
  if R has EVALAB R
  and R has SETCAT
eval : (%,Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,List Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean
  if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
less? : (%,NonNegativeInteger) -> Boolean
latex : % -> String if R has SETCAT
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean
  if R has SETCAT
  and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?=? : (%,%) -> Boolean if R has SETCAT
?~=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p19) CoercibleTo(Vector(R:Type)):

```
coerce : % -> Vector R
```

These exports come from (p100) FullyRetractableTo(R:SetCategory):

```
coerce : R -> % if R has SETCAT
coerce : Fraction Integer -> %
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
retract : % -> Integer
  if and(has(R,RetractableTo Integer),
        has(R,SetCategory))
retract : % -> R if R has SETCAT
retract : % -> Fraction Integer
  if and(has(R,RetractableTo Fraction Integer),
        has(R,SetCategory))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(R,RetractableTo Integer),
        has(R,SetCategory))
retractIfCan : % -> Union(R,"failed") if R has SETCAT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(R,RetractableTo Fraction Integer),
        has(R,SetCategory))
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
0 : () -> % if R has CABMON
subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
zero? : % -> Boolean if R has CABMON
?+? : (%,%) -> % if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has CABMON
?*? : (Integer,%) -> % if R has RING
?-? : (%,%) -> % if R has RING
-? : % -> % if R has RING
```

These exports come from (p1053) DifferentialExtension(R:Ring):

```
1 : () -> % if R has RING
D : (%,(R -> R)) -> % if R has RING
D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
D : % -> % if and(has(R,DifferentialRing),has(R,Ring))
D : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : % -> %
```

```
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
one? : % -> Boolean if R has RING
recip : % -> Union(%,"failed") if R has RING
?*? : (%,%) -> % if R has RING
?**? : (%,PositiveInteger) -> % if R has RING
?**? : (%,NonNegativeInteger) -> % if R has RING
?^? : (%,PositiveInteger) -> % if R has RING
?^? : (%,NonNegativeInteger) -> % if R has RING
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p188) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
```

These exports come from (p971) OrderedRing():

```
abs : % -> % if R has ORDRING
max : (%,%) -> % if R has ORDRING or R has OAMONS
min : (%,%) -> % if R has ORDRING or R has OAMONS
negative? : % -> Boolean if R has ORDRING
positive? : % -> Boolean if R has ORDRING
sign : % -> Integer if R has ORDRING
?<? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?<=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
```

These exports come from (p841) OrderedAbelianMonoidSup():

```
sup : (%,%) -> % if R has OAMONS
```

⟨*category DIRPCAT DirectProductCategory*⟩≡

```
)abbrev category DIRPCAT DirectProductCategory
-- all direct product category domains must be compiled
-- without subsumption, set SourceLevelSubset to EQUAL
```

```
--)bo $noSubsumption := true

--% DirectProductCategory

++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: DirectProduct
++ Also See: VectorCategory
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category represents a finite cartesian product of a given type.
++ Many categorical properties are preserved under this construction.

DirectProductCategory(dim:NonNegativeInteger, R:Type): Category ==
  Join(IndexedAggregate(Integer, R), CoercibleTo Vector R) with
         finiteAggregate
           ++ attribute to indicate an aggregate of finite size
         directProduct: Vector R -> %
           ++ directProduct(v) converts the vector v to become
           ++ a direct product. Error: if the length of v is
           ++ different from dim.
         if R has SetCategory then FullyRetractableTo R
         if R has Ring then
           BiModule(R, R)
           DifferentialExtension R
           FullyLinearlyExplicitRingOver R
           unitVector: PositiveInteger -> %
             ++ unitVector(n) produces a vector with 1 in position n and
             ++ zero elsewhere.
           dot: (%, %) -> R
             ++ dot(x,y) computes the inner product of the vectors x and y.
         if R has AbelianSemiGroup then AbelianSemiGroup
         if R has CancellationAbelianMonoid then CancellationAbelianMonoid
         if R has Monoid then
            Monoid
           _* : (R, %) -> %
             ++ r * y multiplies the element r times each component of the
             ++ vector y.
           _* : (%, R) -> %
             ++ y*r multiplies each component of the vector y by the element r.
         if R has Finite then Finite
         if R has CommutativeRing then
```

```
          Algebra R
          CommutativeRing
         if R has unitsKnown then unitsKnown
         if R has OrderedRing then OrderedRing
         if R has OrderedAbelianMonoidSup then OrderedAbelianMonoidSup
         if R has Field then VectorSpace R
add
     if R has Ring then
       equation2R: Vector % -> Matrix R

       coerce(n:Integer):%          == n::R::%

       characteristic()             == characteristic()$R

       differentiate(z:%, d:R -> R) == map(d, z)

       equation2R v ==
         ans:Matrix(R) := new(dim, #v, 0)
         for i in minRowIndex ans .. maxRowIndex ans repeat
           for j in minColIndex ans .. maxColIndex ans repeat
             qsetelt_!(ans, i, j, qelt(qelt(v, j), i))
         ans

       reducedSystem(m:Matrix %):Matrix(R) ==
         empty? m => new(0, 0, 0)
         reduce(vertConcat, [equation2R row(m, i)
                for i in minRowIndex m .. maxRowIndex m])$List(Matrix R)

       reducedSystem(m:Matrix %, v:Vector %):
         Record(mat:Matrix R, vec:Vector R) ==
           vh:Vector(R) :=
             empty? v => empty()
             rh := reducedSystem(v::Matrix %)@Matrix(R)
             column(rh, minColIndex rh)
           [reducedSystem(m)@Matrix(R), vh]

     if R has Finite then size == size$R ** dim

     if R has Field then
       x / b       == x * inv b

       dimension() == dim::CardinalNumber
```

⟨*DIRPCAT.dotabb*⟩≡

```
"DIRPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
"DIRPCAT" -> "IXAGG"
"DIRPCAT" -> "KOERCE"
"DIRPCAT" -> "FRETRCT"
"DIRPCAT" -> "BMODULE"
"DIRPCAT" -> "DIFEXT"
"DIRPCAT" -> "FLINEXP"
"DIRPCAT" -> "FINITE"
"DIRPCAT" -> "ORDRING"
"DIRPCAT" -> "OAMONS"
```

⟨*DIRPCAT.dotfull*⟩≡

```
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "IndexedAggregate(a:SetCategory,b:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "CoercibleTo(a:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyRetractableTo(a:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "BiModule(a:Ring,b:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "DifferentialExtension(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "Finite()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedRing()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedAbelianMonoidSup()"
```

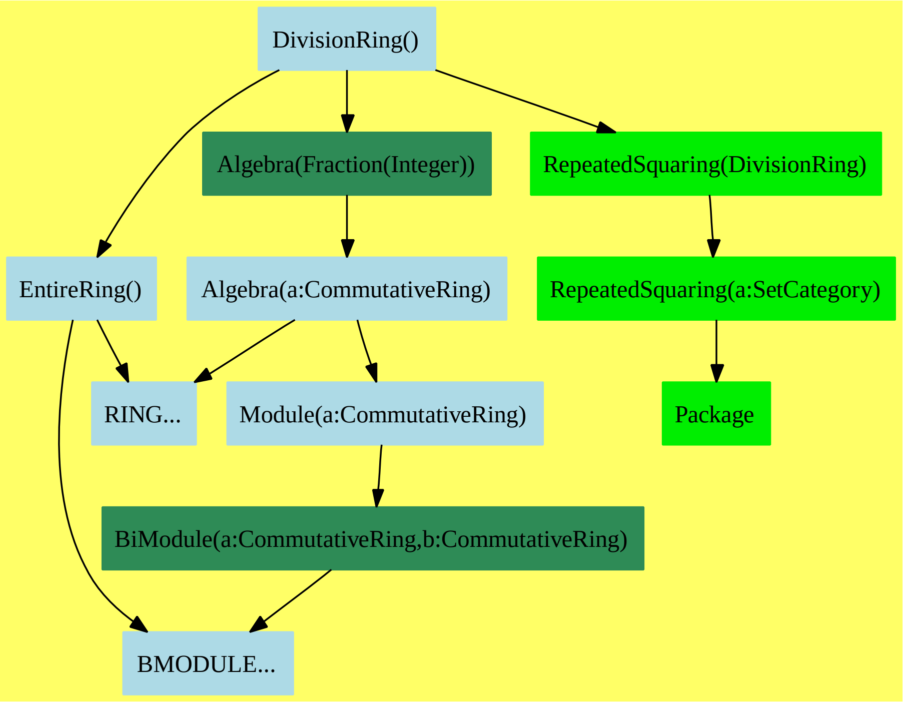
⟨*DIRPCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DirectProductCategory(a:NonNegativeInteger,b:Type)" [color=lightblue];
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "BMODULE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "KOERCE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "DIFEXT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FINITE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FLINEXP..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FRETRCT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "IXAGG..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "OAMONS..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "ORDRING..."

"BMODULE..." [color=lightblue];
"KOERCE..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FINITE..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"IXAGG..." [color=lightblue];
"OAMONS..." [color=lightblue];
"ORDRING..." [color=lightblue];

}
```

12.0.161 DivisionRing (DIVRING)


DivisionRing()
Algebra(Fraction(Integer))
RepeatedSquaring(DivisionRing)
EntireRing()
Algebra(a:CommutativeRing)
RepeatedSquaring(a:SetCategory)
RING...
Module(a:CommutativeRing)
Package
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*DivisionRing.input*⟩≡

```
)set break resume
)sys rm -f DivisionRing.output
)spool DivisionRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DivisionRing
--R DivisionRing  is a category constructor
--R Abbreviation for DivisionRing is DIVRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIVRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
```

```
--R ?-? : (%,%) -> %                        -? : % -> %
--R ?=? : (%,%) -> Boolean                  1 : () -> %
--R 0 : () -> %                             ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %          coerce : Fraction Integer -> %
--R coerce : Integer -> %                   coerce : % -> OutputForm
--R hash : % -> SingleInteger                inv : % -> %
--R latex : % -> String                     one? : % -> Boolean
--R recip : % -> Union(%,"failed")          sample : () -> %
--R zero? : % -> Boolean                    ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*DivisionRing.help*⟩≡

```
====================================================================
DivisionRing examples
====================================================================

See Also:
o )show DivisionRing
```

See:

⇒ "Field" (FIELD) 16.0.185 on page 1347
⇐ "Algebra" (ALGEBRA) 11.0.152 on page 1045
⇐ "EntireRing" (ENTIRER) 10.0.139 on page 944

Exports:

1	0	characteristic	coerce	hash
inv	latex	one?	recip	sample
subtractIfCan	zero?	?^?	?~=?	?**?
?*?	?+?	?-?	-?	?=?

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p944) EntireRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1045) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
```

⟨*category DIVRING DivisionRing*⟩≡

```
)abbrev category DIVRING DivisionRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A division ring (sometimes called a skew field),
++ i.e. a not necessarily commutative ring where
++ all non-zero elements have multiplicative inverses.

DivisionRing(): Category ==
 Join(EntireRing, Algebra Fraction Integer) with
      "**": (%,Integer) -> %
          ++ x**n returns x raised to the integer power n.
      "^" : (%,Integer) -> %
          ++ x^n returns x raised to the integer power n.
      inv : % -> %
          ++ inv x returns the multiplicative inverse of x.
          ++ Error: if x is 0.
-- Q-algebra is a lie, should be conditional on characteristic 0,
-- but knownInfo cannot handle the following commented
--    if % has CharacteristicZero then Algebra Fraction Integer
    add
      n: Integer
      x: %
      _^(x:%, n:Integer):% == x ** n
      import RepeatedSquaring(%)
      x ** n: Integer ==
         zero? n => 1
         zero? x =>
            n<0 => error "division by zero"
            x
         n<0 =>
            expt(inv x,(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)
--    if % has CharacteristicZero() then
      q:Fraction(Integer) * x:% == numer(q) * inv(denom(q)::%) * x
```

⟨*DIVRING.dotabb*⟩≡

```
"DIVRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
"DIVRING" -> "ENTIRER"
"DIVRING" -> "ALGEBRA"
```

⟨*DIVRING.dotfull*⟩≡

```
"DivisionRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
"DivisionRing()" -> "EntireRing()"
"DivisionRing()" -> "Algebra(Fraction(Integer))"
"DivisionRing()" -> "RepeatedSquaring(DivisionRing)"
```

⟨*DIVRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DivisionRing()" [color=lightblue];
"DivisionRing()" -> "EntireRing()"
"DivisionRing()" -> "Algebra(Fraction(Integer))"
"DivisionRing()" -> "RepeatedSquaring(DivisionRing)"

"RepeatedSquaring(DivisionRing)" [color="#00EE00"];
"RepeatedSquaring(DivisionRing)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(Fraction(Integer))" [color=seagreen];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

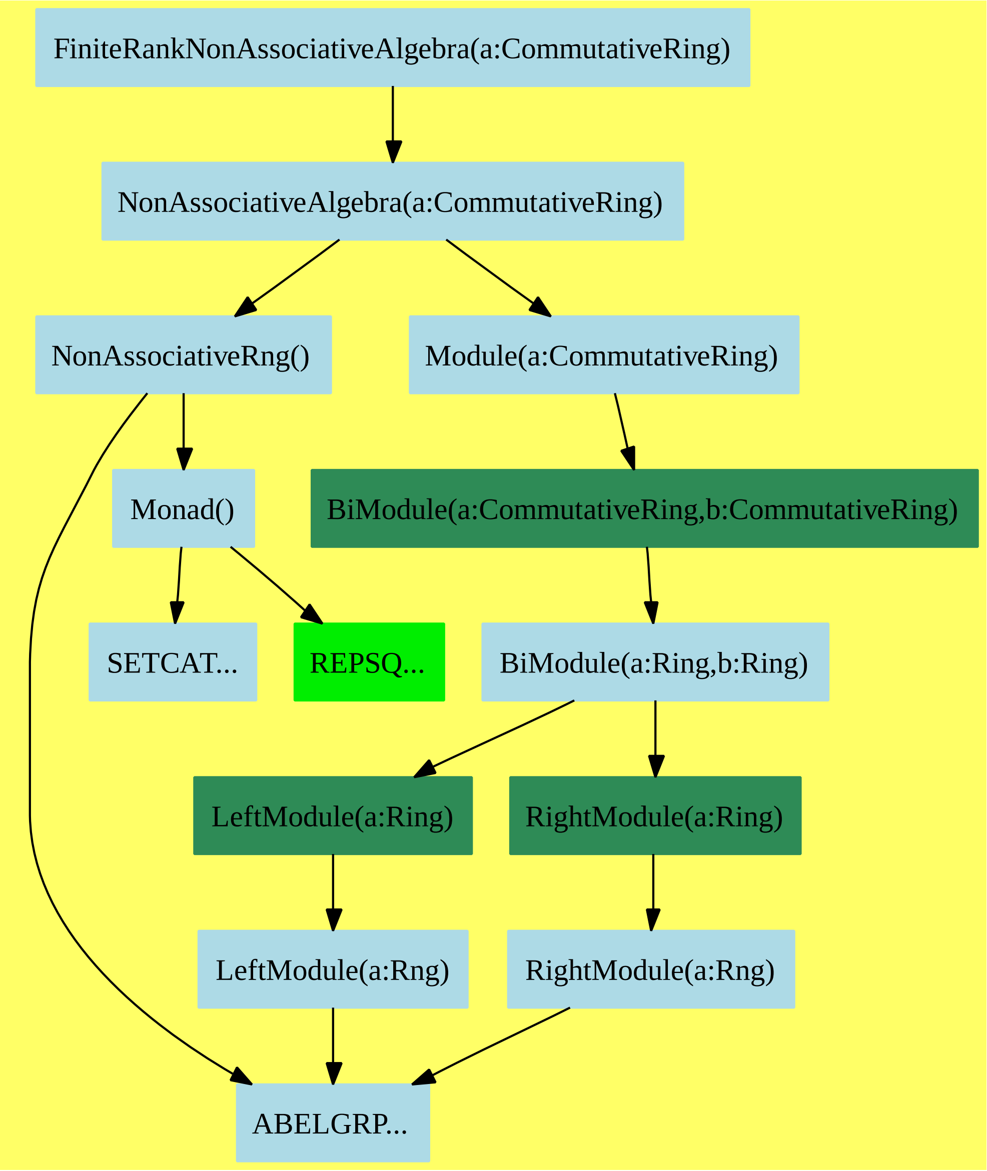
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

12.0.162 FiniteRankNonAssociativeAlgebra (FINAALG)


FiniteRankNonAssociativeAlgebra(a:CommutativeRing)
NonAssociativeAlgebra(a:CommutativeRing)
NonAssociativeRng()
Module(a:CommutativeRing)
Monad()
BiModule(a:CommutativeRing,b:CommutativeRing)
SETCAT...
REPSQ...
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*FiniteRankNonAssociativeAlgebra.input*⟩≡

```
)set break resume
)sys rm -f FiniteRankNonAssociativeAlgebra.output
)spool FiniteRankNonAssociativeAlgebra.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show FiniteRankNonAssociativeAlgebra
--R FiniteRankNonAssociativeAlgebra R: CommutativeRing  is a category constructor
--R Abbreviation for FiniteRankNonAssociativeAlgebra is FINAALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINAALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           alternative? : () -> Boolean
--R antiAssociative? : () -> Boolean      antiCommutative? : () -> Boolean
--R antiCommutator : (%,%) -> %           associative? : () -> Boolean
--R associator : (%,%,%) -> %             coerce : % -> OutputForm
--R commutative? : () -> Boolean          commutator : (%,%) -> %
--R flexible? : () -> Boolean             hash : % -> SingleInteger
--R jacobiIdentity? : () -> Boolean       jordanAdmissible? : () -> Boolean
--R jordanAlgebra? : () -> Boolean        latex : % -> String
--R leftAlternative? : () -> Boolean      leftDiscriminant : Vector % -> R
--R leftNorm : % -> R                     leftTrace : % -> R
--R lieAdmissible? : () -> Boolean        lieAlgebra? : () -> Boolean
--R powerAssociative? : () -> Boolean     rank : () -> PositiveInteger
--R rightAlternative? : () -> Boolean     rightDiscriminant : Vector % -> R
--R rightNorm : % -> R                    rightTrace : % -> R
--R sample : () -> %                      someBasis : () -> Vector %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R associatorDependence : () -> List Vector R if R has INTDOM
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : (%,Vector %) -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R noncommutativeJordanAlgebra? : () -> Boolean
```

```
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : (%,Vector %) -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R h
--R structuralConstants : Vector % -> Vector Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteRankNonAssociativeAlgebra.help*⟩≡

```
====================================================================
FiniteRankNonAssociativeAlgebra examples
====================================================================

See Also:
o )show FiniteRankNonAssociativeAlgebra
```

See:

⇒ “FramedNonAssociativeAlgebra” (FRNAALG) 13.0.172 on page 1230
⇐ “NonAssociativeAlgebra” (NAALG) 11.0.157 on page 1082

Exports:

0
alternative?
antiAssociative?
antiCommutative?
antiCommutator
associative?
associator
associatorDependence
coerce
commutative?
commutator
conditionsForIdempotents
coordinates
flexible?
hash
jacobiIdentity?
jordanAdmissible?
jordanAlgebra?
latex
leftAlternative?
leftCharacteristicPolynomial
leftDiscriminant
leftMinimalPolynomial
leftNorm
leftPower
leftRecip
leftRegularRepresentation
leftTrace
leftTraceMatrix
leftUnit
leftUnits
lieAdmissible?
lieAlgebra?
noncommutativeJordanAlgebra?
plenaryPower
powerAssociative?
rank
recip
represents
rightAlternative?
rightCharacteristicPolynomial
rightDiscriminant
rightMinimalPolynomial
rightNorm
rightPower
rightRecip
rightRegularRepresentation
rightTrace
rightTraceMatrix
rightUnit
rightUnits
sample
someBasis
structuralConstants
subtractIfCan
unit
zero?
?*?
?**?
?+?
?-?
-?
?=?
?~=?

Attributes Exported:

- if R has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (%,Vector %) -> Vector R
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () -> Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
powerAssociative? : () -> Boolean
rank : () -> PositiveInteger
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () -> Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
someBasis : () -> Vector %
unit : () -> Union(%,"failed") if R has INTDOM
```

These are implemented by this category:

```
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
associative? : () -> Boolean
associatorDependence : () -> List Vector R if R has INTDOM
commutative? : () -> Boolean
coordinates : (Vector %,Vector %) -> Matrix R
flexible? : () -> Boolean
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
leftDiscriminant : Vector % -> R
leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
leftNorm : % -> R
leftRecip : % -> Union(%,"failed") if R has INTDOM
leftRegularRepresentation : (%,Vector %) -> Matrix R
leftTrace : % -> R
leftTraceMatrix : Vector % -> Matrix R
lieAdmissible? : () -> Boolean
lieAlgebra? : () -> Boolean
noncommutativeJordanAlgebra? : () -> Boolean
recip : % -> Union(%,"failed") if R has INTDOM
represents : (Vector R,Vector %) -> %
rightAlternative? : () -> Boolean
rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
rightDiscriminant : Vector % -> R
rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
```

```
rightNorm : % -> R
rightRecip : % -> Union(%,"failed") if R has INTDOM
rightRegularRepresentation : (%,Vector %) -> Matrix R
rightTrace : % -> R
rightTraceMatrix : Vector % -> Matrix R
structuralConstants : Vector % -> Vector Matrix R
```

These exports come from (p1082) NonAssociativeAlgebra(R:CommutativeRing):

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
plenaryPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category FINAALG FiniteRankNonAssociativeAlgebra*⟩≡

```
)abbrev category FINAALG FiniteRankNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 12 June 1991
++ Basic Operations: +,-,*,**, someBasis
++ Related Constructors: FramedNonAssociativeAlgebra, FramedAlgebra,
++   FiniteRankAssociativeAlgebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra, basis
++ References:
++   R.D. Schafer: An Introduction to Nonassociative Algebras
++   Academic Press, New York, 1966
```

```
++
++   R. Wisbauer: Bimodule Structure of Algebra
++   Lecture Notes Univ. Duesseldorf 1991
++ Description:
++ A FiniteRankNonAssociativeAlgebra is a non associative algebra over
++ a commutative ring R which is a free \spad{R}-module of finite rank.

FiniteRankNonAssociativeAlgebra(R:CommutativeRing):
 Category == NonAssociativeAlgebra R with
    someBasis : () -> Vector %
      ++ someBasis() returns some \spad{R}-module basis.
    rank : () -> PositiveInteger
      ++ rank() returns the rank of the algebra as \spad{R}-module.
    conditionsForIdempotents: Vector % -> List Polynomial R
      ++ conditionsForIdempotents([v1,...,vn]) determines a complete list
      ++ of polynomial equations for the coefficients of idempotents
      ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
    structuralConstants: Vector % -> Vector Matrix R
      ++ structuralConstants([v1,v2,...,vm]) calculates the structural
      ++ constants \spad{[(gammaijk) for k in 1..m]} defined by
      ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijm * vm},
      ++ where \spad{[v1,...,vm]} is an \spad{R}-module basis
      ++ of a subalgebra.
    leftRegularRepresentation: (% , Vector %) -> Matrix R
      ++ leftRegularRepresentation(a,[v1,...,vn]) returns the matrix of
      ++ the linear map defined by left multiplication by \spad{a}
      ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.
    rightRegularRepresentation: (% , Vector %) -> Matrix R
      ++ rightRegularRepresentation(a,[v1,...,vn]) returns the matrix of
      ++ the linear map defined by right multiplication by \spad{a}
      ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.
    leftTrace: %  -> R
      ++ leftTrace(a) returns the trace of the left regular representation
      ++ of \spad{a}.
    rightTrace: %  -> R
      ++ rightTrace(a) returns the trace of the right regular representation
      ++ of \spad{a}.
    leftNorm: %  -> R
      ++ leftNorm(a) returns the determinant of the left regular
      ++ representation of \spad{a}.
    rightNorm: %  -> R
      ++ rightNorm(a) returns the determinant of the right regular
      ++ representation of \spad{a}.
    coordinates: (%, Vector %) -> Vector R
      ++ coordinates(a,[v1,...,vn]) returns the coordinates of \spad{a}
      ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
```

```
coordinates: (Vector %, Vector %) -> Matrix R
  ++ coordinates([a1,...,am],[v1,...,vn]) returns a matrix whose
  ++ i-th row is formed by the coordinates of \spad{ai}
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
represents: (Vector R, Vector %) -> %
  ++ represents([a1,...,am],[v1,...,vm]) returns the linear
  ++ combination \spad{a1*vm + ... + an*vm}.
leftDiscriminant: Vector % -> R
  ++ leftDiscriminant([v1,...,vn]) returns  the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the left trace of the product
  ++ \spad{vi*vj}. Note that this is the same as
  ++ \spad{determinant(leftTraceMatrix([v1,...,vn]))}.
rightDiscriminant: Vector % -> R
  ++ rightDiscriminant([v1,...,vn]) returns  the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the right trace of the product
  ++ \spad{vi*vj}. Note that this is the same as
  ++ \spad{determinant(rightTraceMatrix([v1,...,vn]))}.
leftTraceMatrix: Vector % -> Matrix R
  ++ leftTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the left trace of the product \spad{vi*vj}.
rightTraceMatrix: Vector % -> Matrix R
  ++ rightTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the right trace of the product \spad{vi*vj}.
leftCharacteristicPolynomial: % -> SparseUnivariatePolynomial R
  ++ leftCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the left regular representation of \spad{a}
  ++ with respect to any basis.
rightCharacteristicPolynomial: % -> SparseUnivariatePolynomial R
  ++ rightCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the right regular representation of \spad{a}
  ++ with respect to any basis.

--we not necessarily have a unit
--if R has CharacteristicZero then CharacteristicZero
--if R has CharacteristicNonZero then CharacteristicNonZero

commutative?:()-> Boolean
  ++ commutative?() tests if multiplication in the algebra
  ++ is commutative.
antiCommutative?:()-> Boolean
  ++ antiCommutative?() tests if \spad{a*a = 0}
  ++ for all \spad{a} in the algebra.
```

```
    ++ Note that this implies \spad{a*b + b*a = 0} for all
    ++ \spad{a} and \spad{b}.
  associative?:()-> Boolean
    ++ associative?() tests if multiplication in algebra
    ++ is associative.
  antiAssociative?:()-> Boolean
    ++ antiAssociative?() tests if multiplication in algebra
    ++ is anti-associative, i.e. \spad{(a*b)*c + a*(b*c) = 0}
    ++ for all \spad{a},b,c in the algebra.
  leftAlternative?: ()-> Boolean
    ++ leftAlternative?() tests if \spad{2*associator(a,a,b) = 0}
    ++ for all \spad{a}, b in the algebra.
    ++ Note that we only can test this; in general we don't know
    ++ whether \spad{2*a=0} implies \spad{a=0}.
  rightAlternative?: ()-> Boolean
    ++ rightAlternative?() tests if \spad{2*associator(a,b,b) = 0}
    ++ for all \spad{a}, b in the algebra.
    ++ Note that we only can test this; in general we don't know
    ++ whether \spad{2*a=0} implies \spad{a=0}.
  flexible?: ()->  Boolean
    ++ flexible?() tests if \spad{2*associator(a,b,a) = 0}
    ++ for all \spad{a}, b in the algebra.
    ++ Note that we only can test this; in general we don't know
    ++ whether \spad{2*a=0} implies \spad{a=0}.
  alternative?: ()-> Boolean
    ++ alternative?() tests if
    ++ \spad{2*associator(a,a,b) = 0 = 2*associator(a,b,b)}
    ++ for all \spad{a}, b in the algebra.
    ++ Note that we only can test this; in general we don't know
    ++ whether \spad{2*a=0} implies \spad{a=0}.
  powerAssociative?:()-> Boolean
    ++ powerAssociative?() tests if all subalgebras
    ++ generated by a single element are associative.
  jacobiIdentity?:() -> Boolean
    ++ jacobiIdentity?() tests if \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
    ++ for all \spad{a},b,c in the algebra. For example, this holds
    ++ for crossed products of 3-dimensional vectors.
  lieAdmissible?: () -> Boolean
    ++ lieAdmissible?() tests if the algebra defined by the commutators
    ++ is a Lie algebra, i.e. satisfies the Jacobi identity.
    ++ The property of anticommutativity follows from definition.
  jordanAdmissible?: () -> Boolean
    ++ jordanAdmissible?() tests if 2 is invertible in the
    ++ coefficient domain and the multiplication defined by
    ++ \spad{(1/2)(a*b+b*a)} determines a
    ++ Jordan algebra, i.e. satisfies the Jordan identity.
  ++ The property of \spadatt{commutative("*")}
  ++ follows from by definition.
noncommutativeJordanAlgebra?: () -> Boolean
  ++ noncommutativeJordanAlgebra?() tests if the algebra
  ++ is flexible and Jordan admissible.
jordanAlgebra?:() -> Boolean
  ++ jordanAlgebra?() tests if the algebra is commutative,
  ++ characteristic is not 2, and \spad{(a*b)*a**2 - a*(b*a**2) = 0}
  ++ for all \spad{a},b,c in the algebra (Jordan identity).
  ++ Example:
  ++ for every associative algebra \spad{(A,+,@)} we can construct a
  ++ Jordan algebra \spad{(A,+,*)}, where \spad{a*b := (a@b+b@a)/2}.
lieAlgebra?:() -> Boolean
  ++ lieAlgebra?() tests if the algebra is anticommutative
  ++ and \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
  ++ for all \spad{a},b,c in the algebra (Jacobi identity).
  ++ Example:
  ++ for every associative algebra \spad{(A,+,@)} we can construct a
  ++ Lie algebra \spad{(A,+,*)}, where \spad{a*b := a@b-b@a}.

if R has IntegralDomain then
  -- we not neccessarily have a unit, hence we don't inherit
  -- the next 3 functions anc hence copy them from MonadWithUnit:
  recip: % -> Union(%,"failed")
    ++ recip(a) returns an element, which is both a left and a right
    ++ inverse of \spad{a},
    ++ or \spad{"failed"} if there is no unit element, if such an
    ++ element doesn't exist or cannot be determined (see unitsKnown).
  leftRecip: % -> Union(%,"failed")
    ++ leftRecip(a) returns an element, which is a left inverse of
    ++ \spad{a}, or \spad{"failed"} if there is no unit element, if such
    ++ an element doesn't exist or cannot be determined (see unitsKnown).
  rightRecip: % -> Union(%,"failed")
    ++ rightRecip(a) returns an element, which is a right inverse of
    ++ \spad{a},
    ++ or \spad{"failed"} if there is no unit element, if such an
    ++ element doesn't exist or cannot be determined (see unitsKnown).
  associatorDependence:() -> List Vector R
    ++ associatorDependence() looks for the associator identities, i.e.
    ++ finds a basis of the solutions of the linear combinations of the
    ++ six permutations of \spad{associator(a,b,c)} which yield 0,
    ++ for all \spad{a},b,c in the algebra.
    ++ The order of the permutations is \spad{123 231 312 132 321 213}.
  leftMinimalPolynomial : % -> SparseUnivariatePolynomial R
    ++ leftMinimalPolynomial(a) returns the polynomial determined by the
    ++ smallest non-trivial linear combination of left powers of
```

```
        ++ \spad{a}. Note that the polynomial never has a constant term as in
        ++ general the algebra has no unit.
      rightMinimalPolynomial : % -> SparseUnivariatePolynomial R
        ++ rightMinimalPolynomial(a) returns the polynomial determined by the
        ++ smallest non-trivial linear
        ++ combination of right powers of \spad{a}.
        ++ Note that the polynomial never has a constant term as in general
        ++ the algebra has no unit.
      leftUnits:() -> Union(Record(particular: %, basis: List %), "failed")
        ++ leftUnits() returns the affine space of all left units of the
        ++ algebra, or \spad{"failed"} if there is none.
      rightUnits:() -> Union(Record(particular: %, basis: List %), "failed")
        ++ rightUnits() returns the affine space of all right units of the
        ++ algebra, or \spad{"failed"} if there is none.
      leftUnit:() -> Union(%, "failed")
        ++ leftUnit() returns a left unit of the algebra
        ++ (not necessarily unique), or \spad{"failed"} if there is none.
      rightUnit:() -> Union(%, "failed")
        ++ rightUnit() returns a right unit of the algebra
        ++ (not necessarily unique), or \spad{"failed"} if there is none.
      unit:() -> Union(%, "failed")
        ++ unit() returns a unit of the algebra (necessarily unique),
        ++ or \spad{"failed"} if there is none.
      -- we not necessarily have a unit, hence we can't say anything
      -- about characteristic
      -- if R has CharacteristicZero then CharacteristicZero
      -- if R has CharacteristicNonZero then CharacteristicNonZero
      unitsKnown
        ++ unitsKnown means that \spadfun{recip} truly yields reciprocal
        ++ or \spad{"failed"} if not a unit,
        ++ similarly for \spadfun{leftRecip} and
        ++ \spadfun{rightRecip}. The reason is that we use left, respectively
        ++ right, minimal polynomials to decide this question.
  add
    --n := rank()
    --b := someBasis()
    --gamma : Vector Matrix R := structuralConstants b
    -- here is a problem: there seems to be a problem having local
    -- variables in the capsule of a category, furthermore
    -- see the commented code of conditionsForIdempotents, where
    -- we call structuralConstants, which also doesn't work
    -- at runtime, i.e. is not properly inherited, hence for
    -- the moment we put the code for
    -- conditionsForIdempotents, structuralConstants, unit, leftUnit,
    -- rightUnit into the domain constructor ALGSC
    V  ==> Vector
```

```
M  ==> Matrix
REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)


SUP ==>  SparseUnivariatePolynomial
NNI ==>  NonNegativeInteger
-- next 2 functions: use a general characteristicPolynomial
leftCharacteristicPolynomial a ==
   n := rank()$%
   ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
   mb : Matrix SUP R := zero(n,n)
   for i in 1..n repeat
     for j in 1..n repeat
       mb(i,j):=
         i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
         monomial(ma(i,j),1)$SUP(R)
   determinant mb

rightCharacteristicPolynomial a ==
   n := rank()$%
   ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
   mb : Matrix SUP R := zero(n,n)
   for i in 1..n repeat
     for j in 1..n repeat
       mb(i,j):=
         i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
         monomial(ma(i,j),1)$SUP(R)
   determinant mb

leftTrace a ==
  t : R := 0
  ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
  for i in 1..rank()$% repeat
    t := t + elt(ma,i,i)
  t

rightTrace a ==
  t : R := 0
  ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
  for i in 1..rank()$% repeat
    t := t + elt(ma,i,i)
  t

leftNorm a == determinant leftRegularRepresentation(a,someBasis()$%)
```

```
rightNorm a == determinant rightRegularRepresentation(a,someBasis()$%)

antiAssociative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        not zero? ( (b.i*b.j)*b.k + b.i*(b.j*b.k) )  =>
          messagePrint("algebra is not anti-associative")$OutputForm
          return false
  messagePrint("algebra is anti-associative")$OutputForm
  true

jordanAdmissible?() ==
  b := someBasis()
  n := rank()
  recip(2 * 1$R) case "failed" =>
    messagePrint("this algebra is not Jordan admissible, " _
     "as 2 is not invertible in the ground ring")$OutputForm
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for l in 1..n repeat
       not zero? ( _
         antiCommutator(antiCommutator(b.i,b.j),_
                        antiCommutator(b.l,b.k)) + _
         antiCommutator(antiCommutator(b.l,b.j),_
                        antiCommutator(b.k,b.i)) + _
         antiCommutator(antiCommutator(b.k,b.j),_
                        antiCommutator(b.i,b.l))   _
                  ) =>
           messagePrint(_
                     "this algebra is not Jordan admissible")$OutputForm
           return false
  messagePrint("this algebra is Jordan admissible")$OutputForm
  true

lieAdmissible?() ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? (commutator(commutator(b.i,b.j),b.k) _
            + commutator(commutator(b.j,b.k),b.i) _
            + commutator(commutator(b.k,b.i),b.j))   =>
      messagePrint("this algebra is not Lie admissible")$OutputForm
      return false
  messagePrint("this algebra is Lie admissible")$OutputForm
  true

-- conditionsForIdempotents b  ==
--   n := rank()
--   gamma : Vector Matrix R := structuralConstants b
--   listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
--   symbolsForCoef : Vector Symbol :=
--     [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
--   conditions : List Polynomial R := []
--  for k in 1..n repeat
--    xk := symbolsForCoef.k
--    p : Polynomial R :=  monomial( - 1$Polynomial(R), [xk], [1] )
--    for i in 1..n repeat
--      for j in 1..n repeat
--        xi := symbolsForCoef.i
--        xj := symbolsForCoef.j
--        p := p + monomial(_
--          elt((gamma.k),i,j) :: Polynomial(R), [xi,xj], [1,1])
--    conditions := cons(p,conditions)
--  conditions

structuralConstants b ==
  --n := rank()
  -- be careful with the possibility that b is not a basis
  m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
  sC : Vector Matrix R := [new(m,m,0$R) for k in 1..m]
  for i in 1..m repeat
    for j in 1..m repeat
      covec : Vector R := coordinates(b.i * b.j, b)
      for k in 1..m repeat
         setelt( sC.k, i, j, covec.k )
  sC

if R has IntegralDomain then

  leftRecip x ==
    zero? x => "failed"
    lu := leftUnit()
    lu case "failed" => "failed"
    b := someBasis()
    xx : % := (lu :: %)
```

```
      k  : PositiveInteger := 1
      cond : Matrix R := coordinates(xx,b) :: Matrix(R)
      listOfPowers : List % := [xx]
      while rank(cond) = k repeat
        k := k+1
        xx := xx*x
        listOfPowers := cons(xx,listOfPowers)
        cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
      vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
      invC := recip vectorOfCoef.1
      invC case "failed" => "failed"
      invCR : R :=  - (invC :: R)
      reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
       2..maxIndex vectorOfCoef for power in reverse listOfPowers])

    rightRecip x ==
      zero? x => "failed"
      ru := rightUnit()
      ru case "failed" => "failed"
      b := someBasis()
      xx : % := (ru :: %)
      k  : PositiveInteger := 1
      cond : Matrix R := coordinates(xx,b) :: Matrix(R)
      listOfPowers : List % := [xx]
      while rank(cond) = k repeat
        k := k+1
        xx := x*xx
        listOfPowers := cons(xx,listOfPowers)
        cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
      vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
      invC := recip vectorOfCoef.1
      invC case "failed" => "failed"
      invCR : R :=  - (invC :: R)
      reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
       2..maxIndex vectorOfCoef for power in reverse listOfPowers])

    recip x ==
      lrx := leftRecip x
      lrx case "failed" => "failed"
      rrx := rightRecip x
      rrx case "failed" => "failed"
      (lrx :: %) ^= (rrx :: %)  => "failed"
      lrx :: %

    leftMinimalPolynomial x ==
      zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
```

```
  b := someBasis()
  xx : % := x
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  while rank(cond) = k repeat
    k := k+1
    xx := x*xx
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
  res : SparseUnivariatePolynomial R := 0
  for i in 1..k repeat
    res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
  res

rightMinimalPolynomial x ==
  zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
  b := someBasis()
  xx : % := x
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  while rank(cond) = k repeat
    k := k+1
    xx := xx*x
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
  res : SparseUnivariatePolynomial R := 0
  for i in 1..k repeat
    res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
  res

associatorDependence() ==
  n := rank()
  b := someBasis()
  cond : Matrix(R) := new(n**4,6,0$R)$Matrix(R)
  z : Integer := 0
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     a123 : Vector R := coordinates(associator(b.i,b.j,b.k),b)
     a231 : Vector R := coordinates(associator(b.j,b.k,b.i),b)
     a312 : Vector R := coordinates(associator(b.k,b.i,b.j),b)
     a132 : Vector R := coordinates(associator(b.i,b.k,b.j),b)
     a321 : Vector R := coordinates(associator(b.k,b.j,b.i),b)
     a213 : Vector R := coordinates(associator(b.j,b.i,b.k),b)
     for r in 1..n repeat
      z:= z+1
```

```
          setelt(cond,z,1,elt(a123,r))
          setelt(cond,z,2,elt(a231,r))
          setelt(cond,z,3,elt(a312,r))
          setelt(cond,z,4,elt(a132,r))
          setelt(cond,z,5,elt(a321,r))
          setelt(cond,z,6,elt(a213,r))
      nullSpace(cond)

    jacobiIdentity?()  ==
      n := rank()
      b := someBasis()
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
          not zero? ((b.i*b.j)*b.k + (b.j*b.k)*b.i + (b.k*b.i)*b.j) =>
            messagePrint("Jacobi identity does not hold")$OutputForm
            return false
      messagePrint("Jacobi identity holds")$OutputForm
      true

    lieAlgebra?()  ==
      not antiCommutative?() =>
        messagePrint("this is not a Lie algebra")$OutputForm
        false
      not jacobiIdentity?() =>
        messagePrint("this is not a Lie algebra")$OutputForm
        false
      messagePrint("this is a Lie algebra")$OutputForm
      true

    jordanAlgebra?()  ==
      b := someBasis()
      n := rank()
      recip(2 * 1$R) case "failed" =>
        messagePrint("this is not a Jordan algebra, as 2 is not " _
         "invertible in the ground ring")$OutputForm
        false
      not commutative?() =>
        messagePrint("this is not a Jordan algebra")$OutputForm
        false
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         for l in 1..n repeat
           not zero? (associator(b.i,b.j,b.l*b.k)+_
               associator(b.l,b.j,b.k*b.i)+associator(b.k,b.j,b.i*b.l)) =>
          messagePrint("not a Jordan algebra")$OutputForm
          return false
     messagePrint("this is a Jordan algebra")$OutputForm
     true

   noncommutativeJordanAlgebra?() ==
     b := someBasis()
     n := rank()
     recip(2 * 1$R) case "failed" =>
      messagePrint("this is not a noncommutative Jordan algebra,_
as 2 is not invertible in the ground ring")$OutputForm
      false
     not flexible?()$% =>
      messagePrint("this is not a noncommutative Jordan algebra,_
as it is not flexible")$OutputForm
      false
     not jordanAdmissible?()$% =>
      messagePrint("this is not a noncommutative Jordan algebra,_
as it is not Jordan admissible")$OutputForm
      false
     messagePrint("this is a noncommutative Jordan algebra")$OutputForm
     true

   antiCommutative?() ==
     b := someBasis()
     n := rank()
     for i in 1..n repeat
       for j in i..n repeat
         not zero? (i=j => b.i*b.i; b.i*b.j + b.j*b.i) =>
           messagePrint("algebra is not anti-commutative")$OutputForm
           return false
     messagePrint("algebra is anti-commutative")$OutputForm
     true

   commutative?() ==
     b := someBasis()
     n := rank()
     for i in 1..n repeat
      for j in i+1..n repeat
        not zero? commutator(b.i,b.j) =>
          messagePrint("algebra is not commutative")$OutputForm
          return false
     messagePrint("algebra is commutative")$OutputForm
     true

   associative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? associator(b.i,b.j,b.k) =>
       messagePrint("algebra is not associative")$OutputForm
       return false
  messagePrint("algebra is associative")$OutputForm
  true

leftAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not left alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,a,b) = 0")$OutputForm
  true

rightAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not right alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,b) = 0")$OutputForm
  true

flexible?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.k,b.j,b.i)) =>
       messagePrint("algebra is not flexible")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,a) = 0")$OutputForm
  true
```

```
alternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not alternative")$OutputForm
       return false
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,b) = 0 " _
               "=  2*associator(a,a,b) = 0")$OutputForm
  true

leftDiscriminant v == determinant leftTraceMatrix v
rightDiscriminant v == determinant rightTraceMatrix v

coordinates(v:Vector %, b:Vector %) ==
  m := new(#v, #b, 0)$Matrix(R)
  for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates(qelt(v, i), b))
  m

represents(v, b) ==
  m := minIndex v - 1
  reduce(_+,[v(i+m) * b(i+m) for i in 1..maxIndex b])

leftTraceMatrix v ==
  matrix [[leftTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
           for i in minIndex v .. maxIndex v]$List(List R)

rightTraceMatrix v ==
  matrix [[rightTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
           for i in minIndex v .. maxIndex v]$List(List R)

leftRegularRepresentation(x, b) ==
  m := minIndex b - 1
  matrix
   [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)

rightRegularRepresentation(x, b) ==
  m := minIndex b - 1
  matrix
```

```
        [parts coordinates(b(i+m)*x,b) for i in 1..rank()]$List(List R)
```

⟨*FINAALG.dotabb*⟩≡

```
"FINAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
"FINAALG" -> "NAALG"
```

⟨*FINAALG.dotfull*⟩≡

```
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"
```

⟨*FINAALG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

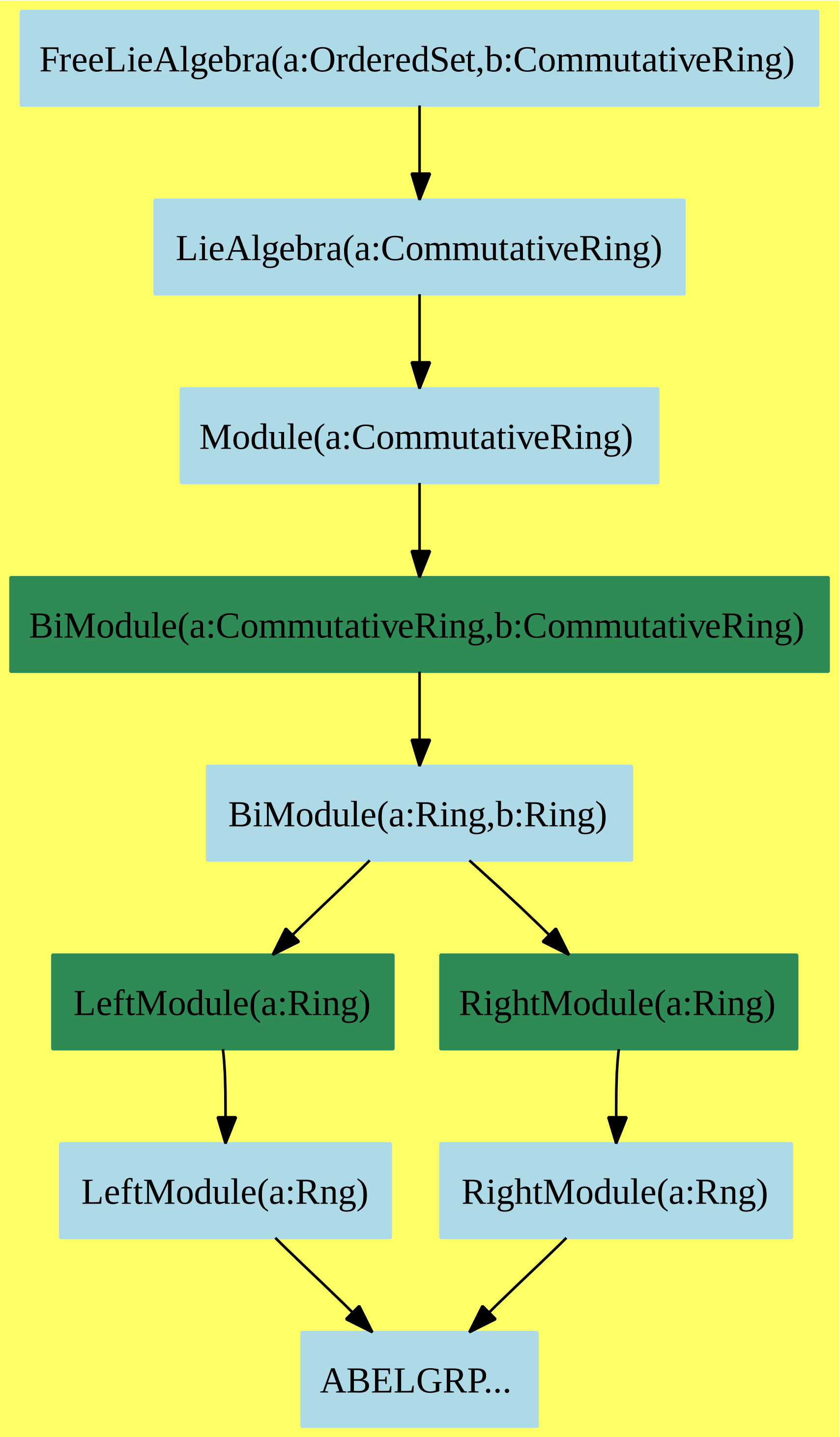
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

12.0.163 FreeLieAlgebra (FLALG)


FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)
LieAlgebra(a:CommutativeRing)
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*FreeLieAlgebra.input*⟩≡
```
)set break resume
)sys rm -f FreeLieAlgebra.output
)spool FreeLieAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeLieAlgebra
--R FreeLieAlgebra(VarSet: OrderedSet,R: CommutativeRing)  is a category constructor
--R Abbreviation for FreeLieAlgebra is FLALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R LiePoly : LyndonWord VarSet -> %      0 : () -> %
--R coerce : VarSet -> %                  coerce : % -> OutputForm
--R construct : (%,%) -> %                degree : % -> NonNegativeInteger
--R eval : (%,VarSet,%) -> %              hash : % -> SingleInteger
--R latex : % -> String                   mirror : % -> %
--R sample : () -> %                      varList : % -> List VarSet
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R coef : (XRecursivePolynomial(VarSet,R),%) -> R
--R coerce : % -> XRecursivePolynomial(VarSet,R)
--R coerce : % -> XDistributedPolynomial(VarSet,R)
--R eval : (%,List VarSet,List %) -> %
--R lquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
--R rquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*FreeLieAlgebra.help*⟩≡
```
====================================================================
FreeLieAlgebra examples
====================================================================

See Also:
o )show FreeLieAlgebra
```

See:

⇐ "LieAlgebra" (LIECAT) 11.0.155 on page 1067

Exports:

0	coef	coerce	construct	degree
eval	hash	latex	LiePoly	lquo
mirror	rquo	sample	subtractIfCan	trunc
varList	zero?	?~=?	?*?	?/?
?+?	?-?	-?	?=?	

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (XRecursivePolynomial(VarSet,R),%) -> R
coerce : VarSet -> %
coerce : % -> XRecursivePolynomial(VarSet,R)
coerce : % -> XDistributedPolynomial(VarSet,R)
degree : % -> NonNegativeInteger
eval : (%,List VarSet,List %) -> %
eval : (%,VarSet,%) -> %
LiePoly : LyndonWord VarSet -> %
lquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
mirror : % -> %
rquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
trunc : (%,NonNegativeInteger) -> %
varList : % -> List VarSet
```

These exports come from (p1067) LieAlgebra(CommutativeRing):

```
0 : () -> %
coerce : % -> OutputForm
construct : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?/? : (%,R) -> % if R has FIELD
```

```
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category FLALG FreeLieAlgebra*⟩≡

```
)abbrev category FLALG FreeLieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr)
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of free Lie algebras.
++ It is used by domains of non-commutative algebra:
++ \spadtype{LiePolynomial} and
++ \spadtype{XPBWPolynomial}.

FreeLieAlgebra(VarSet:OrderedSet, R:CommutativeRing) :Category == _
 CatDef where
   XRPOLY  ==> XRecursivePolynomial(VarSet,R)
   XDPOLY  ==> XDistributedPolynomial(VarSet,R)
   RN      ==> Fraction Integer
   LWORD   ==> LyndonWord(VarSet)

   CatDef ==  Join(LieAlgebra(R)) with
     coef      : (XRPOLY , $) -> R
       ++ \axiom{coef(x,y)} returns the scalar product of \axiom{x} by
       ++ \axiom{y}, the set of words being regarded as an orthogonal basis.
     coerce    : VarSet -> $
       ++ \axiom{coerce(x)} returns \axiom{x} as a Lie polynomial.
     coerce    : $ -> XDPOLY
       ++ \axiom{coerce(x)} returns \axiom{x} as distributed polynomial.
     coerce    : $ -> XRPOLY
       ++ \axiom{coerce(x)} returns \axiom{x} as a recursive polynomial.
     degree    : $ -> NonNegativeInteger
```

```
    ++ \axiom{degree(x)} returns the greatest length of a word in the
    ++ support of \axiom{x}.
  --if R has Module(RN) then
  --  Hausdorff : ($,$,PositiveInteger) -> $
  lquo      : (XRPOLY , $) -> XRPOLY
    ++ \axiom{lquo(x,y)} returns the left simplification of \axiom{x}
    ++ by \axiom{y}.
  rquo      : (XRPOLY , $) -> XRPOLY
    ++ \axiom{rquo(x,y)} returns the right simplification of \axiom{x}
    ++ by \axiom{y}.
  LiePoly   : LWORD -> $
    ++ \axiom{LiePoly(l)} returns the bracketed form of \axiom{l} as
    ++ a Lie polynomial.
  mirror    : $ -> $
    ++ \axiom{mirror(x)} returns \axiom{Sum(r_i mirror(w_i))}
    ++ if \axiom{x} is \axiom{Sum(r_i w_i)}.
  trunc     : ($, NonNegativeInteger) -> $
    ++ \axiom{trunc(p,n)} returns the polynomial \axiom{p}
    ++ truncated at order \axiom{n}.
  varList   : $ -> List VarSet
    ++ \axiom{varList(x)} returns the list of distinct entries
    ++ of \axiom{x}.
  eval      : ($, VarSet, $) -> $
    ++ \axiom{eval(p, x, v)} replaces \axiom{x} by \axiom{v}
    ++ in \axiom{p}.
  eval      : ($, List VarSet, List $) -> $
    ++ \axiom{eval(p, [x1,...,xn], [v1,...,vn])} replaces \axiom{xi}
    ++ by \axiom{vi} in \axiom{p}.
```

⟨*FLALG.dotabb*⟩≡

```
"FLALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FLALG" -> "LIECAT"
```

⟨*FLALG.dotfull*⟩≡

```
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"
```

⟨*FLALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" [color=lightblue];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

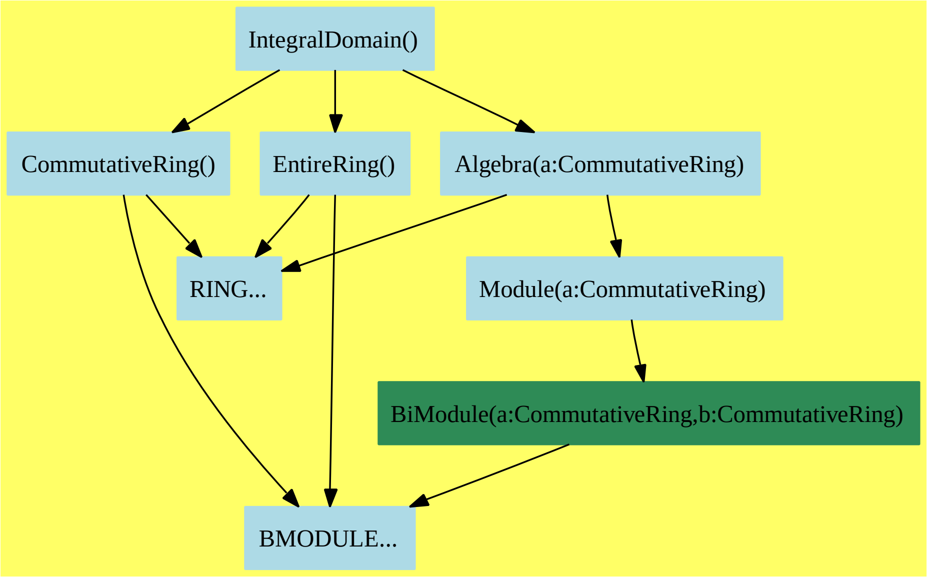
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

12.0.164 IntegralDomain (INTDOM)


IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*IntegralDomain.input*⟩≡

```
)set break resume
)sys rm -f IntegralDomain.output
)spool IntegralDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegralDomain
--R IntegralDomain  is a category constructor
--R Abbreviation for IntegralDomain is INTDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INTDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
```

```
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*IntegralDomain.help*⟩≡

```
====================================================================
IntegralDomain examples
====================================================================

See Also:
o )show IntegralDomain
```

All Commutative Rings are Integral Domains.
⇐ “CommutativeRing” (COMRING) 10.0.137 on page 932 Integral Domains are a subset of Unique Factorization domains.
⇒ “UniqueFactorizationDomain” (UFD) 14.0.179 on page 1294.

See:

⇒ “FortranMachineTypeCategory” (FMTC) 13.0.171 on page 1223
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “GcdDomain” (GCDDOM) 13.0.173 on page 1246
⇒ “OrderedIntegralDomain” (OINTDOM) 13.0.174 on page 1253
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “CommutativeRing” (COMRING) 10.0.137 on page 932
⇐ “EntireRing” (ENTIRER) 10.0.139 on page 944

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	one?	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation ” $*$ ” : $(D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

Attributes Used:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

These are directly exported but not implemented:

```
exquo : (%,%) -> Union(%,"failed")
```

These are implemented by this category:

```
associates? : (%,%) -> Boolean
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

These exports come from (p932) CommutativeRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

TPDHERE: Should we construct this coercion?

These exports come from (p1045) Algebra(a:IntegralDomain):

```
coerce : % -> %
```

⟨*category INTDOM IntegralDomain*⟩≡

```
)abbrev category INTDOM IntegralDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References: Davenport & Trager I
++ Description:
++ The category of commutative integral domains, i.e. commutative
++ rings with no zero divisors.
++
++ Conditional attributes\br
++ canonicalUnitNormal\tab{5}the canonical field is the same for
++ all associates\br
++ canonicalsClosed\tab{5}the product of two canonicals is itself canonical

IntegralDomain(): Category ==
  Join(CommutativeRing, Algebra(%), EntireRing) with
    "exquo": (%,%) -> Union(%,"failed")
        ++ exquo(a,b) either returns an element c such that
        ++ \spad{c*b=a} or "failed" if no such element can be found.
    unitNormal: % -> Record(unit:%,canonical:%,associate:%)
        ++ unitNormal(x) tries to choose a canonical element
        ++ from the associate class of x.
        ++ The attribute canonicalUnitNormal, if asserted, means that
        ++ the "canonical" element is the same across all associates of x
        ++ if \spad{unitNormal(x) = [u,c,a]} then
        ++ \spad{u*c = x}, \spad{a*u = 1}.
    unitCanonical: % -> %
        ++ \spad{unitCanonical(x)} returns \spad{unitNormal(x).canonical}.
    associates?: (%,%) -> Boolean
        ++ associates?(x,y) tests whether x and y are associates, i.e.
        ++ differ by a unit factor.
    unit?: % -> Boolean
        ++ unit?(x) tests whether x is a unit, i.e. is invertible.
 add
      x,y: %

      UCA ==> Record(unit:%,canonical:%,associate:%)
      if not (% has Field) then
        unitNormal(x) == [1$%,x,1$%]$UCA -- the non-canonical definition
      unitCanonical(x) == unitNormal(x).canonical -- always true
      recip(x) == if zero? x then "failed" else _exquo(1$%,x)
      unit?(x) == (recip x case "failed" => false; true)
      if % has canonicalUnitNormal then
         associates?(x,y) ==
           (unitNormal x).canonical = (unitNormal y).canonical
       else
         associates?(x,y) ==
           zero? x => zero? y
           zero? y => false
```

```
            x exquo y case "failed" => false
            y exquo x case "failed" => false
            true
```

⟨*INTDOM.dotabb*⟩≡

```
"INTDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"INTDOM" -> "COMRING"
"INTDOM" -> "ALGEBRA"
"INTDOM" -> "ENTIRER"
```

⟨*INTDOM.dotfull*⟩≡

```
"IntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"
```

⟨*INTDOM.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

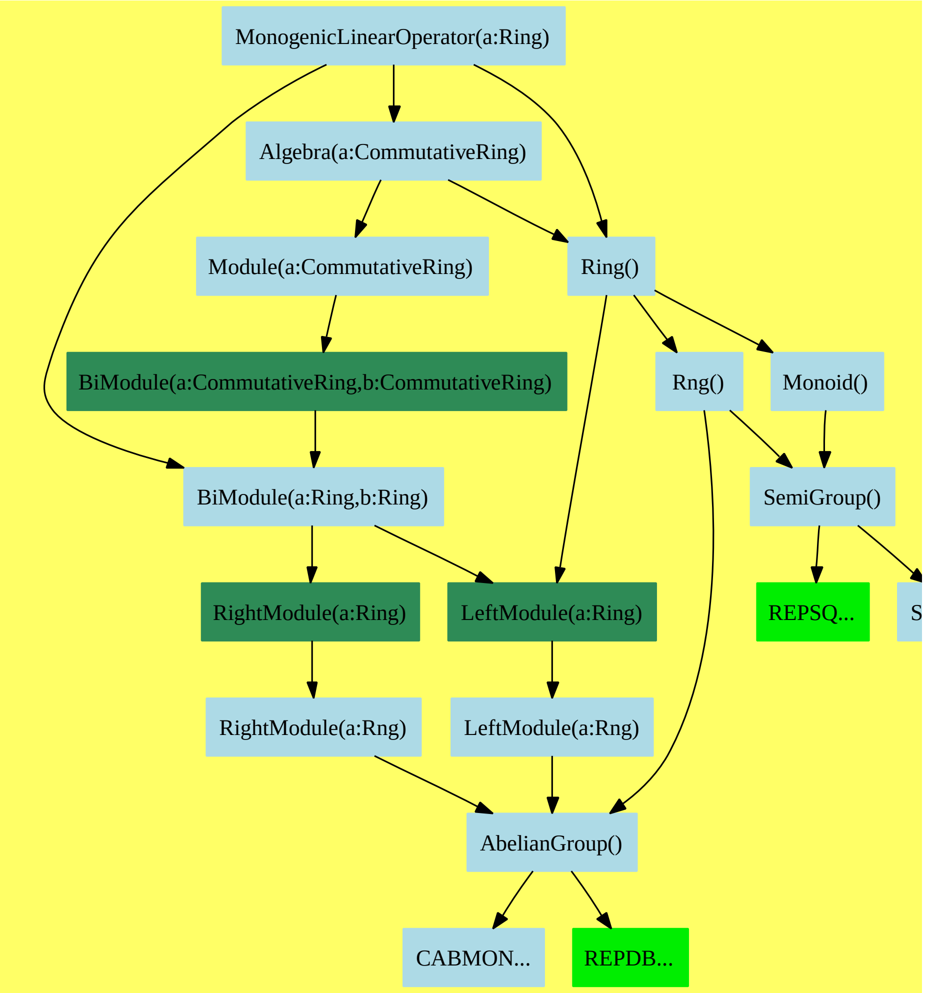
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

12.0.165 MonogenicLinearOperator (MLO)


MonogenicLinearOperator(a:Ring)
Algebra(a:CommutativeRing)
Module(a:CommutativeRing)
Ring()
BiModule(a:CommutativeRing,b:CommutativeRing)
Rng()
Monoid()
BiModule(a:Ring,b:Ring)
SemiGroup()
RightModule(a:Ring)
LeftModule(a:Ring)
REPSQ...
S
RightModule(a:Rng)
LeftModule(a:Rng)
AbelianGroup()
CABMON...
REPDB...

⟨*MonogenicLinearOperator.input*⟩≡

```
)set break resume
)sys rm -f MonogenicLinearOperator.output
)spool MonogenicLinearOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonogenicLinearOperator
--R MonogenicLinearOperator R: Ring  is a category constructor
```

```
--R Abbreviation for MonogenicLinearOperator is MLO
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MLO
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> R           one? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : R -> % if R has COMRING
--R minimumDegree : % -> NonNegativeInteger
--R monomial : (R,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*MonogenicLinearOperator.help*⟩≡

```
====================================================================
MonogenicLinearOperator examples
====================================================================

See Also:
o )show MonogenicLinearOperator
```

See:

⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “BiModule” (BMODULE) 9.0.122 on page 800
⇐ “Ring” (RING) 9.0.129 on page 854

Exports:

0
1
characteristic
coefficient
coerce
degree
hash
latex
leadingCoefficient
minimumDegree
monomial
one?
recip
reductum
sample
subtractIfCan
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
minimumDegree : % -> NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
```

```
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p1045) Algebra(R:CommutativeRing):

```
coerce : R -> % if R has COMRING
```

⟨*category MLO MonogenicLinearOperator*⟩≡

```
)abbrev category MLO MonogenicLinearOperator
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains:  NonCommutativeOperatorDivision
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This is the category of linear operator rings with one generator.
++ The generator is not named by the category but can always be
++ constructed as \spad{monomial(1,1)}.
++
++ For convenience, call the generator \spad{G}.
++ Then each value is equal to
++ \spad{sum(a(i)*G**i, i = 0..n)}
++ for some unique \spad{n} and \spad{a(i)} in \spad{R}.
++
++ Note that multiplication is not necessarily commutative.
++ In fact,  if \spad{a} is in \spad{R}, it is quite normal
++ to have \spad{a*G \^= G*a}.
```

```
MonogenicLinearOperator(R): Category == Defn where
    E ==> NonNegativeInteger
    R: Ring
    Defn == Join(Ring, BiModule(R,R)) with
        if R has CommutativeRing then Algebra(R)
        degree: $ -> E
            ++ degree(l) is \spad{n} if
            ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
        minimumDegree: $ -> E
            ++ minimumDegree(l) is the smallest \spad{k} such that
            ++ \spad{a(k) \^= 0} if
            ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
        leadingCoefficient: $ -> R
            ++ leadingCoefficient(l) is \spad{a(n)} if
            ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
        reductum: $ -> $
            ++ reductum(l) is \spad{l - monomial(a(n),n)} if
            ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
        coefficient: ($, E) -> R
            ++ coefficient(l,k) is \spad{a(k)} if
            ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
        monomial: (R, E) -> $
            ++ monomial(c,k) produces c times the k-th power of
            ++ the generating operator, \spad{monomial(1,1)}.
```

⟨MLO.dotabb⟩≡

```
"MLO"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MLO" -> "BMODULE"
"MLO" -> "RING"
"MLO" -> "ALGEBRA"
```

⟨MLO.dotfull⟩≡

```
"MonogenicLinearOperator(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"
```

⟨*MLO.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MonogenicLinearOperator(a:Ring)" [color=lightblue];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

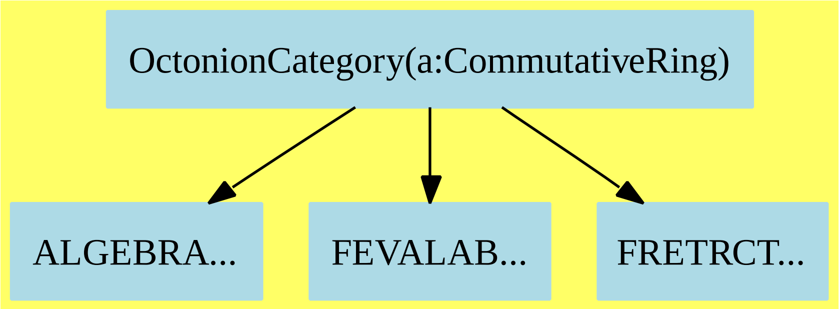
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

12.0.166 OctonionCategory (OC)



⟨*OctonionCategory.input*⟩≡

```
  )set break resume
  )sys rm -f OctonionCategory.output
  )spool OctonionCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show OctonionCategory
  --R OctonionCategory R: CommutativeRing  is a category constructor
  --R Abbreviation for OctonionCategory is OC
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for OC
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
  --R 1 : () -> %                           0 : () -> %
  --R ?^? : (%,PositiveInteger) -> %        abs : % -> R if R has RNS
  --R coerce : R -> %                       coerce : Integer -> %
  --R coerce : % -> OutputForm              conjugate : % -> %
  --R hash : % -> SingleInteger             imagE : % -> R
  --R imagI : % -> R                        imagJ : % -> R
  --R imagK : % -> R                        imagi : % -> R
  --R imagj : % -> R                        imagk : % -> R
  --R inv : % -> % if R has FIELD           latex : % -> String
  --R map : ((R -> R),%) -> %               norm : % -> R
  --R octon : (R,R,R,R,R,R,R,R) -> %        one? : % -> Boolean
```

```
--R real : % -> R                         recip : % -> Union(%,"failed")
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R convert : % -> InputForm if R has KONVERT INFORM
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Equation R) -> % if R has EVALAB R
--R eval : (%,Equation R) -> % if R has EVALAB R
--R eval : (%,R,R) -> % if R has EVALAB R
--R eval : (%,List R,List R) -> % if R has EVALAB R
--R index : PositiveInteger -> % if R has FINITE
--R lookup : % -> PositiveInteger if R has FINITE
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R random : () -> % if R has FINITE
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R size : () -> NonNegativeInteger if R has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨OctonionCategory.help⟩≡

```
====================================================================
OctonionCategory examples
====================================================================

See Also:
o )show OctonionCategory
```

See:

⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “FullyEvalableOver” (FEVALAB) 4.0.44 on page 178
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100

Exports:

0	1	abs	characteristic	charthRoot
coerce	conjugate	convert	eval	hash
imagE	imagI	imagJ	imagK	imagi
imagj	imagk	index	inv	latex
lookup	map	max	min	norm
octon	one?	random	rational	rational?
rationalIfCan	real	recip	retract	retractIfCan
sample	size	subtractIfCan	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?	?>?
?>=?	?.?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagi : % -> R
imagj : % -> R
imagk : % -> R
imagE : % -> R
imagI : % -> R
imagJ : % -> R
imagK : % -> R
octon : (R,R,R,R,R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
```

```
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean if R has ORDSET
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
```

These exports come from (p1045) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p100) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p178) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p188) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

⟨*category OC OctonionCategory*⟩≡

```
)abbrev category OC OctonionCategory
++ Author: R. Wisbauer, J. Grabmeier
++ Date Created: 05 September 1990
++ Date Last Updated: 19 September 1990
++ Basic Operations: _+, _*, octon, real, imagi, imagj, imagk,
++  imagE, imagI, imagJ, imagK
++ Related Constructors: QuaternionCategory
++ Also See:
++ AMS Classifications:
++ Keywords: octonion, non-associative algebra, Cayley-Dixon
++ References: e.g. I.L Kantor, A.S. Solodovnikov:
++  Hypercomplex Numbers, Springer Verlag Heidelberg, 1989,
++  ISBN 0-387-96980-2
++ Description:
++ OctonionCategory gives the categorial frame for the
++ octonions, and eight-dimensional non-associative algebra,
++ doubling the the quaternions in the same way as doubling
++ the Complex numbers to get the quaternions.

OctonionCategory(R: CommutativeRing): Category ==
  -- we are cheating a little bit, algebras in \Language{}
-- are mainly considered to be associative, but that's not
-- an attribute and we can't guarantee that there is no piece
-- of code which implicitly
-- uses this. In a later version we shall properly combine
-- all this code in the context of general, non-associative
-- algebras, which are meanwhile implemented in \Language{}
Join(Algebra R, FullyRetractableTo R, FullyEvalableOver R) with
   conjugate: % -> %
     ++ conjugate(o) negates the imaginary parts i,j,k,E,I,J,K of octonian o.
   real:    % -> R
     ++ real(o) extracts real part of octonion o.
   imagi:   % -> R
     ++ imagi(o) extracts the i part of octonion o.
   imagj:   % -> R
     ++ imagj(o) extracts the j part of octonion o.
   imagk:   % -> R
     ++ imagk(o) extracts the k part of octonion o.
   imagE:   % -> R
     ++ imagE(o) extracts the imaginary E part of octonion o.
   imagI:   % -> R
     ++ imagI(o) extracts the imaginary I part of octonion o.
   imagJ:   % -> R
     ++ imagJ(o) extracts the imaginary J part of octonion o.
   imagK:   % -> R
     ++ imagK(o) extracts the imaginary K part of octonion o.
   norm:    % -> R
     ++ norm(o) returns the norm of an octonion, equal to
     ++ the sum of the squares
     ++ of its coefficients.
   octon: (R,R,R,R,R,R,R,R) -> %
     ++ octon(re,ri,rj,rk,rE,rI,rJ,rK) constructs an octonion
     ++ from scalars.
   if R has Finite then Finite
   if R has OrderedSet then OrderedSet
   if R has ConvertibleTo InputForm then ConvertibleTo InputForm
   if R has CharacteristicZero then CharacteristicZero
   if R has CharacteristicNonZero then CharacteristicNonZero
   if R has RealNumberSystem then
     abs:   % -> R
       ++ abs(o) computes the absolute value of an octonion, equal to
       ++ the square root of the \spadfunFrom{norm}{Octonion}.
   if R has IntegerNumberSystem then
     rational?    : % -> Boolean
       ++ rational?(o) tests if o is rational, i.e. that all seven
       ++ imaginary parts are 0.
     rational     : % -> Fraction Integer
```

```
        ++ rational(o) returns the real part if all seven
        ++ imaginary parts are 0.
        ++ Error: if o is not rational.
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(o) returns the real part if
        ++ all seven imaginary parts are 0, and "failed" otherwise.
    if R has Field then
      inv : % -> %
        ++ inv(o) returns the inverse of o if it exists.
add
    characteristic() ==
      characteristic()$R

    conjugate x ==
      octon(real x, - imagi x, - imagj x, - imagk x, - imagE x,_
      - imagI x, - imagJ x, - imagK x)

    map(fn, x)       ==
      octon(fn real x,fn imagi x,fn imagj x,fn imagk x, fn imagE x,_
      fn imagI x, fn imagJ x,fn imagK x)

    norm x ==
      real x * real x + imagi x * imagi x + _
      imagj x * imagj x + imagk x * imagk x + _
      imagE x * imagE x + imagI x * imagI x + _
      imagJ x * imagJ x + imagK x * imagK x

    x = y            ==
      (real x = real y) and (imagi x = imagi y) and _
      (imagj x = imagj y) and (imagk x = imagk y) and _
      (imagE x = imagE y) and (imagI x = imagI y) and _
      (imagJ x = imagJ y) and (imagK x = imagK y)

    x + y            ==
      octon(real x + real y, imagi x + imagi y,_
      imagj x + imagj y, imagk x + imagk y,_
      imagE x + imagE y, imagI x + imagI y,_
      imagJ x + imagJ y, imagK x + imagK y)

    - x              ==
      octon(- real x, - imagi x, - imagj x, - imagk x,_
      - imagE x, - imagI x, - imagJ x, - imagK x)

    r:R * x:%        ==
      octon(r * real x, r * imagi x, r * imagj x, r * imagk x,_
      r * imagE x, r * imagI x, r * imagJ x, r * imagK x)
      n:Integer * x:%  ==
        octon(n * real x, n * imagi x, n * imagj x, n * imagk x,_
        n * imagE x, n * imagI x, n * imagJ x, n * imagK x)

      coerce(r:R)      ==
        octon(r,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

      coerce(n:Integer)      ==
        octon(n :: R,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

      zero? x ==
        zero? real x and zero? imagi x and _
        zero? imagj x and zero? imagk x and _
        zero? imagE x and zero? imagI x and _
        zero? imagJ x and zero? imagK x

      retract(x):R ==
        not (zero? imagi x and zero? imagj x and zero? imagk x and _
        zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
          error "Cannot retract octonion."
        real x

      retractIfCan(x):Union(R,"failed") ==
        not (zero? imagi x and zero? imagj x and zero? imagk x and _
        zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
          "failed"
        real x

      coerce(x:%):OutputForm ==
          part,z : OutputForm
          y : %
          zero? x => (0$R) :: OutputForm
          not zero?(real x) =>
            y := octon(0$R,imagi(x),imagj(x),imagk(x),imagE(x),
              imagI(x),imagJ(x),imagK(x))
            zero? y => real(x) :: OutputForm
            (real(x) :: OutputForm) + (y :: OutputForm)
          -- we know that the real part is 0
          not zero?(imagi(x)) =>
            y := octon(0$R,0$R,imagj(x),imagk(x),imagE(x),
              imagI(x),imagJ(x),imagK(x))
            z :=
              part := "i"::Symbol::OutputForm
--              one? imagi(x) => part
              (imagi(x) = 1) => part
```

```
             (imagi(x) :: OutputForm) * part
           zero? y => z
           z + (y :: OutputForm)
         -- we know that the real part and i part are 0
         not zero?(imagj(x)) =>
           y := octon(0$R,0$R,0$R,imagk(x),imagE(x),
             imagI(x),imagJ(x),imagK(x))
           z :=
             part := "j"::Symbol::OutputForm
--             one? imagj(x) => part
             (imagj(x) = 1) => part
             (imagj(x) :: OutputForm) * part
           zero? y => z
           z + (y :: OutputForm)
         -- we know that the real part and i and j parts are 0
         not zero?(imagk(x)) =>
           y := octon(0$R,0$R,0$R,0$R,imagE(x),
             imagI(x),imagJ(x),imagK(x))
           z :=
             part := "k"::Symbol::OutputForm
--             one? imagk(x) => part
             (imagk(x) = 1) => part
             (imagk(x) :: OutputForm) * part
           zero? y => z
           z + (y :: OutputForm)
         -- we know that the real part,i,j,k parts are 0
         not zero?(imagE(x)) =>
           y := octon(0$R,0$R,0$R,0$R,0$R,
             imagI(x),imagJ(x),imagK(x))
           z :=
             part := "E"::Symbol::OutputForm
--             one? imagE(x) => part
             (imagE(x) = 1) => part
             (imagE(x) :: OutputForm) * part
           zero? y => z
           z + (y :: OutputForm)
         -- we know that the real part,i,j,k,E parts are 0
         not zero?(imagI(x)) =>
           y := octon(0$R,0$R,0$R,0$R,0$R,0$R,imagJ(x),imagK(x))
           z :=
             part := "I"::Symbol::OutputForm
--             one? imagI(x) => part
             (imagI(x) = 1) => part
             (imagI(x) :: OutputForm) * part
           zero? y => z
           z + (y :: OutputForm)
```

```
        -- we know that the real part,i,j,k,E,I parts are 0
        not zero?(imagJ(x)) =>
          y := octon(0$R,0$R,0$R,0$R,0$R,0$R,0$R,imagK(x))
          z :=
            part := "J"::Symbol::OutputForm
--            one? imagJ(x) => part
            (imagJ(x) = 1) => part
            (imagJ(x) :: OutputForm) * part
          zero? y => z
          z + (y :: OutputForm)
        -- we know that the real part,i,j,k,E,I,J parts are 0
        part := "K"::Symbol::OutputForm
--        one? imagK(x) => part
        (imagK(x) = 1) => part
        (imagK(x) :: OutputForm) * part

    if R has Field then
      inv x ==
        (norm x) = 0 => error "This octonion is not invertible."
        (inv norm x) * conjugate x

    if R has ConvertibleTo InputForm then
      convert(x:%):InputForm ==
        l : List InputForm := [convert("octon" :: Symbol),
          convert(real x)$R, convert(imagi x)$R, convert(imagj x)$R,_
            convert(imagk x)$R, convert(imagE x)$R,_
            convert(imagI x)$R, convert(imagJ x)$R,_
            convert(imagK x)$R]
        convert(l)$InputForm

    if R has OrderedSet then
      x < y ==
        real x = real y =>
         imagi x = imagi y =>
          imagj x = imagj y =>
           imagk x = imagk y =>
            imagE x = imagE y =>
             imagI x = imagI y =>
              imagJ x = imagJ y =>
               imagK x < imagK y
              imagJ x < imagJ y
             imagI x < imagI y
            imagE x < imagE y
           imagk x < imagk y
          imagj x < imagj y
         imagi x < imagi y
```

```
          real x < real y

      if R has RealNumberSystem then
        abs x == sqrt norm x

      if R has IntegerNumberSystem then
        rational? x ==
          (zero? imagi x) and (zero? imagj x) and (zero? imagk x) and _
          (zero? imagE x) and (zero? imagI x) and (zero? imagJ x) and _
          (zero? imagK x)

        rational  x ==
          rational? x => rational real x
          error "Not a rational number"

        rationalIfCan x ==
          rational? x => rational real x
          "failed"
```

⟨*OC.dotabb*⟩≡

```
"OC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OC" -> "ALGEBRA"
"OC" -> "FEVALAB"
"OC" -> "FRETRCT"
```

⟨*OC.dotfull*⟩≡

```
"OctonionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OctonionCategory(a:CommutativeRing)" -> "Algebra(a:CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" -> "FullyEvalableOver(CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

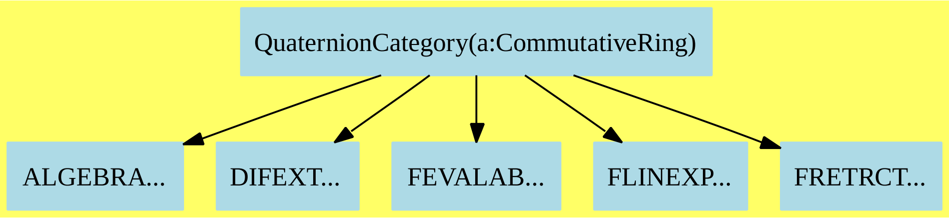
⟨*OC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OctonionCategory(a:CommutativeRing)" [color=lightblue];
"OctonionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"OctonionCategory(a:CommutativeRing)" -> "FEVALAB..."
"OctonionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

12.0.167 QuaternionCategory (QUATCAT)



⟨*QuaternionCategory.input*⟩≡

```
)set break resume
)sys rm -f QuaternionCategory.output
)spool QuaternionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuaternionCategory
--R QuaternionCategory R: CommutativeRing  is a category constructor
--R Abbreviation for QuaternionCategory is QUATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QUATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                D : % -> % if R has DIFRING
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       abs : % -> R if R has RNS
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             conjugate : % -> %
--R hash : % -> SingleInteger            imagI : % -> R
--R imagJ : % -> R                       imagK : % -> R
--R inv : % -> % if R has FIELD          latex : % -> String
--R map : ((R -> R),%) -> %              norm : % -> R
--R one? : % -> Boolean                  quatern : (R,R,R,R) -> %
--R real : % -> R                        recip : % -> Union(%,"failed")
--R retract : % -> R                     sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has FIELD
```

```
--R ?*? : (%,Fraction Integer) -> % if R has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R ?^? : (%,Integer) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
--R convert : % -> InputForm if R has KONVERT INFORM
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Equation R) -> % if R has EVALAB R
--R eval : (%,Equation R) -> % if R has EVALAB R
--R eval : (%,R,R) -> % if R has EVALAB R
--R eval : (%,List R,List R) -> % if R has EVALAB R
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
```

```
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*QuaternionCategory.help*⟩≡

```
====================================================================
QuaternionCategory examples
====================================================================

See Also:
o )show QuaternionCategory
```

See:

⇐ "Algebra" (ALGEBRA) 11.0.152 on page 1045
⇐ "DifferentialExtension" (DIFEXT) 11.0.153 on page 1053
⇐ "FullyEvalableOver" (FEVALAB) 4.0.44 on page 178
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.154 on page 1060
⇐ "FullyRetractableTo" (FRETRCT) 3.0.27 on page 100

Exports:

0	1	abs	characteristic
charthRoot	coerce	conjugate	convert
D	differentiate	eval	hash
imagI	imagJ	imagK	inv
latex	map	max	min
norm	one?	quatern	rational
rational?	rationalIfCan	real	recip
reducedSystem	retract	retractIfCan	sample
subtractIfCan	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?
?>?	?>=?	?.?	

Attributes Exported:

- if #1 has EntireRing then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagI : % -> R
imagJ : % -> R
imagK : % -> R
quatern : (R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
```

```
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
differentiate : (%,(R -> R)) -> %
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
one? : % -> Boolean
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1045) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p100) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p1053) DifferentialExtension(R:CommutativeRing):

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
?*? : (%,%) -> %
```

These exports come from (p178) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R)
where R:CommutativeRing:

```
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1116) DivisionRing():

```
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

⟨*category QUATCAT QuaternionCategory*⟩≡

```
)abbrev category QUATCAT QuaternionCategory
++ Author: Robert S. Sutor
++ Date Created: 23 May 1990
++ Change History:
++   10 September 1990
++ Basic Operations: (Algebra)
++   abs, conjugate, imagI, imagJ, imagK, norm, quatern, rational,
++   rational?, real
++ Related Constructors: Quaternion, QuaternionCategoryFunctions2
++ Also See: DivisionRing
++ AMS Classifications: 11R52
++ Keywords: quaternions, division ring, algebra
++ Description:
++ \spadtype{QuaternionCategory} describes the category of quaternions
++ and implements functions that are not representation specific.

QuaternionCategory(R: CommutativeRing): Category ==
  Join(Algebra R, FullyRetractableTo R, DifferentialExtension R,
   FullyEvalableOver R, FullyLinearlyExplicitRingOver R) with

     conjugate: $ -> $
       ++ conjugate(q) negates the imaginary parts of quaternion \spad{q}.
     imagI:   $ -> R
       ++ imagI(q) extracts the imaginary i part of quaternion \spad{q}.
     imagJ:   $ -> R
       ++ imagJ(q) extracts the imaginary j part of quaternion \spad{q}.
     imagK:   $ -> R
       ++ imagK(q) extracts the imaginary k part of quaternion \spad{q}.
     norm:    $ -> R
       ++ norm(q) computes the norm of \spad{q} (the sum of the
       ++ squares of the components).
     quatern: (R,R,R,R) -> $
       ++ quatern(r,i,j,k) constructs a quaternion from scalars.
     real:    $ -> R
       ++ real(q) extracts the real part of quaternion \spad{q}.
```

```
    if R has EntireRing then EntireRing
    if R has OrderedSet then OrderedSet
    if R has Field then DivisionRing
    if R has ConvertibleTo InputForm then ConvertibleTo InputForm
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has RealNumberSystem then
      abs    : $ -> R
        ++ abs(q) computes the absolute value of quaternion \spad{q}
        ++ (sqrt of norm).
    if R has IntegerNumberSystem then
      rational?    : $ -> Boolean
        ++ rational?(q) returns {\it true} if all the imaginary
        ++ parts of \spad{q} are zero and the real part can be
        ++ converted into a rational number, and {\it false}
        ++ otherwise.
      rational     : $ -> Fraction Integer
        ++ rational(q) tries to convert \spad{q} into a
        ++ rational number. Error: if this is not
        ++ possible. If \spad{rational?(q)} is true, the
        ++ conversion will be done and the rational number returned.
      rationalIfCan: $ -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(q) returns \spad{q} as a rational number,
        ++ or "failed" if this is not possible.
        ++ Note that if \spad{rational?(q)} is true, the conversion
        ++ can be done and the rational number will be returned.

add

      characteristic() ==
        characteristic()$R

      conjugate x      ==
        quatern(real x, - imagI x, - imagJ x, - imagK x)

      map(fn, x)       ==
        quatern(fn real x, fn imagI x, fn imagJ x, fn imagK x)

      norm x ==
        real x * real x + imagI x * imagI x +
          imagJ x * imagJ x + imagK x * imagK x

      x = y            ==
        (real x = real y) and (imagI x = imagI y) and
          (imagJ x = imagJ y) and (imagK x = imagK y)
        x + y             ==
          quatern(real x + real y, imagI x + imagI y,
            imagJ x + imagJ y, imagK x + imagK y)

        x - y             ==
          quatern(real x - real y, imagI x - imagI y,
            imagJ x - imagJ y, imagK x - imagK y)

        - x               ==
          quatern(- real x, - imagI x, - imagJ x, - imagK x)

        r:R * x:$         ==
          quatern(r * real x, r * imagI x, r * imagJ x, r * imagK x)

        n:Integer * x:$  ==
          quatern(n * real x, n * imagI x, n * imagJ x, n * imagK x)

        differentiate(x:$, d:R -> R) ==
          quatern(d real x, d imagI x, d imagJ x, d imagK x)

        coerce(r:R)       ==
          quatern(r,0$R,0$R,0$R)

        coerce(n:Integer)       ==
          quatern(n :: R,0$R,0$R,0$R)

        one? x ==
--            one? real x and zero? imagI x and
          (real x) = 1 and zero? imagI x and
            zero? imagJ x and zero? imagK x

        zero? x ==
          zero? real x and zero? imagI x and
            zero? imagJ x and zero? imagK x

        retract(x):R ==
          not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
            error "Cannot retract quaternion."
          real x

        retractIfCan(x):Union(R,"failed") ==
          not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
            "failed"
          real x
        coerce(x:$):OutputForm ==
          part,z : OutputForm
          y : $
          zero? x => (0$R) :: OutputForm
          not zero?(real x) =>
            y := quatern(0$R,imagI(x),imagJ(x),imagK(x))
            zero? y => real(x) :: OutputForm
            (real(x) :: OutputForm) + (y :: OutputForm)
          -- we know that the real part is 0
          not zero?(imagI(x)) =>
            y := quatern(0$R,0$R,imagJ(x),imagK(x))
            z :=
              part := "i"::Symbol::OutputForm
--              one? imagI(x) => part
              (imagI(x) = 1) => part
              (imagI(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part and i part are 0
          not zero?(imagJ(x)) =>
            y := quatern(0$R,0$R,0$R,imagK(x))
            z :=
              part := "j"::Symbol::OutputForm
--              one? imagJ(x) => part
              (imagJ(x) = 1) => part
              (imagJ(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part and i and j parts are 0
          part := "k"::Symbol::OutputForm
--          one? imagK(x) => part
          (imagK(x) = 1) => part
          (imagK(x) :: OutputForm) * part

        if R has Field then
          inv x ==
            norm x = 0 => error "This quaternion is not invertible."
            (inv norm x) * conjugate x

        if R has ConvertibleTo InputForm then
          convert(x:$):InputForm ==
            l : List InputForm := [convert("quatern" :: Symbol),
              convert(real x)$R, convert(imagI x)$R, convert(imagJ x)$R,
                convert(imagK x)$R]
            convert(l)$InputForm
```

```
      if R has OrderedSet then
        x < y ==
          real x = real y =>
            imagI x = imagI y =>
              imagJ x = imagJ y =>
                imagK x < imagK y
              imagJ x < imagJ y
            imagI x < imagI y
          real x < real y

      if R has RealNumberSystem then
        abs x == sqrt norm x

      if R has IntegerNumberSystem then
        rational? x ==
          (zero? imagI x) and (zero? imagJ x) and (zero? imagK x)

        rational  x ==
          rational? x => rational real x
          error "Not a rational number"

        rationalIfCan x ==
          rational? x => rational real x
          "failed"
```

⟨*QUATCAT.dotabb*⟩≡

```
  "QUATCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
  "QUATCAT" -> "ALGEBRA"
  "QUATCAT" -> "DIFEXT"
  "QUATCAT" -> "FEVALAB"
  "QUATCAT" -> "FLINEXP"
  "QUATCAT" -> "FRETRCT"
```

⟨*QUATCAT.dotfull*⟩≡

```
"QuaternionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
"QuaternionCategory(a:CommutativeRing)" ->
   "Algebra(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "DifferentialExtension(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyEvalableOver(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

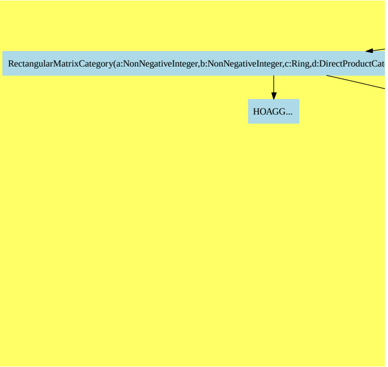
⟨*QUATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QuaternionCategory(a:CommutativeRing)" [color=lightblue];
"QuaternionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"QuaternionCategory(a:CommutativeRing)" -> "DIFEXT..."
"QuaternionCategory(a:CommutativeRing)" -> "FEVALAB..."
"QuaternionCategory(a:CommutativeRing)" -> "FLINEXP..."
"QuaternionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

12.0.168 SquareMatrixCategory (SMATCAT)


RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCat
HOAGG...

We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The SquareMatrix domain is for square matrices of fixed dimension.

⟨*SquareMatrixCategory.input*⟩≡

```
  )set break resume
  )sys rm -f SquareMatrixCategory.output
  )spool SquareMatrixCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show SquareMatrixCategory
  --R SquareMatrixCategory(ndim: NonNegativeInteger,R: Ring,Row: DirectProductCateg
  --R Abbreviation for SquareMatrixCategory is SMATCAT
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for SMATCAT
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (Row,%) -> Row                  ?*? : (%,Col) -> Col
  --R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
```

```
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : % -> % if R has DIFRING           D : (%,(R -> R)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        antisymmetric? : % -> Boolean
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              column : (%,Integer) -> Col
--R copy : % -> %                         diagonal : % -> Row
--R diagonal? : % -> Boolean              diagonalMatrix : List R -> %
--R diagonalProduct : % -> R              elt : (%,Integer,Integer) -> R
--R elt : (%,Integer,Integer,R) -> R      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             latex : % -> String
--R listOfLists : % -> List List R        map : ((R -> R),%) -> %
--R map : (((R,R) -> R),%,%) -> %         matrix : List List R -> %
--R maxColIndex : % -> Integer            maxRowIndex : % -> Integer
--R minColIndex : % -> Integer            minRowIndex : % -> Integer
--R ncols : % -> NonNegativeInteger       nrows : % -> NonNegativeInteger
--R one? : % -> Boolean                   qelt : (%,Integer,Integer) -> R
--R recip : % -> Union(%,"failed")        retract : % -> R
--R row : (%,Integer) -> Row              sample : () -> %
--R scalarMatrix : R -> %                 square? : % -> Boolean
--R symmetric? : % -> Boolean             trace : % -> R
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
```

```
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,(R -> R)) -> %
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R inverse : % -> Union(%,"failed") if R has FIELD
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List Col if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate
--R rank : % -> NonNegativeInteger if R has INTDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

⟨*SquareMatrixCategory.help*⟩≡

```
====================================================================
SquareMatrixCategory examples
====================================================================

See Also:
o )show SquareMatrixCategory
```

See:

⇐ "BiModule" (BMODULE) 9.0.122 on page 800
⇐ "DifferentialExtension" (DIFEXT) 11.0.153 on page 1053
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.154 on page 1060
⇐ "FullyRetractableTo" (FRETRCT) 3.0.27 on page 100
⇐ "MatrixCategory" (MATCAT) 6.0.88 on page 458
⇐ "RectangularMatrixCategory" (RMATCAT) 10.0.147 on page 994

Exports:

0
1
antisymmetric?
any?
characteristic
coerce
column
copy
count
D
determinant
differentiate
diagonal
diagonal?
diagonalMatrix
diagonalProduct
elt
empty
empty?
eq?
eval
every?
exquo
hash
inverse
latex
less?
listOfLists
map
map!
matrix
maxColIndex
maxRowIndex
member?
members
minColIndex
minordet
minRowIndex
more?
ncols
nrows
nullSpace
nullity
one?
parts
qelt
rank
recip
reducedSystem
retract
retractIfCan
row
rowEchelon
sample
scalarMatrix
size?
square?
subtractIfCan
symmetric?
trace
zero?
#?
?^?
?*?
?**?
?+?
?-?
-?
?=?
?~=?
?/?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **nil**

TPDHERE: How did MATCAT get in the type tower?

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
```

These are directly exported but not implemented:

```
diagonalMatrix : List R -> %
minordet : % -> R if R has commutative *
scalarMatrix : R -> %
?*? : (Row,%) -> Row
?*? : (%,Col) -> Col
```

These are implemented by this category:

```
coerce : R -> %
diagonal : % -> Row
diagonalProduct : % -> R
differentiate : (%,(R -> R)) -> %
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
retract : % -> R
retractIfCan : % -> Union(R,"failed")
trace : % -> R
?**? : (%,Integer) -> % if R has FIELD
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p1053) DifferentialExtension(R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from
(p994) RectangularMatrixCategory(ndim,ndim,R,Row,Col)
where ndim:NonNegativeInteger,R:Ring,Row:DirectProductCategory(ndim,R)
Col:DirectProductCategory(ndim,R):

```
antisymmetric? : % -> Boolean
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
diagonal? : % -> Boolean
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
less? : (%,NonNegativeInteger) -> Boolean
listOfLists : % -> List List R
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
matrix : List List R -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer,Integer) -> R
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
size? : (%,NonNegativeInteger) -> Boolean
square? : % -> Boolean
symmetric? : % -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p100) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

⟨*category SMATCAT SquareMatrixCategory*⟩≡

```
)abbrev category SMATCAT SquareMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: SquareMatrix(ndim,R)
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{SquareMatrixCategory} is a general square matrix category which
++ allows different representations and indexing schemes.  Rows and
++ columns may be extracted with rows returned as objects of
++ type Row and colums returned as objects of type Col.

SquareMatrixCategory(ndim,R,Row,Col): Category == Definition where
  ndim : NonNegativeInteger
  R    : Ring
  Row  : DirectProductCategory(ndim,R)
  Col  : DirectProductCategory(ndim,R)
  I ==> Integer

  Definition ==> Join(DifferentialExtension R, BiModule(R, R),_
                      RectangularMatrixCategory(ndim,ndim,R,Row,Col),_
                      FullyRetractableTo R,_
                      FullyLinearlyExplicitRingOver R) with
    if R has CommutativeRing then Module(R)
    scalarMatrix: R -> %
      ++ \spad{scalarMatrix(r)} returns an n-by-n matrix with r's on the
      ++ diagonal and zeroes elsewhere.
    diagonalMatrix: List R -> %
      ++ \spad{diagonalMatrix(l)} returns a diagonal matrix with the elements
      ++ of l on the diagonal.
    diagonal: % -> Row
      ++ \spad{diagonal(m)} returns a row consisting of the elements on the
      ++ diagonal of the matrix m.
    trace: % -> R
      ++ \spad{trace(m)} returns the trace of the matrix m. this is the sum
      ++ of the elements on the diagonal of the matrix m.
    diagonalProduct: % -> R
      ++ \spad{diagonalProduct(m)} returns the product of the elements on the
      ++ diagonal of the matrix m.
    "*": (%,Col) -> Col
      ++ \spad{x * c} is the product of the matrix x and the column vector c.
      ++ Error: if the dimensions are incompatible.
    "*": (Row,%) -> Row
      ++ \spad{r * x} is the product of the row vector r and the matrix x.
      ++ Error: if the dimensions are incompatible.

--% Linear algebra
```

```
     if R has commutative("*") then
       Algebra R
       determinant: % -> R
         ++ \spad{determinant(m)} returns the determinant of the matrix m.
       minordet: % -> R
         ++ \spad{minordet(m)} computes the determinant of the matrix m
         ++ using minors.
     if R has Field then
       inverse: % -> Union(%,"failed")
         ++ \spad{inverse(m)} returns the inverse of the matrix m, if that
         ++ matrix is invertible and returns "failed" otherwise.
       "**": (%,Integer) -> %
         ++ \spad{m**n} computes an integral power of the matrix m.
         ++ Error: if the matrix is not invertible.

    add
     minr ==> minRowIndex
     maxr ==> maxRowIndex
     minc ==> minColIndex
     maxc ==> maxColIndex
     mini ==> minIndex
     maxi ==> maxIndex

     positivePower:(%,Integer) -> %
     positivePower(x,n) ==
--       one? n => x
       (n = 1) => x
       odd? n => x * positivePower(x,n - 1)
       y := positivePower(x,n quo 2)
       y * y

     x:% ** n:NonNegativeInteger ==
       zero? n => scalarMatrix 1
       positivePower(x,n)

     coerce(r:R) == scalarMatrix r

     equation2R: Vector % -> Matrix R

     differentiate(x:%,d:R -> R) == map(d,x)

     diagonal x ==
       v:Vector(R) := new(ndim,0)
       for i in minr x .. maxr x
         for j in minc x .. maxc x
```

```
      for k in minIndex v .. maxIndex v repeat
        qsetelt_!(v, k, qelt(x, i, j))
  directProduct v

retract(x:%):R ==
  diagonal? x => retract diagonal x
  error "Not retractable"

retractIfCan(x:%):Union(R, "failed") ==
  diagonal? x => retractIfCan diagonal x
  "failed"

equation2R v ==
  ans:Matrix(Col) := new(ndim,#v,0)
  for i in minr ans .. maxr ans repeat
    for j in minc ans .. maxc ans repeat
      qsetelt_!(ans, i, j, column(qelt(v, j), i))
  reducedSystem ans

reducedSystem(x:Matrix %):Matrix(R) ==
  empty? x => new(0,0,0)
  reduce(vertConcat, [equation2R row(x, i)
                         for i in minr x .. maxr x])$List(Matrix R)

reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  vh:Vector(R) :=
    empty? v => new(0,0)
    rh := reducedSystem(v::Matrix %)@Matrix(R)
    column(rh, minColIndex rh)
  [reducedSystem(m)@Matrix(R), vh]

trace x ==
  tr : R := 0
  for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
    tr := tr + x(i,j)
  tr

diagonalProduct x ==
  pr : R := 1
  for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
    pr := pr * x(i,j)
  pr

if R has Field then
```

```
    x:% ** n:Integer ==
      zero? n => scalarMatrix 1
      positive? n => positivePower(x,n)
      (xInv := inverse x) case "failed" =>
        error "**: matrix must be invertible"
      positivePower(xInv :: %,-n)
```

⟨*SMATCAT.dotabb*⟩≡

```
"SMATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SMATCAT" -> "BMODULE"
"SMATCAT" -> "DIFEXT"
"SMATCAT" -> "FLINEXP"
"SMATCAT" -> "FRETRCT"
"SMATCAT" -> "RMATCAT"
```

⟨*SMATCAT.dotfull*⟩≡

```
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "FullyLinearlyExplicitRingOver(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "FullyRetractableTo(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d
```

⟨*SMATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 [color=lightblue];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "FLINEXP..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "FRETRCT..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProd

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."
```

```
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
  -> "HOAGG..."

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

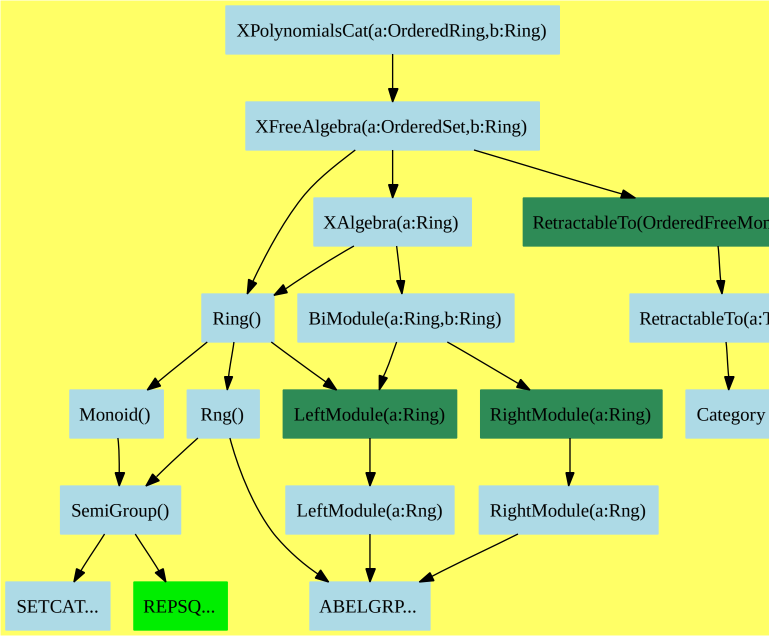
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

}
```

12.0.169 XPolynomialsCat (XPOLYC)


XPolynomialsCat(a:OrderedRing,b:Ring)
XFreeAlgebra(a:OrderedSet,b:Ring)
XAlgebra(a:Ring)
RetractableTo(OrderedFreeMon
Ring()
BiModule(a:Ring,b:Ring)
RetractableTo(a:T
Monoid()
Rng()
LeftModule(a:Ring)
RightModule(a:Ring)
Category
SemiGroup()
LeftModule(a:Rng)
RightModule(a:Rng)
SETCAT...
REPSQ...
ABELGRP...

⟨*XPolynomialsCat.input*⟩≡

```
)set break resume
)sys rm -f XPolynomialsCat.output
)spool XPolynomialsCat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XPolynomialsCat
--R XPolynomialsCat(vl: OrderedSet,R: Ring)  is a category constructor
--R Abbreviation for XPolynomialsCat is XPOLYC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XPOLYC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (vl,%) -> %                    ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                     ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
```

```
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coef : (%,%) -> R                     coerce : vl -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              constant : % -> R
--R constant? : % -> Boolean               degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger              latex : % -> String
--R lquo : (%,%) -> %                     lquo : (%,vl) -> %
--R map : ((R -> R),%) -> %               mirror : % -> %
--R monomial? : % -> Boolean               one? : % -> Boolean
--R quasiRegular : % -> %                 quasiRegular? : % -> Boolean
--R recip : % -> Union(%,"failed")        rquo : (%,%) -> %
--R rquo : (%,vl) -> %                    sample : () -> %
--R varList : % -> List vl                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid vl) -> R
--R coerce : OrderedFreeMonoid vl -> %
--R lquo : (%,OrderedFreeMonoid vl) -> %
--R maxdeg : % -> OrderedFreeMonoid vl
--R mindeg : % -> OrderedFreeMonoid vl
--R mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R monom : (OrderedFreeMonoid vl,R) -> %
--R retract : % -> OrderedFreeMonoid vl
--R retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
--R rquo : (%,OrderedFreeMonoid vl) -> %
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R sh : (%,%) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

⟨*XPolynomialsCat.help*⟩≡

```
====================================================================
XPolynomialsCat examples
====================================================================

See Also:
o )show XPolynomialsCat
```

See:

⇐ "XFreeAlgebra" (XFALG) 11.0.159 on page 1094

Exports:

0	1	characteristic	coef	coerce
constant	constant?	degree	hash	latex
lquo	map	maxdeg	mindeg	mindegTerm
mirror	monom	monomial?	one?	quasiRegular
quasiRegular?	recip	retract	retractIfCan	rquo
sample	sh	subtractIfCan	trunc	varList
zero?	?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?	

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : % -> NonNegativeInteger
maxdeg : % -> OrderedFreeMonoid vl
trunc : (%,NonNegativeInteger) -> %
```

These exports come from (p77) Aggregate():

These exports come from (p1094) XFreeAlgebra(vl:OrderedSet,R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
coerce : % -> OutputForm
coerce : R -> %
coerce : OrderedFreeMonoid vl -> %
coerce : Integer -> %
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
hash : % -> SingleInteger
latex : % -> String
lquo : (%,OrderedFreeMonoid vl) -> %
lquo : (%,%) -> %
lquo : (%,vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
monomial? : % -> Boolean
one? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
recip : % -> Union(%,"failed")
retract : % -> OrderedFreeMonoid vl
retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sample : () -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
subtractIfCan : (%,%) -> Union(%,"failed")
varList : % -> List vl
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (vl,%) -> %
```

These exports come from (p130) SetCategory():

⟨*category XPOLYC XPolynomialsCat*⟩≡

```
)abbrev category XPOLYC XPolynomialsCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The Category of polynomial rings with non-commutative variables.
++ The coefficient ring may be non-commutative too.
++ However coefficients commute with vaiables.

XPolynomialsCat(vl:OrderedSet,R:Ring):Category == Export where
  WORD ==> OrderedFreeMonoid(vl)

  Export == XFreeAlgebra(vl,R) with
    maxdeg: % -> WORD
      ++ \spad{maxdeg(p)} returns the greatest leading word in the
      ++ support of \spad{p}.
    degree: % -> NonNegativeInteger
      ++ \spad{degree(p)} returns the degree of \spad{p}.
      ++  Note that the degree of a word is its length.
    trunc : (% , NonNegativeInteger) -> %
      ++  \spad{trunc(p,n)} returns the polynomial \spad{p} truncated
      ++ at order \spad{n}.
```

⟨*XPOLYC.dotabb*⟩≡

```
"XPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPOLYC" -> "XFALG"
```

⟨*XPOLYC.dotfull*⟩≡

```
"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"
```

⟨*XPOLYC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"
"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

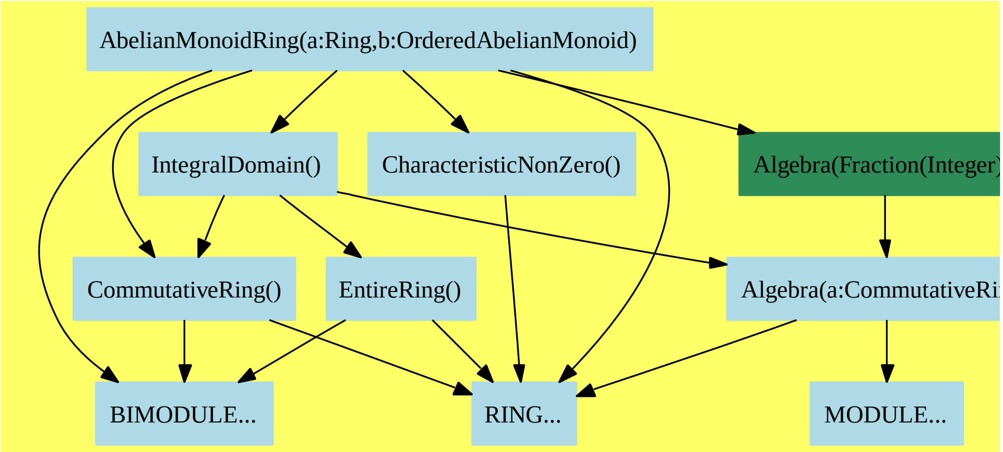
"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 13

Category Layer 12

13.0.170 AbelianMonoidRing (AMR)


AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)
IntegralDomain()
CharacteristicNonZero()
Algebra(Fraction(Integer)
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRi
BIMODULE...
RING...
MODULE...

⟨*AbelianMonoidRing.input*⟩≡

```
)set break resume
)sys rm -f AbelianMonoidRing.output
)spool AbelianMonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianMonoidRing
--R AbelianMonoidRing(R: Ring,E: OrderedAbelianMonoid)  is a category constructor
--R Abbreviation for AbelianMonoidRing is AMR
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AMR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> E                       hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : R -> % if R has COMRING
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*AbelianMonoidRing.help*⟩≡

```
====================================================================
AbelianMonoidRing examples
====================================================================

See Also:
o )show AbelianMonoidRing
```

See:

⇒ "FiniteAbelianMonoidRing" (FAMR) 14.0.175 on page 1259
⇒ "PowerSeriesCategory" (PSCAT) 14.0.177 on page 1280
⇐ "BiModule" (BMODULE) 9.0.122 on page 800
⇐ "Ring" (RING) 9.0.129 on page 854

Exports:

0	1	associates?	characteristic
charthRoot	coefficient	coerce	degree
exquo	hash	latex	leadingCoefficient
leadingMonomial	map	monomial	monomial?
one?	recip	reductum	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if \$ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- if \$ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,E) -> R
degree : % -> E
leadingCoefficient : % -> R
leadingMonomial : % -> %
monomial : (R,E) -> %
reductum : % -> %
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
map : ((R -> R),%) -> %
monomial? : % -> Boolean
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
```

These exports come from (p854) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p800) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p1153) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
coerce : % -> % if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p932) CommutativeRing():

```
coerce : R -> % if R has COMRING
```

These exports come from (p1045) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
```

⟨*category AMR AbelianMonoidRing*⟩≡

```
)abbrev category AMR AbelianMonoidRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Abelian monoid ring elements (not necessarily of finite support)
++ of this ring are of the form formal SUM (r_i * e_i)
++ where the r_i are coefficents and the e_i, elements of the
++ ordered abelian monoid, are thought of as exponents or monomials.
++ The monomials commute with each other, and with
++ the coefficients (which themselves may or may not be commutative).
++ See \spadtype{FiniteAbelianMonoidRing} for the case of finite support
++ a useful common model for polynomials and power series.
++ Conceptually at least, only the non-zero terms are ever operated on.

AbelianMonoidRing(R:Ring, E:OrderedAbelianMonoid): Category ==
     Join(Ring,BiModule(R,R)) with
  leadingCoefficient: % -> R
    ++ leadingCoefficient(p) returns the coefficient highest
    ++ degree term of p.
  leadingMonomial: % -> %
    ++ leadingMonomial(p) returns the monomial of p with the highest degree.
  degree: % -> E
    ++ degree(p) returns the maximum of the exponents of the terms of p.
  map: (R -> R, %) -> %
    ++ map(fn,u) maps function fn onto the coefficients
    ++ of the non-zero monomials of u.
  monomial?: % -> Boolean
    ++ monomial?(p) tests if p is a single monomial.
  monomial: (R,E) -> %
    ++ monomial(r,e) makes a term from a coefficient r and an exponent e.
  reductum: % -> %
    ++ reductum(u) returns u minus its leading monomial
    ++ returns zero if handed the zero element.
  coefficient: (%,E) -> R
```

```
      ++ coefficient(p,e) extracts the coefficient of the monomial with
      ++ exponent e from polynomial p, or returns zero if exponent
      ++ is not present.
    if R has Field then "/": (%,R) -> %
      ++ p/c divides p by the coefficient c.
    if R has CommutativeRing then
       CommutativeRing
       Algebra R
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has IntegralDomain then IntegralDomain
    if R has Algebra Fraction Integer then Algebra Fraction Integer
   add
    monomial? x == zero? reductum x

    map(fn:R -> R, x: %) ==
          -- this default definition assumes that reductum is cheap
       zero? x => 0
       r:=fn leadingCoefficient x
       zero? r => map(fn,reductum x)
       monomial(r, degree x) + map(fn,reductum x)

    if R has Algebra Fraction Integer then
      q:Fraction(Integer) * p:% == map(x1 +-> q * x1, p)
```

⟨*AMR.dotabb*⟩≡

```
  "AMR"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
  "AMR" -> "RING"
  "AMR" -> "BMODULE"
  "AMR" -> "INTDOM"
  "AMR" -> "CHARNZ"
  "AMR" -> "COMRING"
  "AMR" -> "ALGEBRA"
```

⟨*AMR.dotfull*⟩≡

```
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "Ring()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BiModule(a:Ring,b:OrderedAbelianMonoid)"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"
```

⟨*AMR.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

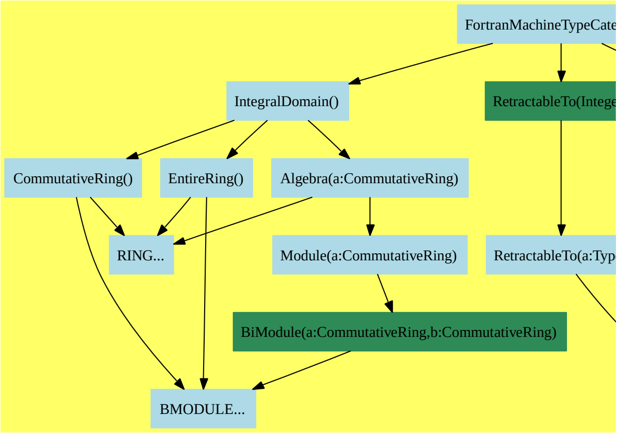
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

13.0.171 FortranMachineTypeCategory (FMTC)


FortranMachineTypeCate
IntegralDomain()
RetractableTo(Intege
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
RetractableTo(a:Typ
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*FortranMachineTypeCategory.input*⟩≡

```
)set break resume
)sys rm -f FortranMachineTypeCategory.output
)spool FortranMachineTypeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranMachineTypeCategory
--R FortranMachineTypeCategory  is a category constructor
--R Abbreviation for FortranMachineTypeCategory is FMTC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMTC
--R
--R-------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : Integer -> %                 coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
```

```
--R retract : % -> Integer                sample : () -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FortranMachineTypeCategory.help*⟩≡

```
====================================================================
FortranMachineTypeCategory examples
====================================================================

See Also:
o )show FortranMachineTypeCategory
```

See:

⇐ "IntegralDomain" (INTDOM) 12.0.164 on page 1153
⇐ "OrderedSet" (ORDSET) 4.0.56 on page 242
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	max	min
one?	recip	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal	zero?
?~=?	?^?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?			

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D,D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1153) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
coerce : % -> %
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
```

```
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p62) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

⟨*category FMTC FortranMachineTypeCategory*⟩≡

```
)abbrev category FMTC FortranMachineTypeCategory
++ Author: Mike Dewar
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, MachineInteger, MachineFloat, MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A category of domains which model machine arithmetic
++ used by machines in the AXIOM-NAG link.

FortranMachineTypeCategory():Category == Join(IntegralDomain,OrderedSet,
                                              RetractableTo(Integer) )
```

⟨*FMTC.dotabb*⟩≡

```
"FMTC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FMTC" -> "INTDOM"
"FMTC" -> "ORDSET"
"FMTC" -> "RETRACT"
```

⟨*FMTC.dotfull*⟩≡

```
"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"
```

⟨*FMTC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"

"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

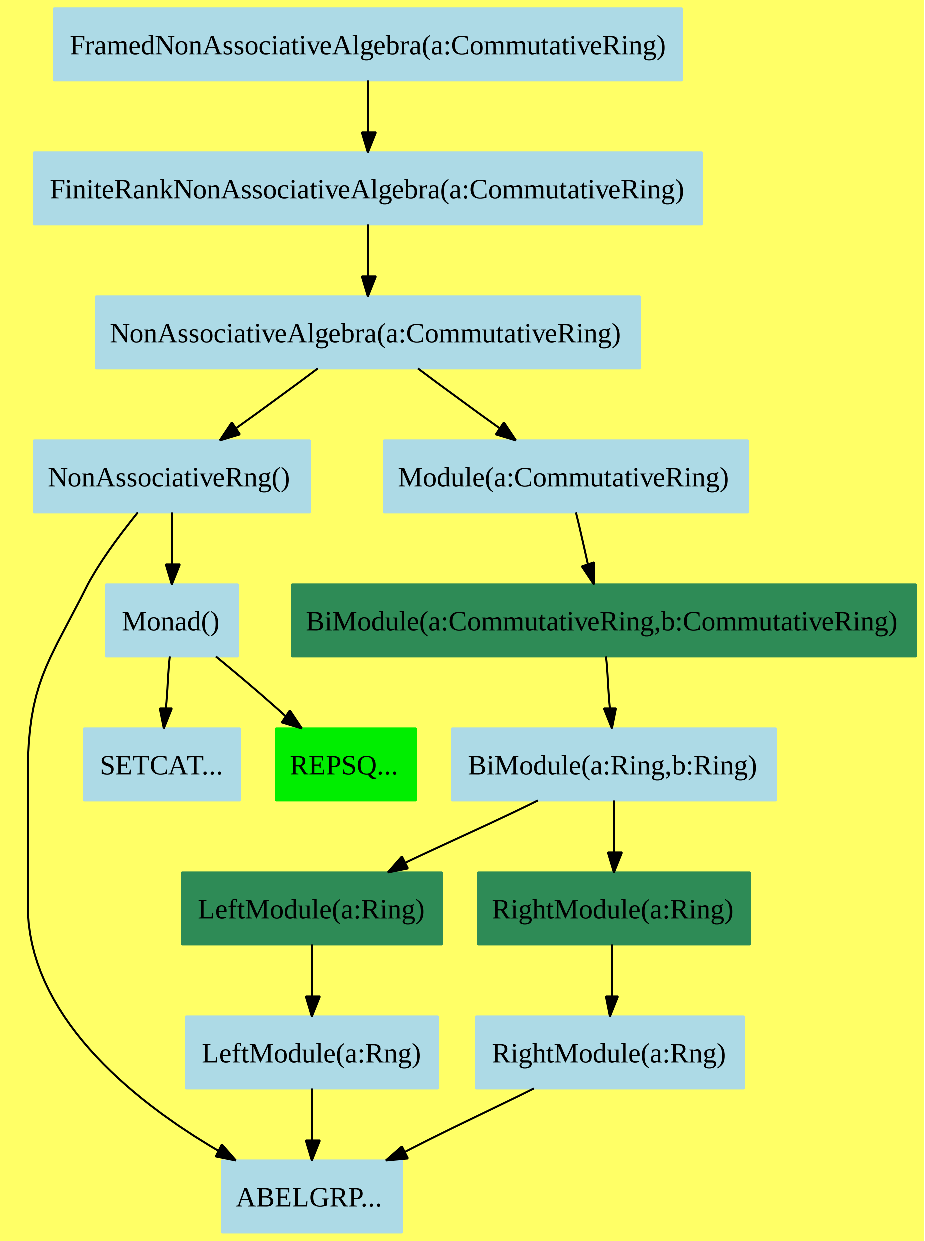
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

13.0.172 FramedNonAssociativeAlgebra (FRNAALG)


FramedNonAssociativeAlgebra(a:CommutativeRing)
FiniteRankNonAssociativeAlgebra(a:CommutativeRing)
NonAssociativeAlgebra(a:CommutativeRing)
NonAssociativeRng()
Module(a:CommutativeRing)
Monad()
BiModule(a:CommutativeRing,b:CommutativeRing)
SETCAT...
REPSQ...
BiModule(a:Ring,b:Ring)
LeftModule(a:Ring)
RightModule(a:Ring)
LeftModule(a:Rng)
RightModule(a:Rng)
ABELGRP...

⟨*FramedNonAssociativeAlgebra.input*⟩≡

```
)set break resume
)sys rm -f FramedNonAssociativeAlgebra.output
```

```
)spool FramedNonAssociativeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedNonAssociativeAlgebra
--R FramedNonAssociativeAlgebra R: CommutativeRing  is a category constructor
--R Abbreviation for FramedNonAssociativeAlgebra is FRNAALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRNAALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           alternative? : () -> Boolean
--R antiAssociative? : () -> Boolean      antiCommutative? : () -> Boolean
--R antiCommutator : (%,%) -> %           apply : (Matrix R,%) -> %
--R associative? : () -> Boolean          associator : (%,%,%) -> %
--R basis : () -> Vector %                coerce : % -> OutputForm
--R commutative? : () -> Boolean          commutator : (%,%) -> %
--R convert : Vector R -> %               convert : % -> Vector R
--R coordinates : % -> Vector R           ?.? : (%,Integer) -> R
--R flexible? : () -> Boolean             hash : % -> SingleInteger
--R jacobiIdentity? : () -> Boolean       jordanAdmissible? : () -> Boolean
--R jordanAlgebra? : () -> Boolean        latex : % -> String
--R leftAlternative? : () -> Boolean      leftDiscriminant : () -> R
--R leftDiscriminant : Vector % -> R      leftNorm : % -> R
--R leftTrace : % -> R                    leftTraceMatrix : () -> Matrix R
--R lieAdmissible? : () -> Boolean        lieAlgebra? : () -> Boolean
--R powerAssociative? : () -> Boolean     rank : () -> PositiveInteger
--R represents : Vector R -> %            rightAlternative? : () -> Boolean
--R rightDiscriminant : () -> R           rightDiscriminant : Vector % -> R
--R rightNorm : % -> R                    rightTrace : % -> R
--R rightTraceMatrix : () -> Matrix R     sample : () -> %
--R someBasis : () -> Vector %            zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R associatorDependence : () -> List Vector R if R has INTDOM
--R conditionsForIdempotents : () -> List Polynomial R
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
```

```
--R coordinates : (%,Vector %) -> Vector R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has F
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : % -> Matrix R
--R leftRegularRepresentation : (%,Vector %) -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R ha
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has 
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : % -> Matrix R
--R rightRegularRepresentation : (%,Vector %) -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R h
--R structuralConstants : () -> Vector Matrix R
--R structuralConstants : Vector % -> Vector Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*FramedNonAssociativeAlgebra.help*⟩≡

```
====================================================================
FramedNonAssociativeAlgebra examples
====================================================================

See Also:
o )show FramedNonAssociativeAlgebra
```

See:

⇐ “FiniteRankNonAssociativeAlgebra” (FINAALG) 12.0.162 on page 1122

Exports:

0
alternative?
antiAssociative?
antiCommutative?
antiCommutator
apply
associative?
associator
associatorDependence
basis
coerce
commutative?
commutator
conditionsForIdempotents
convert
coordinates
flexible?
hash
jacobiIdentity?
jordanAdmissible?
jordanAlgebra?
latex
leftAlternative?
leftCharacteristicPolynomial
leftDiscriminant
leftMinimalPolynomial
leftNorm
leftPower
leftRankPolynomial
leftRecip
leftRegularRepresentation
leftTrace
leftTraceMatrix
leftUnit
leftUnits
lieAdmissible?
lieAlgebra?
noncommutativeJordanAlgebra?
plenaryPower
powerAssociative?
rank
recip
represents
rightAlternative?
rightCharacteristicPolynomial
rightDiscriminant
rightMinimalPolynomial
rightNorm
rightPower
rightRankPolynomial
rightRecip
rightRegularRepresentation
rightTrace
rightTraceMatrix
rightUnit
rightUnits
sample
someBasis
structuralConstants
subtractIfCan
unit
zero?
?*?
?**?
?+?
?-?
-?
?=?
?.?
?~=?

Attributes exported:

- if $ has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (Matrix R,%) -> %
basis : () -> Vector %
convert : Vector R -> %
?.? : (%,Integer) -> R
```

These are implemented by this category:

```
conditionsForIdempotents : () -> List Polynomial R
convert : % -> Vector R
coordinates : % -> Vector R
coordinates : Vector % -> Matrix R
leftDiscriminant : () -> R
leftRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
leftRegularRepresentation : % -> Matrix R
leftTraceMatrix : () -> Matrix R
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
represents : Vector R -> %
rightDiscriminant : () -> R
rightRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
rightRegularRepresentation : % -> Matrix R
rightTraceMatrix : () -> Matrix R
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
structuralConstants : () -> Vector Matrix R
unit : () -> Union(%,"failed") if R has INTDOM
```

These exports come from (p1122) FiniteRankNonAssociativeAlgebra(R) where R:CommutativeRing:

```
0 : () -> %
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
antiCommutator : (%,%) -> %
```

```
associative? : () -> Boolean
associator : (%,%,%) -> %
associatorDependence : () -> List Vector R
    if R has INTDOM
coerce : % -> OutputForm
commutative? : () -> Boolean
commutator : (%,%) -> %
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : (%,Vector %) -> Vector R
flexible? : () -> Boolean
hash : % -> SingleInteger
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
latex : % -> String
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % ->
  SparseUnivariatePolynomial R
leftDiscriminant : Vector % -> R
leftMinimalPolynomial : % ->
  SparseUnivariatePolynomial R
    if R has INTDOM
leftNorm : % -> R
leftPower : (%,PositiveInteger) -> %
leftRecip : % -> Union(%,"failed") if R has INTDOM
leftRegularRepresentation : (%,Vector %) -> Matrix R
leftTrace : % -> R
leftTraceMatrix : Vector % -> Matrix R
lieAdmissible? : () -> Boolean
lieAlgebra? : () -> Boolean
noncommutativeJordanAlgebra? : () -> Boolean
plenaryPower : (%,PositiveInteger) -> %
powerAssociative? : () -> Boolean
rank : () -> PositiveInteger
recip : % -> Union(%,"failed") if R has INTDOM
represents : (Vector R,Vector %) -> %
rightAlternative? : () -> Boolean
rightCharacteristicPolynomial : % ->
  SparseUnivariatePolynomial R
rightDiscriminant : Vector % -> R
rightMinimalPolynomial : % ->
  SparseUnivariatePolynomial R
    if R has INTDOM
rightNorm : % -> R
rightPower : (%,PositiveInteger) -> %
rightRecip : % -> Union(%,"failed") if R has INTDOM
rightRegularRepresentation : (%,Vector %) -> Matrix R
rightTrace : % -> R
rightTraceMatrix : Vector % -> Matrix R
```

```
sample : () -> %
someBasis : () -> Vector %
structuralConstants : Vector % -> Vector Matrix R
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category FRNAALG FramedNonAssociativeAlgebra*⟩≡

```
)abbrev category FRNAALG FramedNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: +,-,*,**,basis
++ Related Constructors: FiniteRankNonAssociativeAlgebra, FramedAlgebra,
++   FiniteRankAssociativeAlgebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra, basis
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++ FramedNonAssociativeAlgebra(R) is a
++ \spadtype{FiniteRankNonAssociativeAlgebra} (i.e. a non associative
++ algebra over R which is a free \spad{R}-module of finite rank)
++ over a commutative ring R together with a fixed \spad{R}-module basis.

FramedNonAssociativeAlgebra(R:CommutativeRing):
        Category == FiniteRankNonAssociativeAlgebra(R) with
    basis: () -> Vector %
      ++ basis() returns the fixed \spad{R}-module basis.
    coordinates: % -> Vector R
      ++ coordinates(a) returns the coordinates of \spad{a}
      ++ with respect to the
      ++ fixed \spad{R}-module basis.
    coordinates: Vector % -> Matrix R
      ++ coordinates([a1,...,am]) returns a matrix whose i-th row
```

```
  ++ is formed by the coordinates of \spad{ai} with respect to the
  ++ fixed \spad{R}-module basis.
elt : (%,Integer) -> R
  ++ elt(a,i) returns the i-th coefficient of \spad{a} with respect
  ++ to the fixed \spad{R}-module basis.
structuralConstants:() -> Vector Matrix R
  ++ structuralConstants() calculates the structural constants
  ++ \spad{[(gammaijk) for k in 1..rank()]} defined by
  ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijn * vn},
  ++ where \spad{v1},...,\spad{vn} is the fixed \spad{R}-module basis.
conditionsForIdempotents: () -> List Polynomial R
  ++ conditionsForIdempotents() determines a complete list
  ++ of polynomial equations for the coefficients of idempotents
  ++ with respect to the fixed \spad{R}-module basis.
represents: Vector R -> %
  ++ represents([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
  ++ where \spad{v1}, ..., \spad{vn} are the elements of the
  ++ fixed \spad{R}-module basis.
convert: % -> Vector R
  ++ convert(a) returns the coordinates of \spad{a} with respect to the
  ++ fixed \spad{R}-module basis.
convert: Vector R -> %
  ++ convert([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
  ++ where \spad{v1}, ..., \spad{vn} are the elements of the
  ++ fixed \spad{R}-module basis.
leftDiscriminant : () -> R
  ++ leftDiscriminant() returns the
  ++ determinant of the \spad{n}-by-\spad{n}
  ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
  ++ is given by the left trace of the product \spad{vi*vj}, where
  ++ \spad{v1},...,\spad{vn} are the
  ++ elements of the fixed \spad{R}-module basis.
  ++ Note that the same as \spad{determinant(leftTraceMatrix())}.
rightDiscriminant : () -> R
  ++ rightDiscriminant() returns the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is
  ++ given by the right trace of the product \spad{vi*vj}, where
  ++ \spad{v1},...,\spad{vn} are the elements of
  ++ the fixed \spad{R}-module basis.
  ++ Note that the same as \spad{determinant(rightTraceMatrix())}.
leftTraceMatrix : () -> Matrix R
  ++ leftTraceMatrix() is the \spad{n}-by-\spad{n}
  ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
  ++ is given by left trace of the product \spad{vi*vj},
  ++ where \spad{v1},...,\spad{vn} are the
```

```
      ++ elements of the fixed \spad{R}-module basis.
    rightTraceMatrix : () -> Matrix R
      ++ rightTraceMatrix() is the \spad{n}-by-\spad{n}
      ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
      ++ is given by the right trace of the product \spad{vi*vj}, where
      ++ \spad{v1},...,\spad{vn} are the elements
      ++ of the fixed \spad{R}-module basis.
    leftRegularRepresentation : % -> Matrix R
      ++ leftRegularRepresentation(a) returns the matrix of the linear
      ++ map defined by left multiplication by \spad{a} with respect
      ++ to the fixed \spad{R}-module basis.
    rightRegularRepresentation : % -> Matrix R
      ++ rightRegularRepresentation(a) returns the matrix of the linear
      ++ map defined by right multiplication by \spad{a} with respect
      ++ to the fixed \spad{R}-module basis.
    if R has Field then
      leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
        ++ leftRankPolynomial() calculates the left minimal polynomial
        ++ of the generic element in the algebra,
        ++ defined by the same structural
        ++ constants over the polynomial ring in symbolic coefficients with
        ++ respect to the fixed basis.
      rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
        ++ rightRankPolynomial() calculates the right minimal polynomial
        ++ of the generic element in the algebra,
        ++ defined by the same structural
        ++ constants over the polynomial ring in symbolic coefficients with
        ++ respect to the fixed basis.
    apply: (Matrix R, %) -> %
      ++ apply(m,a) defines a left operation of n by n matrices
      ++ where n is the rank of the algebra in terms of matrix-vector
      ++ multiplication, this is a substitute for a left module structure.
      ++ Error: if shape of matrix doesn't fit.
    --attributes
      --separable <=> discriminant() ^= 0
  add

    V  ==> Vector
    M  ==> Matrix
    P  ==> Polynomial
    F  ==> Fraction
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)
    CVMP ==> CoerceVectorMatrixPackage(R)

    --GA ==> GenericNonAssociativeAlgebra(R,rank()$%,_
-- [random()$Character :: String :: Symbol for i in 1..rank()$%], _
-- structuralConstants()$%)
--y : GA := generic()
if R has Field then
  leftRankPolynomial() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
    symbolsForCoef : Vector Symbol :=
      [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
    xx : M P R
    mo : P R
    x : M P R := new(1,n,0)
    for i in 1..n repeat
      mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
      qsetelt_!(x,1,i,mo)
    y : M P R := copy x
    k  : PositiveInteger := 1
    cond : M P R := copy x
    -- multiplication in the generic algebra means using
    -- the structural matrices as bilinear forms.
    -- left multiplication by x, we prepare for that:
    genGamma : V M P R :=  coerceP$CVMP gamma
    x := reduce(horizConcat,[x*genGamma(i) for i in 1..#genGamma])
    while rank(cond) = k repeat
      k := k+1
      for i in 1..n repeat
        setelt(xx,[1],[i],x*transpose y)
      y := copy xx
      cond := horizConcat(cond, xx)
    vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
    res : SparseUnivariatePolynomial P R := 0
    for i in 1..k repeat
     res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial P R)
    res

  rightRankPolynomial() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
    symbolsForCoef : Vector Symbol :=
      [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
    xx : M P R
    mo : P R
```

```
      x : M P R := new(1,n,0)
      for i in 1..n repeat
        mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
        qsetelt_!(x,1,i,mo)
      y : M P R := copy x
      k  : PositiveInteger := 1
      cond : M P R := copy x
      -- multiplication in the generic algebra means using
      -- the structural matrices as bilinear forms.
      -- left multiplication by x, we prepare for that:
      genGamma : V M P R :=  coerceP$CVMP gamma
      x := _
       reduce(horizConcat,[genGamma(i)*transpose x for i in 1..#genGamma])
      while rank(cond) = k repeat
        k := k+1
        for i in 1..n repeat
          setelt(xx,[1],[i],y * transpose x)
        y := copy xx
        cond := horizConcat(cond, xx)
      vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
      res : SparseUnivariatePolynomial P R := 0
      for i in 1..k repeat
       res := _
        res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial  P R)
      res

    leftUnitsInternal : () -> REC
    leftUnitsInternal() ==
      n := rank()
      b := basis()
      gamma : Vector Matrix R := structuralConstants b
      cond : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
      rhs : Vector(R) := new(n**2,0$R)$Vector(R)
      z : Integer := 0
      addOn : R := 0
      for k in 1..n repeat
       for i in 1..n repeat
         z := z+1   -- index for the rows
         addOn :=
           k=i => 1
           0
         setelt(rhs,z,addOn)$Vector(R)
         for j in 1..n repeat  -- index for the columns
           setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
      solve(cond,rhs)$LSMP
```

```
leftUnit() ==
  res : REC := leftUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no left unit")$OutputForm
    "failed"
  represents (res.particular :: V R)

leftUnits() ==
  res : REC := leftUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no left unit")$OutputForm
    "failed"
  [represents(res.particular :: V R)$%, _
    map(represents, res.basis)$ListFunctions2(Vector R, %) ]

rightUnitsInternal : () -> REC
rightUnitsInternal() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  condo : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
  rhs : Vector(R) := new(n**2,0$R)$Vector(R)
  z : Integer := 0
  addOn : R := 0
  for k in 1..n repeat
   for i in 1..n repeat
     z := z+1   -- index for the rows
     addOn :=
       k=i => 1
       0
     setelt(rhs,z,addOn)$Vector(R)
     for j in 1..n repeat  -- index for the columns
       setelt(condo,z,j,elt(gamma.k,i,j))$Matrix(R)
  solve(condo,rhs)$LSMP

rightUnit() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no right unit")$OutputForm
    "failed"
  represents (res.particular :: V R)

rightUnits() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
```

```
          messagePrint("this algebra has no right unit")$OutputForm
          "failed"
        [represents(res.particular :: V R)$%, _
          map(represents, res.basis)$ListFunctions2(Vector R, %) ]

      unit() ==
        n := rank()
        b := basis()
        gamma : Vector Matrix R := structuralConstants b
        cond : Matrix(R) := new(2*n**2,n,0$R)$Matrix(R)
        rhs : Vector(R) := new(2*n**2,0$R)$Vector(R)
        z : Integer := 0
        u : Integer := n*n
        addOn : R := 0
        for k in 1..n repeat
         for i in 1..n repeat
           z := z+1   -- index for the rows
           addOn :=
             k=i => 1
             0
           setelt(rhs,z,addOn)$Vector(R)
           setelt(rhs,u,addOn)$Vector(R)
           for j in 1..n repeat  -- index for the columns
             setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
             setelt(cond,u,j,elt(gamma.k,i,j))$Matrix(R)
        res : REC := solve(cond,rhs)$LSMP
        res.particular case "failed" =>
          messagePrint("this algebra has no unit")$OutputForm
          "failed"
        represents (res.particular :: V R)
    apply(m:Matrix(R),a:%) ==
      v : Vector R := coordinates(a)
      v := m *$Matrix(R) v
      convert v


    structuralConstants()   == structuralConstants basis()
    conditionsForIdempotents() == conditionsForIdempotents basis()
    convert(x:%):Vector(R)  == coordinates(x, basis())
    convert(v:Vector R):%   == represents(v, basis())
    leftTraceMatrix()       == leftTraceMatrix basis()
    rightTraceMatrix()      == rightTraceMatrix basis()
    leftDiscriminant()      == leftDiscriminant basis()
    rightDiscriminant()     == rightDiscriminant basis()
    leftRegularRepresentation x == leftRegularRepresentation(x, basis())
    rightRegularRepresentation x == rightRegularRepresentation(x, basis())
    coordinates x           == coordinates(x, basis())
    represents(v:Vector R):%== represents(v, basis())

    coordinates(v:Vector %) ==
      m := new(#v, rank(), 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates qelt(v, i))
      m
```

⟨*FRNAALG.dotabb*⟩≡

```
"FRNAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
"FRNAALG" -> "FINAALG"
```

⟨*FRNAALG.dotfull*⟩≡

```
"FramedNonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
"FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
    "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
```

⟨*FRNAALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FramedNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
    "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

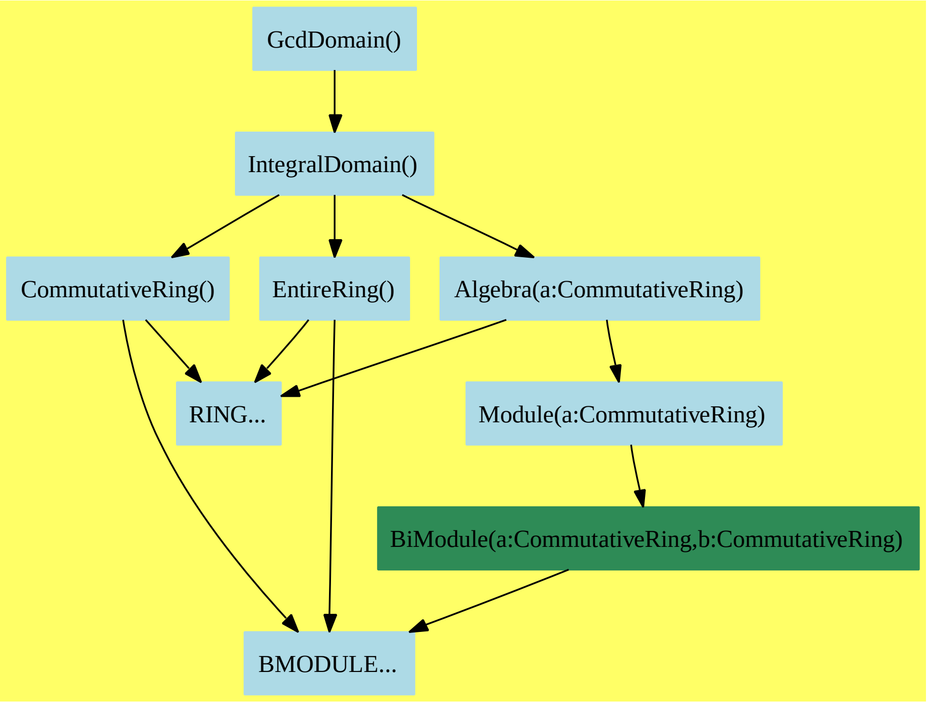
"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

13.0.173 GcdDomain (GCDDOM)


GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*GcdDomain.input*⟩≡

```
  )set break resume
  )sys rm -f GcdDomain.output
  )spool GcdDomain.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show GcdDomain
  --R GcdDomain  is a category constructor
  --R Abbreviation for GcdDomain is GCDDOM
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for GCDDOM
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       associates? : (%,%) -> Boolean
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R latex : % -> String                  lcm : List % -> %
--R lcm : (%,%) -> %                     one? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*GcdDomain.help*⟩≡

```
====================================================================
GcdDomain examples
====================================================================

See Also:
o )show GcdDomain
```

See:

⇒ “IntervalCategory” (INTCAT) 14.0.176 on page 1270
⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇒ “PrincipalIdealDomain” (PID) 14.0.178 on page 1288
⇒ “UniqueFactorizationDomain” (UFD) 14.0.179 on page 1294
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇐ “IntegralDomain” (INTDOM) 12.0.164 on page 1153

Exports:

0
1
associates?
characteristic
coerce
exquo
gcd
gcdPolynomial
hash
latex
lcm
one?
recip
sample
subtractIfCan
unit?
unitCanonical
unitNormal
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
gcd : (%,%) -> %
```

These are implemented by this category:

```
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : (%,%) -> %
lcm : List % -> %
```

These exports come from (p1153) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category GCDDOM GcdDomain*⟩≡

```
)abbrev category GCDDOM GcdDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: Davenport & Trager 1
++ Description:
++ This category describes domains where
++ \spadfun{gcd} can be computed but where there is no guarantee
++ of the existence of \spadfun{factor} operation for factorisation
++ into irreducibles. However, if such a \spadfun{factor} operation exist,
++ factorization will be unique up to order and units.
```

```
GcdDomain(): Category == IntegralDomain with
    gcd: (%,%) -> %
      ++ gcd(x,y) returns the greatest common divisor of x and y.
      -- gcd(x,y) = gcd(y,x) in the presence of canonicalUnitNormal,
      -- but not necessarily elsewhere
    gcd: List(%) -> %
      ++ gcd(l) returns the common gcd of the elements in the list l.
    lcm: (%,%) -> %
      ++ lcm(x,y) returns the least common multiple of x and y.
      -- lcm(x,y) = lcm(y,x) in the presence of canonicalUnitNormal,
      -- but not necessarily elsewhere
    lcm: List(%) -> %
      ++ lcm(l) returns the least common multiple of the elements of
      ++ the list l.
    gcdPolynomial: (SparseUnivariatePolynomial %, _
                    SparseUnivariatePolynomial %) -> _
                       SparseUnivariatePolynomial %
      ++ gcdPolynomial(p,q) returns the greatest common divisor (gcd) of
      ++ univariate polynomials over the domain
  add
   lcm(x: %,y: %) ==
     y = 0 => 0
     x = 0 => 0
     LCM : Union(%,"failed") := y exquo gcd(x,y)
     LCM case % =>  x * LCM
     error "bad gcd in lcm computation"
   lcm(l:List %) == reduce(lcm,l,1,0)
   gcd(l:List %) == reduce(gcd,l,0,1)
   SUP ==> SparseUnivariatePolynomial
   gcdPolynomial(p1,p2) ==
     zero? p1 => unitCanonical p2
     zero? p2 => unitCanonical p1
     c1:= content(p1); c2:= content(p2)
     p1:= (p1 exquo c1)::SUP %
     p2:= (p2 exquo c2)::SUP %
     if (e1:=minimumDegree p1) > 0 then p1:=(p1 exquo monomial(1,e1))::SUP %
     if (e2:=minimumDegree p2) > 0 then p2:=(p2 exquo monomial(1,e2))::SUP %
     e1:=min(e1,e2); c1:=gcd(c1,c2)
     p1:=
        degree p1 = 0 or degree p2 = 0 => monomial(c1,0)
        p:= subResultantGcd(p1,p2)
        degree p = 0 => monomial(c1,0)
        c2:= gcd(leadingCoefficient p1,leadingCoefficient p2)
        unitCanonical(_
          c1 * primitivePart(((c2*p) exquo leadingCoefficient p)::SUP %))
        zero? e1 => p1
        monomial(1,e1)*p1
```

⟨*GCDDOM.dotabb*⟩≡

```
  "GCDDOM"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
  "GCDDOM" -> "INTDOM"
```

⟨*GCDDOM.dotfull*⟩≡

```
  "GcdDomain()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
  "GcdDomain()" -> "IntegralDomain()"
```

⟨*GCDDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

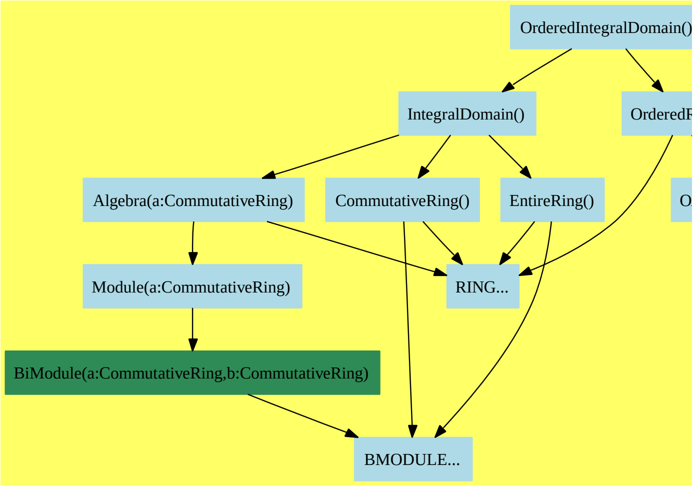
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

13.0.174 OrderedIntegralDomain (OINTDOM)


OrderedIntegralDomain()
IntegralDomain()
OrderedR
Algebra(a:CommutativeRing)
CommutativeRing()
EntireRing()
O
Module(a:CommutativeRing)
RING...
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*OrderedIntegralDomain.input*⟩≡

```
)set break resume
)sys rm -f OrderedIntegralDomain.output
)spool OrderedIntegralDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedIntegralDomain
--R OrderedIntegralDomain  is a category constructor
--R Abbreviation for OrderedIntegralDomain is OINTDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OINTDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        abs : % -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
```

```
--R max : (%,%) -> %                          min : (%,%) -> %
--R negative? : % -> Boolean                  one? : % -> Boolean
--R positive? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                          sign : % -> Integer
--R unit? : % -> Boolean                      unitCanonical : % -> %
--R zero? : % -> Boolean                      ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*OrderedIntegralDomain.help*⟩≡

```
====================================================================
OrderedIntegralDomain examples
====================================================================

See Also:
o )show OrderedIntegralDomain
```

See:

⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇐ “IntegralDomain” (INTDOM) 12.0.164 on page 1153
⇐ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇐ “OrderedRing” (ORDRING) 10.0.144 on page 971

Exports:

1	0	abs	associates?	characteristic
coerce	exquo	hash	latex	max
min	negative?	one?	positive?	recip
sample	sign	subtractIfCan	unit?	unitCanonical
unitNormal	zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?	?^?	?~=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1153) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
```

```
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p971) OrderedRing():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category OINTDOM OrderedIntegralDomain*⟩≡

```
)abbrev category OINTDOM OrderedIntegralDomain
++ Author: JH Davenport (after L Gonzalez-Vega)
++ Date Created: 30.1.96
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Description:
++ The category of ordered commutative integral domains, where ordering
++ and the arithmetic operations are compatible

OrderedIntegralDomain(): Category ==
  Join(IntegralDomain, OrderedRing)
```

⟨*OINTDOM.dotabb*⟩≡
```
"OINTDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OINTDOM" -> "INTDOM"
"OINTDOM" -> "ORDRING"
```

⟨*OINTDOM.dotfull*⟩≡
```
"OrderedIntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OrderedIntegralDomain()" -> "IntegralDomain()"
"OrderedIntegralDomain()" -> "OrderedRing()"
```

⟨*OINTDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedIntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OrderedIntegralDomain()" -> "IntegralDomain()"
"OrderedIntegralDomain()" -> "OrderedRing()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OAGROUP..."
"OrderedRing()" -> "RING..."
"OrderedRing()" -> "MONOID..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

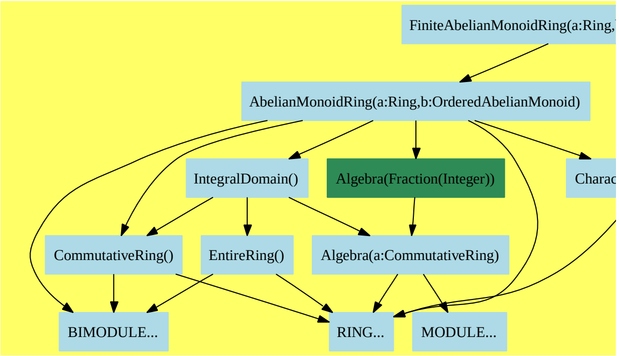
"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"OAGROUP..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MONOID..." [color=lightblue];
}
```

Chapter 14

Category Layer 13

14.0.175 FiniteAbelianMonoidRing (FAMR)


FiniteAbelianMonoidRing(a:Ring,
AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)
IntegralDomain()
Algebra(Fraction(Integer))
Charac
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
BIMODULE...
RING...
MODULE...

⟨*FiniteAbelianMonoidRing.input*⟩≡

```
)set break resume
)sys rm -f FiniteAbelianMonoidRing.output
)spool FiniteAbelianMonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteAbelianMonoidRing
--R FiniteAbelianMonoidRing(R: Ring,E: OrderedAbelianMonoid)  is a category constructor
--R Abbreviation for FiniteAbelianMonoidRing is FAMR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAMR
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> E                       ground : % -> R
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R mapExponents : ((E -> E),%) -> %      minimumDegree : % -> E
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R one? : % -> Boolean                   pomopo! : (%,R,E,%) -> %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT or R has ALGEBRA FRA
--R coerce : % -> % if R has INTDOM
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R numberOfMonomials : % -> NonNegativeInteger
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
```

```
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteAbelianMonoidRing.help*⟩≡

```
====================================================================
FiniteAbelianMonoidRing examples
====================================================================

See Also:
o )show FiniteAbelianMonoidRing
```

See:

⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇐ “AbelianMonoidRing” (AMR) 13.0.170 on page 1213
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100

Exports:

0	1	associates?	binomThmExpt
characteristic	charthRoot	coefficient	coefficients
coerce	content	degree	exquo
ground	ground?	hash	latex
leadingCoefficient	leadingMonomial	map	mapExponents
minimumDegree	monomial	monomial?	numberOfMonomials
one?	pomopo!	primitivePart	recip
reductum	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if $ has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation " $*$ " : $(D, D)- > D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
minimumDegree : % -> E
numberOfMonomials : % -> NonNegativeInteger
```

These are implemented by this category:

```
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficients : % -> List R
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
```

```
ground : % -> R
ground? : % -> Boolean
mapExponents : ((E -> E),%) -> %
pomopo! : (%,R,E,%) -> %
primitivePart : % -> % if R has GCDDOM
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p1213) AbelianMonoidRing(R,E)
where R:Ring and E:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coefficient : (%,E) -> R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
degree : % -> E
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal :
   % -> Record(unit: %,canonical: %,associate: %)
   if R has INTDOM
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p100) FullyRetractableTo(R:Ring):

```
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
```

⟨*category FAMR FiniteAbelianMonoidRing*⟩≡

```
)abbrev category FAMR FiniteAbelianMonoidRing
++ Author:
++ Date Created:
++ Date Last Updated: 14.08.2000 Exported pomopo! and binomThmExpt [MMM]
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category is similar to AbelianMonoidRing, except that the sum is
++ assumed to be finite. It is a useful model for polynomials,
++ but is somewhat more general.

FiniteAbelianMonoidRing(R:Ring, E:OrderedAbelianMonoid): Category ==
   Join(AbelianMonoidRing(R,E),FullyRetractableTo R) with
    ground?: % -> Boolean
      ++ ground?(p) tests if polynomial p is a member of the
      ++ coefficient ring.
      -- can't be defined earlier, since a power series
      -- might not know if there were other terms or not
    ground: % -> R
      ++ ground(p) retracts polynomial p to the coefficient ring.
```

```
  coefficients: % -> List R
    ++ coefficients(p) gives the list of non-zero coefficients
    ++ of polynomial p.
  numberOfMonomials: % -> NonNegativeInteger
    ++ numberOfMonomials(p) gives the number of non-zero monomials
    ++ in polynomial p.
  minimumDegree: % -> E
    ++ minimumDegree(p) gives the least exponent of a non-zero term
    ++ of polynomial p. Error: if applied to 0.
  mapExponents: (E -> E, %) -> %
    ++ mapExponents(fn,u) maps function fn onto the exponents
    ++ of the non-zero monomials of polynomial u.
  pomopo!: (%,R,E,%) -> %
    ++ \spad{pomopo!(p1,r,e,p2)} returns \spad{p1 + monomial(e,r) * p2}
    ++ and may use \spad{p1} as workspace. The constaant \spad{r} is
    ++ assumed to be nonzero.
  if R has CommutativeRing then
     binomThmExpt: (%,%,NonNegativeInteger) -> %
       ++ \spad{binomThmExpt(p,q,n)} returns \spad{(x+y)^n}
       ++ by means of the binomial theorem trick.
  if R has IntegralDomain then
     "exquo": (%,R) -> Union(%,"failed")
     ++ exquo(p,r) returns the exact quotient of polynomial p by r,
     ++ or "failed" if none exists.
  if R has GcdDomain then
     content: % -> R
       ++ content(p) gives the gcd of the coefficients of polynomial p.
     primitivePart: % -> %
       ++ primitivePart(p) returns the unit normalized form of polynomial p
       ++ divided by the content of p.
add
  pomopo!(p1,r,e,p2) == p1 + r * mapExponents(x1+->x1+e,p2)

  if R has CommutativeRing then
     binomThmExpt(x,y,nn) ==
             nn = 0 => 1$%
             ans,xn,yn: %
             bincoef: Integer
             powl: List(%):= [x]
             for i in 2..nn repeat powl:=[x * powl.first, :powl]
             yn:=y; ans:=powl.first; i:=1; bincoef:=nn
             for xn in powl.rest repeat
                ans:= bincoef * xn * yn + ans
                bincoef:= (nn-i) * bincoef quo (i+1);  i:= i+1
                -- last I and BINCOEF unused
                yn:= y * yn
```

```
           ans + yn
ground? x ==
  retractIfCan(x)@Union(R,"failed") case "failed" => false
  true

ground x == retract(x)@R

mapExponents (fn:E -> E, x: %) ==
     -- this default definition assumes that reductum is cheap
   zero? x => 0
   monomial(leadingCoefficient x,fn degree x)+mapExponents(fn,reductum x)

coefficients x ==
  zero? x => empty()
  concat(leadingCoefficient x, coefficients reductum x)

if R has Field then
   x/r == map(x1+->x1/r,x)

if R has IntegralDomain then
   x exquo r ==
      -- probably not a very good definition in most special cases
      zero? x => 0
      ans:% :=0
      t:=leadingCoefficient x exquo r
      while not (t case "failed") and not zero? x repeat
        ans:=ans+monomial(t::R,degree x)
        x:=reductum x
        if not zero? x then t:=leadingCoefficient x exquo r
      t case "failed" => "failed"
      ans

if R has GcdDomain then
   content x ==       -- this assumes  reductum is cheap
      zero? x => 0
      r:=leadingCoefficient x
      x:=reductum x
--        while not zero? x and not one? r repeat
      while not zero? x and not (r = 1) repeat
        r:=gcd(r,leadingCoefficient x)
        x:=reductum x
      r

   primitivePart x ==
      zero? x => x
      c := content x
```

```
          unitCanonical((x exquo c)::%)
```

⟨*FAMR.dotabb*⟩≡

```
"FAMR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
"FAMR" -> "AMR"
"FAMR" -> "FRETRCT"
```

⟨*FAMR.dotfull*⟩≡

```
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "FullyRetractableTo(a:Ring)"

"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FAMR"];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)" ->
    "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
```

⟨*FAMR.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "FullyRetractableTo(a:Ring)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RETRACT..."

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

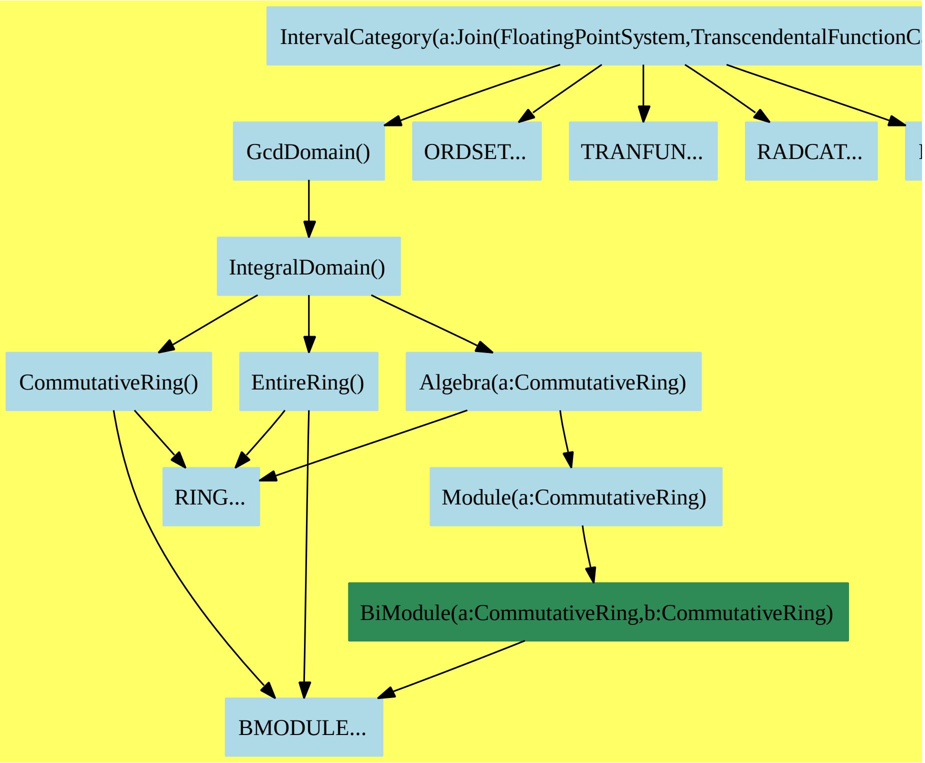
"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"RETRACT..." [color=lightblue];
"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

14.0.176 IntervalCategory (INTCAT)


IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionC
GcdDomain()
ORDSET...
TRANFUN...
RADCAT...
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*IntervalCategory.input*⟩≡

```
)set break resume
)sys rm -f IntervalCategory.output
)spool IntervalCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntervalCategory
--R IntervalCategory R: Join(FloatingPointSystem,TranscendentalFunctionCategory)
--R Abbreviation for IntervalCategory is INTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
```

```
--R ?**? : (%,%) -> %                     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        acos : % -> %
--R acosh : % -> %                        acot : % -> %
--R acoth : % -> %                        acsc : % -> %
--R acsch : % -> %                        asec : % -> %
--R asech : % -> %                        asin : % -> %
--R asinh : % -> %                        associates? : (%,%) -> Boolean
--R atan : % -> %                         atanh : % -> %
--R coerce : Integer -> %                 coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R contains? : (%,R) -> Boolean          cos : % -> %
--R cosh : % -> %                         cot : % -> %
--R coth : % -> %                         csc : % -> %
--R csch : % -> %                         exp : % -> %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inf : % -> R
--R interval : Fraction Integer -> %      interval : R -> %
--R interval : (R,R) -> %                 latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R log : % -> %                          max : (%,%) -> %
--R min : (%,%) -> %                      negative? : % -> Boolean
--R nthRoot : (%,Integer) -> %            one? : % -> Boolean
--R pi : () -> %                          positive? : % -> Boolean
--R qinterval : (R,R) -> %                recip : % -> Union(%,"failed")
--R retract : % -> Integer                sample : () -> %
--R sec : % -> %                          sech : % -> %
--R sin : % -> %                          sinh : % -> %
--R sqrt : % -> %                         sup : % -> R
--R tan : % -> %                          tanh : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R width : % -> R                        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

```
--R
--E 1

)spool
)lisp (bye)
```

⟨*IntervalCategory.help*⟩≡

```
====================================================================
IntervalCategory examples
====================================================================

See Also:
o )show IntervalCategory
```

See:

⇐ “GcdDomain” (GCDDOM) 13.0.173 on page 1246
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242
⇐ “RadicalCategory” (RADCAT) 2.0.17 on page 59
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135

Exports:

0 1 acos acosh acot
acoth acsc acsch asec asech
asin asinh associates? atan atanh
characteristic coerce contains? cos cosh
cot coth csc csch exp
exquo gcd gcdPolynomial hash inf
interval latex lcm log max
min negative? nthRoot one? pi
positive? qinterval recip retract retractIfCan
sample sec sech sin sinh
sqrt subtractIfCan sup tan tanh
unit? unitCanonical unitNormal width zero?
?*? ?**? ?+? ?-? -?
?<? ?<=? ?=? ?>? ?>=?
?^? ?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **approximate** means “is an approximation to the real numbers”.

TPDHERE: Note that the signature `coerce : Integer -> %` shows up twice.

These are directly exported but not implemented:

```
contains? : (%,R) -> Boolean
inf : % -> R
```

```
interval : (R,R) -> %
interval : R -> %
interval : Fraction Integer -> %
negative? : % -> Boolean
positive? : % -> Boolean
qinterval : (R,R) -> %
sup : % -> R
width : % -> R
```

These exports come from (p1246) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p135) TranscendentalFunctionCategory():

```
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
cot : % -> %
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
?**? : (%,%) -> %
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p62) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

⟨*category INTCAT IntervalCategory*⟩≡

```
)abbrev category INTCAT IntervalCategory
++ Author: Mike Dewar
++ Date Created: November 1996
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category implements of interval arithmetic and transcendental
++ functions over intervals.

IntervalCategory(R: Join(FloatingPointSystem,TranscendentalFunctionCategory)):
 Category == _
  Join(GcdDomain, OrderedSet, TranscendentalFunctionCategory, _
       RadicalCategory, RetractableTo(Integer)) with
  approximate
  interval : (R,R) -> %
    ++ interval(inf,sup) creates a new interval, either \axiom{[inf,sup]} if
    ++ \axiom{inf <= sup} or \axiom{[sup,in]} otherwise.
  qinterval : (R,R) -> %
    ++ qinterval(inf,sup) creates a new interval \axiom{[inf,sup]}, without
    ++ checking the ordering on the elements.
  interval : R -> %
    ++ interval(f) creates a new interval around f.
  interval : Fraction Integer -> %
    ++ interval(f) creates a new interval around f.
  inf : % -> R
    ++ inf(u) returns the infinum of \axiom{u}.
  sup : % -> R
    ++ sup(u) returns the supremum of \axiom{u}.
  width : % -> R
    ++ width(u) returns \axiom{sup(u) - inf(u)}.
  positive? : % -> Boolean
    ++ positive?(u) returns \axiom{true} if every element of u is positive,
    ++ \axiom{false} otherwise.
  negative? : % -> Boolean
    ++ negative?(u) returns \axiom{true} if every element of u is negative,
    ++ \axiom{false} otherwise.
  contains? : (%,R) -> Boolean
    ++ contains?(i,f) returns true if \axiom{f} is contained within the
    ++ interval \axiom{i}, false otherwise.
```

⟨*INTCAT.dotabb*⟩≡

```
"INTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"INTCAT" -> "GCDDOM"
"INTCAT" -> "ORDSET"
"INTCAT" -> "RADCAT"
"INTCAT" -> "RETRACT"
"INTCAT" -> "TRANFUN"
```

⟨*INTCAT.dotfull*⟩≡

```
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "OrderedSet()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TranscendentalFunctionCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RadicalCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RetractableTo(Integer)"
```

⟨*INTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "ORDSET..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TRANFUN..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RADCAT..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RETRACT..."

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

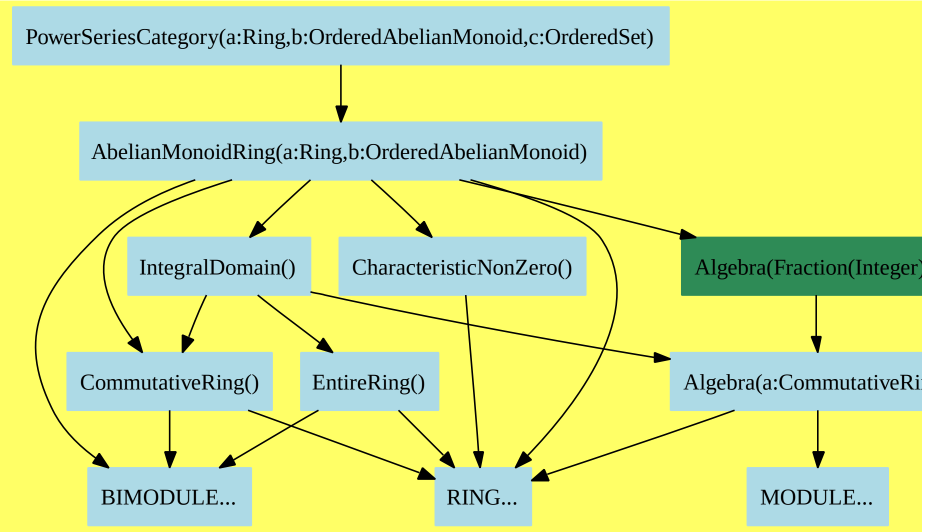
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"ORDSET..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

14.0.177 PowerSeriesCategory (PSCAT)


PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)
IntegralDomain()
CharacteristicNonZero()
Algebra(Fraction(Integer)
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRi
BIMODULE...
RING...
MODULE...

⟨*PowerSeriesCategory.input*⟩≡

```
)set break resume
)sys rm -f PowerSeriesCategory.output
)spool PowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PowerSeriesCategory
--R PowerSeriesCategory(Coef: Ring,Expon: OrderedAbelianMonoid,Var: OrderedSet)
--R Abbreviation for PowerSeriesCategory is PSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,Expon) -> Coef
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Expon
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial : (%,Var,Expon) -> %
--R monomial : (Coef,Expon) -> %          monomial? : % -> Boolean
--R one? : % -> Boolean                   pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      variables : % -> List Var
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R monomial : (%,List Var,List Expon) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*PowerSeriesCategory.help*⟩≡

```
====================================================================
PowerSeriesCategory examples
====================================================================

See Also:
o )show PowerSeriesCategory
```

See:

⇒ “MultivariateTaylorSeriesCategory” (MTSCAT) 15.0.182 on page 1316
⇒ “UnivariatePowerSeriesCategory” (UPSCAT) 15.0.184 on page 1334
⇐ “AbelianMonoidRing” (AMR) 13.0.170 on page 1213

Exports:

0	1	associates?	characteristic	charthRoot
coefficient	coerce	complete	degree	exquo
hash	latex	leadingCoefficient	leadingMonomial	map
monomial	monomial?	one?	pole?	recip
reductum	sample	subtractIfCan	variables	unit?
unitCanonical	unitNormal	zero?	?*?	?**?
?+?	?-?	-?	?=?	?~=?
?/?	?^?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
complete : % -> %
degree : % -> Expon
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
monomial : (%,List Var,List Expon) -> %
monomial : (%,Var,Expon) -> %
pole? : % -> Boolean
variables : % -> List Var
```

These are implemented by this category:

```
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p1213) AbelianMonoidRing(Coef,Expon) where Coef:Ring and Expon:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (Coef,Expon) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category PSCAT PowerSeriesCategory*⟩≡

```
)abbrev category PSCAT PowerSeriesCategory
```

```
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 25 February 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: power series
++ Examples:
++ References:
++ Description:
++ \spadtype{PowerSeriesCategory} is the most general power series
++ category with exponents in an ordered abelian monoid.

PowerSeriesCategory(Coef,Expon,Var): Category == Definition where
  Coef  : Ring
  Expon : OrderedAbelianMonoid
  Var   : OrderedSet
  I   ==> Integer
  RN  ==> Fraction Integer

  Definition ==> AbelianMonoidRing(Coef,Expon) with
    monomial: (%,Var,Expon) -> %
      ++ \spad{monomial(a,x,n)} computes \spad{a*x**n}.
    monomial: (%,List Var,List Expon) -> %
      ++ \spad{monomial(a,[x1,..,xk],[n1,..,nk])} computes
      ++ \spad{a * x1**n1 * .. * xk**nk}.
    leadingMonomial: % -> %
      ++ leadingMonomial(f) returns the monomial of \spad{f} of lowest order.
    leadingCoefficient: % -> Coef
      ++ leadingCoefficient(f) returns the coefficient of the lowest order
      ++ term of \spad{f}
    degree : % -> Expon
      ++ degree(f) returns the exponent of the lowest order term of \spad{f}.
    variables: % -> List Var
      ++ \spad{variables(f)} returns a list of the variables occuring in the
      ++ power series f.
    pole?: % -> Boolean
      ++ \spad{pole?(f)} determines if the power series f has a pole.
    complete: % -> %
      ++ \spad{complete(f)} causes all terms of f to be computed.
      ++ Note that this results in an infinite loop
      ++ if f has infinitely many terms.

   add
    n:I    * ps:% == (zero? n => 0; map((r1:Coef):Coef +-> n * r1,ps))
    r:Coef * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

    ps:% * r:Coef == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    - ps          == map((r1:Coef):Coef +-> -r1,ps)

    if Coef has Algebra Fraction Integer then
      r:RN * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

      ps:% * r:RN == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    if Coef has Field then
      ps:% / r:Coef == map((r1:Coef):Coef +-> r1 / r,ps)
```

⟨*PSCAT.dotabb*⟩≡

```
"PSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PSCAT" -> "AMR"
```

⟨*PSCAT.dotfull*⟩≡

```
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
```

⟨*PSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

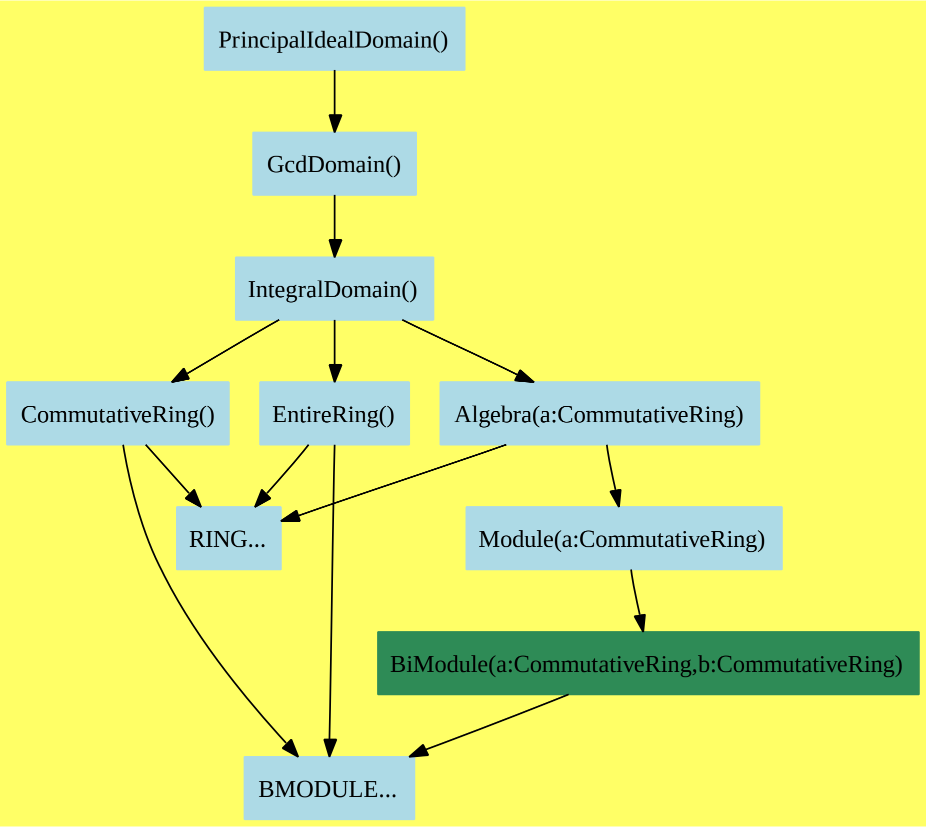
"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

14.0.178 PrincipalIdealDomain (PID)


PrincipalIdealDomain()
GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*PrincipalIdealDomain.input*⟩≡

```
)set break resume
)sys rm -f PrincipalIdealDomain.output
)spool PrincipalIdealDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrincipalIdealDomain
--R PrincipalIdealDomain  is a category constructor
--R Abbreviation for PrincipalIdealDomain is PID
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PID
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PrincipalIdealDomain.help*⟩≡

```
====================================================================
PrincipalIdealDomain examples
====================================================================

See Also:
o )show PrincipalIdealDomain
```

See:

Unique Factorization Domains are a subset of Principal Ideal Domains.
⇐ "UniqueFactorizationDomain" (UFD) 14.0.179 on page 1294 Principal Ideal Domains are a subset of Euclidean Domains.
⇒ "EuclideanDomain" (EUCDOM) 15.0.181 on page 1307

⇐ "GcdDomain" (GCDDOM) 13.0.173 on page 1246

Exports:

0	1	associates?	characteristic	coerce
expressIdealMember	exquo	gcd	gcdPolynomial	hash
latex	lcm	one?	principalIdeal	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?~=?	?*?	?**?	?^?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
```

These exports come from (p1246) GcdDomain():

```
associates? : (%,%) -> Boolean
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : List % -> %
lcm : (%,%) -> %
unit? : % -> Boolean
```

```
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category PID PrincipalIdealDomain*⟩≡

```
)abbrev category PID PrincipalIdealDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of constructive principal ideal domains, i.e.
++ where a single generator can be constructively found for
++ any ideal given by a finite set of generators.
++ Note that this constructive definition only implies that
++ finitely generated ideals are principal. It is not clear
++ what we would mean by an infinitely generated ideal.

PrincipalIdealDomain(): Category == GcdDomain with
```

```
        principalIdeal: List % -> Record(coef:List %,generator:%)
           ++ principalIdeal([f1,...,fn]) returns a record whose
           ++ generator component is a generator of the ideal
           ++ generated by \spad{[f1,...,fn]} whose coef component satisfies
           ++ \spad{generator = sum (input.i * coef.i)}
        expressIdealMember: (List %,%) -> Union(List %,"failed")
           ++ expressIdealMember([f1,...,fn],h) returns a representation
           ++ of h as a linear combination of the fi or "failed" if h
           ++ is not in the ideal generated by the fi.
```

⟨*PID.dotabb*⟩≡

```
  "PID"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
  "PID" -> "GCDDOM"
```

⟨*PID.dotfull*⟩≡

```
  "PrincipalIdealDomain()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
  "PrincipalIdealDomain()" -> "GcdDomain()"
```

⟨*PID.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

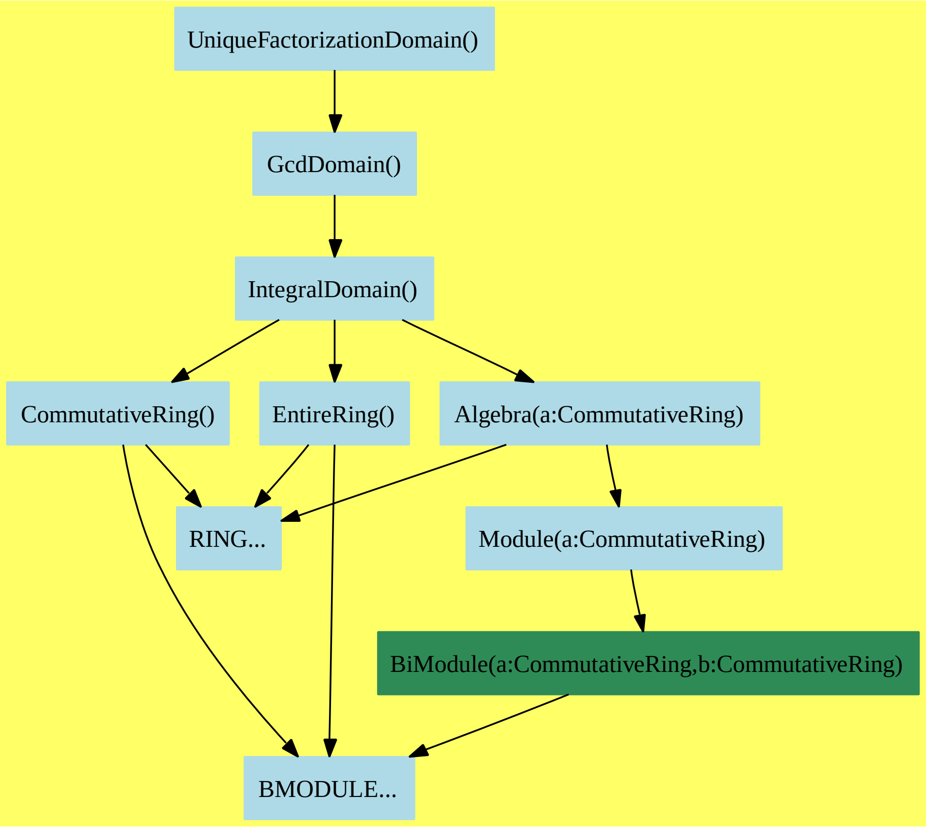
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

14.0.179 UniqueFactorizationDomain (UFD)


UniqueFactorizationDomain()
GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BiModule(a:CommutativeRing,b:CommutativeRing)
BMODULE...

⟨*UniqueFactorizationDomain.input*⟩≡

```
)set break resume
)sys rm -f UniqueFactorizationDomain.output
)spool UniqueFactorizationDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UniqueFactorizationDomain
--R UniqueFactorizationDomain  is a category constructor
--R Abbreviation for UniqueFactorizationDomain is UFD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UFD
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*UniqueFactorizationDomain.help*⟩≡

```
====================================================================
UniqueFactorizationDomain examples
====================================================================

See Also:
o )show UniqueFactorizationDomain
```

All Integral Domains are UniqueFactorizationDomains.
⇐ “IntegralDomain” (INTDOM) 12.0.164 on page 1153. Unique Factorization Domains are a subset of Principal Ideal Domains.
⇒ “PrincipalIdealDomain” (PID) 14.0.178 on page 1288

See:

⇒ “Field” (FIELD) 16.0.185 on page 1347
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “PolynomialFactorizationExplicit” (PFECAT) 15.0.183 on page 1325
⇐ “GcdDomain” (GCDDOM) 13.0.173 on page 1246

Exports:

0
exquo
latex
sample
unitCanonical
?+?
?^?
1
factor
lcm
squareFree
unitNormal
?-?
associates?
gcd
one?
squareFreePart
zero?
-?
characteristic
gcdPolynomial
prime?
subtractIfCan
?*?
?=?
coerce
hash
recip
unit?
?**?
?~=?

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factor : % -> Factored %
squareFree : % -> Factored %
```

These are implemented by this category:

```
prime? : % -> Boolean
squareFreePart : % -> %
```

These exports come from (p1246) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category UFD UniqueFactorizationDomain*⟩≡

```
)abbrev category UFD UniqueFactorizationDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
```

```
++ References:
++ Description:
++ A constructive unique factorization domain, i.e. where
++ we can constructively factor members into a product of
++ a finite number of irreducible elements.

UniqueFactorizationDomain(): Category == GcdDomain with
   prime?: % -> Boolean
     ++ prime?(x) tests if x can never be written as the product of two
     ++ non-units of the ring,
     ++ i.e., x is an irreducible element.
   squareFree    : % -> Factored(%)
     ++ squareFree(x) returns the square-free factorization of x
     ++ i.e. such that the factors are pairwise relatively prime
     ++ and each has multiple prime factors.
   squareFreePart: % -> %
     ++ squareFreePart(x) returns a product of prime factors of
     ++ x each taken with multiplicity one.
   factor: % -> Factored(%)
     ++ factor(x) returns the factorization of x into irreducibles.
 add
  squareFreePart x ==
    unit(s := squareFree x) * _*/[f.factor for f in factors s]

  prime? x == # factorList factor x = 1
```

⟨*UFD.dotabb*⟩≡

```
"UFD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UFD" -> "GCDDOM"
```

⟨*UFD.dotfull*⟩≡

```
"UniqueFactorizationDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UniqueFactorizationDomain()" -> "GcdDomain()"
```

⟨*UFD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

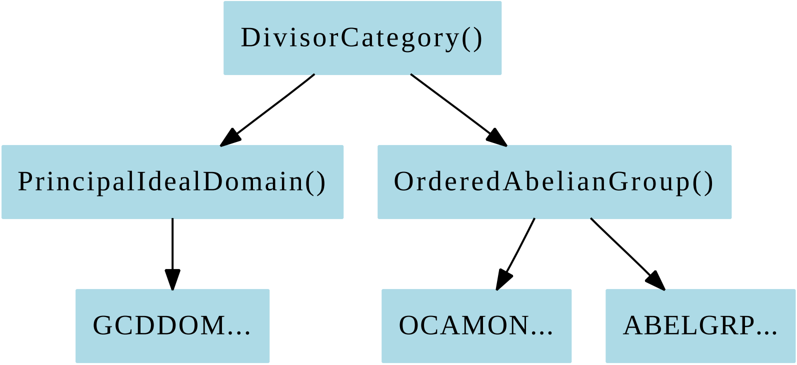
"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

Chapter 15

Category Layer 14

15.0.180 DivisorCategory (DIVCAT)


DivisorCategory()
PrincipalIdealDomain()
OrderedAbelianGroup()
GCDDOM...
OCAMON...
ABELGRP...

⟨*DivisorCategory.input*⟩≡
```
)set break resume
)sys rm -f DivisorCategory.output
)spool DivisorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DivisorCategory
--R DivisorCategory S: SetCategory  is a category constructor
--R Abbreviation for DivisorCategory is DIVCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIVCAT
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,S) -> %               ?*? : (Integer,%) -> %
--R ?*? : (%,Integer) -> %               ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?+? : (S,%) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               0 : () -> %
--R coefficient : (S,%) -> Integer       coerce : S -> %
--R coerce : % -> OutputForm             collect : % -> %
--R concat : (%,%) -> %                  degree : % -> Integer
--R divOfPole : % -> %                   divOfZero : % -> %
--R effective? : % -> Boolean            hash : % -> SingleInteger
--R incr : % -> %                        latex : % -> String
--R mapGen : ((S -> S),%) -> %           nthCoef : (%,Integer) -> Integer
--R nthFactor : (%,Integer) -> S         retract : % -> S
--R sample : () -> %                     size : % -> NonNegativeInteger
--R split : % -> List %                  supp : % -> List S
--R suppOfPole : % -> List S             suppOfZero : % -> List S
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R highCommonTerms : (%,%) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: Integer)
--R
--E 1

)spool
)lisp (bye)
```

⟨*DivisorCategory.help*⟩≡

```
====================================================================
DivisorCategory examples
====================================================================

See Also:
o )show DivisorCategory
```

See:

⇐ “OrderedAbelianGroup” (OAGROUP) 9.0.126 on page 836
⇐ “PrincipalIdealDomain” (PID) 14.0.178 on page 1288

Exports:

0	-?	?*?	?+?
?-?	?<=?	?=?	?~=?
coefficient	coerce	collect	concat
degree	divOfPole	divOfZero	effective?
hash	highCommonTerms	incr	latex
mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size
split	subtractIfCan	supp	suppOfPole
suppOfZero	terms	zero?	

These operations are exported but not implemented:

```
?<=? : (%,%) -> Boolean
collect : % -> %
concat : (%,%) -> %
degree : % -> Integer
divOfPole : % -> %
divOfZero : % -> %
effective? : % -> Boolean
incr : % -> %
split : % -> List %
supp : % -> List S
suppOfPole : % -> List S
suppOfZero : % -> List S
```

These operations come from (p576) AbelianGroup:

```
0 : () -> %
-? : % -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
```

These operations come from (p966) Module(Integer):

```
?*? : (Integer,%) -> %
?*? : (%,Integer) -> %
```

These operations come from (p622) FreeAbelianMonoidCategory(S: SetCategory,E: CancellationAbelianMonoid)

```
?*? : (E,S) -> %
?+? : (S,%) -> %
coefficient : (S,%) -> E
coerce : S -> %
highCommonTerms : (%,%) -> % if E has OAMON
mapCoef : ((E -> E),%) -> %
mapGen : ((S -> S),%) -> %
nthCoef : (%,Integer) -> E
nthFactor : (%,Integer) -> S
retract : % -> S
retractIfCan : % -> Union(S,"failed")
size : % -> NonNegativeInteger
terms : % -> List Record(gen: S,exp: E)
```

⟨*category DIVCAT DivisorCategory*⟩≡

```
)abbrev category DIVCAT DivisorCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description: This category exports the function for domains
DivisorCategory(S:SetCategory):Category == Exports where

 INT     ==> Integer
 BOOLEAN ==> Boolean
 LIST    ==> List

 Exports ==> _
  Join(AbelianGroup,Module Integer,FreeAbelianMonoidCategory(S,Integer)) with

    degree: % -> INT
      ++ degree(d) returns the degree of the divisor d

    split: % -> List %
      ++ split(d) splits the divisor d. For example,
      ++ split( 2 p1 + 3p2 ) returns the list [ 2 p1, 3 p2 ].

    "<=" : (%,%) -> BOOLEAN

    collect: % -> %
      ++ collect collects the duplicative points in the divisor.
    concat: (%,%) -> %
      ++ concat(a,b) concats the divisor a and b
      ++ without collecting the duplicative points.

    effective?: % -> BOOLEAN
      ++ effective?(d) returns true if d >= 0.

    supp: % -> LIST(S)
      ++ supp(d) returns the support of the divisor d.

    suppOfZero: % -> LIST(S)
      ++ suppOfZero(d) returns the elements of the support of d that
      ++ have a positive coefficient.

    suppOfPole: % -> LIST(S)
      ++ suppOfZero(d) returns the elements of the support of d that
      ++ have a negative coefficient.

    divOfZero: % -> %
      ++ divOfZero(d) returns the positive part of d.

    divOfPole: % -> %
      ++ divOfPole(d) returns the negative part of d.

    incr: % -> %
```

⟨*DIVCAT.dotabb*⟩≡

```
"DIVCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"OAGROUP" [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"PID" [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"DIVCAT" -> "OAGROUP"
"DIVCAT" -> "PID"
```

⟨*DIVCAT.dotfull*⟩≡

```
"DivisorCategory()" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"DivisorCategory()" -> "PrincipalIdealDomain()"
```

⟨*DIVCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DivisorCategory()" [color=lightblue];
"DivisorCategory()" -> "PrincipalIdealDomain()"
"DivisorCategory()" -> "OrderedAbelianGroup()"

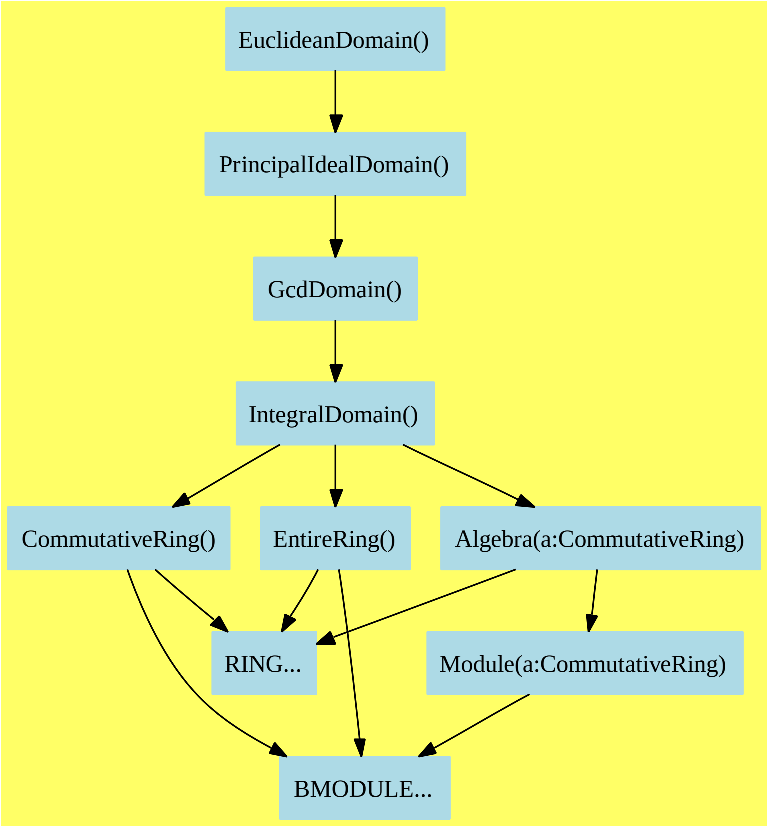
"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OCAMON..."
"OrderedAbelianGroup()" -> "ABELGRP..."

"GCDDOM..." [color=lightblue];
"OCAMON..." [color=lightblue];
"ABELGRP..." [color=lightblue];

}
```

15.0.181 EuclideanDomain (EUCDOM)


EuclideanDomain()
PrincipalIdealDomain()
GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BMODULE...

⟨*EuclideanDomain.input*⟩≡

```
)set break resume
)sys rm -f EuclideanDomain.output
)spool EuclideanDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EuclideanDomain
--R EuclideanDomain  is a category constructor
--R Abbreviation for EuclideanDomain is EUCDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for EUCDOM
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      one? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R sizeLess? : (%,%) -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*EuclideanDomain.help*⟩≡

```
====================================================================
EuclideanDomain examples
====================================================================

See Also:
o )show EuclideanDomain
```

Principal Ideal Domains are a subset of Euclidean Domains.
⇐ “PrincipalIdealDomain” (PID) 14.0.178 on page 1288. Euclidean Domains are a subset of Fields.
⇒ “Field” (FIELD) 16.0.185 on page 1347

See:

⇒ “Field” (FIELD) 16.0.185 on page 1347
⇒ “IntegerNumberSystem” (INS) 16.0.186 on page 1355
⇒ “PAdicIntegerCategory” (PADICCT) 16.0.188 on page 1377
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇐ “PrincipalIdealDomain” (PID) 14.0.178 on page 1288

Exports:

1	0	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
gcd	gcdPolynomial	hash	latex	lcm
multiEuclidean	one?	principalIdeal	recip	sample
sizeLess?	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?quo?	?rem?	?~=?	?*?	?**?
?^?				

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
```

These are implemented by this category:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
gcd : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1288) PrincipalIdealDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category EUCDOM EuclideanDomain*⟩≡

```
)abbrev category EUCDOM EuclideanDomain
++ Author:
```

```
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A constructive euclidean domain, i.e. one can divide producing
++ a quotient and a remainder where the remainder is either zero
++ or is smaller (\spadfun{euclideanSize}) than the divisor.
++
++ Conditional attributes\br
++ \tab{5}multiplicativeValuation\tab{5}Size(a*b)=Size(a)*Size(b)\br
++ \tab{5}additiveValuation\tab{11}Size(a*b)=Size(a)+Size(b)

EuclideanDomain(): Category == PrincipalIdealDomain with
   sizeLess?: (%,%) -> Boolean
     ++ sizeLess?(x,y) tests whether x is strictly
     ++ smaller than y with respect to the
     ++ \spadfunFrom{euclideanSize}{EuclideanDomain}.
   euclideanSize: % -> NonNegativeInteger
     ++ euclideanSize(x) returns the euclidean size of the element x.
     ++ Error: if x is zero.
   divide: (%,%) -> Record(quotient:%,remainder:%)
     ++ divide(x,y) divides x by y producing a record containing a
     ++ \spad{quotient} and \spad{remainder},
     ++ where the remainder is smaller (see
     ++ \spadfunFrom{sizeLess?}{EuclideanDomain}) than the divisor y.
   "quo" : (%,%) -> %
     ++ x quo y is the same as \spad{divide(x,y).quotient}.
     ++ See \spadfunFrom{divide}{EuclideanDomain}.
   "rem": (%,%) -> %
     ++ x rem y is the same as \spad{divide(x,y).remainder}.
     ++ See \spadfunFrom{divide}{EuclideanDomain}.
   extendedEuclidean: (%,%) -> Record(coef1:%,coef2:%,generator:%)
     -- formerly called princIdeal
     ++ extendedEuclidean(x,y) returns a record rec where
     ++ \spad{rec.coef1*x+rec.coef2*y = rec.generator} and
     ++ rec.generator is a gcd of x and y.
     ++ The gcd is unique only
     ++ up to associates if \spadatt{canonicalUnitNormal} is not asserted.
     ++ \spadfun{principalIdeal} provides a version of this operation
     ++ which accepts an arbitrary length list of arguments.
   extendedEuclidean: (%,%,%) -> Union(Record(coef1:%,coef2:%),"failed")
```

```
      -- formerly called expressIdealElt
      ++ extendedEuclidean(x,y,z) either returns a record rec
      ++ where \spad{rec.coef1*x+rec.coef2*y=z} or returns "failed"
      ++ if z cannot be expressed as a linear combination of x and y.
    multiEuclidean: (List %,%) -> Union(List %,"failed")
      ++ multiEuclidean([f1,...,fn],z) returns a list of coefficients
      ++ \spad{[a1, ..., an]} such that
      ++ \spad{ z / prod fi = sum aj/fj}.
      ++ If no such list of coefficients exists, "failed" is returned.
   add
       x,y,z: %
       l: List %
       sizeLess?(x,y) ==
             zero? y => false
             zero? x => true
             euclideanSize(x)<euclideanSize(y)
       x quo y == divide(x,y).quotient --divide must be user-supplied
       x rem y == divide(x,y).remainder
       x exquo y ==
          zero? x => 0
          zero? y => "failed"
          qr:=divide(x,y)
          zero?(qr.remainder) => qr.quotient
          "failed"
       gcd(x,y) ==                --Euclidean Algorithm
          x:=unitCanonical x
          y:=unitCanonical y
          while not zero? y repeat
             (x,y):= (y,x rem y)
             y:=unitCanonical y   -- this doesn't affect the
                                  -- correctness of Euclid's algorithm,
                                  -- but
                                  -- a) may improve performance
                                  -- b) ensures gcd(x,y)=gcd(y,x)
                                  --    if canonicalUnitNormal
          x
       IdealElt ==> Record(coef1:%,coef2:%,generator:%)
       unitNormalizeIdealElt(s:IdealElt):IdealElt ==
          (u,c,a):=unitNormal(s.generator)
--          one? a => s
          (a = 1) => s
          [a*s.coef1,a*s.coef2,c]$IdealElt
       extendedEuclidean(x,y) ==         --Extended Euclidean Algorithm
          s1:=unitNormalizeIdealElt([1$%,0$%,x]$IdealElt)
          s2:=unitNormalizeIdealElt([0$%,1$%,y]$IdealElt)
          zero? y => s1
   zero? x => s2
   while not zero?(s2.generator) repeat
      qr:= divide(s1.generator, s2.generator)
      s3:=[s1.coef1 - qr.quotient * s2.coef1,
           s1.coef2 - qr.quotient * s2.coef2, qr.remainder]$IdealElt
      s1:=s2
      s2:=unitNormalizeIdealElt s3
   if not(zero?(s1.coef1)) and not sizeLess?(s1.coef1,y)
     then
        qr:= divide(s1.coef1,y)
        s1.coef1:= qr.remainder
        s1.coef2:= s1.coef2 + qr.quotient * x
        s1 := unitNormalizeIdealElt s1
   s1

TwoCoefs ==> Record(coef1:%,coef2:%)
extendedEuclidean(x,y,z) ==
   zero? z => [0,0]$TwoCoefs
   s:= extendedEuclidean(x,y)
   (w:= z exquo s.generator) case "failed" => "failed"
   zero? y =>
      [s.coef1 * w, s.coef2 * w]$TwoCoefs
   qr:= divide((s.coef1 * w), y)
   [qr.remainder, s.coef2 * w + qr.quotient * x]$TwoCoefs
principalIdeal l ==
   l = [] => error "empty list passed to principalIdeal"
   rest l = [] =>
        uca:=unitNormal(first l)
        [[uca.unit],uca.canonical]
   rest rest l = [] =>
       u:= extendedEuclidean(first l,second l)
       [[u.coef1, u.coef2], u.generator]
   v:=principalIdeal rest l
   u:= extendedEuclidean(first l,v.generator)
   [[u.coef1,:[u.coef2*vv for vv in v.coef]],u.generator]
expressIdealMember(l,z) ==
   z = 0 => [0 for v in l]
   pid := principalIdeal l
   (q := z exquo (pid.generator)) case "failed" => "failed"
   [q*v for v in pid.coef]
multiEuclidean(l,z) ==
   n := #l
   zero? n => error "empty list passed to multiEuclidean"
   n = 1 => [z]
   l1 := copy l
   l2 := split!(l1, n quo 2)
```

```
          u:= extendedEuclidean(*/l1, */l2, z)
          u case "failed" => "failed"
          v1 := multiEuclidean(l1,u.coef2)
          v1 case "failed" => "failed"
          v2 := multiEuclidean(l2,u.coef1)
          v2 case "failed" => "failed"
          concat(v1,v2)
```

⟨*EUCDOM.dotabb*⟩≡

```
"EUCDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"EUCDOM" -> "PID"
```

⟨*EUCDOM.dotfull*⟩≡

```
"EuclideanDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"EuclideanDomain()" -> "PrincipalIdealDomain()"
```

⟨*EUCDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

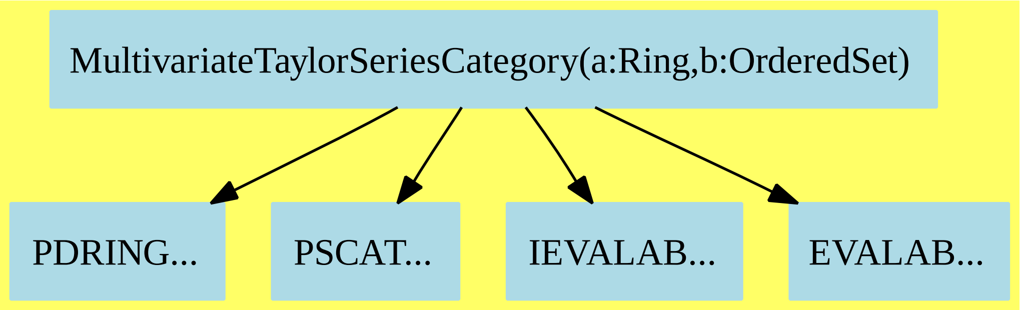
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

15.0.182 MultivariateTaylorSeriesCategory (MTSCAT)



⟨*MultivariateTaylorSeriesCategory.input*⟩≡

```
  )set break resume
  )sys rm -f MultivariateTaylorSeriesCategory.output
  )spool MultivariateTaylorSeriesCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show MultivariateTaylorSeriesCategory
  --R MultivariateTaylorSeriesCategory(Coef: Ring,Var: OrderedSet)  is a category c
  --R Abbreviation for MultivariateTaylorSeriesCategory is MTSCAT
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for MTSCAT
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (%,Coef) -> %                   ?*? : (Coef,%) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
  --R D : (%,List Var) -> %                 D : (%,Var) -> %
  --R 1 : () -> %                           0 : () -> %
  --R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
  --R coerce : % -> OutputForm              complete : % -> %
  --R differentiate : (%,Var) -> %          eval : (%,List %,List %) -> %
  --R eval : (%,%,%) -> %                   eval : (%,Equation %) -> %
  --R eval : (%,List Equation %) -> %       eval : (%,List Var,List %) -> %
  --R eval : (%,Var,%) -> %                 hash : % -> SingleInteger
  --R latex : % -> String                   leadingCoefficient : % -> Coef
  --R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
  --R monomial? : % -> Boolean              one? : % -> Boolean
```

```
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R variables : % -> List Var             zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : (%,List Var,List NonNegativeInteger) -> %
--R D : (%,Var,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,List Var,List NonNegativeInteger) -> %
--R coefficient : (%,Var,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents Var) -> Coef
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : % -> % if Coef has INTDOM
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R degree : % -> IndexedExponents Var
--R differentiate : (%,List Var,List NonNegativeInteger) -> %
--R differentiate : (%,Var,NonNegativeInteger) -> %
--R differentiate : (%,List Var) -> %
--R exp : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R integrate : (%,Var) -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List Var,List NonNegativeInteger) -> %
--R monomial : (%,Var,NonNegativeInteger) -> %
--R monomial : (Coef,IndexedExponents Var) -> %
--R monomial : (%,Var,IndexedExponents Var) -> %
--R monomial : (%,List Var,List IndexedExponents Var) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Var,NonNegativeInteger) -> NonNegativeInteger
--R order : (%,Var) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDO
--R
--E 1

)spool
)lisp (bye)
```

⟨*MultivariateTaylorSeriesCategory.help*⟩≡

```
====================================================================
MultivariateTaylorSeriesCategory examples
====================================================================

See Also:
o )show MultivariateTaylorSeriesCategory
```

See:

⇐ “Evalable” (EVALAB) 3.0.25 on page 91
⇐ “InnerEvalable” (IEVALAB) 2.0.12 on page 39
⇐ “PartialDifferentialRing” (PDRING) 10.0.145 on page 977
⇐ “PowerSeriesCategory” (PSCAT) 14.0.177 on page 1280

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	charthRoot	coefficient	coerce	complete
cos	cosh	cot	coth	csc
csch	D	degree	differentiate	eval
exp	exquo	extend	hash	integrate
latex	leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	nthRoot	order	one?
pi	pole?	polynomial	recip	reductum
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	tan	tanh	unit?
unitCanonical	unitNormal	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?/?		

Attributes Exported:

- if $ has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation ” $*$ ” : $(D,D)->D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1*x=x$ for all x.
- **rightUnitary** is true if $x*1=x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Var,NonNegativeInteger) -> %
coefficient : (%,List Var,List NonNegativeInteger) -> %
extend : (%,NonNegativeInteger) -> %
integrate : (%,Var) -> % if Coef has ALGEBRA FRAC INT
monomial : (%,Var,NonNegativeInteger) -> %
monomial : (%,List Var,List NonNegativeInteger) -> %
```

```
order : (%,Var,NonNegativeInteger) -> NonNegativeInteger
order : (%,Var) -> NonNegativeInteger
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
```

These exports come from (p977) PartialDifferentialRing(OrderedSet):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,Var) -> %
D : (%,List Var) -> %
D : (%,List Var,List NonNegativeInteger) -> %
D : (%,Var,NonNegativeInteger) -> %
differentiate : (%,Var) -> %
differentiate : (%,List Var,List NonNegativeInteger) -> %
differentiate : (%,Var,NonNegativeInteger) -> %
differentiate : (%,List Var) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1280) PowerSeriesCategory(A,B,C)
where A:Ring, B:IndexedExponents(OrderedSet) and C:OrderedSet:

```
associates? : (%,%) -> Boolean if Coef has INTDOM
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,IndexedExponents Var) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
```

```
complete : % -> %
degree : % -> IndexedExponents Var
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,IndexedExponents Var) -> %
monomial? : % -> Boolean
monomial : (%,List Var,List IndexedExponents Var) -> %
monomial : (%,Var,IndexedExponents Var) -> %
pole? : % -> Boolean
reductum : % -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variables : % -> List Var
?*? : (%,Coef) -> %
?*? : (Coef,%) -> %
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
```

These exports come from (p39) InnerEvalable(OrderedSet,

```
eval : (%,Var,%) -> %
eval : (%,List Var,List %) -> %
```

These exports come from (p91) Evalable(

```
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p135) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
```

```
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

⟨*category MTSCAT MultivariateTaylorSeriesCategory*⟩≡

```
)abbrev category MTSCAT MultivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 March 1990
++ Date Last Updated: 6 March 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++ \spadtype{MultivariateTaylorSeriesCategory} is the most general
++ multivariate Taylor series category.

MultivariateTaylorSeriesCategory(Coef,Var): Category == Definition where
  Coef  : Ring
  Var   : OrderedSet
  L   ==> List
  NNI ==> NonNegativeInteger

  Definition ==> Join(PartialDifferentialRing Var,_
                    PowerSeriesCategory(Coef,IndexedExponents Var,Var),_
                    InnerEvalable(Var,%),Evalable %) with
```

```
coefficient: (%,Var,NNI) -> %
  ++ \spad{coefficient(f,x,n)} returns the coefficient of \spad{x^n} in f.
coefficient: (%,L Var,L NNI) -> %
  ++ \spad{coefficient(f,[x1,x2,...,xk],[n1,n2,...,nk])} returns the
  ++ coefficient of \spad{x1^n1 * ... * xk^nk} in f.
extend: (%,NNI) -> %
  ++ \spad{extend(f,n)} causes all terms of f of degree
  ++ \spad{<= n} to be computed.
monomial: (%,Var,NNI) -> %
  ++ \spad{monomial(a,x,n)} returns  \spad{a*x^n}.
monomial: (%,L Var,L NNI) -> %
  ++ \spad{monomial(a,[x1,x2,...,xk],[n1,n2,...,nk])} returns
  ++ \spad{a * x1^n1 * ... * xk^nk}.
order: (%,Var) -> NNI
  ++ \spad{order(f,x)} returns the order of f viewed as a series in x
  ++ may result in an infinite loop if f has no non-zero terms.
order: (%,Var,NNI) -> NNI
  ++ \spad{order(f,x,n)} returns \spad{min(n,order(f,x))}.
polynomial: (%,NNI) -> Polynomial Coef
  ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
  ++ of all terms of f of degree \spad{<= k}.
polynomial: (%,NNI,NNI) -> Polynomial Coef
  ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
  ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.
if Coef has Algebra Fraction Integer then
  integrate: (%,Var) -> %
    ++ \spad{integrate(f,x)} returns the anti-derivative of the power
    ++ series \spad{f(x)} with respect to the variable x with constant
    ++ coefficient 1.  We may integrate a series when we can divide
    ++ coefficients by integers.
  RadicalCategory
    --++ We provide rational powers when we can divide coefficients
    --++ by integers.
  TranscendentalFunctionCategory
    --++ We provide transcendental functions when we can divide
    --++ coefficients by integers.
```

⟨*MTSCAT.dotabb*⟩≡

```
"MTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
"MTSCAT" -> "PDRING"
"MTSCAT" -> "PSCAT"
"MTSCAT" -> "IEVALAB"
"MTSCAT" -> "EVALAB"
```

⟨*MTSCAT.dotfull*⟩≡
```
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
    "PartialDifferentialRing(a:OrderedSet)"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
```

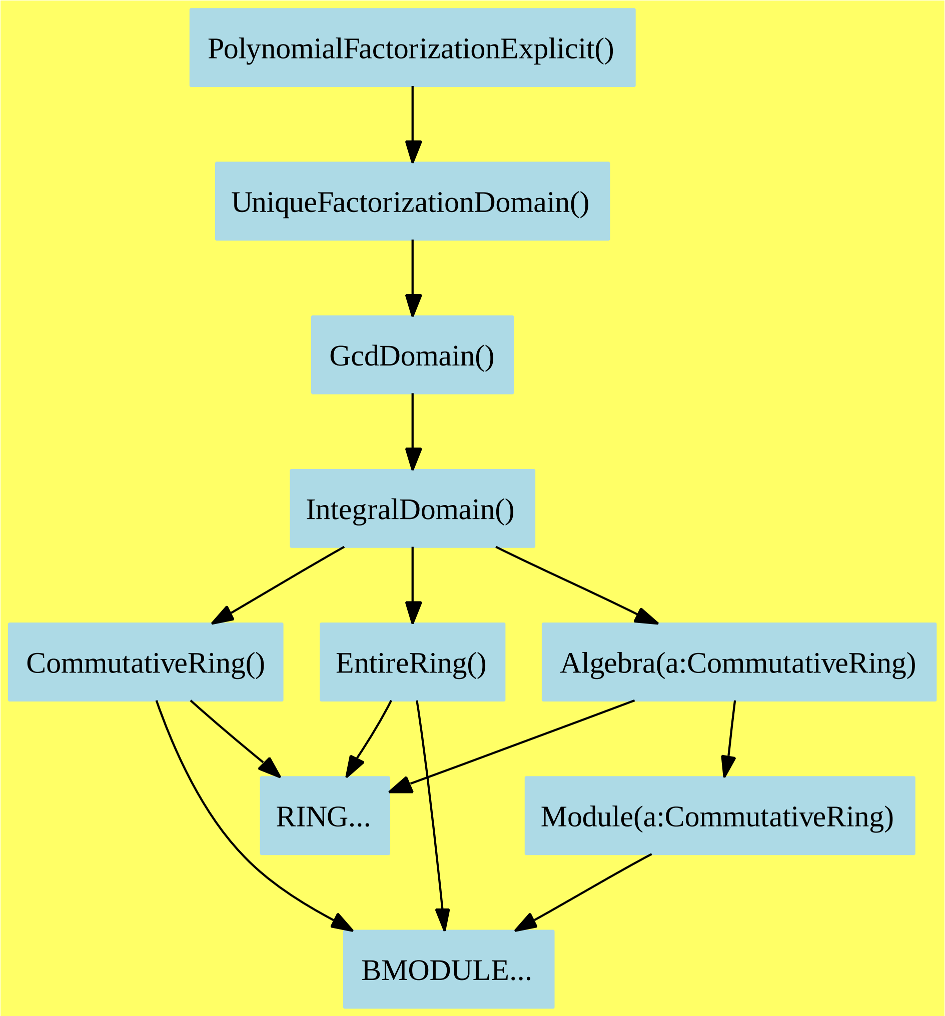
⟨*MTSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" [color=lightblue];
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
    "PDRING..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "PSCAT..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "IEVALAB..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "EVALAB..."

"PDRING..." [color=lightblue];
"PSCAT..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"EVALAB..." [color=lightblue];

}
```

15.0.183 PolynomialFactorizationExplicit (PFECAT)


PolynomialFactorizationExplicit()
UniqueFactorizationDomain()
GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BMODULE...

⟨*PolynomialFactorizationExplicit.input*⟩≡

```
)set break resume
)sys rm -f PolynomialFactorizationExplicit.output
)spool PolynomialFactorizationExplicit.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialFactorizationExplicit
```

```
--R PolynomialFactorizationExplicit  is a category constructor
--R Abbreviation for PolynomialFactorizationExplicit is PFECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PFECAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ
--R exquo : (%,%) -> Union(%,"failed")
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUniv
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivari
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PolynomialFactorizationExplicit.help*⟩≡

```
====================================================================
PolynomialFactorizationExplicit examples
====================================================================

See Also:
o )show PolynomialFactorizationExplicit
```

See:

⇒ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇐ “UniqueFactorizationDomain” (UFD) 14.0.179 on page 1294

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	exquo	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	hash	latex
lcm	one?	prime?
recip	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitNormal
unitCanonical	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?		

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation ” $*$ ” : $(D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionP : Matrix % -> Union(Vector %,"failed")
    if $ has CHARNZ
factorPolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed") if $ has CHARNZ
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
```

These exports come from (p1294) UniqueFactorizationDomain():

```
factor : % -> Factored %
squareFree : % -> Factored %
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
prime? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
unitCanonical : % -> %
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

```
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category PFECAT PolynomialFactorizationExplicit*⟩≡

```
)abbrev category PFECAT PolynomialFactorizationExplicit
++ Author: James Davenport
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This is the category of domains that know "enough" about
++ themselves in order to factor univariate polynomials over themselves.
++ This will be used in future releases for supporting factorization
++ over finitely generated coefficient fields, it is not yet available
++ in the current release of axiom.

PolynomialFactorizationExplicit(): Category == Definition where
  P ==> SparseUnivariatePolynomial %
  Definition ==>
   UniqueFactorizationDomain with
     squareFreePolynomial: P -> Factored(P)
       ++ squareFreePolynomial(p) returns the
       ++ square-free factorization of the
       ++ univariate polynomial p.
     factorPolynomial: P -> Factored(P)
       ++ factorPolynomial(p) returns the factorization
       ++ into irreducibles of the univariate polynomial p.
     factorSquareFreePolynomial: P -> Factored(P)
       ++ factorSquareFreePolynomial(p) factors the
       ++ univariate polynomial p into irreducibles
       ++ where p is known to be square free
       ++ and primitive with respect to its main variable.
     gcdPolynomial: (P, P) -> P
       ++ gcdPolynomial(p,q) returns the gcd of the univariate
       ++ polynomials p qnd q.
       -- defaults to Euclidean, but should be implemented via
       -- modular or p-adic methods.
     solveLinearPolynomialEquation: (List P, P) -> Union(List P,"failed")
       ++ solveLinearPolynomialEquation([f1, ..., fn], g)
   ++ (where the fi are relatively prime to each other)
   ++ returns a list of ai such that
   ++ \spad{g/prod fi = sum ai/fi}
   ++ or returns "failed" if no such list of ai's exists.
 if % has CharacteristicNonZero then
   conditionP: Matrix % -> Union(Vector %,"failed")
     ++ conditionP(m) returns a vector of elements, not all zero,
     ++ whose \spad{p}-th powers (p is the characteristic of the domain)
     ++ are a solution of the homogenous linear system represented
     ++ by m, or "failed" is there is no such vector.
   charthRoot: % -> Union(%,"failed")
     ++ charthRoot(r) returns the \spad{p}-th root of r, or "failed"
     ++ if none exists in the domain.
     -- this is a special case of conditionP, but often the one we want
add
    gcdPolynomial(f,g) ==
       zero? f => g
       zero? g => f
       cf:=content f
       if not one? cf then f:=(f exquo cf)::P
       cg:=content g
       if not one? cg then g:=(g exquo cg)::P
       ans:=subResultantGcd(f,g)$P
       gcd(cf,cg)*(ans exquo content ans)::P
    if % has CharacteristicNonZero then
      charthRoot f ==
         -- to take p'th root of f, solve the system X-fY=0,
         -- so solution is [x,y]
         -- with x^p=X and y^p=Y, then (x/y)^p = f
         zero? f => 0
         m:Matrix % := matrix [[1,-f]]
         ans:= conditionP m
         ans case "failed" => "failed"
         (ans.1) exquo (ans.2)
    if % has Field then
      solveLinearPolynomialEquation(lf,g) ==
        multiEuclidean(lf,g)$P
    else solveLinearPolynomialEquation(lf,g) ==
           LPE ==> LinearPolynomialEquationByFractions %
           solveLinearPolynomialEquationByFractions(lf,g)$LPE
```

⟨*PFECAT.dotabb*⟩≡

```
"PFECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PFECAT" -> "UFD"
```

⟨*PFECAT.dotfull*⟩≡

```
"PolynomialFactorizationExplicit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"
```

⟨*PFECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialFactorizationExplicit()" [color=lightblue];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

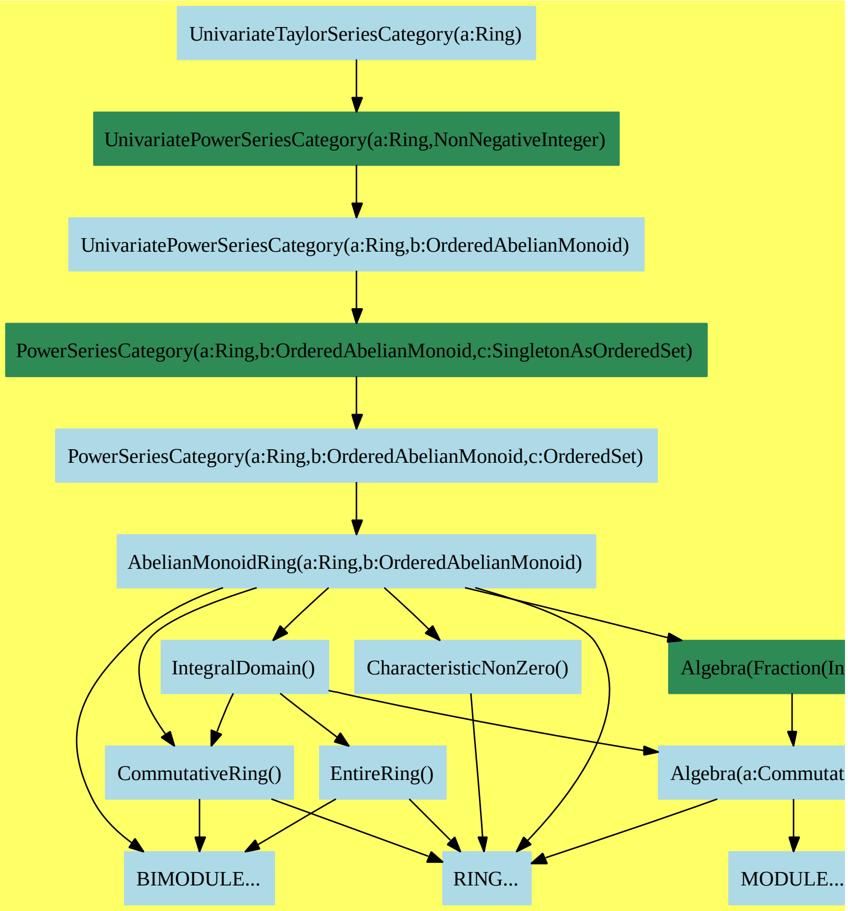
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

15.0.184 UnivariatePowerSeriesCategory (UPSCAT)


UnivariateTaylorSeriesCategory(a:Ring)
UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)
IntegralDomain()
CharacteristicNonZero()
Algebra(Fraction(In
CommutativeRing()
EntireRing()
Algebra(a:Commutat
BIMODULE...
RING...
MODULE...

⟨*UnivariatePowerSeriesCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariatePowerSeriesCategory.output
)spool UnivariatePowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePowerSeriesCategory
--R UnivariatePowerSeriesCategory(Coef: Ring,Expon: OrderedAbelianMonoid)  is a c
--R Abbreviation for UnivariatePowerSeriesCategory is UPSCAT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficient : (%,Expon) -> Coef       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> Expon                   ?.? : (%,Expon) -> Coef
--R extend : (%,Expon) -> %               hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Expon) -> %          monomial? : % -> Boolean
--R one? : % -> Boolean                   order : (%,Expon) -> Expon
--R order : % -> Expon                    pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      truncate : (%,Expon,Expon) -> %
--R truncate : (%,Expon) -> %             variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Expon,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Expon,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Expon,Coef) -> Coef
--R D : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Expon,Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has *: (Expon,
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Expon) -> Coef if Coef has **: (Coef,Expon) -> Coef and Coef has coerce
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Expon,Coef) -
--R differentiate : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Expon,Co
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRI
--R ?.? : (%,%) -> % if Expon has SGROUP
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Expon) -> Coef
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R monomial : (%,List SingletonAsOrderedSet,List Expon) -> %
--R monomial : (%,SingletonAsOrderedSet,Expon) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> Stream Record(k: Expon,c: Coef)
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDO
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariatePowerSeriesCategory.help*⟩≡

```
====================================================================
UnivariatePowerSeriesCategory examples
====================================================================

See Also:
o )show UnivariatePowerSeriesCategory
```

See:

⇒ “LocalPowerSeriesCategory” (LOCPOWC) 16.0.187 on page 1368
⇒ “UnivariateLaurentSeriesCategory” (ULSCAT) 17.0.202 on page 1595
⇒ “UnivariatePuiseuxSeriesCategory” (UPXSCAT) 17.0.203 on page 1608
⇒ “UnivariateTaylorSeriesCategory” (UTSCAT) 16.0.190 on page 1408
⇐ “PowerSeriesCategory” (PSCAT) 14.0.177 on page 1280

Exports:

0	1	approximate	associates?	center
characteristic	charthRoot	coefficient	coerce	complete
D	degree	differentiate	eval	exquo
extend	hash	latex	leadingCoefficient	leadingMonomial
map	monomial	monomial?	multiplyExponents	one?
order	pole?	recip	reductum	sample
subtractIfCan	truncate	terms	unit?	unitCanonical
unitNormal	variable	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?.?	?~=?	?/?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.

These are directly exported but not implemented:

```
approximate : (%,Expon) -> Coef
    if Coef has **: (Coef,Expon) -> Coef
    and Coef has coerce: Symbol -> Coef
center : % -> Coef
eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Expon) -> Coef
extend : (%,Expon) -> %
multiplyExponents : (%,PositiveInteger) -> %
order : % -> Expon
order : (%,Expon) -> Expon
```

```
terms : % -> Stream Record(k: Expon,c: Coef)
truncate : (%,Expon) -> %
truncate : (%,Expon,Expon) -> %
variable : % -> Symbol
?.? : (%,Expon) -> Coef
```

These are implemented by this category:

```
degree : % -> Expon
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
monomial : (%,SingletonAsOrderedSet,Expon) -> %
reductum : % -> %
variables : % -> List SingletonAsOrderedSet
```

These exports come from (p1280) PowerSeriesCategory(C,E,S)
where C:Ring, E:OrderedAbelianMonoid, S:SingletonAsOrderedSet:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (%,List Var,List Expon) -> %
monomial : (Coef,Expon) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
pole? : % -> Boolean
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
```

```
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p32) Eltable(%,%):

```
?.? : (%,%) -> % if Expon has SGROUP
```

These exports come from (p938) DifferentialRing():

```
D : % -> %
    if Coef has *: (Expon,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (Expon,Coef) -> Coef
```

These exports come from (p977) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
```

⟨*category UPSCAT UnivariatePowerSeriesCategory*⟩≡

```
)abbrev category UPSCAT UnivariatePowerSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{UnivariatePowerSeriesCategory} is the most general
++ univariate power series category with exponents in an ordered
++ abelian monoid.
++ Note that this category exports a substitution function if it is
++ possible to multiply exponents.
++ Also note that this category exports a derivative operation if it is
++ possible to multiply coefficients by exponents.

UnivariatePowerSeriesCategory(Coef,Expon): Category == Definition where
  Coef   : Ring
  Expon  : OrderedAbelianMonoid
  Term ==> Record(k:Expon,c:Coef)

  Definition ==> PowerSeriesCategory(Coef,Expon,SingletonAsOrderedSet) with

    terms: % -> Stream Term
      ++ \spad{terms(f(x))} returns a stream of non-zero terms, where a
      ++ a term is an exponent-coefficient pair.  The terms in the stream
      ++ are ordered by increasing order of exponents.
    --series: Stream Term -> %
      --++ \spad{series(st)} creates a series from a stream of non-zero terms,
      --++ where a term is an exponent-coefficient pair.  The terms in the
      --++ stream should be ordered by increasing order of exponents.
```

```
 elt: (%,Expon) -> Coef
   ++ \spad{elt(f(x),r)} returns the coefficient of the term of degree r in
   ++ \spad{f(x)}.  This is the same as the function \spadfun{coefficient}.
 variable: % -> Symbol
   ++ \spad{variable(f)} returns the (unique) power series variable of
   ++ the power series f.
 center: % -> Coef
   ++ \spad{center(f)} returns the point about which the series f is
   ++ expanded.
 multiplyExponents: (%,PositiveInteger) -> %
   ++ \spad{multiplyExponents(f,n)} multiplies all exponents of the power
   ++ series f by the positive integer n.
 order: % -> Expon
   ++ \spad{order(f)} is the degree of the lowest order non-zero term in f.
   ++ This will result in an infinite loop if f has no non-zero terms.
 order: (%,Expon) -> Expon
   ++ \spad{order(f,n) = min(m,n)}, where m is the degree of the
   ++ lowest order non-zero term in f.
 truncate: (%,Expon) -> %
   ++ \spad{truncate(f,k)} returns a (finite) power series consisting of
   ++ the sum of all terms of f of degree \spad{<= k}.
 truncate: (%,Expon,Expon) -> %
   ++ \spad{truncate(f,k1,k2)} returns a (finite) power
   ++ series consisting of
   ++ the sum of all terms of f of degree d with \spad{k1 <= d <= k2}.
 if Coef has coerce: Symbol -> Coef then
   if Coef has "**":(Coef,Expon) -> Coef then
     approximate: (%,Expon) -> Coef
       ++ \spad{approximate(f)} returns a truncated power series with the
       ++ series variable viewed as an element of the coefficient domain.
 extend: (%,Expon) -> %
   ++ \spad{extend(f,n)} causes all terms of f of degree <= n
   ++ to be computed.
 if Expon has SemiGroup then Eltable(%,%)
 if Coef has "*": (Expon,Coef) -> Coef then
   DifferentialRing
   --!! DifferentialExtension Coef
   if Coef has PartialDifferentialRing Symbol then
     PartialDifferentialRing Symbol
 if Coef has "**": (Coef,Expon) -> Coef then
   eval: (%,Coef) -> Stream Coef
     ++ \spad{eval(f,a)} evaluates a power series at a value in the
     ++ ground ring by returning a stream of partial sums.

add
 degree f == order f
```

```
      leadingCoefficient f == coefficient(f,order f)

      leadingMonomial f ==
        ord := order f
        monomial(coefficient(f,ord),ord)

      monomial(f:%,listVar:List SingletonAsOrderedSet,listExpon:List Expon) ==
        empty? listVar or not empty? rest listVar =>
          error "monomial: variable list must have exactly one entry"
        empty? listExpon or not empty? rest listExpon =>
          error "monomial: exponent list must have exactly one entry"
        f * monomial(1,first listExpon)

      monomial(f:%,v:SingletonAsOrderedSet,n:Expon) ==
        f * monomial(1,n)

      reductum f == f - leadingMonomial f

      variables f == list create()
```

⟨*UPSCAT.dotabb*⟩≡

```
"UPSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UPSCAT" -> "PSCAT"
```

⟨*UPSCAT.dotfull*⟩≡

```
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
```

⟨*UPSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

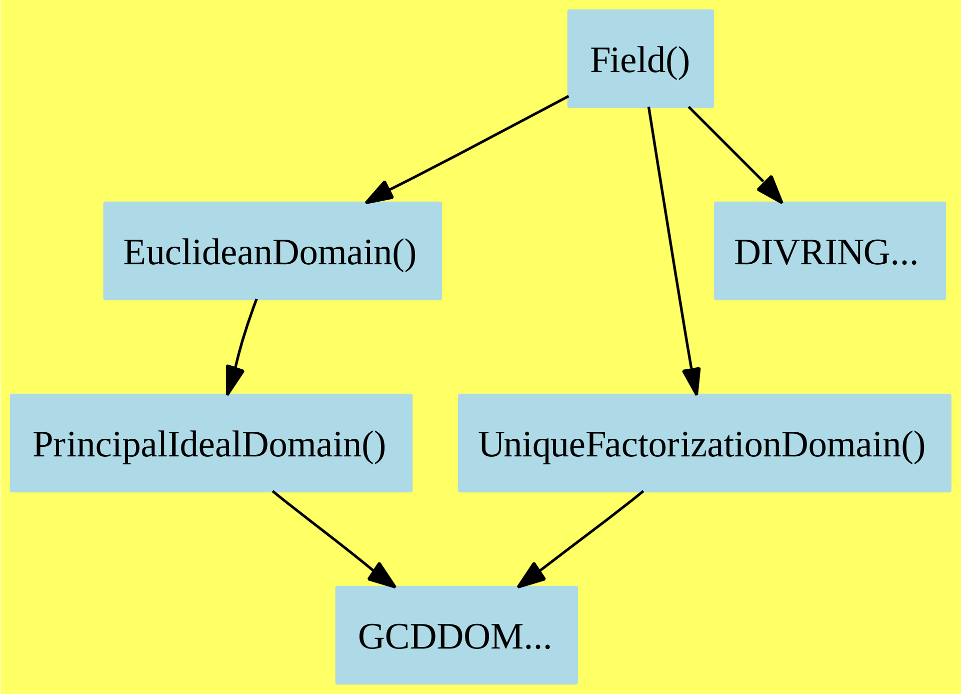
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

Chapter 16

Category Layer 15

16.0.185 Field (FIELD)


Field()
EuclideanDomain()
DIVRING...
PrincipalIdealDomain()
UniqueFactorizationDomain()
GCDDOM...

⟨*Field.input*⟩≡

```
)set break resume
)sys rm -f Field.output
)spool Field.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Field
--R Field  is a category constructor
--R Abbreviation for Field is FIELD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FIELD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inv : % -> %
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*Field.help*⟩≡

```
====================================================================
Field examples
====================================================================

See Also:
o )show Field
```

Euclidean Domains are a subset of Fields.
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307
⇒ “PseudoAlgebraicClosureOfPerfectFieldCategory” (PACPERC) 17.0.197 on page 1509

See:

⇒ “AlgebraicallyClosedField” (ACF) 17.0.191 on page 1425
⇒ “ExtensionField” (XF) 18.0.206 on page 1662
⇒ “FieldOfPrimeCharacteristic” (FPC) 17.0.193 on page 1457
⇒ “FiniteRankAlgebra” (FINRALG) 17.0.194 on page 1465
⇒ “FunctionSpace” (FS) 17.0.195 on page 1472
⇒ “QuotientFieldCategory” (QFCAT) 17.0.198 on page 1516
⇒ “RealClosedField” (RCFIELD) 17.0.199 on page 1531
⇒ “RealNumberSystem” (RNS) 17.0.200 on page 1543
⇒ “UnivariateLaurentSeriesCategory” (ULSCAT) 17.0.202 on page 1595
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.204 on page 1620
⇒ “UnivariatePuiseuxSeriesCategory” (UPXSCAT) 17.0.203 on page 1608
⇐ “DivisionRing” (DIVRING) 12.0.161 on page 1116
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307
⇐ “UniqueFactorizationDomain” (UFD) 14.0.179 on page 1294

Exports:

0	1	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash	inv
latex	lcm	multiEuclidean	one?	prime?
principalIdeal	recip	sample	sizeLess?	squareFree
squareFreePart	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?	?-?
-?	?/?	?=?	?^?	?quo?
?rem?	?~=?			

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
associates? : (%,%) -> Boolean
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
inv : % -> %
prime? : % -> Boolean
squareFree : % -> Factored %
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
```

These exports come from (p1307) EuclideanDomain():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
```

```
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1294) UniqueFactorizationDomain():

```
squareFreePart : % -> %
```

These exports come from (p1116) DivisionRing():

```
coerce : Fraction Integer -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
```

⟨*category FIELD Field*⟩≡

```
)abbrev category FIELD Field
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of commutative fields, i.e. commutative rings
++ where all non-zero elements have multiplicative inverses.
++ The \spadfun{factor} operation while trivial is useful to have defined.
++
++ Axioms\br
++ \tab{5}\spad{a*(b/a) = b}\br
++ \tab{5}\spad{inv(a) = 1/a}
```

```
  Field(): Category == Join(EuclideanDomain,UniqueFactorizationDomain,
    DivisionRing) with
        "/": (%,%) -> %
          ++ x/y divides the element x by the element y.
          ++ Error: if y is 0.
        canonicalUnitNormal  ++ either 0 or 1.
        canonicalsClosed     ++ since \spad{0*0=0}, \spad{1*1=1}
      add
        x,y: %
        n: Integer
        UCA ==> Record(unit:%,canonical:%,associate:%)
        unitNormal(x) ==
            if zero? x then [1$%,0$%,1$%]$UCA else [x,1$%,inv(x)]$UCA
        unitCanonical(x) == if zero? x then x else 1
        associates?(x,y) == if zero? x then zero? y else not(zero? y)
        inv x ==((u:=recip x) case "failed" => error "not invertible"; u)
        x exquo y == (y=0 => "failed"; x / y)
        gcd(x,y) == 1
        euclideanSize(x) == 0
        prime? x == false
        squareFree x == x::Factored(%)
        factor x == x::Factored(%)
        x / y == (zero? y => error "catdef: division by zero"; x * inv(y))
        divide(x,y) == [x / y,0]
```

⟨*FIELD.dotabb*⟩≡

```
  "FIELD"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
  "FIELD" -> "EUCDOM"
  "FIELD" -> "UFD"
  "FIELD" -> "DIVRING"
```

⟨*FIELD.dotfull*⟩≡

```
  "Field()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
  "Field()" -> "EuclideanDomain()"
  "Field()" -> "UniqueFactorizationDomain()"
  "Field()" -> "DivisionRing()"
```

⟨*FIELD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

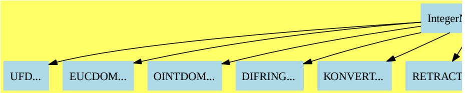
"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
}
```

16.0.186 IntegerNumberSystem (INS)



⟨*IntegerNumberSystem.input*⟩≡

```
)set break resume
)sys rm -f IntegerNumberSystem.output
)spool IntegerNumberSystem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegerNumberSystem
--R IntegerNumberSystem  is a category constructor
--R Abbreviation for IntegerNumberSystem is INS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                        ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %          ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                        ?-? : (%,%) -> %
--R -? : % -> %                             ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean                 ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                  ?>=? : (%,%) -> Boolean
--R D : % -> %                              D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                             0 : () -> %
--R ?^? : (%,PositiveInteger) -> %          abs : % -> %
--R addmod : (%,%,%) -> %                   associates? : (%,%) -> Boolean
--R base : () -> %                          binomial : (%,%) -> %
--R bit? : (%,%) -> Boolean                 coerce : Integer -> %
--R coerce : % -> %                         coerce : Integer -> %
--R coerce : % -> OutputForm                convert : % -> DoubleFloat
--R convert : % -> Float                    convert : % -> Pattern Integer
--R convert : % -> InputForm                convert : % -> Integer
--R copy : % -> %                           dec : % -> %
--R differentiate : % -> %                  even? : % -> Boolean
--R factor : % -> Factored %                factorial : % -> %
--R gcd : List % -> %                       gcd : (%,%) -> %
--R hash : % -> %                           hash : % -> SingleInteger
--R inc : % -> %                            init : () -> %
--R invmod : (%,%) -> %                     latex : % -> String
```

```
--R lcm : List % -> %                     lcm : (%,%) -> %
--R length : % -> %                       mask : % -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R mulmod : (%,%,%) -> %                 negative? : % -> Boolean
--R odd? : % -> Boolean                   one? : % -> Boolean
--R permutation : (%,%) -> %              positive? : % -> Boolean
--R positiveRemainder : (%,%) -> %        powmod : (%,%,%) -> %
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R random : % -> %                       random : () -> %
--R rational : % -> Fraction Integer      rational? : % -> Boolean
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> Integer                sample : () -> %
--R shift : (%,%) -> %                    sign : % -> Integer
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               submod : (%,%,%) -> %
--R symmetricRemainder : (%,%) -> %       unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMa
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R rationalIfCan : % -> Union(Fraction Integer,"failed")
--R reducedSystem : Matrix % -> Matrix Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*IntegerNumberSystem.help*⟩≡
```
====================================================================
IntegerNumberSystem examples
====================================================================

See Also:
o )show IntegerNumberSystem
```

See:

⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “CombinatorialFunctionCategory” (CFCAT) 2.0.7 on page 22
⇐ “ConvertibleTo” (KONVERT) 2.0.8 on page 25
⇐ “DifferentialRing” (DIFRING) 10.0.138 on page 938
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.0.142 on page 961
⇐ “OrderedIntegralDomain” (OINTDOM) 13.0.174 on page 1253
⇐ “Patternable” (PATAB) 2.0.15 on page 53
⇐ “RealConstant” (REAL) 3.0.32 on page 122
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “StepThrough” (STEP) 4.0.64 on page 283
⇐ “UniqueFactorizationDomain” (UFD) 14.0.179 on page 1294

Exports:

0	1	abs	addmod
associates?	base	binomial	bit?
characteristic	coerce	convert	copy
D	dec	differentiate	divide
euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	factor	factorial	gcd
gcdPolynomial	hash	inc	init
invmod	latex	lcm	length
mask	max	min	mulmod
multiEuclidean	negative?	nextItem	odd?
one?	patternMatch	permutation	positive?
positiveRemainder	powmod	prime?	principalIdeal
random	rational	rational?	rationalIfCan
recip	reducedSystem	retract	retractIfCan
sample	shift	sign	sizeLess?
squareFree	squareFreePart	submod	subtractIfCan
symmetricRemainder	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
addmod : (%,%,%) -> %
base : () -> %
dec : % -> %
hash : % -> %
inc : % -> %
length : % -> %
mulmod : (%,%,%) -> %
odd? : % -> Boolean
positiveRemainder : (%,%) -> %
random : () -> %
random : % -> %
shift : (%,%) -> %
submod : (%,%,%) -> %
```

These are implemented by this category:

```
binomial : (%,%) -> %
bit? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
convert : % -> Float
convert : % -> DoubleFloat
convert : % -> InputForm
convert : % -> Pattern Integer
copy : % -> %
differentiate : % -> %
euclideanSize : % -> NonNegativeInteger
even? : % -> Boolean
factor : % -> Factored %
factorial : % -> %
init : () -> %
invmod : (%,%) -> %
mask : % -> %
nextItem : % -> Union(%,"failed")
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
```

```
   PatternMatchResult(Integer,%)
permutation : (%,%) -> %
positive? : % -> Boolean
powmod : (%,%,%) -> %
prime? : % -> Boolean
rational : % -> Fraction Integer
rational? : % -> Boolean
rationalIfCan : % -> Union(Fraction Integer,"failed")
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
squareFree : % -> Factored %
symmetricRemainder : (%,%) -> %
```

These exports come from (p1294) UniqueFactorizationDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

```
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1307) EuclideanDomain():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1253) OrderedIntegralDomain():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p938) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p25) ConvertibleTo(Integer):

```
convert : % -> Integer
```

These exports come from (p961) LinearlyExplicitRingOver(Integer):

```
reducedSystem : Matrix % -> Matrix Integer
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
```

⟨*category INS IntegerNumberSystem*⟩≡

```
)abbrev category INS IntegerNumberSystem
++ Author: Stephen M. Watt
++ Date Created: January 1988
++ Change History:
++ Description:
++ An \spad{IntegerNumberSystem} is a model for the integers.

IntegerNumberSystem(): Category ==
  Join(UniqueFactorizationDomain, EuclideanDomain, OrderedIntegralDomain,
         DifferentialRing, ConvertibleTo Integer, RetractableTo Integer,
           LinearlyExplicitRingOver Integer, ConvertibleTo InputForm,
             ConvertibleTo Pattern Integer, PatternMatchable Integer,
               CombinatorialFunctionCategory, RealConstant,
                 CharacteristicZero, StepThrough) with
   odd?     : % -> Boolean
      ++ odd?(n) returns true if and only if n is odd.
   even?    : % -> Boolean
      ++ even?(n) returns true if and only if n is even.
   multiplicativeValuation
      ++ euclideanSize(a*b) returns \spad{euclideanSize(a)*euclideanSize(b)}.
   base     : () -> %
      ++ base() returns the base for the operations of
      ++ \spad{IntegerNumberSystem}.
   length   : % -> %
      ++ length(a) length of \spad{a} in digits.
   shift    : (%, %) -> %
      ++ shift(a,i) shift \spad{a} by i digits.
   bit?     : (%, %) -> Boolean
      ++ bit?(n,i) returns true if and only if i-th bit of n is a 1.
   positiveRemainder     : (%, %) -> %
      ++ positiveRemainder(a,b) (where \spad{b > 1}) yields r
      ++ where \spad{0 <= r < b} and \spad{r == a rem b}.
   symmetricRemainder     : (%, %) -> %
      ++ symmetricRemainder(a,b) (where \spad{b > 1}) yields r
      ++ where \spad{ -b/2 <= r < b/2 }.
   rational?: % -> Boolean
      ++ rational?(n) tests if n is a rational number
      ++ (see \spadtype{Fraction Integer}).
   rational : % -> Fraction Integer
      ++ rational(n) creates a rational number
      ++ (see \spadtype{Fraction Integer})..
   rationalIfCan: % -> Union(Fraction Integer, "failed")
      ++ rationalIfCan(n) creates a rational number, or returns "failed"
      ++ if this is not possible.
   random   : () -> %
      ++ random() creates a random element.
   random   : % -> %
      ++ random(a) creates a random element from 0 to \spad{n-1}.
```

```
     hash      : % -> %
        ++ hash(n) returns the hash code of n.
     copy      : % -> %
        ++ copy(n) gives a copy of n.
     inc       : % -> %
        ++ inc(x) returns \spad{x + 1}.
     dec       : % -> %
        ++ dec(x) returns \spad{x - 1}.
     mask      : % -> %
        ++ mask(n) returns \spad{2**n-1} (an n bit mask).
     addmod    : (%,%,%) -> %
        ++ addmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a+b mod p}.
     submod    : (%,%,%) -> %
        ++ submod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a-b mod p}.
     mulmod    : (%,%,%) -> %
        ++ mulmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a*b mod p}.
     powmod    : (%,%,%) -> %
        ++ powmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a**b mod p}.
     invmod    : (%,%) -> %
        ++ invmod(a,b), \spad{0<=a<b>1}, \spad{(a,b)=1} means \spad{1/a mod b}.
     canonicalUnitNormal
--     commutative("*")     -- follows from the above

   add
     characteristic()          == 0

     differentiate x           == 0

     even? x                   == not odd? x

     positive? x               == x > 0

     copy x                    == x

     bit?(x, i)                == odd? shift(x, -i)

     mask n                    == dec shift(1, n)

     rational? x               == true

     euclideanSize(x)          ==
          x=0 => error "euclideanSize called on zero"
          x<0 => (-convert(x)@Integer)::NonNegativeInteger
          convert(x)@Integer::NonNegativeInteger

     convert(x:%):Float        == (convert(x)@Integer)::Float
convert(x:%):DoubleFloat  == (convert(x)@Integer)::DoubleFloat

convert(x:%):InputForm   == convert(convert(x)@Integer)

retract(x:%):Integer     == convert(x)@Integer

convert(x:%):Pattern(Integer)== convert(x)@Integer ::Pattern(Integer)

factor x          == factor(x)$IntegerFactorizationPackage(%)

squareFree x      == squareFree(x)$IntegerFactorizationPackage(%)

prime? x          == prime?(x)$IntegerPrimesPackage(%)

factorial x       == factorial(x)$IntegerCombinatoricFunctions(%)

binomial(n, m)    == binomial(n, m)$IntegerCombinatoricFunctions(%)

permutation(n, m) == permutation(n,m)$IntegerCombinatoricFunctions(%)

retractIfCan(x:%):Union(Integer, "failed") == convert(x)@Integer

init() == 0

-- iterates in order 0,1,-1,2,-2,3,-3,...
nextItem(n) ==
  zero? n => 1
  n>0 => -n
  1-n

patternMatch(x, p, l) ==
  patternMatch(x, p, l)$PatternMatchIntegerNumberSystem(%)

rational(x:%):Fraction(Integer) ==
  (convert(x)@Integer)::Fraction(Integer)

rationalIfCan(x:%):Union(Fraction Integer, "failed") ==
  (convert(x)@Integer)::Fraction(Integer)

symmetricRemainder(x, n) ==
   r := x rem n
   r = 0 => r
   if n < 0 then n:=-n
   r > 0 =>
      2 * r > n => r - n
```

```
         r
      2*r + n <= 0 => r + n
      r

   invmod(a, b) ==
      if negative? a then a := positiveRemainder(a, b)
      c := a; c1:% := 1
      d := b; d1:% := 0
      while not zero? d repeat
         q := c quo d
         r := c-q*d
         r1 := c1-q*d1
         c := d; c1 := d1
         d := r; d1 := r1
--       not one? c => error "inverse does not exist"
      not (c = 1) => error "inverse does not exist"
      negative? c1 => c1 + b
      c1

   powmod(x, n, p) ==
      if negative? x then x := positiveRemainder(x, p)
      zero? x => 0
      zero? n => 1
      y:% := 1
      z := x
      repeat
         if odd? n then y := mulmod(y, z, p)
         zero?(n := shift(n, -1)) => return y
         z := mulmod(z, z, p)
```

⟨*INS.dotabb*⟩≡

```
"INS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"INS" -> "UFD"
"INS" -> "EUCDOM"
"INS" -> "OINTDOM"
"INS" -> "DIFRING"
"INS" -> "KONVERT"
"INS" -> "RETRACT"
"INS" -> "LINEXP"
"INS" -> "PATMAB"
"INS" -> "CFCAT"
"INS" -> "REAL"
"INS" -> "CHARZ"
"INS" -> "STEP"
```

⟨*INS.dotfull*⟩≡

```
"IntegerNumberSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"IntegerNumberSystem()" -> "UniqueFactorizationDomain()"
"IntegerNumberSystem()" -> "EuclideanDomain()"
"IntegerNumberSystem()" -> "OrderedIntegralDomain()"
"IntegerNumberSystem()" -> "DifferentialRing()"
"IntegerNumberSystem()" -> "ConvertibleTo(Integer)"
"IntegerNumberSystem()" -> "ConvertibleTo(InputForm)"
"IntegerNumberSystem()" -> "ConvertibleTo(Pattern(Integer))"
"IntegerNumberSystem()" -> "RetractableTo(Integer)"
"IntegerNumberSystem()" -> "LinearlyExplicitRingOver(Integer)"
"IntegerNumberSystem()" -> "PatternMatchable(Integer)"
"IntegerNumberSystem()" -> "CombinatorialFunctionCategory()"
"IntegerNumberSystem()" -> "RealConstant()"
"IntegerNumberSystem()" -> "CharacteristicZero()"
"IntegerNumberSystem()" -> "StepThrough()"
```

⟨*INS.dotpic*⟩≡

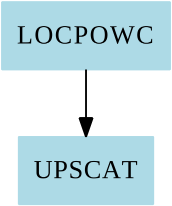
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntegerNumberSystem()" [color=lightblue];
"IntegerNumberSystem()" -> "UFD..."
"IntegerNumberSystem()" -> "EUCDOM..."
"IntegerNumberSystem()" -> "OINTDOM..."
"IntegerNumberSystem()" -> "DIFRING..."
"IntegerNumberSystem()" -> "KONVERT..."
"IntegerNumberSystem()" -> "RETRACT..."
"IntegerNumberSystem()" -> "LINEXP..."
"IntegerNumberSystem()" -> "PATMAB..."
"IntegerNumberSystem()" -> "CFCAT..."
"IntegerNumberSystem()" -> "REAL..."
"IntegerNumberSystem()" -> "CHARZ..."
"IntegerNumberSystem()" -> "STEP..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"OINTDOM..." [color=lightblue];
"DIFRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"RETRACT..." [color=lightblue];
"LINEXP..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CFCAT..." [color=lightblue];
"REAL..." [color=lightblue];
"CHARZ..." [color=lightblue];
"STEP..." [color=lightblue];

}
```

16.0.187 LocalPowerSeriesCategory (LOCPOWC)



⟨*LocalPowerSeriesCategory.input*⟩≡

```
)set break resume
)sys rm -f LocalPowerSeriesCategory.output
)spool LocalPowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LocalPowerSeriesCategory
--R LocalPowerSeriesCategory K: Field  is a category constructor
--R Abbreviation for LocalPowerSeriesCategory is LOCPOWC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LOCPOWC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,K) -> %                       ?*? : (K,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                       -? : % -> %
--R ?/? : (%,%) -> %                       ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,Integer) -> %                 ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        center : % -> K
--R coefOfFirstNonZeroTerm : % -> K       coefficient : (%,Integer) -> K
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Integer
--R delay : (() -> %) -> %                ?.? : (%,Integer) -> K
--R extend : (%,Integer) -> %             factor : % -> Factored %
--R filterUpTo : (%,Integer) -> %         findCoef : (%,Integer) -> K
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inv : % -> %
```

```
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      leadingCoefficient : % -> K
--R leadingMonomial : % -> %              map : ((K -> K),%) -> %
--R monomial : (K,Integer) -> %           monomial? : % -> Boolean
--R one? : % -> Boolean                   order : % -> Integer
--R order : % -> Integer                  order : (%,Integer) -> Integer
--R pole? : % -> Boolean                  posExpnPart : % -> %
--R prime? : % -> Boolean                 printInfo : () -> Boolean
--R printInfo : Boolean -> Boolean        ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R ?rem? : (%,%) -> %                    removeFirstZeroes : % -> %
--R removeZeroes : % -> %                 removeZeroes : (Integer,%) -> %
--R sample : () -> %                      sbt : (%,%) -> %
--R series : (Integer,K,%) -> %           shift : (%,Integer) -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               truncate : (%,Integer) -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,K) -> % if K has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if K has PDRING SYMBOL and K has *: (I
--R D : (%,Symbol,NonNegativeInteger) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -
--R D : (%,List Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -> K
--R D : (%,Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -> K
--R D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R D : % -> % if K has *: (Integer,K) -> K
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> K if K has **: (K,Integer) -> K and K has coerce: Symbol -
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if K has CHARNZ
--R coerce : % -> Stream Record(k: Integer,c: K)
--R coerce : Stream Record(k: Integer,c: K) -> %
--R coerce : K -> % if K has COMRING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if K has PDRING SYMBOL and
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if K has PDRING SYMBOL and K has *:
--R differentiate : (%,List Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) ->
--R differentiate : (%,Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -> K
--R differentiate : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R differentiate : % -> % if K has *: (Integer,K) -> K
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,K) -> Stream K if K has **: (K,Integer) -> K
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial2series : (List %,List NonNegativeInteger,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R multiplyExponents : (%,PositiveInteger) -> %
--R orderIfNegative : % -> Union(Integer,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> Stream Record(k: Integer,c: K)
--R truncate : (%,Integer,Integer) -> %
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*LocalPowerSeriesCategory.help*⟩≡

```
====================================================================
LocalPowerSeriesCategory examples
====================================================================

See Also:
o )show LocalPowerSeriesCategory
```

See:

⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307
⇐ “UnivariatePowerSeriesCategory” (UPSCAT) 15.0.184 on page 1334

Exports:

0	1	-?
?**?	?+?	?-?
?.?	?/?	?=?
?^?	?~=?	?quo?
?rem?	D	approximate
associates?	center	characteristic
charthRoot	coefOfFirstNonZeroTerm	coefficient
coerce	complete	degree
delay	differentiate	divide
euclideanSize	eval	expressIdealMember
exquo	extend	extendedEuclidean
factor	filterUpTo	findCoef
gcd	gcdPolynomial	hash
inv	latex	lcm
leadingCoefficient	leadingMonomial	map
monomial	monomial2series	monomial?
multiEuclidean	multiplyExponents	one?
order	orderIfNegative	pole?
posExpnPart	prime?	principalIdeal
printInfo	recip	reductum
removeFirstZeroes	removeZeroes	sample
sbt	series	shift
sizeLess?	squareFree	squareFreePart
subtractIfCan	terms	truncate
unit?	unitCanonical	unitNormal
variable	variables	zero?

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.

- **rightUnitary** is true if $x * 1 = x$ for all x.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

These are directly exported but not implemented

```
coefOfFirstNonZeroTerm : % -> K
coerce : % -> Stream Record(k: Integer,c: K)
coerce : Stream Record(k: Integer,c: K) -> %
delay : (() -> %) -> %
filterUpTo : (%,Integer) -> %
findCoef : (%,Integer) -> K
monomial2series : (List %,List NonNegativeInteger,Integer) -> %
order : % -> Integer
orderIfNegative : % -> Union(Integer,"failed")
posExpnPart : % -> %
printInfo : () -> Boolean
printInfo : Boolean -> Boolean
removeFirstZeroes : % -> %
removeZeroes : % -> %
removeZeroes : (Integer,%) -> %
sbt : (%,%) -> %
series : (Integer,K,%) -> %
shift : (%,Integer) -> %
```

These exports come from (p1334) UnivariatePowerSeriesCategory():

```
-? : % -> %
0 : () -> %
1 : () -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?.? : (%,%) -> % if Integer has SGROUP
?.? : (%,Integer) -> K
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
D : % -> % if K has *: (Integer,K) -> K
D : (%,List Symbol) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,List Symbol,List NonNegativeInteger) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
D : (%,Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,Symbol,NonNegativeInteger) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
approximate : (%,Integer) -> K
    if K has **: (K,Integer) -> K and K has coerce: Symbol -> K
associates? : (%,%) -> Boolean
center : % -> K
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if K has CHARNZ
coefficient : (%,Integer) -> K
coerce : % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : K -> % if K has COMRING
complete : % -> %
degree : % -> Integer
differentiate : % -> % if K has *: (Integer,K) -> K
differentiate : (%,List Symbol) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,NonNegativeInteger) -> %
   if K has *: (Integer,K) -> K
differentiate : (%,Symbol) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,Symbol,NonNegativeInteger) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
eval : (%,K) -> Stream K if K has **: (K,Integer) -> K
exquo : (%,%) -> Union(%,"failed")
extend : (%,Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> K
leadingMonomial : % -> %
map : ((K -> K),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (K,Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
```

```
order : % -> Integer
order : (%,Integer) -> Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Integer,c: K)
truncate : (%,Integer) -> %
truncate : (%,Integer,Integer) -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
```

These exports come from (p1347) Field():

```
?**? : (%,Integer) -> %
?*? : (%,K) -> %
?*? : (K,%) -> %
?/? : (%,K) -> % if K has FIELD
?^? : (%,Integer) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
   SparseUnivariatePolynomial %
inv : % -> %
lcm : (%,%) -> %
lcm : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
```

⟨*category LOCPOWC LocalPowerSeriesCategory*⟩≡

```
)abbrev category LOCPOWC LocalPowerSeriesCategory
LocalPowerSeriesCategory(K:Field):Category == Implementation where
```

```
INT   ==> Integer
TERM  ==> Record(k:INT,c:K)
SER   ==> Stream(TERM)
NNI   ==> NonNegativeInteger

Implementation ==>
  Join(Field,UnivariatePowerSeriesCategory(K,INT))  with

    order: % -> Integer
      ++ order(s) returns the order of s.

    findCoef: (%,Integer) -> K

    coerce: SER -> %

    coerce: % -> SER

    posExpnPart: % -> %
      ++ posExpnPart(s) returns the series s less the terms with
      ++ negative exponant.

    orderIfNegative: % -> Union(Integer,"failed")

    removeFirstZeroes: % -> %

    sbt: (%,%) -> %

    delay: ( () -> % ) -> %
      ++ delay delayed the computation of the next term of the series given
      ++ by the input function.

    monomial2series: (List %, List NNI, INT) -> %
      ++ monomial2series(ls,le,n) returns
      ++ t**n * reduce("*",[s ** e for s in ls for e in le])

    removeZeroes: (INT,%) -> %
      ++ removeZeroes(n,s) removes the zero terms in the first n terms of s.

    removeZeroes: % -> %
      ++ removeZeroes(s) removes the zero terms in  s.

    series: (INT,K,%) -> %
      ++ series(e,c,s) create the series c*t**e + s.

    shift: (%,INT) -> %
```

```
          ++ shift(s,n) returns t**n * s

        filterUpTo : (%,INT)  -> %
          ++ filterUpTo(s,n) returns the series consisting of the terms
          ++ of s having degree strictly less than n.

        coefOfFirstNonZeroTerm: % -> K
          ++ coefOfFirstNonZeroTerm(s) returns the first non zero coefficient
          ++ of the series.

        printInfo: Boolean -> Boolean
          ++ printInfo(b) set a flag such that when true (b <- true) prints
          ++ some information during some critical computation.

        printInfo: () -> Boolean
          ++ returns the value of the \spad{printInfo} flag.
```

⟨*LOCPOWC.dotabb*⟩≡

```
"LOCPOWC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCPOWC"];
"LOCPOWC" -> "UPSCAT"
```

⟨*LOCPOWC.dotfull*⟩≡

```
"LocalPowerSeriesCategory(f:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCPOWC"];
"LocalPowerSeriesCategory(f:Field)" ->
  "UnivariatePowerSeriesCategory(c:Ring,e:OrderedAbelianMonoid)"
```

⟨*LOCPOWC.dotpic*⟩≡

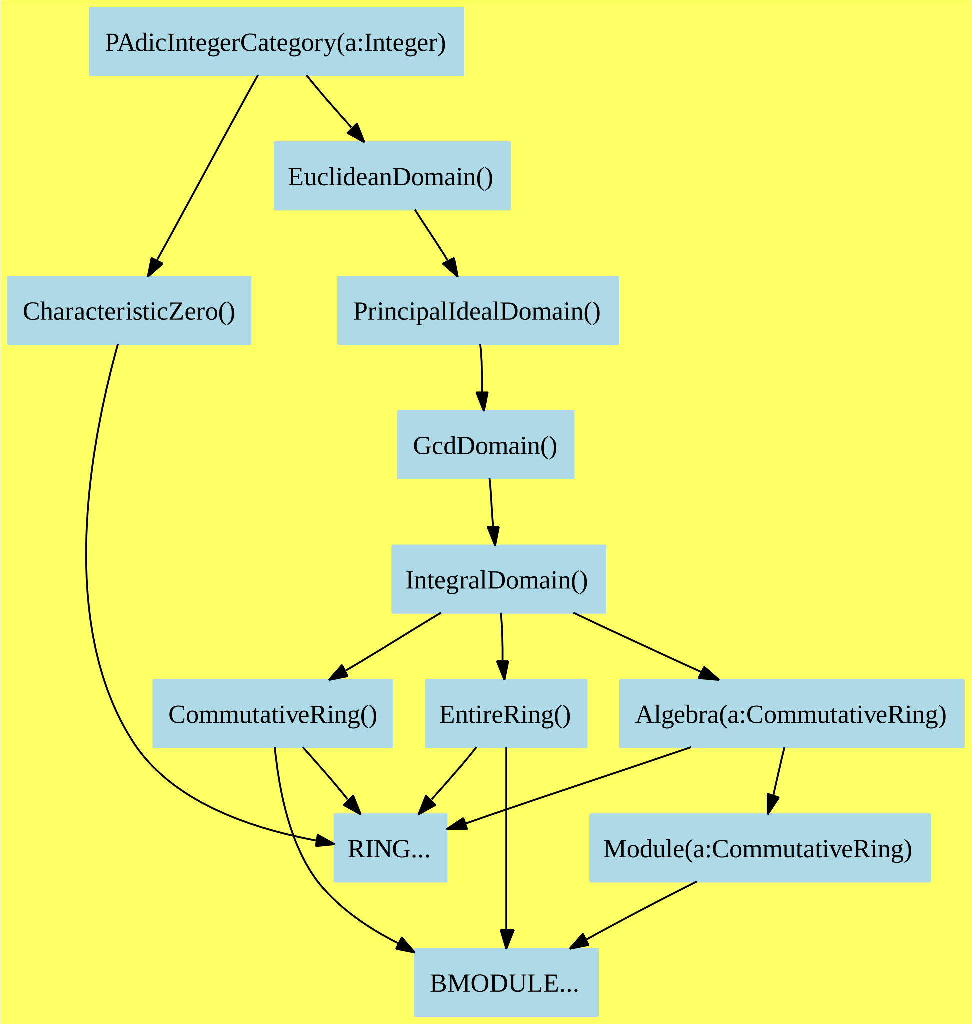
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LocalPowerSeriesCategory(f:Field)" [color=lightblue];
"LocalPowerSeriesCategory(f:Field)" ->
  "UnivariatePowerSeriesCategory(c:Ring,e:OrderedAbelianMonoid)"

"Ring" [color=lightblue];
"OrderedAbelianMonoid"  [color=lightblue];

}
```

16.0.188 PAdicIntegerCategory (PADICCT)


PAdicIntegerCategory(a:Integer)
EuclideanDomain()
CharacteristicZero()
PrincipalIdealDomain()
GcdDomain()
IntegralDomain()
CommutativeRing()
EntireRing()
Algebra(a:CommutativeRing)
RING...
Module(a:CommutativeRing)
BMODULE...

⟨*PAdicIntegerCategory.input*⟩≡

```
)set break resume
)sys rm -f PAdicIntegerCategory.output
)spool PAdicIntegerCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicIntegerCategory
```

```
--R PAdicIntegerCategory p: Integer  is a category constructor
--R Abbreviation for PAdicIntegerCategory is PADICCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PADICCT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R digits : % -> Stream Integer          extend : (%,Integer) -> %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R moduloP : % -> Integer                modulus : () -> Integer
--R one? : % -> Boolean                   order : % -> NonNegativeInteger
--R ?quo? : (%,%) -> %                    quotientByP : % -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R sqrt : (%,Integer) -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R root : (SparseUnivariatePolynomial Integer,Integer) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*PAdicIntegerCategory.help*⟩≡

```
====================================================================
PAdicIntegerCategory examples
====================================================================

See Also:
o )show PAdicIntegerCategory
```

See:

⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	moduloP
modulus	multiEuclidean	one?	order
principalIdeal	quotientByP	recip	root
sample	sizeLess?	sqrt	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?*?	?**?	?+?	?-?
-?	?=?	?quo?	?rem?
?˜=?	?ˆ?		

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
approximate : (%,Integer) -> Integer
complete : % -> %
digits : % -> Stream Integer
extend : (%,Integer) -> %
moduloP : % -> Integer
modulus : () -> Integer
order : % -> NonNegativeInteger
quotientByP : % -> %
root : (SparseUnivariatePolynomial Integer,Integer) -> %
sqrt : (%,Integer) -> %
```

These exports come from (p1307) EuclideanDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

⟨*category PADICCT PAdicIntegerCategory*⟩≡

```
)abbrev category PADICCT PAdicIntegerCategory
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ This is the category of stream-based representations of
++ the p-adic integers.

PAdicIntegerCategory(p): Category == Definition where
  p   :   Integer
  I   ==> Integer
  NNI ==> NonNegativeInteger
  ST  ==> Stream
  SUP ==> SparseUnivariatePolynomial

  Definition ==> Join(EuclideanDomain,CharacteristicZero) with
    digits: % -> ST I
      ++ \spad{digits(x)} returns a stream of p-adic digits of x.
    order: % -> NNI
      ++ \spad{order(x)} returns the exponent of the highest power of p
      ++ dividing x.
    extend: (%,I) -> %
      ++ \spad{extend(x,n)} forces the computation of digits up to order n.
    complete: % -> %
      ++ \spad{complete(x)} forces the computation of all digits.
    modulus: () -> I
      ++ \spad{modulus()} returns the value of p.
    moduloP: % -> I
      ++ \spad{modulo(x)} returns a, where \spad{x = a + b p}.
    quotientByP: % -> %
      ++ \spad{quotientByP(x)} returns b, where \spad{x = a + b p}.
    approximate: (%,I) -> I
      ++ \spad{approximate(x,n)} returns an integer y such that
      ++ \spad{y = x (mod p^n)}
      ++ when n is positive, and 0 otherwise.
    sqrt: (%,I) -> %
      ++ \spad{sqrt(b,a)} returns a square root of b.
      ++ Argument \spad{a} is a square root of b \spad{(mod p)}.
    root: (SUP I,I) -> %
      ++ \spad{root(f,a)} returns a root of the polynomial \spad{f}.
      ++ Argument \spad{a} must be a root of \spad{f} \spad{(mod p)}.
```

⟨*PADICCT.dotabb*⟩≡

```
"PADICCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PADICCT" -> "CHARZ"
"PADICCT" -> "EUCDOM"
```

⟨*PADICCT.dotfull*⟩≡

```
"PAdicIntegerCategory(a:Integer)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"
```

⟨*PADICCT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PAdicIntegerCategory(a:Integer)" [color=lightblue];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

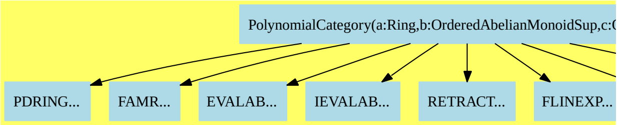
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

16.0.189 PolynomialCategory (POLYCAT)



⟨*PolynomialCategory.input*⟩≡

```
  )set break resume
  )sys rm -f PolynomialCategory.output
  )spool PolynomialCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show PolynomialCategory
  --R PolynomialCategory(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet)  is a category
  --R Abbreviation for PolynomialCategory is POLYCAT
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for POLYCAT
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?=? : (%,%) -> Boolean
  --R D : (%,List VarSet) -> %              D : (%,VarSet) -> %
  --R 1 : () -> %                           0 : () -> %
  --R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
  --R coefficients : % -> List R            coerce : VarSet -> %
  --R coerce : R -> %                       coerce : Integer -> %
  --R coerce : % -> OutputForm              degree : % -> E
  --R differentiate : (%,VarSet) -> %       eval : (%,VarSet,%) -> %
  --R eval : (%,VarSet,R) -> %              eval : (%,List %,List %) -> %
  --R eval : (%,%,%) -> %                   eval : (%,Equation %) -> %
  --R eval : (%,List Equation %) -> %       ground : % -> R
  --R ground? : % -> Boolean                hash : % -> SingleInteger
  --R latex : % -> String                   leadingCoefficient : % -> R
  --R leadingMonomial : % -> %              map : ((R -> R),%) -> %
  --R mapExponents : ((E -> E),%) -> %      minimumDegree : % -> E
  --R monomial : (R,E) -> %                 monomial? : % -> Boolean
  --R monomials : % -> List %               one? : % -> Boolean
  --R pomopo! : (%,R,E,%) -> %              primitiveMonomials : % -> List %
```

```
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> VarSet                 retract : % -> R
--R sample : () -> %                      variables : % -> List VarSet
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List VarSet,List NonNegativeInteger) -> %
--R D : (%,VarSet,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R
--R coefficient : (%,List VarSet,List NonNegativeInteger) -> %
--R coefficient : (%,VarSet,NonNegativeInteger) -> %
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT or R has ALGEBRA FRA
--R coerce : % -> % if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,Characteristic
--R content : (%,VarSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if VarSet has KONVERT INFORM and R has KONVERT INFOR
--R convert : % -> Pattern Integer if VarSet has KONVERT PATTERN INT and R has KO
--R convert : % -> Pattern Float if VarSet has KONVERT PATTERN FLOAT and R has KO
--R degree : (%,List VarSet) -> List NonNegativeInteger
--R degree : (%,VarSet) -> NonNegativeInteger
--R differentiate : (%,List VarSet,List NonNegativeInteger) -> %
--R differentiate : (%,VarSet,NonNegativeInteger) -> %
--R differentiate : (%,List VarSet) -> %
--R discriminant : (%,VarSet) -> % if R has COMRING
--R eval : (%,List VarSet,List %) -> %
--R eval : (%,List VarSet,List R) -> %
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
```

```
--R isExpt : % -> Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(VarSet,"failed")
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List VarSet) -> List NonNegativeInteger
--R minimumDegree : (%,VarSet) -> NonNegativeInteger
--R monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List VarSet,List NonNegativeInteger) -> %
--R monomial : (%,VarSet,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,VarSet) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(VarSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyn
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomi
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List VarSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,VarSet) -> SparseUnivariatePolynomial %
--R
--E 1
)spool
)lisp (bye)
```

⟨PolynomialCategory.help⟩≡

```
====================================================================
PolynomialCategory examples
====================================================================

See Also:
o )show PolynomialCategory
```

See:

⇒ "DifferentialPolynomialCategory" (DPOLCAT) 17.0.192 on page 1439
⇒ "RecursivePolynomialCategory" (RPOLCAT) 17.0.201 on page 1552
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.204 on page 1620

⇐ "InnerEvalable" (IEVALAB) 2.0.12 on page 39
⇐ "Evalable" (EVALAB) 3.0.25 on page 91
⇐ "FiniteAbelianMonoidRing" (FAMR) 14.0.175 on page 1259
⇐ "GcdDomain" (GCDDOM) 13.0.173 on page 1246
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.154 on page 1060
⇐ "OrderedSet" (ORDSET) 4.0.56 on page 242
⇐ "PartialDifferentialRing" (PDRING) 10.0.145 on page 977
⇐ "PolynomialFactorizationExplicit" (PFECAT) 15.0.183 on page 1325
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0
1
associates?
binomThmExpt
characteristic
charthRoot
coefficient
coefficients
coerce
conditionP
content
convert
D
degree
differentiate
discriminant
eval
exquo
factor
factorPolynomial
factorSquareFreePolynomial
gcd
gcdPolynomial
ground
ground?
hash
isExpt
isPlus
isTimes
latex
lcm
leadingCoefficient
leadingMonomial
mainVariable
map
mapExponents
max
min
minimumDegree
monicDivide
monomial
monomial?
monomials
multivariate
numberOfMonomials
one?
patternMatch
pomopo!
prime?
primitiveMonomials
primitivePart
recip
reducedSystem
reductum
resultant
retract
retractIfCan
sample
solveLinearPolynomialEquation
squareFree
squareFreePart
squareFreePolynomial
subtractIfCan
totalDegree
unit?
unitCanonical
unitNormal
univariate
variables
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?<?
?<=?
?>?
?>=?

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.

- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : (%,VarSet) -> NonNegativeInteger
degree : (%,List VarSet) -> List NonNegativeInteger
mainVariable : % -> Union(VarSet,"failed")
minimumDegree : (%,List VarSet) -> List NonNegativeInteger
minimumDegree : (%,VarSet) -> NonNegativeInteger
monomial : (%,VarSet,NonNegativeInteger) -> %
multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
univariate : (%,VarSet) -> SparseUnivariatePolynomial %
univariate : % -> SparseUnivariatePolynomial R
variables : % -> List VarSet
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient : (%,VarSet,NonNegativeInteger) -> %
coefficient : (%,List VarSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
content : (%,VarSet) -> % if R has GCDDOM
convert : % -> Pattern Integer
    if VarSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if VarSet has KONVERT PATTERN FLOAT
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if VarSet has KONVERT INFORM
    and R has KONVERT INFORM
discriminant : (%,VarSet) -> % if R has COMRING
eval : (%,List Equation %) -> %
factor : % -> Factored % if R has PFECAT
factorPolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
factorSquareFreePolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
```

```
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if R has GCDDOM
isExpt : % ->
  Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
monomial : (%,List VarSet,List NonNegativeInteger) -> %
monomials : % -> List %
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if VarSet has PATMAB INT
      and R has PATMAB INT
patternMatch :
   (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if VarSet has PATMAB FLOAT
       and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : % -> % if R has GCDDOM
primitivePart : (%,VarSet) -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,VarSet) -> % if R has COMRING
retract : % -> VarSet
retractIfCan : % -> Union(VarSet,"failed")
solveLinearPolynomialEquation :
   (List SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       Union(List SparseUnivariatePolynomial %,"failed")
         if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
totalDegree : % -> NonNegativeInteger
totalDegree : (%,List VarSet) -> NonNegativeInteger
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p977) PartialDifferentialRing(VarSet) where VarSet:OrderedSet:

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,List VarSet) -> %
```

```
D : (%,VarSet) -> %
D : (%,List VarSet,List NonNegativeInteger) -> %
D : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,VarSet) -> %
differentiate : (%,List VarSet,List NonNegativeInteger) -> %
differentiate : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,List VarSet) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1259) FiniteAbelianMonoidRing(R,E) where R:Ring and E:OrderedAbelianMonoidSup:

```
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
degree : % -> E
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
ground : % -> R
ground? : % -> Boolean
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
```

```
mapExponents : ((E -> E),%) -> %
minimumDegree : % -> E
monomial : (R,E) -> %
monomial? : % -> Boolean
numberOfMonomials : % -> NonNegativeInteger
pomopo! : (%,R,E,%) -> %
reductum : % -> %
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p91) Evalable(PolynomialCategory(...)):

```
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p39) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:Ring

```
eval : (%,VarSet,R) -> %
eval : (%,List VarSet,List R) -> %
```

These exports come from (p39) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:PolynomialCategory(...):

```
eval : (%,VarSet,%) -> %
eval : (%,List VarSet,List %) -> %
```

These exports come from (p62) RetractableTo(VarSet)
where VarSet:OrderedSet:

```
coerce : VarSet -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R)
where R:Ring:

```
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
      if R has LINEXP INT
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1246) GcdDomain():

```
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
```

These exports come from (p1325) PolynomialFactorizationExplicit():

```
prime? : % -> Boolean if R has PFECAT
squareFreePolynomial : SparseUnivariatePolynomial % ->
   Factored SparseUnivariatePolynomial %
     if R has PFECAT
```

⟨*category POLYCAT PolynomialCategory*⟩≡

```
)abbrev category POLYCAT PolynomialCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, differentiate, eval
++ Related Constructors: Polynomial, DistributedMultivariatePolynomial
++ Also See: UnivariatePolynomialCategory
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category for general multi-variate polynomials over a ring
++ R, in variables from VarSet, with exponents from the
++ \spadtype{OrderedAbelianMonoidSup}.

PolynomialCategory(R:Ring, E:OrderedAbelianMonoidSup, VarSet:OrderedSet):
     Category ==
  Join(PartialDifferentialRing VarSet, FiniteAbelianMonoidRing(R, E),
       Evalable %, InnerEvalable(VarSet, R),
       InnerEvalable(VarSet, %), RetractableTo VarSet,
       FullyLinearlyExplicitRingOver R) with
    degree : (%,VarSet) -> NonNegativeInteger
      ++ degree(p,v) gives the degree of polynomial p with respect
      ++ to the variable v.
    degree : (%,List(VarSet)) -> List(NonNegativeInteger)
      ++ degree(p,lv) gives the list of degrees of polynomial p
      ++ with respect to each of the variables in the list lv.
    coefficient: (%,VarSet,NonNegativeInteger) -> %
      ++ coefficient(p,v,n) views the polynomial p as a univariate
      ++ polynomial in v and returns the coefficient of the \spad{v**n} term.
    coefficient: (%,List VarSet,List NonNegativeInteger) -> %
      ++ coefficient(p, lv, ln) views the polynomial p as a polynomial
      ++ in the variables of lv and returns the coefficient of the term
      ++ \spad{lv**ln}, i.e. \spad{prod(lv_i ** ln_i)}.
    monomials: % -> List %
      ++ monomials(p) returns the list of non-zero monomials of
      ++ polynomial p, i.e.
      ++ \spad{monomials(sum(a_(i) X^(i))) = [a_(1) X^(1),...,a_(n) X^(n)]}.
    univariate   : (%,VarSet) -> SparseUnivariatePolynomial(%)
      ++ univariate(p,v) converts the multivariate polynomial p
      ++ into a univariate polynomial in v, whose coefficients are still
      ++ multivariate polynomials (in all the other variables).
    univariate   : % -> SparseUnivariatePolynomial(R)
      ++ univariate(p) converts the multivariate polynomial p,
      ++ which should actually involve only one variable,
      ++ into a univariate polynomial
      ++ in that variable, whose coefficients are in the ground ring.
      ++ Error: if polynomial is genuinely multivariate
    mainVariable  : % -> Union(VarSet,"failed")
      ++ mainVariable(p) returns the biggest variable which actually
      ++ occurs in the polynomial p, or "failed" if no variables are
      ++ present.
      ++ fails precisely if polynomial satisfies ground?
    minimumDegree : (%,VarSet) -> NonNegativeInteger
      ++ minimumDegree(p,v) gives the minimum degree of polynomial p
      ++ with respect to v, i.e. viewed a univariate polynomial in v
    minimumDegree : (%,List(VarSet)) -> List(NonNegativeInteger)
      ++ minimumDegree(p, lv) gives the list of minimum degrees of the
      ++ polynomial p with respect to each of the variables in the list lv
    monicDivide : (%,%,VarSet) -> Record(quotient:%,remainder:%)
      ++ monicDivide(a,b,v) divides the polynomial a by the polynomial b,
      ++ with each viewed as a univariate polynomial in v returning
  ++ both the quotient and remainder.
  ++ Error: if b is not monic with respect to v.
monomial : (%,VarSet,NonNegativeInteger) -> %
  ++ monomial(a,x,n) creates the monomial \spad{a*x**n} where \spad{a} is
  ++ a polynomial, x is a variable and n is a nonnegative integer.
monomial : (%,List VarSet,List NonNegativeInteger) -> %
  ++ monomial(a,[v1..vn],[e1..en]) returns \spad{a*prod(vi**ei)}.
multivariate : (SparseUnivariatePolynomial(R),VarSet) -> %
  ++ multivariate(sup,v) converts an anonymous univariable
  ++ polynomial sup to a polynomial in the variable v.
multivariate : (SparseUnivariatePolynomial(%),VarSet) -> %
  ++ multivariate(sup,v) converts an anonymous univariable
  ++ polynomial sup to a polynomial in the variable v.
isPlus: % -> Union(List %, "failed")
  ++ isPlus(p) returns \spad{[m1,...,mn]} if polynomial
  ++ \spad{p = m1 + ... + mn} and
  ++ \spad{n >= 2} and each mi is a nonzero monomial.
isTimes: % -> Union(List %, "failed")
  ++ isTimes(p) returns \spad{[a1,...,an]} if polynomial
  ++ \spad{p = a1 ... an} and \spad{n >= 2}, and, for each i,
  ++ ai is either a nontrivial constant in R or else of the
  ++ form \spad{x**e}, where \spad{e > 0} is an integer
  ++ and x in a member of VarSet.
isExpt: % -> Union(Record(var:VarSet, exponent:NonNegativeInteger),_
                   "failed")
  ++ isExpt(p) returns \spad{[x, n]} if polynomial p has the
  ++ form \spad{x**n} and \spad{n > 0}.
totalDegree : % -> NonNegativeInteger
  ++ totalDegree(p) returns the largest sum over all monomials
  ++ of all exponents of a monomial.
totalDegree : (%,List VarSet) -> NonNegativeInteger
  ++ totalDegree(p, lv) returns the maximum sum (over all monomials
  ++ of polynomial p) of the variables in the list lv.
variables : % -> List(VarSet)
  ++ variables(p) returns the list of those variables actually
  ++ appearing in the polynomial p.
primitiveMonomials: % -> List %
  ++ primitiveMonomials(p) gives the list of monomials of the
  ++ polynomial p with their coefficients removed. Note that
  ++ \spad{primitiveMonomials(sum(a_(i) X^(i))) = [X^(1),...,X^(n)]}.
if R has OrderedSet  then OrderedSet
-- OrderedRing view removed to allow EXPR to define abs
--if R has OrderedRing then OrderedRing
if (R has ConvertibleTo InputForm) and
   (VarSet has ConvertibleTo InputForm) then
     ConvertibleTo InputForm
```

```
if (R has ConvertibleTo Pattern Integer) and
   (VarSet has ConvertibleTo Pattern Integer) then
     ConvertibleTo Pattern Integer
if (R has ConvertibleTo Pattern Float) and
   (VarSet has ConvertibleTo Pattern Float) then
     ConvertibleTo Pattern Float
if (R has PatternMatchable Integer) and
   (VarSet has PatternMatchable Integer) then
     PatternMatchable Integer
if (R has PatternMatchable Float) and
   (VarSet has PatternMatchable Float) then
     PatternMatchable Float
if R has CommutativeRing then
  resultant : (%,%,VarSet) -> %
     ++ resultant(p,q,v) returns the resultant of the polynomials
     ++ p and q with respect to the variable v.
  discriminant : (%,VarSet) -> %
     ++ discriminant(p,v) returns the disriminant of the polynomial p
     ++ with respect to the variable v.
if R has GcdDomain then
  GcdDomain
  content: (%,VarSet) -> %
    ++ content(p,v) is the gcd of the coefficients of the polynomial p
    ++ when p is viewed as a univariate polynomial with respect to the
    ++ variable v.
    ++ Thus, for polynomial 7*x**2*y + 14*x*y**2, the gcd of the
    ++ coefficients with respect to x is 7*y.
  primitivePart: % -> %
    ++ primitivePart(p) returns the unitCanonical associate of the
    ++ polynomial p with its content divided out.
  primitivePart: (%,VarSet) -> %
    ++ primitivePart(p,v) returns the unitCanonical associate of the
    ++ polynomial p with its content with respect to the variable v
    ++ divided out.
  squareFree: % -> Factored %
    ++ squareFree(p) returns the square free factorization of the
    ++ polynomial p.
  squareFreePart: % -> %
    ++ squareFreePart(p) returns product of all the irreducible factors
    ++ of polynomial p each taken with multiplicity one.

-- assertions
if R has canonicalUnitNormal then canonicalUnitNormal
        ++ we can choose a unique representative for each
        ++ associate class.
        ++ This normalization is chosen to be normalization of
```

```
            ++ leading coefficient (by default).
    if R has PolynomialFactorizationExplicit then
       PolynomialFactorizationExplicit
 add
    p:%
    v:VarSet
    ln:List NonNegativeInteger
    lv:List VarSet
    n:NonNegativeInteger
    pp,qq:SparseUnivariatePolynomial %

    eval(p:%, l:List Equation %) ==
      empty? l => p
      for e in l repeat
        retractIfCan(lhs e)@Union(VarSet,"failed") case "failed" =>
             error "cannot find a variable to evaluate"
      lvar:=[retract(lhs e)@VarSet for e in l]
      eval(p, lvar,[rhs e for e in l]$List(%))

    monomials p ==
--    zero? p => empty()
--    concat(leadingMonomial p, monomials reductum p)
--    replaced by sequential version for efficiency, by WMSIT, 7/30/90
      ml:= empty$List(%)
      while p ^= 0 repeat
        ml:=concat(leadingMonomial p, ml)
        p:= reductum p
      reverse ml

    isPlus p ==
      empty? rest(l := monomials p) => "failed"
      l

    isTimes p ==
      empty?(lv := variables p) or not monomial? p => "failed"
      l := [monomial(1, v, degree(p, v)) for v in lv]
--      one?(r := leadingCoefficient p) =>
      ((r := leadingCoefficient p) = 1) =>
        empty? rest lv => "failed"
        l
      concat(r::%, l)

    isExpt p ==
      (u := mainVariable p) case "failed" => "failed"
      p = monomial(1, u::VarSet, d := degree(p, u::VarSet)) =>
        [u::VarSet, d]
```

```
  "failed"

coefficient(p,v,n) == coefficient(univariate(p,v),n)

coefficient(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in coefficient"
   empty? ln  => error "mismatched lists in coefficient"
   coefficient(coefficient(univariate(p,first lv),first ln),
               rest lv,rest ln)

monomial(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in monomial"
   empty? ln  => error "mismatched lists in monomial"
   monomial(monomial(p,first lv, first ln),rest lv, rest ln)

retract(p:%):VarSet ==
  q := mainVariable(p)::VarSet
  q::% = p => q
  error "Polynomial is not a single variable"

retractIfCan(p:%):Union(VarSet, "failed") ==
  ((q := mainVariable p) case VarSet) and (q::VarSet::% = p) => q
  "failed"

mkPrim(p:%):% == monomial(1,degree p)

primitiveMonomials p == [mkPrim q for q in monomials p]

totalDegree p ==
    ground? p => 0
    u := univariate(p, mainVariable(p)::VarSet)
    d: NonNegativeInteger := 0
    while u ^= 0 repeat
      d := max(d, degree u + totalDegree leadingCoefficient u)
      u := reductum u
    d

totalDegree(p,lv) ==
    ground? p => 0
    u := univariate(p, v:=(mainVariable(p)::VarSet))
    d: NonNegativeInteger := 0
    w: NonNegativeInteger := 0
    if member?(v, lv) then w:=1
    while u ^= 0 repeat
      d := max(d, w*(degree u) + totalDegree(leadingCoefficient u,lv))
      u := reductum u
    d

if R has CommutativeRing then
    resultant(p1,p2,mvar) ==
      resultant(univariate(p1,mvar),univariate(p2,mvar))

    discriminant(p,var) ==
      discriminant(univariate(p,var))

if R has IntegralDomain then
  allMonoms(l:List %):List(%) ==
    removeDuplicates_! concat [primitiveMonomials p for p in l]

  P2R(p:%, b:List E, n:NonNegativeInteger):Vector(R) ==
    w := new(n, 0)$Vector(R)
    for i in minIndex w .. maxIndex w for bj in b repeat
      qsetelt_!(w, i, coefficient(p, bj))
    w

  eq2R(l:List %, b:List E):Matrix(R) ==
    matrix [[coefficient(p, bj) for p in l] for bj in b]

  reducedSystem(m:Matrix %):Matrix(R) ==
    l := listOfLists m
    b := removeDuplicates_!
                       concat [allMonoms r for r in l]$List(List(%))
    d := [degree bj for bj in b]
    mm := eq2R(first l, d)
    l := rest l
    while not empty? l repeat
      mm := vertConcat(mm, eq2R(first l, d))
      l := rest l
    mm

  reducedSystem(m:Matrix %, v:Vector %):
   Record(mat:Matrix R, vec:Vector R) ==
    l := listOfLists m
    r := entries v
    b : List % := removeDuplicates_! concat(allMonoms r,
                      concat [allMonoms s for s in l]$List(List(%)))
    d := [degree bj for bj in b]
    n := #d
```

```
         mm := eq2R(first l, d)
         w := P2R(first r, d, n)
         l := rest l
         r := rest r
         while not empty? l repeat
           mm := vertConcat(mm, eq2R(first l, d))
           w := concat(w, P2R(first r, d, n))
           l := rest l
           r := rest r
         [mm, w]

    if R has PolynomialFactorizationExplicit then
       -- we might be in trouble if its actually only
       -- a univariate polynomial category - have to remember to
       -- over-ride these in UnivariatePolynomialCategory

       PFBR ==>PolynomialFactorizationByRecursion(R,E,VarSet,%)

       gcdPolynomial(pp,qq) ==
          gcdPolynomial(pp,qq)$GeneralPolynomialGcdPackage(E,VarSet,R,%)

       solveLinearPolynomialEquation(lpp,pp) ==
         solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBR

       factorPolynomial(pp) ==
         factorByRecursion(pp)$PFBR

       factorSquareFreePolynomial(pp) ==
         factorSquareFreeByRecursion(pp)$PFBR

       factor p ==
         v:Union(VarSet,"failed"):=mainVariable p
         v case "failed" =>
           ansR:=factor leadingCoefficient p
           makeFR(unit(ansR)::%,
                  [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
         up:SparseUnivariatePolynomial %:=univariate(p,v)
         ansSUP:=factorByRecursion(up)$PFBR
         makeFR(multivariate(unit(ansSUP),v),
                [[ww.flg,multivariate(ww.fctr,v),ww.xpnt]
                 for ww in factorList ansSUP])
       if R has CharacteristicNonZero then
          mat: Matrix %

          conditionP mat ==
            ll:=listOfLists transpose mat  --hence each list corresponds to a
```

```
                                     --column, i.e. to one variable
        llR:List List R := [ empty() for z in first ll]
        monslist:List List % := empty()
        ch:=characteristic()$%
        for l in ll repeat
            mons:= "setUnion"/[primitiveMonomials u for u in l]
            redmons:List % :=[]
            for m in mons repeat
                vars:=variables m
                degs:=degree(m,vars)
                deg1:List NonNegativeInteger
                deg1:=[ ((nd:=d:Integer exquo ch:Integer)
                           case "failed" => return "failed" ;
                            nd::Integer::NonNegativeInteger)
                       for d in degs ]
                redmons:=[monomial(1,vars,deg1),:redmons]
                llR:=[[ground coefficient(u,vars,degs),:v]_
                        for u in l for v in llR]
            monslist:=[redmons,:monslist]
        ans:=conditionP transpose matrix llR
        ans case "failed" => "failed"
        i:NonNegativeInteger:=0
        [ +/[m*(ans.(i:=i+1))::% for m in mons ]
          for mons in monslist]

if R has CharacteristicNonZero then
      charthRootlv:(%,List VarSet,NonNegativeInteger) ->_
                                                  Union(%,"failed")
      charthRoot p ==
        vars:= variables p
        empty? vars =>
          ans := charthRoot ground p
          ans case "failed" => "failed"
          ans::R::%
        ch:=characteristic()$%
        charthRootlv(p,vars,ch)

      charthRootlv(p,vars,ch) ==
        empty? vars =>
          ans := charthRoot ground p
          ans case "failed" => "failed"
          ans::R::%
        v:=first vars
        vars:=rest vars
        d:=degree(p,v)
        ans:% := 0
        while (d>0) repeat
           (dd:=(d::Integer exquo ch::Integer)) case "failed" =>
                  return "failed"
           cp:=coefficient(p,v,d)
           p:=p-monomial(cp,v,d)
           ansx:=charthRootlv(cp,vars,ch)
           ansx case "failed" => return "failed"
           d:=degree(p,v)
           ans:=ans+monomial(ansx,v,dd::Integer::NonNegativeInteger)
        ansx:=charthRootlv(p,vars,ch)
        ansx case "failed" => return "failed"
        return ans+ansx

monicDivide(p1,p2,mvar) ==
   result:=monicDivide(univariate(p1,mvar),univariate(p2,mvar))
   [multivariate(result.quotient,mvar),
    multivariate(result.remainder,mvar)]

if R has GcdDomain then
  if R has EuclideanDomain and R has CharacteristicZero then
   squareFree p == squareFree(p)$MultivariateSquareFree(E,VarSet,R,%)
  else
    squareFree p == squareFree(p)$PolynomialSquareFree(VarSet,E,R,%)

  squareFreePart p ==
    unit(s := squareFree p) * */[f.factor for f in factors s]

  content(p,v) == content univariate(p,v)

  primitivePart p ==
    zero? p => p
    unitNormal((p exquo content p) ::%).canonical

  primitivePart(p,v) ==
    zero? p => p
    unitNormal((p exquo content(p,v)) ::%).canonical

if R has OrderedSet then
  p:% < q:% ==
    (dp:= degree p) < (dq := degree q) => (leadingCoefficient q) > 0
    dq < dp => (leadingCoefficient p) < 0
    leadingCoefficient(p - q) < 0

  if (R has PatternMatchable Integer) and
     (VarSet has PatternMatchable Integer) then
       patternMatch(p:%, pat:Pattern Integer,
         l:PatternMatchResult(Integer, %)) ==
           patternMatch(p, pat,
             l)$PatternMatchPolynomialCategory(Integer,E,VarSet,R,%)

     if (R has PatternMatchable Float) and
        (VarSet has PatternMatchable Float) then
          patternMatch(p:%, pat:Pattern Float,
           l:PatternMatchResult(Float, %)) ==
             patternMatch(p, pat,
               l)$PatternMatchPolynomialCategory(Float,E,VarSet,R,%)

   if (R has ConvertibleTo Pattern Integer) and
      (VarSet has ConvertibleTo Pattern Integer) then
        convert(x:%):Pattern(Integer) ==
          map(convert, convert,
             x)$PolynomialCategoryLifting(E,VarSet,R,%,Pattern Integer)

   if (R has ConvertibleTo Pattern Float) and
      (VarSet has ConvertibleTo Pattern Float) then
        convert(x:%):Pattern(Float) ==
          map(convert, convert,
           x)$PolynomialCategoryLifting(E, VarSet, R, %, Pattern Float)

   if (R has ConvertibleTo InputForm) and
      (VarSet has ConvertibleTo InputForm) then
        convert(p:%):InputForm ==
          map(convert, convert,
                 p)$PolynomialCategoryLifting(E,VarSet,R,%,InputForm)
```

⟨*POLYCAT.dotabb*⟩≡

```
  "POLYCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
  "POLYCAT" -> "PDRING"
  "POLYCAT" -> "FAMR"
  "POLYCAT" -> "EVALAB"
  "POLYCAT" -> "IEVALAB"
  "POLYCAT" -> "RETRACT"
  "POLYCAT" -> "FLINEXP"
  "POLYCAT" -> "ORDSET"
  "POLYCAT" -> "GCDDOM"
  "POLYCAT" -> "PFECAT"
```

⟨*POLYCAT.dotfull*⟩≡

```
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PartialDifferentialRing(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "Evalable(PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RetractableTo(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "OrderedSet()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GcdDomain()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PolynomialFactorizationExplicit()"

"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

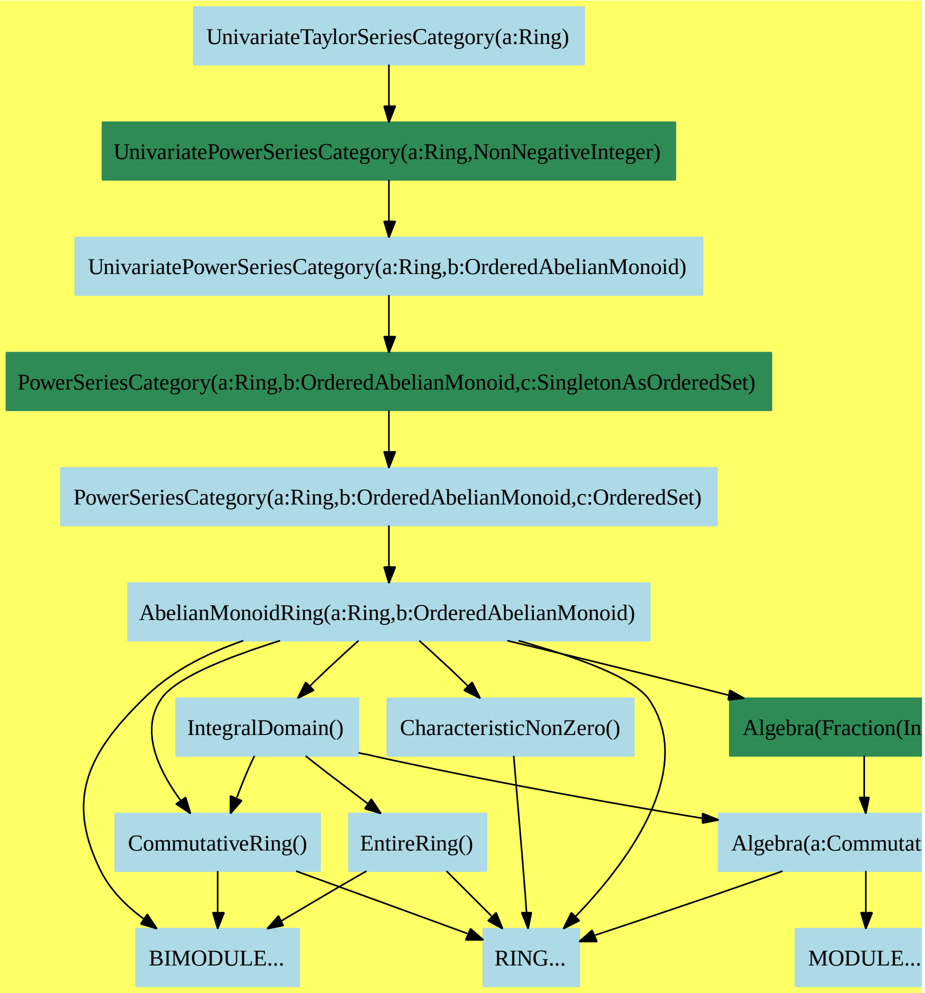
⟨*POLYCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

16.0.190 UnivariateTaylorSeriesCategory (UTSCAT)


UnivariateTaylorSeriesCategory(a:Ring)
UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)
IntegralDomain()
CharacteristicNonZero()
Algebra(Fraction(In
CommutativeRing()
EntireRing()
Algebra(a:Commutat
BIMODULE...
RING...
MODULE...

⟨*UnivariateTaylorSeriesCategory.input*⟩≡
```
  )set break resume
  )sys rm -f UnivariateTaylorSeriesCategory.output
  )spool UnivariateTaylorSeriesCategory.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show UnivariateTaylorSeriesCategory
  --R UnivariateTaylorSeriesCategory Coef: Ring  is a category constructor
```

```
--R Abbreviation for UnivariateTaylorSeriesCategory is UTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficients : % -> Stream Coef       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> NonNegativeInteger      hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R order : % -> NonNegativeInteger       pole? : % -> Boolean
--R quoByVar : % -> %                     recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R series : Stream Coef -> %             variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (NonNegativeInteger,Coef)
--R D : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (NonNegativeInteger,C
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has *: (NonNeg
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) -
--R asec : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeIntege
--R differentiate : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (No
--R differentiate : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRI
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> C
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R integrate : (%,Symbol) -> % if Coef has ACFS INT and Coef has PRIMCAT and Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariateTaylorSeriesCategory.help*⟩≡

```
====================================================================
UnivariateTaylorSeriesCategory examples
====================================================================

See Also:
o )show UnivariateTaylorSeriesCategory
```

See:

⇐ “RadicalCategory” (RADCAT) 2.0.17 on page 59
⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135
⇐ “UnivariatePowerSeriesCategory” (UPSCAT) 15.0.184 on page 1334

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	exp	exquo	extend
hash	integrate	latex	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiplyCoefficients	multiplyExponents	nthRoot
one?	order	pi	pole?
polynomial	quoByVar	recip	reductum
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	variable
variables	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?/?	?.?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
coefficients : % -> Stream Coef
integrate : (%,Symbol) -> %
    if Coef has ACFS INT
    and Coef has PRIMCAT
    and Coef has TRANFUN
    and Coef has ALGEBRA FRAC INT
    or Coef has variables: Coef -> List Symbol
    and Coef has integrate: (Coef,Symbol) -> Coef
    and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
quoByVar : % -> %
series : Stream Coef -> %
series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
```

These are implemented by this category:

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
coerce : % -> OutputForm
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
zero? : % -> Boolean
?**? : (%,Coef) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1334) UnivariatePowerSeriesCategory(Coef,NNI) where Coef:Ring and NNI:NonNegativeInteger:

```
0 : () -> %
1 : () -> %
approximate : (%,NonNegativeInteger) -> Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
    and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,NonNegativeInteger) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
complete : % -> %
D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
degree : % -> NonNegativeInteger
```

```
extend : (%,NonNegativeInteger) -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
eval : (%,Coef) -> Stream Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : % -> NonNegativeInteger
order : (%,NonNegativeInteger) -> NonNegativeInteger
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
truncate : (%,NonNegativeInteger) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if NonNegativeInteger has SGROUP
?.? : (%,NonNegativeInteger) -> Coef
```

These exports come from (p135) TranscendentalFunctionCategory():

```
pi : () -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
```

⟨*category UTSCAT UnivariateTaylorSeriesCategory*⟩≡

```
)abbrev category UTSCAT UnivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 26 May 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Taylor, linebacker
++ Examples:
++ References:
++ Description:
++ \spadtype{UnivariateTaylorSeriesCategory} is the category of Taylor
++ series in one variable.

UnivariateTaylorSeriesCategory(Coef): Category == Definition where
  Coef  : Ring
  I    ==> Integer
  L    ==> List
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  RN   ==> Fraction Integer
  STTA ==> StreamTaylorSeriesOperations Coef
  STTF ==> StreamTranscendentalFunctions Coef
  STNC ==> StreamTranscendentalFunctionsNonCommutative Coef
  Term ==> Record(k:NNI,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,NNI) with

    series: Stream Term -> %
      ++ \spad{series(st)} creates a series from a stream of non-zero terms,
      ++ where a term is an exponent-coefficient pair.  The terms in the
      ++ stream should be ordered by increasing order of exponents.
    coefficients: % -> Stream Coef
      ++ \spad{coefficients(a0 + a1 x + a2 x**2 + ...)} returns a stream
  ++ of coefficients: \spad{[a0,a1,a2,...]}. The entries of the stream
  ++ may be zero.
series: Stream Coef -> %
  ++ \spad{series([a0,a1,a2,...])} is the Taylor series
  ++ \spad{a0 + a1 x + a2 x**2 + ...}.
quoByVar: % -> %
  ++ \spad{quoByVar(a0 + a1 x + a2 x**2 + ...)}
  ++ returns \spad{a1 + a2 x + a3 x**2 + ...}
  ++ Thus, this function substracts the constant term and divides by
  ++ the series variable.  This function is used when Laurent series
  ++ are represented by a Taylor series and an order.
multiplyCoefficients: (I -> Coef,%) -> %
  ++ \spad{multiplyCoefficients(f,sum(n = 0..infinity,a[n] * x**n))}
  ++ returns \spad{sum(n = 0..infinity,f(n) * a[n] * x**n)}.
  ++ This function is used when Laurent series are represented by
  ++ a Taylor series and an order.
polynomial: (%,NNI) -> Polynomial Coef
  ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
  ++ of all terms of f of degree \spad{<= k}.
polynomial: (%,NNI,NNI) -> Polynomial Coef
  ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
  ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.

if Coef has Field then
  "**": (%,Coef) -> %
    ++ \spad{f(x) ** a} computes a power of a power series.
    ++ When the coefficient ring is a field, we may raise a series
    ++ to an exponent from the coefficient ring provided that the
    ++ constant coefficient of the series is 1.

if Coef has Algebra Fraction Integer then
  integrate: % -> %
    ++ \spad{integrate(f(x))} returns an anti-derivative of the power
    ++ series \spad{f(x)} with constant coefficient 0.
    ++ We may integrate a series when we can divide coefficients
    ++ by integers.
  if Coef has integrate: (Coef,Symbol) -> Coef and _
     Coef has variables: Coef -> List Symbol then
    integrate: (%,Symbol) -> %
      ++ \spad{integrate(f(x),y)} returns an anti-derivative of the
      ++ power series \spad{f(x)} with respect to the variable \spad{y}.
  if Coef has TranscendentalFunctionCategory and _
     Coef has PrimitiveFunctionCategory and _
     Coef has AlgebraicallyClosedFunctionSpace Integer then
    integrate: (%,Symbol) -> %
      ++ \spad{integrate(f(x),y)} returns an anti-derivative of
```

```
          ++ the power series \spad{f(x)} with respect to the variable
          ++ \spad{y}.
      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.
      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.

   add

     zero? x ==
       empty? (coefs := coefficients x) => true
       (zero? frst coefs) and (empty? rst coefs) => true
       false

--% OutputForms

--  We provide defaulr output functions on UTSCAT using the functions
--  'coefficients', 'center', and 'variable'.

    factorials?: () -> Boolean
    -- check a global Lisp variable
    factorials?() == false

    termOutput: (I,Coef,OUT) -> OUT
    termOutput(k,c,vv) ==
    -- creates a term c * vv ** k
      k = 0 => c :: OUT
      mon := (k = 1 => vv; vv ** (k :: OUT))
--       if factorials?() and k > 1 then
--         c := factorial(k)$IntegerCombinatoricFunctions * c
--         mon := mon / hconcat(k :: OUT,"!" :: OUT)
      c = 1 => mon
      c = -1 => -mon
      (c :: OUT) * mon

    showAll?: () -> Boolean
    -- check a global Lisp variable
    showAll?() == true

    coerce(p:%):OUT ==
      empty? (uu := coefficients p) => (0$Coef) :: OUT
      var := variable p; cen := center p
      vv :=
        zero? cen => var :: OUT
      paren(var :: OUT - cen :: OUT)
    n : NNI ; count : NNI := _$streamCount$Lisp
    l : L OUT := empty()
    for n in 0..count while not empty? uu repeat
      if frst(uu) ^= 0 then
        l := concat(termOutput(n :: I,frst uu,vv),l)
      uu := rst uu
    if showAll?() then
      for n in (count + 1).. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput(n :: I,frst uu,vv),l)
        uu := rst uu
    l :=
      explicitlyEmpty? uu => l
      eq?(uu,rst uu) and frst uu = 0 => l
      concat(prefix("O" :: OUT,[vv ** (n :: OUT)]),l)
    empty? l => (0$Coef) :: OUT
    reduce("+",reverse_! l)

  if Coef has Field then
    (x:%) ** (r:Coef) == series power(r,coefficients x)$STTA

  if Coef has Algebra Fraction Integer then
    if Coef has CommutativeRing then
      (x:%) ** (y:%)    == series(coefficients x **$STTF coefficients y)

      (x:%) ** (r:RN)   == series powern(r,coefficients x)$STTA

      exp x == series exp(coefficients x)$STTF

      log x == series log(coefficients x)$STTF

      sin x == series sin(coefficients x)$STTF

      cos x == series cos(coefficients x)$STTF

      tan x == series tan(coefficients x)$STTF

      cot x == series cot(coefficients x)$STTF

      sec x == series sec(coefficients x)$STTF

      csc x == series csc(coefficients x)$STTF

      asin x == series asin(coefficients x)$STTF
```

```
          acos x == series acos(coefficients x)$STTF

          atan x == series atan(coefficients x)$STTF

          acot x == series acot(coefficients x)$STTF

          asec x == series asec(coefficients x)$STTF

          acsc x == series acsc(coefficients x)$STTF

          sinh x == series sinh(coefficients x)$STTF

          cosh x == series cosh(coefficients x)$STTF

          tanh x == series tanh(coefficients x)$STTF

          coth x == series coth(coefficients x)$STTF

          sech x == series sech(coefficients x)$STTF

          csch x == series csch(coefficients x)$STTF

          asinh x == series asinh(coefficients x)$STTF

          acosh x == series acosh(coefficients x)$STTF

          atanh x == series atanh(coefficients x)$STTF

          acoth x == series acoth(coefficients x)$STTF

          asech x == series asech(coefficients x)$STTF

          acsch x == series acsch(coefficients x)$STTF

        else
          (x:%) ** (y:%) == series(coefficients x **$STNC coefficients y)

          (x:%) ** (r:RN) ==
            coefs := coefficients x
            empty? coefs =>
              positive? r => 0
              zero? r => error "0**0 undefined"
              error "0 raised to a negative power"
--            not one? frst coefs =>
            not (frst coefs = 1) =>
```

```
    error "**: constant coefficient should be 1"
  coefs := concat(0,rst coefs)
  onePlusX := monom(1,0)$STTA + $STTA monom(1,1)$STTA
  ratPow := powern(r,onePlusX)$STTA
  series compose(ratPow,coefs)$STTA

exp x == series exp(coefficients x)$STNC

log x == series log(coefficients x)$STNC

sin x == series sin(coefficients x)$STNC

cos x == series cos(coefficients x)$STNC

tan x == series tan(coefficients x)$STNC

cot x == series cot(coefficients x)$STNC

sec x == series sec(coefficients x)$STNC

csc x == series csc(coefficients x)$STNC

asin x == series asin(coefficients x)$STNC

acos x == series acos(coefficients x)$STNC

atan x == series atan(coefficients x)$STNC

acot x == series acot(coefficients x)$STNC

asec x == series asec(coefficients x)$STNC

acsc x == series acsc(coefficients x)$STNC

sinh x == series sinh(coefficients x)$STNC

cosh x == series cosh(coefficients x)$STNC

tanh x == series tanh(coefficients x)$STNC

coth x == series coth(coefficients x)$STNC

sech x == series sech(coefficients x)$STNC

csch x == series csch(coefficients x)$STNC
```

```
          asinh x == series asinh(coefficients x)$STNC

          acosh x == series acosh(coefficients x)$STNC

          atanh x == series atanh(coefficients x)$STNC

          acoth x == series acoth(coefficients x)$STNC

          asech x == series asech(coefficients x)$STNC

          acsch x == series acsch(coefficients x)$STNC
```

⟨*UTSCAT.dotabb*⟩≡

```
"UTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UTSCAT" -> "UPSCAT"
```

⟨*UTSCAT.dotfull*⟩≡

```
"UnivariateTaylorSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
```

⟨*UTSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateTaylorSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

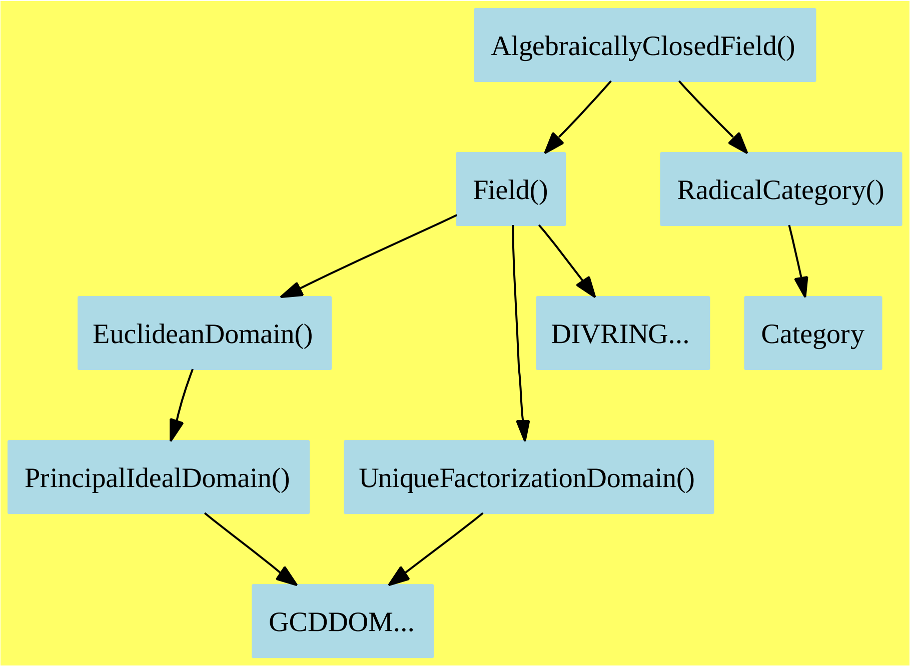
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

Chapter 17

Category Layer 16

17.0.191 AlgebraicallyClosedField (ACF)


AlgebraicallyClosedField()
Field()
RadicalCategory()
EuclideanDomain()
DIVRING...
Category
PrincipalIdealDomain()
UniqueFactorizationDomain()
GCDDOM...

⟨*AlgebraicallyClosedField.input*⟩≡

```
)set break resume
)sys rm -f AlgebraicallyClosedField.output
)spool AlgebraicallyClosedField.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 14
pi:Polynomial(Integer):=-3*x^3+2*x+13
--R
--R
--R            3
--R   (1)  - 3x  + 2x + 13
--R                                                     Type: Polynomial Integer
--E 1

--S 2 of 14
rootOf(pi)
--R
--R
--R   (2)  x
--R                                                        Type: AlgebraicNumber
--E 2

--S 3 of 14
rootsOf(pi)
--R
--R
--R   (3)  [%x0,%x1,- %x1 - %x0]
--R                                                   Type: List AlgebraicNumber
--E 3

--S 4 of 14
zeroOf(pi)
--R
--R
--R   (4)  x
--R                                                        Type: AlgebraicNumber
--E 4

--S 5 of 14
zerosOf(pi)
--R
--R
--R                +-------------+         +-------------+
--R                |       2               |       2
--R             - \|- 27%x3  + 24  - 3%x3 \|- 27%x3  + 24  - 3%x3
--R   (5)  [%x3,-------------------------,-----------------------]
--R                         6                        6
--R                                                   Type: List AlgebraicNumber
```

```
--E 5

--S 6 of 14
sup:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
--R
--R
--R              3
--R   (6)  - 3?  + 2? + 13
--R                                    Type: SparseUnivariatePolynomial Integer
--E 6

--S 7 of 14
rootOf(sup)
--R
--R
--R   (7)  %B
--R                                                        Type: AlgebraicNumber
--E 7

--S 8 of 14
rootOf(sup,x)
--R
--R
--R   (8)  x
--R                                                        Type: AlgebraicNumber
--E 8

--S 9 of 14
rootsOf(sup)
--R
--R
--R   (9)  [%%C0,%%C1,- %%C1 - %%C0]
--R                                                   Type: List AlgebraicNumber
--E 9

--S 10 of 14
rootsOf(sup,x)
--R
--R
--R   (10)  [%x6,%x7,- %x7 - %x6]
--R                                                   Type: List AlgebraicNumber
--E 10

--S 11 of 14
zeroOf(sup)
--R
```

```
--R
--R   (11)  %D
--R                                                        Type: AlgebraicNumber
--E 11

--S 12 of 14
zeroOf(sup,x)
--R
--R
--R   (12)  x
--R                                                        Type: AlgebraicNumber
--E 12

--S 13 of 14
zerosOf(sup)
--R
--R
--R                  +--------------+          +--------------+
--R                  |        2                |        2
--R               - \|- 27%%E0  + 24  - 3%%E0 \|- 27%%E0  + 24  - 3%%E0
--R   (13)  [%%E0,---------------------------,-------------------------]
--R                            6                          6
--R                                                   Type: List AlgebraicNumber
--E 13

--S 14 of 14
zerosOf(sup,x)
--R
--R
--R                 +-------------+         +-------------+
--R                 |       2               |       2
--R              - \|- 27%x9  + 24  - 3%x9 \|- 27%x9  + 24  - 3%x9
--R   (14)  [%x9,-------------------------,-----------------------]
--R                          6                        6
--R                                                   Type: List AlgebraicNumber
--E 14

)spool
)lisp (bye)
```

⟨*AlgebraicallyClosedField.help*⟩≡

```
====================================================================
AlgebraicallyClosedField examples
====================================================================

Given the polynomial:

pi:Polynomial(Integer):=-3*x^3+2*x+13
               3
           - 3x  + 2x + 13


rootOf(pi)
           x


rootsOf(pi)
           [%x0,%x1,- %x1 - %x0]


zeroOf(pi)
           x


zerosOf(pi)
                   +-------------+         +-------------+
                   |       2               |       2
                 - \|- 27%x3  + 24  - 3%x3 \|- 27%x3  + 24  - 3%x3
           [%x3,------------------------,-----------------------]
                              6                        6


These functions can also be applied to Sparse Univariate Polynomials:

sup:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
               3
           - 3?  + 2? + 13


rootOf(sup)
           %B


rootOf(sup,x)
           x
```

```
rootsOf(sup)
           [%%C0,%%C1,- %%C1 - %%C0]


rootsOf(sup,x)
           [%x6,%x7,- %x7 - %x6]


zeroOf(sup)
           %D


zeroOf(sup,x)
        x


zerosOf(sup)
                 +--------------+          +--------------+
                 |        2                |        2
              - \|- 27%%E0  + 24  - 3%%E0 \|- 27%%E0  + 24  - 3%%E0
        [%%E0,---------------------------,-------------------------]
                           6                          6


zerosOf(sup,x)
                +-------------+         +-------------+
                |       2               |       2
             - \|- 27%x9  + 24  - 3%x9 \|- 27%x9  + 24  - 3%x9
        [%x9,-------------------------,-----------------------]
                         6                        6




See Also:
o )show AlgebraicallyClosedField
```

See:

⇒ “AlgebraicallyClosedFunctionSpace” (ACFS) 18.0.205 on page 1645
⇐ “Field” (FIELD) 16.0.185 on page 1347
⇐ “RadicalCategory” (RADCAT) 2.0.17 on page 59

Exports:

0	1	associates?	characteristic
coerce	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor	gcd
gcdPolynomial	hash	inv	latex
lcm	multiEuclidean	nthRoot	one?
prime?	principalIdeal	recip	rootOf
rootsOf	sample	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	zeroOf
zerosOf	?*?	?**?	?+?
?-?	-?	?/?	?=?
?quo?	?rem?	?~=?	?^?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
rootOf : Polynomial % -> %
```

These are implemented by this category:

```
rootOf : SparseUnivariatePolynomial % -> %
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : Polynomial % -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
rootsOf : SparseUnivariatePolynomial % -> List %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zeroOf : Polynomial % -> %
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
```

These exports come from (p1347) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

⟨*category ACF AlgebraicallyClosedField*⟩≡

```
)abbrev category ACF AlgebraicallyClosedField
++ Author: Manuel Bronstein
++ Date Created: 22 Mar 1988
++ Date Last Updated: 27 November 1991
++ Keywords: algebraic, closure, field.
++ Description:
++ Model for algebraically closed fields.

AlgebraicallyClosedField(): Category == Join(Field,RadicalCategory) with
    rootOf: Polynomial $ -> $
      ++ rootOf(p) returns y such that \spad{p(y) = 0}.
      ++ Error: if p has more than one variable y.
      ++
      ++X a:Polynomial(Integer):=-3*x^3+2*x+13
      ++X rootOf(a)
    rootOf: SparseUnivariatePolynomial $ -> $
      ++ rootOf(p) returns y such that \spad{p(y) = 0}.
    ++
    ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
    ++X rootOf(a)
  rootOf: (SparseUnivariatePolynomial $, Symbol) -> $
    ++ rootOf(p, y) returns y such that \spad{p(y) = 0}.
    ++ The object returned displays as \spad{'y}.
    ++
    ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
    ++X rootOf(a,x)
  rootsOf: Polynomial $ -> List $
    ++ rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
    ++ Note that the returned symbols y1,...,yn are bound in the
    ++ interpreter to respective root values.
    ++ Error: if p has more than one variable y.
    ++
    ++X a:Polynomial(Integer):=-3*x^3+2*x+13
    ++X rootsOf(a)
  rootsOf: SparseUnivariatePolynomial $ -> List $
    ++ rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
    ++ Note that the returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.
    ++
    ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
    ++X rootsOf(a)
  rootsOf: (SparseUnivariatePolynomial $, Symbol) -> List $
    ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
    ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
    ++ Note that the returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.
    ++
    ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
    ++X rootsOf(a,x)
  zeroOf: Polynomial $ -> $
    ++ zeroOf(p) returns y such that \spad{p(y) = 0}.
    ++ If possible, y is expressed in terms of radicals.
    ++ Otherwise it is an implicit algebraic quantity.
    ++ Error: if p has more than one variable y.
    ++
    ++X a:Polynomial(Integer):=-3*x^2+2*x-13
    ++X zeroOf(a)
  zeroOf: SparseUnivariatePolynomial $ -> $
    ++ zeroOf(p) returns y such that \spad{p(y) = 0};
    ++ if possible, y is expressed in terms of radicals.
    ++ Otherwise it is an implicit algebraic quantity.
    ++
    ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
```

```
      ++X zeroOf(a)
    zeroOf: (SparseUnivariatePolynomial $, Symbol) -> $
      ++ zeroOf(p, y) returns y such that \spad{p(y) = 0};
      ++ if possible, y is expressed in terms of radicals.
      ++ Otherwise it is an implicit algebraic quantity which
      ++ displays as \spad{'y}.
      ++
      ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
      ++X zeroOf(a,x)
    zerosOf: Polynomial $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible.
      ++ Otherwise they are implicit algebraic quantities.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++ Error: if p has more than one variable y.
      ++
      ++X a:Polynomial(Integer):=-3*x^2+2*x-13
      ++X zerosOf(a)
    zerosOf: SparseUnivariatePolynomial $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible, and otherwise
      ++ as implicit algebraic quantities.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++
      ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
      ++X zerosOf(a)
    zerosOf: (SparseUnivariatePolynomial $, Symbol) -> List $
      ++ zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible, and otherwise
      ++ as implicit algebraic quantities
      ++ which display as \spad{'yi}.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++
      ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
      ++X zerosOf(a,x)
 add
    SUP ==> SparseUnivariatePolynomial $

    assign  : (Symbol, $) -> $
    allroots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $
    binomialRoots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $

    zeroOf(p:SUP)            == assign(x := new(), zeroOf(p, x))
    rootOf(p:SUP)            == assign(x := new(), rootOf(p, x))

    zerosOf(p:SUP)           == zerosOf(p, new())

    rootsOf(p:SUP)           == rootsOf(p, new())

    rootsOf(p:SUP, y:Symbol) == allroots(p, y, rootOf)

    zerosOf(p:SUP, y:Symbol) == allroots(p, y, zeroOf)

    assign(x, f)             == (assignSymbol(x, f, $)$Lisp; f)

    zeroOf(p:Polynomial $) ==
      empty?(l := variables p) => error "zeroOf: constant polynomial"
      zeroOf(univariate p, first l)

    rootOf(p:Polynomial $) ==
      empty?(l := variables p) => error "rootOf: constant polynomial"
      rootOf(univariate p, first l)

    zerosOf(p:Polynomial $) ==
      empty?(l := variables p) => error "zerosOf: constant polynomial"
      zerosOf(univariate p, first l)

    rootsOf(p:Polynomial $) ==
      empty?(l := variables p) => error "rootsOf: constant polynomial"
      rootsOf(univariate p, first l)

    zeroOf(p:SUP, y:Symbol) ==
      zero?(d := degree p) => error "zeroOf: constant polynomial"
      zero? coefficient(p, 0) => 0
      a := leadingCoefficient p
      d = 2 =>
        b := coefficient(p, 1)
        (sqrt(b**2 - 4 * a * coefficient(p, 0)) - b) / (2 * a)
      (r := retractIfCan(reductum p)@Union($,"failed")) case "failed" =>
        rootOf(p, y)
      nthRoot(- (r::$ / a), d)

    binomialRoots(p, y, fn) ==
     -- p = a * x**n + b
      alpha := assign(x := new(y)$Symbol, fn(p, x))
--      one?(n := degree p) =>  [ alpha ]
      ((n := degree p) = 1) =>  [ alpha ]
      cyclo := cyclotomic(n,monomial(1,1)$SUP)_
```

```
                  $NumberTheoreticPolynomialFunctions(SUP)
        beta := assign(x := new(y)$Symbol, fn(cyclo, x))
        [alpha*beta**i for i in 0..(n-1)::NonNegativeInteger]

      import PolynomialDecomposition(SUP,$)

      allroots(p, y, fn) ==
        zero? p => error "allroots: polynomial must be nonzero"
        zero? coefficient(p,0) =>
           concat(0, allroots(p quo monomial(1,1), y, fn))
        zero?(p1:=reductum p) => empty()
        zero? reductum p1 => binomialRoots(p, y, fn)
        decompList := decompose(p)
        # decompList > 1 =>
            h := last decompList
            g := leftFactor(p,h) :: SUP
            groots := allroots(g, y, fn)
            "append"/[allroots(h-r::SUP, y, fn) for r in groots]
        ans := nil()$List($)
        while not ground? p repeat
          alpha := assign(x := new(y)$Symbol, fn(p, x))
          q     := monomial(1, 1)$SUP - alpha::SUP
          if not zero?(p alpha) then
            p   := p quo q
            ans := concat(alpha, ans)
          else while zero?(p alpha) repeat
            p   := (p exquo q)::SUP
            ans := concat(alpha, ans)
        reverse_! ans
```

⟨*ACF.dotabb*⟩≡

```
  "ACF"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
  "ACF" -> "FIELD"
  "ACF" -> "RADCAT"
```

⟨*ACF.dotfull*⟩≡

```
  "AlgebraicallyClosedField()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
  "AlgebraicallyClosedField()" -> "Field()"
  "AlgebraicallyClosedField()" -> "RadicalCategory()"
```

⟨*ACF.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

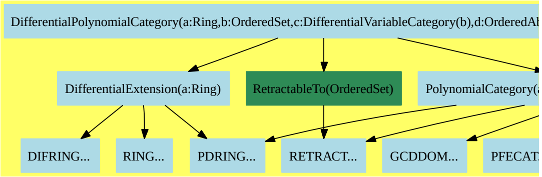
"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];
}
```

17.0.192 DifferentialPolynomialCategory (DPOLCAT)


DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAb
DifferentialExtension(a:Ring)
RetractableTo(OrderedSet)
PolynomialCategory(
DIFRING...
RING...
PDRING...
RETRACT...
GCDDOM...
PFECAT

⟨*DifferentialPolynomialCategory.input*⟩≡

```
)set break resume
)sys rm -f DifferentialPolynomialCategory.output
)spool DifferentialPolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialPolynomialCategory
--R DifferentialPolynomialCategory(R: Ring,S: OrderedSet,V: DifferentialVariableCategory t#
--R Abbreviation for DifferentialPolynomialCategory is DPOLCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DPOLCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> % if R has DIFRING
--R D : (%,List V) -> %                   D : (%,V) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coefficients : % -> List R            coerce : S -> %
--R coerce : V -> %                       coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> E                       differentiate : (%,List V) -> %
--R differentiate : (%,V) -> %            eval : (%,List V,List %) -> %
--R eval : (%,V,%) -> %                   eval : (%,List V,List R) -> %
--R eval : (%,V,R) -> %                   eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %       ground : % -> R
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R initial : % -> %                      isobaric? : % -> Boolean
--R latex : % -> String                   leader : % -> V
```

```
--R leadingCoefficient : % -> R           leadingMonomial : % -> %
--R map : ((R -> R),%) -> %               mapExponents : ((E -> E),%) -> %
--R minimumDegree : % -> E                monomial : (R,E) -> %
--R monomial? : % -> Boolean              monomials : % -> List %
--R one? : % -> Boolean                   order : % -> NonNegativeInteger
--R pomopo! : (%,R,E,%) -> %              primitiveMonomials : % -> List %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> S                      retract : % -> V
--R retract : % -> R                      sample : () -> %
--R separant : % -> %                     variables : % -> List V
--R weight : % -> NonNegativeInteger      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,List V,List NonNegativeInteger) -> %
--R D : (%,V,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R
--R coefficient : (%,List V,List NonNegativeInteger) -> %
--R coefficient : (%,V,NonNegativeInteger) -> %
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT or R has ALGEBRA FRA
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,Characteristic
--R content : (%,V) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if V has KONVERT INFORM and R has KONVERT INFORM
--R convert : % -> Pattern Integer if V has KONVERT PATTERN INT and R has KONVERT
--R convert : % -> Pattern Float if V has KONVERT PATTERN FLOAT and R has KONVERT
--R degree : (%,S) -> NonNegativeInteger
--R degree : (%,List V) -> List NonNegativeInteger
```

```
--R degree : (%,V) -> NonNegativeInteger
--R differentialVariables : % -> List S
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,List V,List NonNegativeInteger) -> %
--R differentiate : (%,V,NonNegativeInteger) -> %
--R discriminant : (%,V) -> % if R has COMRING
--R eval : (%,List S,List R) -> % if R has DIFRING
--R eval : (%,S,R) -> % if R has DIFRING
--R eval : (%,List S,List %) -> % if R has DIFRING
--R eval : (%,S,%) -> % if R has DIFRING
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial 
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(V,"failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : S -> (NonNegativeInteger -> %)
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List V) -> List NonNegativeInteger
--R minimumDegree : (%,V) -> NonNegativeInteger
--R monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
--R monomial : (%,List V,List NonNegativeInteger) -> %
--R monomial : (%,V,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,V) -> %
--R multivariate : (SparseUnivariatePolynomial R,V) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,S) -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Floa
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,V) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(S,"failed")
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUniv
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivari
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List V) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,V) -> SparseUnivariatePolynomial %
--R weight : (%,S) -> NonNegativeInteger
--R weights : (%,S) -> List NonNegativeInteger
--R weights : % -> List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

⟨*DifferentialPolynomialCategory.help*⟩≡

```
====================================================================
DifferentialPolynomialCategory examples
====================================================================

See Also:
o )show DifferentialPolynomialCategory
```

See:

⇐ "DifferentialExtension" (DIFEXT) 11.0.153 on page 1053
⇐ "PolynomialCategory" (POLYCAT) 16.0.189 on page 1385
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0
1
associates?
binomThmExpt
characteristic
charthRoot
coefficient
coefficients
coerce
conditionP
content
convert
D
degree
differentialVariables
differentiate
discriminant
eval
exquo
factor
factorPolynomial
factorSquareFreePolynomial
gcd
gcdPolynomial
ground
ground?
hash
initial
isExpt
isobaric?
isPlus
isTimes
latex
lcm
leader
leadingCoefficient
leadingMonomial
makeVariable
map
mapExponents
max
min
minimumDegree
monicDivide
monomial
monomial?
monomials
multivariate
numberOfMonomials
one?
order
patternMatch
pomopo!
prime?
primitiveMonomials
primitivePart
recip
reducedSystem
reductum
resultant
retract
retractIfCan
sample
separant
solveLinearPolynomialEquation
squareFree
squareFreePart
squareFreePolynomial
subtractIfCan
totalDegree
unit?
unitCanonical
unitNormal
univariate
variables
weight
weights
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?<?
?<=?
?>?
?>=?

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if

and only if `unitCanonical(a) = unitCanonical(b)`.

- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : S -> %
degree : (%,S) -> NonNegativeInteger
differentialVariables : % -> List S
differentiate : (%,(R -> R)) -> %
eval : (%,List S,List R) -> % if R has DIFRING
eval : (%,List S,List %) -> % if R has DIFRING
eval : (%,List Equation %) -> %
initial : % -> %
isobaric? : % -> Boolean
leader : % -> V
makeVariable : S -> (NonNegativeInteger -> %)
makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
order : % -> NonNegativeInteger
order : (%,S) -> NonNegativeInteger
retractIfCan : % -> Union(S,"failed")
separant : % -> %
weight : % -> NonNegativeInteger
weight : (%,S) -> NonNegativeInteger
weights : (%,S) -> List NonNegativeInteger
weights : % -> List NonNegativeInteger
```

These exports come from (p1385) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup,
V:DifferentialVariableCategory(S:OrderedSet):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
```

```
  if and(has($,CharacteristicNonZero),
     has(R,PolynomialFactorizationExplicit))
  or R has CHARNZ
coefficient : (%,List V,List NonNegativeInteger) -> %
coefficient : (%,V,NonNegativeInteger) -> %
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT
  or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : V -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT
  and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT
  and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM
  and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,V) -> %
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
differentiate : (%,List V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,List V,List R) -> %
eval : (%,V,R) -> %
eval : (%,List V,List %) -> %
eval : (%,V,%) -> %
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
```

```
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % ->
  Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT
      and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if V has PATMAB FLOAT
      and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
```

```
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p1053) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p62) RetractableTo(S:OrderedSet):

```
retract : % -> S
```

These exports come from (p39) InnerEvalable(S,R)
where S:OrderedSet, R:Ring:

```
eval : (%,S,R) -> % if R has DIFRING
```

These exports come from (p39) InnerEvalable(S,where S:OrderedSet,

```
eval : (%,S,%) -> % if R has DIFRING
```

These exports come from (p91) Evalable(

⟨*category DPOLCAT DifferentialPolynomialCategory*⟩≡

```
)abbrev category DPOLCAT DifferentialPolynomialCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:PolynomialCategory
++ Related Constructors:DifferentialVariableCategory
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
++ \spadtype{DifferentialPolynomialCategory} is a category constructor
++ specifying basic functions in an ordinary differential polynomial
++ ring with a given ordered set of differential indeterminates.
++ In addition, it implements defaults for the basic functions.
++ The functions \spadfun{order} and \spadfun{weight} are extended
++ from the set of derivatives of differential indeterminates
++ to the set of differential polynomials.  Other operations
++ provided on differential polynomials are
++ \spadfun{leader}, \spadfun{initial},
++ \spadfun{separant}, \spadfun{differentialVariables}, and
++ \spadfun{isobaric?}.   Furthermore, if the ground ring is
++ a differential ring, then evaluation (substitution
++ of differential indeterminates by elements of the ground ring
++ or by differential polynomials) is
++ provided by \spadfun{eval}.
++ A convenient way of referencing derivatives is provided by
++ the functions \spadfun{makeVariable}.
++
++ To construct a domain using this constructor, one needs
++ to provide a ground ring R, an ordered set S of differential
++ indeterminates, a ranking V on the set of derivatives
++ of the differential indeterminates, and a set E of
++ exponents in bijection with the set of differential monomials
++ in the given differential indeterminates.

DifferentialPolynomialCategory(R:Ring,S:OrderedSet,
  V:DifferentialVariableCategory S, E:OrderedAbelianMonoidSup):
            Category ==
  Join(PolynomialCategory(R,E,V),
       DifferentialExtension R, RetractableTo S) with
    -- Examples:
    -- s:=makeVariable('s)
    -- p:= 3*(s 1)**2 + s*(s 2)**3
    --  all functions below have default implementations
    --  using primitives from V

    makeVariable: S -> (NonNegativeInteger -> $)
       ++ makeVariable(s) views s as a differential
       ++ indeterminate,  in such a way that the n-th
       ++ derivative of s may be simply referenced as z.n
       ++ where z :=makeVariable(s).
       ++ Note that In the interpreter, z is
       ++ given as an internal map, which may be ignored.
       -- Example: makeVariable('s); %.5

    differentialVariables: $ ->  List S
      ++ differentialVariables(p) returns a list of differential
      ++ indeterminates occurring in a differential polynomial p.
    order : ($, S) -> NonNegativeInteger
      ++ order(p,s) returns the order of the differential
      ++ polynomial p in differential indeterminate s.
    order : $   -> NonNegativeInteger
      ++ order(p) returns the order of the differential polynomial p,
      ++ which is the maximum number of differentiations of a
      ++ differential indeterminate, among all those appearing in p.
    degree: ($, S) -> NonNegativeInteger
      ++ degree(p, s) returns the maximum degree of
      ++ the differential polynomial p viewed as a differential polynomial
      ++ in the differential indeterminate s alone.
```

```
weights: $ -> List NonNegativeInteger
  ++ weights(p) returns a list of weights of differential monomials
  ++ appearing in differential polynomial p.
weight: $   -> NonNegativeInteger
  ++ weight(p) returns the maximum weight of all differential monomials
  ++ appearing in the differential polynomial p.
weights: ($, S) -> List NonNegativeInteger
  ++ weights(p, s) returns a list of
  ++ weights of differential monomials
  ++ appearing in the differential polynomial p when p is viewed
  ++ as a differential polynomial in the differential indeterminate s
  ++ alone.
weight: ($, S) -> NonNegativeInteger
  ++ weight(p, s) returns the maximum weight of all differential
  ++ monomials appearing in the differential polynomial p
  ++ when p is viewed as a differential polynomial in
  ++ the differential indeterminate s alone.
isobaric?: $ -> Boolean
  ++ isobaric?(p) returns true if every differential monomial appearing
  ++ in the differential polynomial p has same weight,
  ++ and returns false otherwise.
leader: $   -> V
  ++ leader(p) returns the derivative of the highest rank
  ++ appearing in the differential polynomial p
  ++ Note that an error occurs if p is in the ground ring.
initial:$   -> $
  ++ initial(p) returns the
  ++ leading coefficient when the differential polynomial p
  ++ is written as a univariate polynomial in its leader.
separant:$  -> $
  ++ separant(p) returns the
  ++ partial derivative of the differential polynomial p
  ++ with respect to its leader.
if R has DifferentialRing then
  InnerEvalable(S, R)
  InnerEvalable(S, $)
  Evalable $
  makeVariable: $ -> (NonNegativeInteger -> $)
   ++ makeVariable(p) views p as an element of a differential
   ++ ring,  in such a way that the n-th
   ++ derivative of p may be simply referenced as z.n
   ++ where z := makeVariable(p).
   ++ Note that In the interpreter, z is
   ++ given as an internal map, which may be ignored.
   -- Example: makeVariable(p); %.5; makeVariable(%**2); %.2
```

```
add
   p:$
   s:S

   makeVariable s == n +-> makeVariable(s,n)::$

   if R has IntegralDomain then
     differentiate(p:$, d:R -> R) ==
       ans:$ := 0
       l := variables p
       while (u:=retractIfCan(p)@Union(R, "failed")) case "failed" repeat
         t := leadingMonomial p
         lc := leadingCoefficient t
         ans := ans + d(lc)::$ * (t exquo lc)::$
             + +/[differentiate(t, v) * (differentiate v)::$ for v in l]
         p := reductum p
       ans + d(u::R)::$

   order (p:$):NonNegativeInteger ==
     ground? p => 0
     "max"/[order v for v in variables p]

   order (p:$,s:S):NonNegativeInteger ==
     ground? p => 0
     empty? (vv:= [order v for v in variables p | (variable v) = s ]) =>0
     "max"/vv

   degree (p, s) ==
     d:NonNegativeInteger:=0
     for lp in monomials p repeat
       lv:= [v for v in variables lp | (variable v) = s ]
       if not empty? lv then d:= max(d, +/degree(lp, lv))
     d

   weights p ==
     ws:List NonNegativeInteger := nil
     empty? (mp:=monomials p) => ws
     for lp in mp repeat
       lv:= variables lp
       if not empty? lv then
         dv:= degree(lp, lv)
         w:=+/[(weight v) * d _
                for v in lv for d in dv]$(List NonNegativeInteger)
         ws:= concat(ws, w)
     ws
```

```
weight p ==
  empty? (ws:=weights p) => 0
  "max"/ws

weights (p, s) ==
  ws:List NonNegativeInteger := nil
  empty?(mp:=monomials p) => ws
  for lp in mp repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
           for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight (p,s)  ==
  empty? (ws:=weights(p,s)) => 0
  "max"/ws

isobaric? p == (# removeDuplicates weights p) = 1

leader p ==             -- depends on the ranking
  vl:= variables p
  -- it's not enough just to look at leadingMonomial p
  -- the term-ordering need not respect the ranking
  empty? vl => error "leader is not defined "
  "max"/vl

initial p == leadingCoefficient univariate(p,leader p)

separant p == differentiate(p, leader p)

coerce(s:S):$   == s::V::$

retractIfCan(p:$):Union(S, "failed") ==
  (v := retractIfCan(p)@Union(V,"failed")) case "failed" => "failed"
  retractIfCan(v::V)

differentialVariables p ==
  removeDuplicates [variable v for v in variables p]

if R has DifferentialRing then

  makeVariable p == n +-> differentiate(p, n)
```

```
        eval(p:$, sl:List S, rl:List R) ==
          ordp:= order p
          vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                  for s in sl]$List(List V)
          rrl:=nil$List(R)
          for r in rl repeat
            t:= r
            rrl:= concat(rrl,
                  concat(r, [t := differentiate t for i in 1..ordp]))
          eval(p, vl, rrl)

        eval(p:$, sl:List S, rl:List $) ==
          ordp:= order p
          vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                  for s in sl]$List(List V)
          rrl:=nil$List($)
          for r in rl repeat
            t:=r
            rrl:=concat(rrl,
                 concat(r, [t:=differentiate t for i in 1..ordp]))
          eval(p, vl, rrl)

        eval(p:$, l:List Equation $) ==
          eval(p, [retract(lhs e)@S for e in l]$List(S),
                [rhs e for e in l]$List($))
```

⟨*DPOLCAT.dotabb*⟩≡

```
  "DPOLCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
  "DPOLCAT" -> "DIFEXT"
  "DPOLCAT" -> "POLYCAT"
  "DPOLCAT" -> "RETRACT"
```

⟨*DPOLCAT.dotfull*⟩≡

```
  "DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
  "DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
   -> "DifferentialExtension(a:Ring)"
  "DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
   -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  "DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
   -> "RetractableTo(OrderedSet)"
```

⟨*DPOLCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 [color=lightblue];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "DifferentialExtension(a:Ring)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "RetractableTo(OrderedSet)"

"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "RING..."
"DifferentialExtension(a:Ring)" -> "DIFRING..."
"DifferentialExtension(a:Ring)" -> "PDRING..."

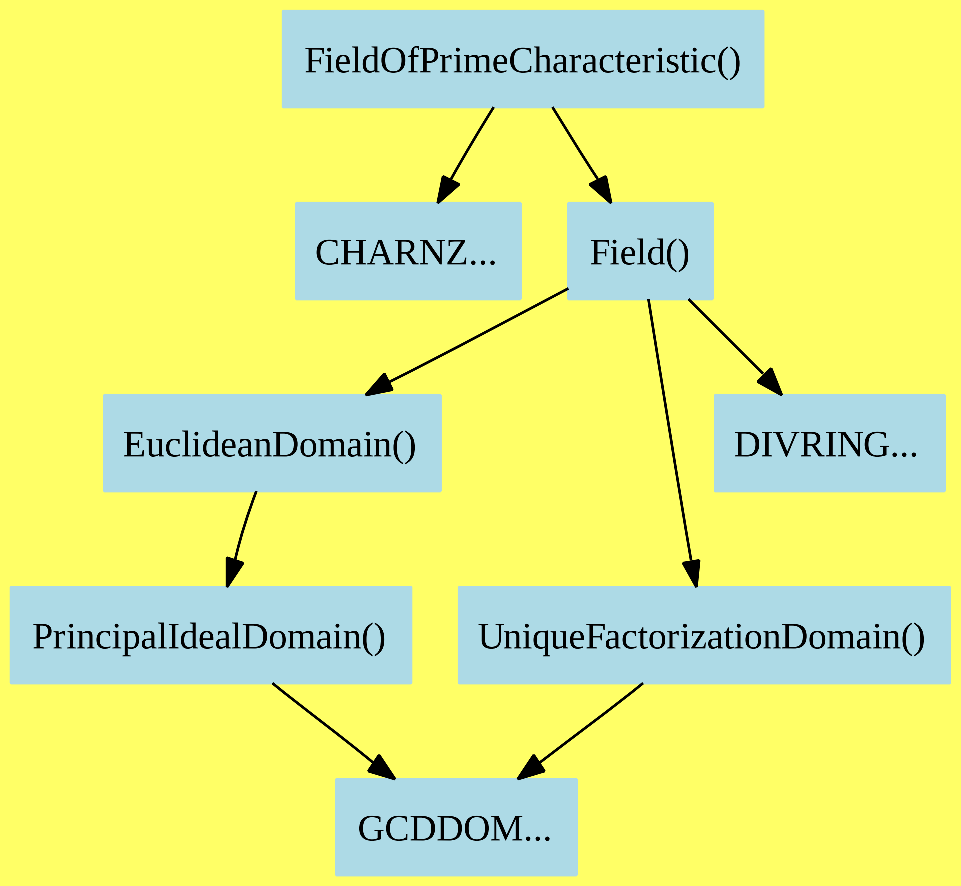
"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RETRACT..."

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."

"DIFRING..." [color=lightblue];
"RING..." [color=lightblue];
"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

17.0.193 FieldOfPrimeCharacteristic (FPC)


FieldOfPrimeCharacteristic()
CHARNZ...
Field()
EuclideanDomain()
DIVRING...
PrincipalIdealDomain()
UniqueFactorizationDomain()
GCDDOM...

⟨*FieldOfPrimeCharacteristic.input*⟩≡
```
)set break resume
)sys rm -f FieldOfPrimeCharacteristic.output
)spool FieldOfPrimeCharacteristic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FieldOfPrimeCharacteristic
--R FieldOfPrimeCharacteristic  is a category constructor
--R Abbreviation for FieldOfPrimeCharacteristic is FPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPC
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       coerce : Fraction Integer -> %
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  lcm : List % -> %
--R lcm : (%,%) -> %                     one? : % -> Boolean
--R prime? : % -> Boolean                primeFrobenius : % -> %
--R ?quo? : (%,%) -> %                   recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                   sample : () -> %
--R sizeLess? : (%,%) -> Boolean         squareFree : % -> Factored %
--R squareFreePart : % -> %              unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1
```

```
)spool
)lisp (bye)
```

⟨*FieldOfPrimeCharacteristic.help*⟩≡

```
====================================================================
FieldOfPrimeCharacteristic examples
====================================================================

See Also:
o )show FieldOfPrimeCharacteristic
```

See:

⇒ "FiniteFieldCategory" (FFIELDC) 18.0.207 on page 1671
⇐ "CharacteristicNonZero" (CHARNZ) 10.0.135 on page 922
⇐ "Field" (FIELD) 16.0.185 on page 1347

Exports:

0	1	associates?	characteristic
charthRoot	coerce	discreteLog	divide
euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash
inv	latex	lcm	multiEuclidean
one?	order	prime?	primeFrobenius
principalIdeal	recip	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?
?/?	?=?	?ˆ?	?rem?
?quo?	?˜=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
order : % -> OnePointCompletion PositiveInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
```

These are implemented by this category:

```
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
```

These exports come from (p1347) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
```

⟨*category FPC FieldOfPrimeCharacteristic*⟩≡

```
)abbrev category FPC FieldOfPrimeCharacteristic
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, finite field, prime characteristic
++ References:
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FieldOfPrimeCharacteristic is the category of fields of prime
++ characteristic, e.g. finite fields, algebraic closures of
++ fields of prime characteristic, transcendental extensions of
++ of fields of prime characteristic.

FieldOfPrimeCharacteristic:Category == _
 Join(Field,CharacteristicNonZero) with
   order: $ -> OnePointCompletion PositiveInteger
     ++ order(a) computes the order of an element in the multiplicative
     ++ group of the field.
     ++ Error: if \spad{a} is 0.
   discreteLog: ($,$) -> Union(NonNegativeInteger,"failed")
     ++ discreteLog(b,a) computes s with \spad{b**s = a} if such an s exists.
   primeFrobenius: $ -> $
     ++ primeFrobenius(a) returns \spad{a**p} where p is the characteristic.
```

```
    primeFrobenius: ($,NonNegativeInteger) -> $
      ++ primeFrobenius(a,s) returns \spad{a**(p**s)} where p
      ++ is the characteristic.
  add
    primeFrobenius(a) == a ** characteristic()
    primeFrobenius(a,s) == a ** (characteristic()**s)
```

⟨*FPC.dotabb*⟩≡

```
"FPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FPC" -> "CHARNZ"
"FPC" -> "FIELD"
```

⟨*FPC.dotfull*⟩≡

```
"FieldOfPrimeCharacteristic()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FieldOfPrimeCharacteristic()" -> "CharacteristicNonZero()"
```

⟨*FPC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

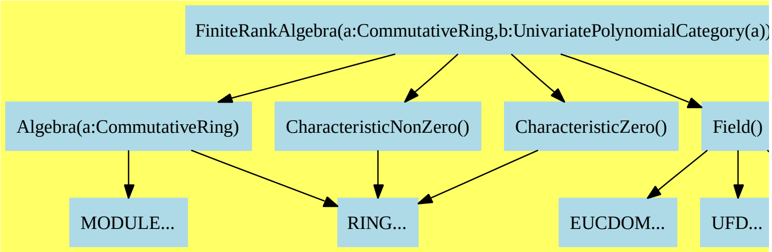
"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"GCDDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"CHARNZ..." [color=lightblue];
}
```

17.0.194 FiniteRankAlgebra (FINRALG)


FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a)
Algebra(a:CommutativeRing)
CharacteristicNonZero()
CharacteristicZero()
Field()
MODULE...
RING...
EUCDOM...
UFD...

⟨*FiniteRankAlgebra.input*⟩≡

```
)set break resume
)sys rm -f FiniteRankAlgebra.output
)spool FiniteRankAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteRankAlgebra
--R FiniteRankAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory t#1)  is a categor
--R Abbreviation for FiniteRankAlgebra is FINRALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINRALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R discriminant : Vector % -> R          hash : % -> SingleInteger
--R latex : % -> String                   norm : % -> R
--R one? : % -> Boolean                   rank : () -> PositiveInteger
--R recip : % -> Union(%,"failed")        sample : () -> %
--R trace : % -> R                        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
```

```
--R characteristicPolynomial : % -> UP
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R minimalPolynomial : % -> UP if R has FIELD
--R regularRepresentation : (%,Vector %) -> Matrix R
--R represents : (Vector R,Vector %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R traceMatrix : Vector % -> Matrix R
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteRankAlgebra.help*⟩≡

```
====================================================================
FiniteRankAlgebra examples
====================================================================

See Also:
o )show FiniteRankAlgebra
```

See:

⇒ “FramedAlgebra” (FRAMALG) 18.0.209 on page 1694
⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “CharacteristicNonZero” (CHARNZ) 10.0.135 on page 922
⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “Field” (FIELD) 16.0.185 on page 1347

Exports:

0
1
characteristic
characteristicPolynomial
charthRoot
coerce
coordinates
discriminant
hash
latex
minimalPolynomial
norm
one?
rank
recip
regularRepresentation
represents
sample
subtractIfCan
trace
traceMatrix
zero?
?**?
?***?
?+?
?-?
-?
?=?
?^?
?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
characteristicPolynomial : % -> UP
coordinates : (%,Vector %) -> Vector R
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
rank : () -> PositiveInteger
trace : % -> R
```

These are implemented by this category:

```
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
traceMatrix : Vector % -> Matrix R
```

These exports come from (p1045) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1347) Field():

```
coerce : R -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p927) CharacteristicZero():

⟨*category FINRALG FiniteRankAlgebra*⟩≡

```
)abbrev category FINRALG FiniteRankAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ A FiniteRankAlgebra is an algebra over a commutative ring R which
++ is a free R-module of finite rank.

FiniteRankAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category == Algebra R with
    rank                    : () -> PositiveInteger
      ++ rank() returns the rank of the algebra.
    regularRepresentation   : (% , Vector %) -> Matrix R
      ++ regularRepresentation(a,basis) returns the matrix of the
      ++ linear map defined by left multiplication by \spad{a} with respect
      ++ to the basis \spad{basis}.
    trace                   : %  -> R
      ++ trace(a) returns the trace of the regular representation
      ++ of \spad{a} with respect to any basis.
    norm                    : %  -> R
      ++ norm(a) returns the determinant of the regular representation
      ++ of \spad{a} with respect to any basis.
    coordinates             : (%, Vector %) -> Vector R
      ++ coordinates(a,basis) returns the coordinates of \spad{a} with
      ++ respect to the basis \spad{basis}.
    coordinates             : (Vector %, Vector %) -> Matrix R
      ++ coordinates([v1,...,vm], basis) returns the coordinates of the
      ++ vi's with to the basis \spad{basis}.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
      ++ function.
    represents              : (Vector R, Vector %) -> %
      ++ represents([a1,..,an],[v1,..,vn]) returns \spad{a1*v1+...+an*vn}.
    discriminant            : Vector % -> R
      ++ discriminant([v1,..,vn]) returns
      ++ \spad{determinant(traceMatrix([v1,..,vn]))}.
    traceMatrix             : Vector % -> Matrix R
      ++ traceMatrix([v1,..,vn]) is the n-by-n matrix ( Tr(vi * vj) )
    characteristicPolynomial: % -> UP
      ++ characteristicPolynomial(a) returns the characteristic
      ++ polynomial of the regular representation of \spad{a} with respect
      ++ to any basis.
    if R has Field then minimalPolynomial : % -> UP
      ++ minimalPolynomial(a) returns the minimal polynomial of \spad{a}.
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero

  add
```

```
    discriminant v == determinant traceMatrix v

    coordinates(v:Vector %, b:Vector %) ==
      m := new(#v, #b, 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates(qelt(v, i), b))
      m

    represents(v, b) ==
      m := minIndex v - 1
      _+/[v(i+m) * b(i+m) for i in 1..rank()]

    traceMatrix v ==
      matrix [[trace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
               for i in minIndex v .. maxIndex v]$List(List R)

    regularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)
```

⟨*FINRALG.dotabb*⟩≡

```
"FINRALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FINRALG" -> "ALGEBRA"
"FINRALG" -> "FIELD"
"FINRALG" -> "CHARNZ"
"FINRALG" -> "CHARZ"
```

⟨*FINRALG.dotfull*⟩≡

```
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"
```

⟨*FINRALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

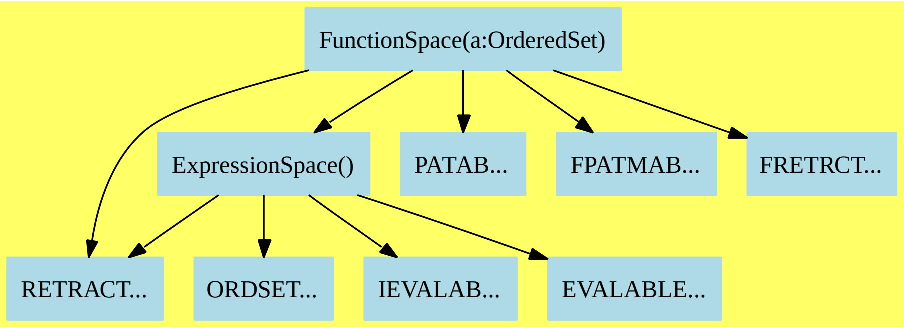
"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

17.0.195 FunctionSpace (FS)



⟨*FunctionSpace.input*⟩≡

```
)set break resume
)sys rm -f FunctionSpace.output
)spool FunctionSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionSpace
--R FunctionSpace R: OrderedSet  is a category constructor
--R Abbreviation for FunctionSpace is FS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FS
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP          ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> % if R has SGROUP           0 : () -> % if R has ABELSG
--R applyQuote : (Symbol,%,%) -> %        applyQuote : (Symbol,%) -> %
--R belong? : BasicOperator -> Boolean    box : List % -> %
--R box : % -> %                          coerce : R -> %
--R coerce : Symbol -> %                  coerce : Kernel % -> %
--R coerce : % -> OutputForm              distribute : (%,%) -> %
--R distribute : % -> %                   elt : (BasicOperator,%,%) -> %
--R elt : (BasicOperator,%) -> %          eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %       eval : (%,Kernel %,%) -> %
--R freeOf? : (%,Symbol) -> Boolean       freeOf? : (%,%) -> Boolean
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             height : % -> NonNegativeInteger
```

```
--R is? : (%,Symbol) -> Boolean           kernel : (BasicOperator,%) -> %
--R kernels : % -> List Kernel %          latex : % -> String
--R map : ((% -> %),Kernel %) -> %        max : (%,%) -> %
--R min : (%,%) -> %                      paren : List % -> %
--R paren : % -> %                        retract : % -> R
--R retract : % -> Symbol                 retract : % -> Kernel %
--R subst : (%,Equation %) -> %           tower : % -> List Kernel %
--R variables : % -> List Symbol          ?~=? : (%,%) -> Boolean
--R ?*? : (%,%) -> % if R has SGROUP
--R ?*? : (PositiveInteger,%) -> % if R has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if R has ABELSG
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (%,Fraction Integer) -> % if R has INTDOM
--R ?*? : (Fraction Integer,%) -> % if R has INTDOM
--R ?**? : (%,PositiveInteger) -> % if R has SGROUP
--R ?**? : (%,NonNegativeInteger) -> % if R has SGROUP
--R ?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?/? : (%,%) -> % if R has GROUP or R has INTDOM
--R ?/? : (SparseMultivariatePolynomial(R,Kernel %),SparseMultivariatePolynomial(R,Kernel %)
--R D : (%,Symbol) -> % if R has RING
--R D : (%,List Symbol) -> % if R has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
--R ?^? : (%,PositiveInteger) -> % if R has SGROUP
--R ?^? : (%,NonNegativeInteger) -> % if R has SGROUP
--R ?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R applyQuote : (Symbol,List %) -> %
--R applyQuote : (Symbol,%,%,%,%) -> %
--R applyQuote : (Symbol,%,%,%) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R characteristic : () -> NonNegativeInteger if R has RING
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Integer -> % if R has RING or R has RETRACT INT
--R coerce : Fraction Integer -> % if R has INTDOM or R has RETRACT INT and R has INTDOM or
--R coerce : Polynomial R -> % if R has RING
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Polynomial R -> % if R has INTDOM
--R coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
--R coerce : Polynomial Fraction R -> % if R has INTDOM
--R coerce : Fraction R -> % if R has INTDOM
--R coerce : SparseMultivariatePolynomial(R,Kernel %) -> % if R has RING
--R commutator : (%,%) -> % if R has GROUP
```

```
--R conjugate : (%,%) -> % if R has GROUP
--R convert : % -> InputForm if R has KONVERT INFORM
--R convert : Factored % -> % if R has INTDOM
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
--R denominator : % -> % if R has INTDOM
--R differentiate : (%,Symbol) -> % if R has RING
--R differentiate : (%,List Symbol) -> % if R has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger if R has INTDOM
--R eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
--R eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> % if R h
--R eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> % if R has RI
--R eval : (%,List BasicOperator,List %,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
--R eval : % -> % if R has KONVERT INFORM
--R eval : (%,List Symbol) -> % if R has KONVERT INFORM
--R eval : (%,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R
--R factor : % -> Factored % if R has INTDOM
--R gcd : (%,%) -> % if R has INTDOM
--R gcd : List % -> % if R has INTDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R inv : % -> % if R has GROUP or R has INTDOM
--R is? : (%,BasicOperator) -> Boolean
```

```
--R isExpt : (%,Symbol) -> Union(Record(var: Kernel %,exponent: Integer),"failed") if R has
--R isExpt : (%,BasicOperator) -> Union(Record(var: Kernel %,exponent: Integer),"failed") if
--R isExpt : % -> Union(Record(var: Kernel %,exponent: Integer),"failed") if R has SGROUP
--R isMult : % -> Union(Record(coef: Integer,var: Kernel %),"failed") if R has ABELSG
--R isPlus : % -> Union(List %,"failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %,exponent: Integer),"failed") if R has RING
--R isTimes : % -> Union(List %,"failed") if R has SGROUP
--R kernel : (BasicOperator,List %) -> %
--R lcm : (%,%) -> % if R has INTDOM
--R lcm : List % -> % if R has INTDOM
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has INTDOM
--R numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
--R numerator : % -> % if R has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R one? : % -> Boolean if R has SGROUP
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R prime? : % -> Boolean if R has INTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has INTDOM
--R ?quo? : (%,%) -> % if R has INTDOM
--R recip : % -> Union(%,"failed") if R has SGROUP
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if and(has(R,LinearlyExplicitRingOver Integer
--R ?rem? : (%,%) -> % if R has INTDOM
--R retract : % -> Fraction Integer if R has RETRACT INT and R has INTDOM or R has RETRACT F
--R retract : % -> Polynomial R if R has RING
--R retract : % -> Fraction Polynomial R if R has INTDOM
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT INT and R has INTD
--R retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
--R retractIfCan : % -> Union(Fraction Polynomial R,"failed") if R has INTDOM
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R sample : () -> % if R has SGROUP or R has ABELSG
--R sizeLess? : (%,%) -> Boolean if R has INTDOM
--R squareFree : % -> Factored % if R has INTDOM
--R squareFreePart : % -> % if R has INTDOM
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial % if R has I
--R zero? : % -> Boolean if R has ABELSG
--R
--E 1

)spool
)lisp (bye)
```

⟨*FunctionSpace.help*⟩≡

```
====================================================================
FunctionSpace examples
====================================================================

See Also:
o )show FunctionSpace
```

See:

⇐ “AbelianGroup” (ABELGRP) 7.0.97 on page 576
⇐ “AbelianMonoid” (ABELMON) 5.0.66 on page 301
⇐ “AbelianSemiGroup” (ABELSG) 4.0.36 on page 141
⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “AlgebraicallyClosedFunctionSpace” (ACFS) 18.0.205 on page 1645
⇐ “CharacteristicNonZero” (CHARNZ) 10.0.135 on page 922
⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “CommutativeRing” (COMRING) 10.0.137 on page 932
⇐ “ConvertibleTo” (KONVERT) 2.0.8 on page 25
⇐ “ExpressionSpace” (ES) 5.0.72 on page 337
⇐ “Field” (FIELD) 16.0.185 on page 1347
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.154 on page 1060
⇐ “FullyPatternMatchable” (FPATMAB) 3.0.28 on page 105
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100
⇐ “Group” (GROUP) 6.0.86 on page 443
⇐ “IntegralDomain” (INTDOM) 12.0.164 on page 1153
⇐ “Monoid” (MONOID) 5.0.76 on page 371
⇐ “PartialDifferentialRing” (PDRING) 10.0.145 on page 977
⇐ “Patternable” (PATAB) 2.0.15 on page 53
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “Ring” (RING) 9.0.129 on page 854
⇐ “SemiGroup” (SGROUP) 4.0.61 on page 269

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
eval	euclideanSize	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	numer
numerator	odd?	one?	operator
operators	paren	patternMatch	prime?
principalIdeal	recip	reducedSystem	retract
retractIfCan	sample	sizeLess?	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	-?	?<?
?<=?	?=?	?>?	?>=?
?~=?	?*?	?**?	?+?
?-?	?/?	?^?	?quo?
?rem?			

Attributes Exported:

- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has IntegralDomain then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if $ has IntegralDomain then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- if $ has Ring or Group then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- if $ has CommutativeRing then leftUnitary where **leftUnitary** is true if $1 * x = x$ for all x.

- if $ has CommutativeRing then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
    if R has RING
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
denom : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has INTDOM
eval : (%,List BasicOperator,List %,Symbol) -> %
    if R has KONVERT INFORM
numer : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has RING
?/? :
  (SparseMultivariatePolynomial(R,Kernel %),
   SparseMultivariatePolynomial(R,Kernel %)) -> %
    if R has INTDOM
?**? : (%,NonNegativeInteger) -> % if R has SGROUP
```

These are implemented by this category:

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
characteristic : () -> NonNegativeInteger if R has RING
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
    if R has INTDOM
    or R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> %
    if R has INTDOM
coerce : Polynomial R -> % if R has RING
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : Factored % -> % if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,List %) -> %
```

```
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,%,Symbol) -> %
    if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> %
    if R has RING
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
ground : % -> R
ground? : % -> Boolean
isExpt : (%,BasicOperator) ->
   Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt : (%,Symbol) ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % ->
  Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
numerator : % -> % if R has RING
operator : BasicOperator -> BasicOperator
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
    if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed")
    if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
univariate : (%,Kernel %) ->
  Fraction SparseUnivariatePolynomial %
```

```
    if R has INTDOM
variables : % -> List Symbol
?*? : (%,R) -> % if R has COMRING
?*? : (R,%) -> % if R has COMRING
```

These exports come from (p337) ExpressionSpace():

```
box : List % -> %
box : % -> %
definingPolynomial : % -> % if $ has RING
distribute : (%,%) -> %
distribute : % -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
hash : % -> SingleInteger
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
latex : % -> String
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial %
    if $ has RING
odd? : % -> Boolean if $ has RETRACT INT
operators : % -> List BasicOperator
paren : List % -> %
paren : % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
```

```
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p105) FullyPatternMatchable(OrderedSet):

```
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if R has PATMAB FLOAT
```

These exports come from (p100) FullyRetractableTo(OrderedSet):

```
coerce : Integer -> % if R has RING or R has RETRACT INT
coerce : R -> %
```

These exports come from (p25) ConvertibleTo(InputForm):

```
convert : % -> InputForm if R has KONVERT INFORM
```

These exports come from (p371) Monoid():

```
1 : () -> % if R has SGROUP
one? : % -> Boolean if R has SGROUP
recip : % -> Union(%,"failed") if R has SGROUP
sample : () -> % if R has SGROUP or R has ABELSG
?^? : (%,NonNegativeInteger) -> % if R has SGROUP
?*? : (%,%) -> % if R has SGROUP
?**? : (%,PositiveInteger) -> % if R has SGROUP
?^? : (%,PositiveInteger) -> % if R has SGROUP
```

These exports come from (p443) Group():

```
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
inv : % -> % if R has GROUP or R has INTDOM
?/? : (%,%) -> % if R has GROUP or R has INTDOM
?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
```

These exports come from (p301) AbelianMonoid():

```
0 : () -> % if R has ABELSG
zero? : % -> Boolean if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has ABELSG
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p576) AbelianGroup():

```
subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
?*? : (Integer,%) -> % if R has ABELGRP
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
```

These exports come from (p977) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has RING
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R) where R:OrderedSet:

```
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R) if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p1153) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
```

These exports come from (p1347) Field():

```
coerce : % -> % if R has INTDOM
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has INTDOM
euclideanSize : % -> NonNegativeInteger if R has INTDOM
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has INTDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has INTDOM
factor : % -> Factored % if R has INTDOM
gcd : (%,%) -> % if R has INTDOM
gcd : List % -> % if R has INTDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has INTDOM
lcm : (%,%) -> % if R has INTDOM
lcm : List % -> % if R has INTDOM
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
prime? : % -> Boolean if R has INTDOM
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has INTDOM
sizeLess? : (%,%) -> Boolean if R has INTDOM
squareFree : % -> Factored % if R has INTDOM
squareFreePart : % -> % if R has INTDOM
?quo? : (%,%) -> % if R has INTDOM
?rem? : (%,%) -> % if R has INTDOM
```

These exports come from (p62) RetractableTo(Integer):

```
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
```

These exports come from (p62) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
    if R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
```

```
?*? : (%,Fraction Integer) -> % if R has INTDOM
?*? : (Fraction Integer,%) -> % if R has INTDOM
```

⟨*category FS FunctionSpace*⟩≡

```
)abbrev category FS FunctionSpace
++ Category for formal functions
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 14 February 1994
++ Keywords: operator, kernel, function.
++ Description:
++ A space of formal functions with arguments in an arbitrary ordered set.

FunctionSpace(R:OrderedSet): Category == Definition where
  OP ==> BasicOperator
  O  ==> OutputForm
  SY ==> Symbol
  N  ==> NonNegativeInteger
  Z  ==> Integer
  K  ==> Kernel %
  Q  ==> Fraction R
  PR ==> Polynomial R
  MP ==> SparseMultivariatePolynomial(R, K)
  QF==> PolynomialCategoryQuotientFunctions(IndexedExponents K,K,R,MP,%)

  ODD  ==> "odd"
  EVEN ==> "even"

  SPECIALDIFF  ==> "%specialDiff"
  SPECIALDISP  ==> "%specialDisp"
  SPECIALEQUAL ==> "%specialEqual"
  SPECIALINPUT ==> "%specialInput"

  Definition ==> Join(ExpressionSpace, RetractableTo SY, Patternable R,
                  FullyPatternMatchable R, FullyRetractableTo R) with
       ground?   : % -> Boolean
         ++ ground?(f) tests if f is an element of R.
       ground    : % -> R
         ++ ground(f) returns f as an element of R.
         ++ An error occurs if f is not an element of R.
       variables : %  -> List SY
         ++ variables(f) returns the list of all the variables of f.
       applyQuote: (SY, %) -> %
         ++ applyQuote(foo, x) returns \spad{'foo(x)}.
       applyQuote: (SY, %, %) -> %
         ++ applyQuote(foo, x, y) returns \spad{'foo(x,y)}.
```

```
applyQuote: (SY, %, %, %) -> %
  ++ applyQuote(foo, x, y, z) returns \spad{'foo(x,y,z)}.
applyQuote: (SY, %, %, %, %) -> %
  ++ applyQuote(foo, x, y, z, t) returns \spad{'foo(x,y,z,t)}.
applyQuote: (SY, List %) -> %
  ++ applyQuote(foo, [x1,...,xn]) returns \spad{'foo(x1,...,xn)}.
if R has ConvertibleTo InputForm then
  ConvertibleTo InputForm
  eval     : (%, SY) -> %
    ++ eval(f, foo) unquotes all the foo's in f.
  eval     : (%, List SY) -> %
    ++ eval(f, [foo1,...,foon]) unquotes all the \spad{fooi}'s in f.
  eval     : % -> %
    ++ eval(f) unquotes all the quoted operators in f.
  eval     : (%, OP, %, SY) -> %
    ++ eval(x, s, f, y) replaces every \spad{s(a)} in x by \spad{f(y)}
    ++ with \spad{y} replaced by \spad{a} for any \spad{a}.
  eval     : (%, List OP, List %, SY) -> %
    ++ eval(x, [s1,...,sm], [f1,...,fm], y) replaces every
    ++ \spad{si(a)} in x by \spad{fi(y)}
    ++ with \spad{y} replaced by \spad{a} for any \spad{a}.
if R has SemiGroup then
  Monoid
  -- the following line is necessary because of a compiler bug
  "**"   : (%, N) -> %
    ++ x**n returns x * x * x * ... * x (n times).
  isTimes: % -> Union(List %, "failed")
    ++ isTimes(p) returns \spad{[a1,...,an]}
    ++ if \spad{p = a1*...*an} and \spad{n > 1}.
  isExpt : % -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0}.
if R has Group then Group
if R has AbelianSemiGroup then
  AbelianMonoid
  isPlus: % -> Union(List %, "failed")
    ++ isPlus(p) returns \spad{[m1,...,mn]}
    ++ if \spad{p = m1 +...+ mn} and \spad{n > 1}.
  isMult: % -> Union(Record(coef:Z, var:K),"failed")
    ++ isMult(p) returns \spad{[n, x]} if \spad{p = n * x}
    ++ and \spad{n <> 0}.
if R has AbelianGroup then AbelianGroup
if R has Ring then
  Ring
  RetractableTo PR
  PartialDifferentialRing SY
  FullyLinearlyExplicitRingOver R
  coerce    : MP -> %
    ++ coerce(p) returns p as an element of %.
  numer     : %  -> MP
    ++ numer(f) returns the
    ++ numerator of f viewed as a polynomial in the kernels over R
    ++ if R is an integral domain. If not, then numer(f) = f viewed
    ++ as a polynomial in the kernels over R.
    -- DO NOT change this meaning of numer!  MB 1/90
  numerator : % -> %
    ++ numerator(f) returns the numerator of \spad{f} converted to %.
  isExpt:(%,OP) -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p,op) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0} and \spad{x = op(a)}.
  isExpt:(%,SY) -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p,f) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0} and \spad{x = f(a)}.
  isPower   : % -> Union(Record(val:%,exponent:Z),"failed")
    ++ isPower(p) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0}.
  eval: (%, List SY, List N, List(% -> %)) -> %
    ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
    ++ every \spad{si(a)**ni} in x by \spad{fi(a)} for any \spad{a}.
  eval: (%, List SY, List N, List(List % -> %)) -> %
    ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
    ++ every \spad{si(a1,...,an)**ni} in x by \spad{fi(a1,...,an)}
    ++ for any a1,...,am.
  eval: (%, SY, N, List % -> %) -> %
    ++ eval(x, s, n, f) replaces every \spad{s(a1,...,am)**n} in x
    ++ by \spad{f(a1,...,am)} for any a1,...,am.
  eval: (%, SY, N, % -> %) -> %
    ++ eval(x, s, n, f) replaces every \spad{s(a)**n} in x
    ++ by \spad{f(a)} for any \spad{a}.
if R has CharacteristicZero then CharacteristicZero
if R has CharacteristicNonZero then CharacteristicNonZero
if R has CommutativeRing then
  Algebra R
if R has IntegralDomain then
  Field
  RetractableTo Fraction PR
  convert   : Factored % -> %
    ++ convert(f1\^e1 ... fm\^em) returns \spad{(f1)\^e1 ... (fm)\^em}
    ++ as an element of %, using formal kernels
    ++ created using a \spadfunFrom{paren}{ExpressionSpace}.
  denom     : %  -> MP
    ++ denom(f) returns the denominator of f viewed as a
```

```
        ++ polynomial in the kernels over R.
      denominator : % -> %
        ++ denominator(f) returns the denominator of \spad{f}
        ++ converted to %.
      "/"       : (MP, MP) -> %
        ++ p1/p2 returns the quotient of p1 and p2 as an element of %.
      coerce    : Q  -> %
        ++ coerce(q) returns q as an element of %.
      coerce    : Polynomial Q -> %
        ++ coerce(p) returns p as an element of %.
      coerce    : Fraction Polynomial Q -> %
        ++ coerce(f) returns f as an element of %.
      univariate: (%, K) -> Fraction SparseUnivariatePolynomial %
        ++ univariate(f, k) returns f viewed as a univariate fraction in k.
      if R has RetractableTo Z then RetractableTo Fraction Z
add
 import BasicOperatorFunctions1(%)

 -- these are needed in Ring only, but need to be declared here
 -- because of compiler bug: if they are declared inside the Ring
 -- case, then they are not visible inside the IntegralDomain case.
 smpIsMult : MP -> Union(Record(coef:Z, var:K),"failed")
 smpret    : MP -> Union(PR, "failed")
 smpeval   : (MP, List K, List %) -> %
 smpsubst  : (MP, List K, List %) -> %
 smpderiv  : (MP, SY) -> %
 smpunq    : (MP, List SY, Boolean) -> %
 kerderiv  : (K, SY)  -> %
 kderiv    : K -> List %
 opderiv   : (OP, N) -> List(List % -> %)
 smp2O     : MP -> O
 bestKernel: List K -> K
 worse?    : (K, K) -> Boolean
 diffArg   : (List %, OP, N) -> List %
 substArg  : (OP, List %, Z, %) -> %
 dispdiff  : List % -> Record(name:O, sub:O, arg:List O, level:N)
 ddiff     : List % -> O
 diffEval  : List % -> %
 dfeval    : (List %, K) -> %
 smprep    : (List SY, List N, List(List % -> %), MP) -> %
 diffdiff  : (List %, SY) -> %
 diffdiff0 : (List %, SY, %, K, List %) -> %
 subs      : (% -> %, K) -> %
 symsub    : (SY, Z) -> SY
 kunq      : (K, List SY, Boolean) -> %
 pushunq   : (List SY, List %) -> List %
```

```
notfound  : (K -> %, List K, K) -> %

equaldiff : (K,K)->Boolean
debugA: (List % ,List %,Boolean) -> Boolean
opdiff := operator("%diff"::SY)$CommonOperators()
opquote := operator("applyQuote"::SY)$CommonOperators

ground? x                == retractIfCan(x)@Union(R,"failed") case R

ground  x                == retract x

coerce(x:SY):%             == kernel(x)@K :: %

retract(x:%):SY            == symbolIfCan(retract(x)@K)::SY

applyQuote(s:SY, x:%)      == applyQuote(s, [x])

applyQuote(s, x, y)        == applyQuote(s, [x, y])

applyQuote(s, x, y, z)     == applyQuote(s, [x, y, z])

applyQuote(s, x, y, z, t)  == applyQuote(s, [x, y, z, t])

applyQuote(s:SY, l:List %) == opquote concat(s::%, l)

belong? op                 == op = opdiff or op = opquote

subs(fn, k) == kernel(operator k,[fn x for x in argument k]$List(%))

operator op ==
  is?(op, "%diff"::SY) => opdiff
  is?(op, "%quote"::SY) => opquote
  error "Unknown operator"

if R has ConvertibleTo InputForm then
  INP==>InputForm
  import MakeUnaryCompiledFunction(%, %, %)
  indiff: List % -> INP
  pint  : List INP-> INP
  differentiand: List % -> %

  differentiand l    == eval(first l, retract(second l)@K, third l)

  pint l  == convert concat(convert("D"::SY)@INP, l)

  indiff l ==
          r2:= convert([convert("::"::SY)@INP,_
                        convert(third l)@INP,_
                        convert("Symbol"::SY)@INP]@List INP)@INP
          pint [convert(differentiand l)@INP, r2]

        eval(f:%, s:SY)            == eval(f, [s])

        eval(f:%, s:OP, g:%, x:SY) == eval(f, [s], [g], x)

        eval(f:%, ls:List OP, lg:List %, x:SY) ==
          eval(f, ls, [compiledFunction(g, x) for g in lg])

        setProperty(opdiff,SPECIALINPUT,_
                     indiff@(List % -> InputForm) pretend None)

      variables x ==
        l := empty()$List(SY)
        for k in tower x repeat
          if ((s := symbolIfCan k) case SY) then l := concat(s::SY, l)
        reverse_! l

      retractIfCan(x:%):Union(SY, "failed") ==
        (k := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
        symbolIfCan(k::K)

      if R has Ring then
        import UserDefinedPartialOrdering(SY)

-- cannot use new()$Symbol because of possible re-instantiation
        gendiff := "%%0"::SY

        characteristic()    == characteristic()$R

        coerce(k:K):%       == k::MP::%

        symsub(sy, i)       == concat(string sy, convert(i)@String)::SY

        numerator x         == numer(x)::%

        eval(x:%, s:SY, n:N, f:% -> %) ==
          eval(x,[s],[n],[(y:List %):% +-> f(first(y))])

        eval(x:%, s:SY, n:N, f:List % -> %) == eval(x, [s], [n], [f])

        eval(x:%, l:List SY, f:List(List % -> %)) == eval(x, l, new(#l, 1), f)
```

```
elt(op:OP, args:List %) ==
  unary? op and ((od? := has?(op, ODD)) or has?(op, EVEN)) and
    leadingCoefficient(numer first args) < 0 =>
      x := op(- first args)
      od? => -x
      x
  elt(op, args)$ExpressionSpace_&(%)

eval(x:%, s:List SY, n:List N, l:List(% -> %)) ==
  eval(x, s, n, [y+-> f(first(y)) for f in l]$List(List % -> %))

-- op(arg)**m ==> func(arg)**(m quo n) * op(arg)**(m rem n)
smprep(lop, lexp, lfunc, p) ==
  (v := mainVariable p) case "failed" => p::%
  symbolIfCan(k := v::K) case SY => p::%
  g := (op := operator k)
     (arg := [eval(a,lop,lexp,lfunc) for a in argument k]$List(%))
  q := map(y+->eval(y::%, lop, lexp, lfunc),
          univariate(p, k))$SparseUnivariatePolynomialFunctions2(MP, %)
  (n := position(name op, lop)) < minIndex lop => q g
  a:%  := 0
  f    := eval((lfunc.n) arg, lop, lexp, lfunc)
  e    := lexp.n
  while q ^= 0 repeat
    m  := degree q
    qr := divide(m, e)
    t1 := f ** (qr.quotient)::N
    t2 := g ** (qr.remainder)::N
    a  := a + leadingCoefficient(q) * t1 * t2
    q  := reductum q
  a

dispdiff l ==
  s := second(l)::O
  t := third(l)::O
  a := argument(k := retract(first l)@K)
  is?(k, opdiff) =>
    rec := dispdiff a
    i   := position(s, rec.arg)
    rec.arg.i := t
    [rec.name,
       hconcat(rec.sub, hconcat(","::SY::O, (i+1-minIndex a)::O)),
                  rec.arg, (zero?(rec.level) => 0; rec.level + 1)]
  i   := position(second l, a)
  m   := [x::O for x in a]$List(O)
  m.i := t
```

```
      [name(operator k)::O, hcconcat(",",::SY::O, (i+1-minIndex a)::O),
                                          m, (empty? rest a => 1; 0)]

    ddiff l ==
      rec := dispdiff l
      opname :=
        zero?(rec.level) => sub(rec.name, rec.sub)
        differentiate(rec.name, rec.level)
      prefix(opname, rec.arg)

    substArg(op, l, i, g) ==
      z := copy l
      z.i := g
      kernel(op, z)


    diffdiff(l, x) ==
      f := kernel(opdiff, l)
      diffdiff0(l, x, f, retract(f)@K, empty())

    diffdiff0(l, x, expr, kd, done) ==
      op  := operator(k := retract(first l)@K)
      gg  := second l
      u   := third l
      arg := argument k
      ans:% := 0
      if (not member?(u,done)) and (ans := differentiate(u,x))^=0 then
        ans := ans * kernel(opdiff,
             [subst(expr, [kd], [kernel(opdiff, [first l, gg, gg])]),
                            gg, u])
      done := concat(gg, done)
      is?(k, opdiff) => ans + diffdiff0(arg, x, expr, k, done)
      for i in minIndex arg .. maxIndex arg for b in arg repeat
        if (not member?(b,done)) and (bp:=differentiate(b,x))^=0 then
          g   := symsub(gendiff, i)::%
          ans := ans + bp * kernel(opdiff, [subst(expr, [kd],
           [kernel(opdiff, [substArg(op, arg, i, g), gg, u])]), g, b])
      ans

    dfeval(l, g) ==
      eval(differentiate(first l, symbolIfCan(g)::SY), g, third l)

    diffEval l ==
      k:K
      g := retract(second l)@K
      ((u := retractIfCan(first l)@Union(K, "failed")) case "failed")
```

```
          or (u case K and symbolIfCan(k := u::K) case SY) => dfeval(l, g)
        op := operator k
        (ud := derivative op) case "failed" =>
             -- possible trouble
             -- make sure it is a dummy var
             dumm:%:=symsub(gendiff,1)::%
             ss:=subst(l.1,l.2=dumm)
             -- output(nl::OutputForm)$OutputPackage
             -- output("fixed"::OutputForm)$OutputPackage
             nl:=[ss,dumm,l.3]
             kernel(opdiff, nl)
        (n := position(second l,argument k)) < minIndex l =>
              dfeval(l,g)
        d := ud::List(List % -> %)
        eval((d.n)(argument k), g, third l)

      diffArg(l, op, i) ==
        n := i - 1 + minIndex l
        z := copy l
        z.n := g := symsub(gendiff, n)::%
        [kernel(op, z), g, l.n]

      opderiv(op, n) ==
--        one? n =>
        (n = 1) =>
          g := symsub(gendiff, n)::%
          [x +-> kernel(opdiff,[kernel(op, g), g, first x])]
        [y +-> kernel(opdiff, diffArg(y, op, i)) for i in 1..n]

      kderiv k ==
        zero?(n := #(args := argument k)) => empty()
        op := operator k
        grad :=
          (u := derivative op) case "failed" => opderiv(op, n)
          u::List(List % -> %)
        if #grad ^= n then grad := opderiv(op, n)
        [g args for g in grad]

    -- SPECIALDIFF contains a map (List %, Symbol) -> %
    -- it is used when the usual chain rule does not apply,
    -- for instance with implicit algebraics.
      kerderiv(k, x) ==
        (v := symbolIfCan(k)) case SY =>
          v::SY = x => 1
          0
        (fn := property(operator k, SPECIALDIFF)) case None =>
```

```
            ((fn::None) pretend ((List %, SY) -> %)) (argument k, x)
          +/[g * differentiate(y,x) for g in kderiv k for y in argument k]

        smpderiv(p, x) ==
          map((s:R):R +-> retract differentiate(s::PR, x), p)::% +
           +/[differentiate(p,k)::% * kerderiv(k, x) for k in variables p]

        coerce(p:PR):% ==
          map(s +-> s::%, r +-> r::%, p)$PolynomialCategoryLifting(
                                          IndexedExponents SY, SY, R, PR, %)

        worse?(k1, k2) ==
          (u := less?(name operator k1,name operator k2)) case "failed" =>
            k1 < k2
          u::Boolean

        bestKernel l ==
          empty? rest l => first l
          a := bestKernel rest l
          worse?(first l, a) => a
          first l

        smp2O p ==
          (r:=retractIfCan(p)@Union(R,"failed")) case R =>r::R::OutputForm
          a :=
            userOrdered?() => bestKernel variables p
            mainVariable(p)::K
          outputForm(map((x:MP):% +-> x::%, univariate(p, a))_
              $SparseUnivariatePolynomialFunctions2(MP, %), a::OutputForm)

        smpsubst(p, lk, lv) ==
          map(x +-> match(lk, lv, x,
              notfound((z:K):%+->subs(s+->subst(s, lk, lv), z), lk, x))_
               $ListToMap(K,%),y+->y::%,p)_
                $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

        smpeval(p, lk, lv) ==
          map(x +-> match(lk, lv, x,
              notfound((z:K):%+->map(s+->eval(s,lk,lv),z),lk,x))_
               $ListToMap(K,%),y+->y::%,p)_
                $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

  -- this is called on k when k is not a member of lk
        notfound(fn, lk, k) ==
          empty? setIntersection(tower(f := k::%), lk) => f
          fn k
```

```
      if R has ConvertibleTo InputForm then
        pushunq(l, arg) ==
           empty? l => [eval a for a in arg]
           [eval(a, l) for a in arg]

        kunq(k, l, givenlist?) ==
          givenlist? and empty? l => k::%
          is?(k, opquote) and
            (member?(s:=retract(first argument k)@SY, l) or empty? l) =>
              interpret(convert(concat(convert(s)@InputForm,
                [convert a for a in pushunq(l, rest argument k)
                   ]@List(InputForm)))@InputForm)$InputFormFunctions1(%)
          (operator k) pushunq(l, argument k)

        smpunq(p, l, givenlist?) ==
          givenlist? and empty? l => p::%
          map(x +-> kunq(x, l, givenlist?), y+->y::%, p)_
            $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

      smpret p ==
        "or"/[symbolIfCan(k) case "failed" for k in variables p] =>
          "failed"
        map(x+->symbolIfCan(x)::SY::PR, y+->y::PR,p)_
          $PolynomialCategoryLifting(IndexedExponents K, K, R, MP, PR)

      isExpt(x:%, op:OP) ==
        (u := isExpt x) case "failed" => "failed"
        is?((u::Record(var:K, exponent:Z)).var, op) => u
        "failed"

      isExpt(x:%, sy:SY) ==
        (u := isExpt x) case "failed" => "failed"
        is?((u::Record(var:K, exponent:Z)).var, sy) => u
        "failed"

      if R has RetractableTo Z then
          smpIsMult p ==
--          (u := mainVariable p) case K and one? degree(q:=univariate(p,u::K))
            (u := mainVariable p) case K and (degree(q:=univariate(p,u::K))=1)
              and zero?(leadingCoefficient reductum q)
                and ((r:=retractIfCan(leadingCoefficient q)@Union(R,"failed"))
                   case R)
                     and (n := retractIfCan(r::R)@Union(Z, "failed")) case Z =>
                       [n::Z, u::K]
            "failed"
```

```
evaluate(opdiff, diffEval)

debugA(a1,a2,t) ==
   -- uncomment for debugging
   -- output(hconcat [a1::OutputForm,_
   --                 a2::OutputForm,t::OutputForm])$OutputPackage
   t

equaldiff(k1,k2) ==
  a1:=argument k1
  a2:=argument k2
  -- check the operator
  res:=operator k1 = operator k2
  not res => debugA(a1,a2,res)
  -- check the evaluation point
  res:= (a1.3 = a2.3)
  not res => debugA(a1,a2,res)
  -- check all the arguments
  res:= (a1.1 = a2.1) and (a1.2 = a2.2)
  res => debugA(a1,a2,res)
  -- check the substituted arguments
  (subst(a1.1,[retract(a1.2)@K],[a2.2]) = a2.1) => debugA(a1,a2,true)
  debugA(a1,a2,false)

setProperty(opdiff,SPECIALEQUAL,
                    equaldiff@((K,K) -> Boolean) pretend None)

setProperty(opdiff, SPECIALDIFF,
                    diffdiff@((List %, SY) -> %) pretend None)

setProperty(opdiff, SPECIALDISP,
                        ddiff@(List % -> OutputForm) pretend None)

if not(R has IntegralDomain) then
  mainKernel x         == mainVariable numer x

  kernels x            == variables numer x

  retract(x:%):R       == retract numer x

  retract(x:%):PR      == smpret(numer x)::PR

  retractIfCan(x:%):Union(R,  "failed") == retract numer x

  retractIfCan(x:%):Union(PR, "failed") == smpret numer x
```

```
    eval(x:%, lk:List K, lv:List %)  == smpeval(numer x, lk, lv)

    subst(x:%, lk:List K, lv:List %) == smpsubst(numer x, lk, lv)

    differentiate(x:%, s:SY)         == smpderiv(numer x, s)

    coerce(x:%):OutputForm           == smp2O numer x

    if R has ConvertibleTo InputForm then
      eval(f:%, l:List SY) == smpunq(numer f, l, true)

      eval f               == smpunq(numer f, empty(), false)

    eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
      smprep(s, n, f, numer x)

    isPlus x ==
      (u := isPlus numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isTimes x ==
      (u := isTimes numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isExpt x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var, r.exponent::Z]

    isPower x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var::%, r.exponent::Z]

    if R has ConvertibleTo Pattern Z then
      convert(x:%):Pattern(Z) == convert numer x

    if R has ConvertibleTo Pattern Float then
      convert(x:%):Pattern(Float) == convert numer x

    if R has RetractableTo Z then
      isMult x == smpIsMult numer x

if R has CommutativeRing then
  r:R * x:% == r::MP::% * x
```

```
    if R has IntegralDomain then
      par   : % -> %

      mainKernel x                    == mainVariable(x)$QF

      kernels x                       == variables(x)$QF

      univariate(x:%, k:K)            == univariate(x, k)$QF

      isPlus x                        == isPlus(x)$QF

      isTimes x                       == isTimes(x)$QF

      isExpt x                        == isExpt(x)$QF

      isPower x                       == isPower(x)$QF

      denominator x                   == denom(x)::%

      coerce(q:Q):%                   == (numer q)::MP / (denom q)::MP

      coerce(q:Fraction PR):%         == (numer q)::% / (denom q)::%

      coerce(q:Fraction Polynomial Q) == (numer q)::% / (denom q)::%

      retract(x:%):PR                == retract(retract(x)@Fraction(PR))

      retract(x:%):Fraction(PR) == smpret(numer x)::PR / smpret(denom x)::PR

      retract(x:%):R == (retract(numer x)@R exquo retract(denom x)@R)::R

      coerce(x:%):OutputForm ==
--        one?(denom x) => smp2O numer x
        ((denom x) = 1) => smp2O numer x
        smp2O(numer x) / smp2O(denom x)

      retractIfCan(x:%):Union(R, "failed") ==
        (n := retractIfCan(numer x)@Union(R, "failed")) case "failed" or
          (d := retractIfCan(denom x)@Union(R, "failed")) case "failed"
            or (r := n::R exquo d::R) case "failed" => "failed"
        r::R

      eval(f:%, l:List SY) ==
        smpunq(numer f, l, true) / smpunq(denom f, l, true)
```

```
if R has ConvertibleTo InputForm then
  eval f ==
    smpunq(numer f, empty(), false) / smpunq(denom f, empty(), false)

  eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
    smprep(s, n, f, numer x) / smprep(s, n, f, denom x)

differentiate(f:%, x:SY) ==
  (smpderiv(numer f, x) * denom(f)::% -
    numer(f)::% * smpderiv(denom f, x))
      / (denom(f)::% ** 2)

eval(x:%, lk:List K, lv:List %) ==
  smpeval(numer x, lk, lv) / smpeval(denom x, lk, lv)

subst(x:%, lk:List K, lv:List %) ==
  smpsubst(numer x, lk, lv) / smpsubst(denom x, lk, lv)

par x ==
  (r := retractIfCan(x)@Union(R, "failed")) case R => x
  paren x

convert(x:Factored %):% ==
  par(unit x) * */[par(f.factor) ** f.exponent for f in factors x]

retractIfCan(x:%):Union(PR, "failed") ==
  (u := retractIfCan(x)@Union(Fraction PR,"failed")) case "failed"
    => "failed"
  retractIfCan(u::Fraction(PR))

retractIfCan(x:%):Union(Fraction PR, "failed") ==
  (n := smpret numer x) case "failed" => "failed"
  (d := smpret denom x) case "failed" => "failed"
  n::PR / d::PR

coerce(p:Polynomial Q):% ==
  map(x+->x::%, y+->y::%,p)_
   $PolynomialCategoryLifting(IndexedExponents SY, SY,
                                               Q, Polynomial Q, %)

if R has RetractableTo Z then
  coerce(x:Fraction Z):% == numer(x)::MP / denom(x)::MP

  isMult x ==
     (u := smpIsMult numer x) case "failed"
        or (v := retractIfCan(denom x)@Union(R, "failed")) case "failed"
```

```
                  or (w := retractIfCan(v::R)@Union(Z, "failed")) case "failed"
                      => "failed"
            r := u::Record(coef:Z, var:K)
            (q := r.coef exquo w::Z) case "failed" => "failed"
            [q::Z, r.var]

      if R has ConvertibleTo Pattern Z then
        convert(x:%):Pattern(Z) == convert(numer x) / convert(denom x)

      if R has ConvertibleTo Pattern Float then
        convert(x:%):Pattern(Float) ==
          convert(numer x) / convert(denom x)
```

⟨*FS.dotabb*⟩≡

```
"FS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
"FS" -> "ES"
"FS" -> "FPATMAB"
"FS" -> "FRETRCT"
"FS" -> "PATAB"
"FS" -> "RETRACT"
"FS" -> "KONVERT"
"FS" -> "MONOID"
"FS" -> "GROUP"
"FS" -> "ABELMON"
"FS" -> "ABELGRP"
"FS" -> "PDRING"
"FS" -> "FLINEXP"
"FS" -> "CHARNZ"
"FS" -> "INTDOM"
"FS" -> "FIELD"
```

⟨*FS.dotfull*⟩≡

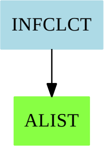
```
"FunctionSpace(a:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
"FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Symbol)"
"FunctionSpace(a:OrderedSet)" -> "Patternable(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "FullyPatternMatchable(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "FullyRetractableTo(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "ConvertibleTo(InputForm)"
"FunctionSpace(a:OrderedSet)" -> "Monoid()"
"FunctionSpace(a:OrderedSet)" -> "Group()"
"FunctionSpace(a:OrderedSet)" -> "AbelianMonoid()"
"FunctionSpace(a:OrderedSet)" -> "AbelianGroup()"
"FunctionSpace(a:OrderedSet)" -> "PartialDifferentialRing(Symbol)"
"FunctionSpace(a:OrderedSet)" -> "FullyLinearlyExplicitRingOver(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "CharacteristicNonZero()"
"FunctionSpace(a:OrderedSet)" -> "IntegralDomain()"
"FunctionSpace(a:OrderedSet)" -> "Field()"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Integer)"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Fraction(Integer))"
```

⟨*FS.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ES..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."
"FunctionSpace(a:OrderedSet)" -> "KONVERT..."
"FunctionSpace(a:OrderedSet)" -> "MONOID..."
"FunctionSpace(a:OrderedSet)" -> "GROUP..."
"FunctionSpace(a:OrderedSet)" -> "ABELMON..."
"FunctionSpace(a:OrderedSet)" -> "ABELGRP..."
"FunctionSpace(a:OrderedSet)" -> "PDRING..."
"FunctionSpace(a:OrderedSet)" -> "FLINEXP..."
"FunctionSpace(a:OrderedSet)" -> "CHARNZ..."
"FunctionSpace(a:OrderedSet)" -> "INTDOM..."
"FunctionSpace(a:OrderedSet)" -> "FIELD..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."

"ES..." [color=lightblue];
"EVALABLE..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"ORDSET..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"MONOID..." [color=lightblue];
"GROUP..." [color=lightblue];
"ABELMON..." [color=lightblue];
"ABELGRP..." [color=lightblue];
"PDRING..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"INTDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"RETRACT..." [color=lightblue];
}
```

17.0.196 InfinitlyClosePointCategory (INFCLCT)



⟨*InfinitlyClosePointCategory.input*⟩≡
```
)set break resume
)sys rm -f InfinitlyClosePointCategory.output
)spool InfinitlyClosePointCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfinitlyClosePointCategory
--I InfinitlyClosePointCategory(K: Field,
--I symb: List Symbol,
--I PolyRing: PolynomialCategory(t#1,t#4,OrderedVariableList t#2),
--I E: DirectProductCategory(# t#2,NonNegativeInteger),
--I ProjPt: ProjectiveSpaceCategory t#1,
--I PCS: LocalPowerSeriesCategory t#1,
--I Plc: PlacesCategory(t#1,t#6),
--I DIVISOR: DivisorCategory t#7,
--I BLMET: BlowUpMethodCategory) is a category constructor
--I Abbreviation for InfinitlyClosePointCategory is INFCLCT
--I This constructor is exposed in this frame.
--I Issue )edit bookvol10.2.pamphlet to see algebra source code for INFCLCT
--I
--I------------------------------- Operations --------------------------------
--I ?=? : (%,%) -> Boolean                  actualExtensionV : % -> K
--I chartV : % -> BLMET                     coerce : % -> OutputForm
--I create : (ProjPt,PolyRing) -> %         degree : % -> PositiveInteger
--I excpDivV : % -> DIVISOR                 hash : % -> SingleInteger
--I latex : % -> String                     localParamV : % -> List PCS
--I localPointV : % -> AffinePlane K        multV : % -> NonNegativeInteger
--I pointV : % -> ProjPt                    setchart! : (%,BLMET) -> BLMET
--I setpoint! : (%,ProjPt) -> ProjPt        symbNameV : % -> Symbol
--I ?~=? : (%,%) -> Boolean
--I create : (ProjPt,
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),
--I  AffinePlane K,
--I  NonNegativeInteger,
--I  BLMET,
--I  NonNegativeInteger,
--I  DIVISOR,
--I  K,
--I  Symbol) -> %
--I curveV : % ->
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--I setcurve! :
--I  (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) ->
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--I setexcpDiv! : (%,DIVISOR) -> DIVISOR
--I setlocalParam! : (%,List PCS) -> List PCS
--I setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
--I setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--I setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--I setsymbName! : (%,Symbol) -> Symbol
--I subMultV : % -> NonNegativeInteger
--I
--E 1

)spool
)lisp (bye)
```

⟨*InfinitlyClosePointCategory.help*⟩≡

```
====================================================================
InfinitlyClosePointCategory examples
====================================================================

See Also:
o )show InfinitlyClosePointCategory
```

⇐ “SetCategoryWithDegree” (SETCATD) 4.0.62 on page 274

Exports:

?=?
?~=?
actualExtensionV
chartV
coerce
create
curveV
degree
excpDivV
hash
latex
localParamV
localPointV
multV
pointV
setchart!
setcurve!
setexcpDiv!
setlocalParam!
setlocalPoint!
setmult!
setpoint!
setsubmult!
setsymbName!
subMultV
symbNameV

These are directly exported but not implemented:

```
actualExtensionV : % -> K
chartV : % -> BLMET
create :
 (ProjPt,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),
  AffinePlane K,NonNegativeInteger,BLMET,NonNegativeInteger,DIVISOR,K,Symbol)
    -> %
create : (ProjPt,PolyRing) -> %
curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
excpDivV : % -> DIVISOR
localParamV : % -> List PCS
localPointV : % -> AffinePlane K
multV : % -> NonNegativeInteger
pointV : % -> ProjPt
setchart! : (%,BLMET) -> BLMET
setcurve! :
  (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) ->
    DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
setexcpDiv! : (%,DIVISOR) -> DIVISOR
setlocalParam! : (%,List PCS) -> List PCS
setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
setpoint! : (%,ProjPt) -> ProjPt
setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
setsymbName! : (%,Symbol) -> Symbol
subMultV : % -> NonNegativeInteger
symbNameV : % -> Symbol
```

These exports come from (p274) SetCategoryWithDegree:

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

⟨*category INFCLCT InfinitlyClosePointCategory*⟩≡

```
)abbrev category INFCLCT InfinitlyClosePointCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description:
++ This category is part of the PAFF package
InfinitlyClosePointCategory(_
     K        :Field,_
     symb     :List(Symbol),_
     PolyRing :PolynomialCategory(K,E,OrderedVariableList(symb)),_
     E        :DirectProductCategory(#symb,NonNegativeInteger),_
     ProjPt   :ProjectiveSpaceCategory(K),_
     PCS      :LocalPowerSeriesCategory(K),_
     Plc      :PlacesCategory(K,PCS),_
     DIVISOR  :DivisorCategory(Plc),_
     BLMET    :BlowUpMethodCategory_
       ):Category == Exports where

 bls      ==> ['X,'Y]
 BlUpRing ==> DistributedMultivariatePolynomial(bls , K)
 AFP      ==> AffinePlane(K)

 Exports ==> SetCategoryWithDegree with

    create:  (ProjPt ,  BlUpRing, AFP , NonNegativeInteger,BLMET, _
              NonNegativeInteger,  DIVISOR,K,Symbol) -> %
      ++ create an infinitly close point

    create:  (ProjPt,PolyRing) -> %

    setpoint_!:  (%,ProjPt) -> ProjPt

    setcurve_!:  (%,BlUpRing) -> BlUpRing

    setlocalPoint_!:   (%,AFP) -> AFP

    setsubmult_! : (%, NonNegativeInteger) -> NonNegativeInteger

    setmult_!:    (%,NonNegativeInteger) -> NonNegativeInteger

    setchart_!:   (%,BLMET) -> BLMET -- CHH

    setexcpDiv_!: (%,DIVISOR) -> DIVISOR

    setlocalParam_!: (%,List PCS) -> List(PCS)
    setsymbName_!: (%,Symbol) -> Symbol

    subMultV: % -> NonNegativeInteger

    localParamV: % -> List PCS

    symbNameV: % -> Symbol

    pointV:  % -> ProjPt
      ++ pointV returns the infinitly close point.

    curveV:  % -> BlUpRing
      ++ curveV(p) returns the defining polynomial of the strict transform
      ++ on which lies the corresponding infinitly close point.

    localPointV: % -> AFP
      ++ localPointV returns the coordinates of the local infinitly
      ++ close point

    multV:       % -> NonNegativeInteger
      ++ multV returns the multiplicity of the infinitly close point.

    chartV:      % -> BLMET -- CHH
      ++ chartV is the chart of the infinitly close point. The first integer
      ++ correspond to variable defining the exceptional line, the last one
      ++ the affine  neighboorhood and the second one is the
      ++ remaining integer. For example [1,2,3] means that
      ++ Z=1, X=X and Y=XY. [2,3,1] means that X=1, Y=Y and Z=YZ.

    excpDivV:    % -> DIVISOR
      ++ excpDivV returns the exceptional divisor of the infinitly close point.

    actualExtensionV: % -> K
```

⟨*INFCLCT.dotabb*⟩≡

```
"INFCLCT" [color=lightblue,href="bookvol10.2.pdf#nameddest=INFCLCT"];
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"INFCLCT" -> "ALIST"
```

⟨*INFCLCT.dotfull*⟩≡

```
"InfinitlyClosePointCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INFCLCT"];
"InfinitlyClosePointCategory" -> "AssocationList(SetCategory,SetCategory)"
```

⟨*INFCLCT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

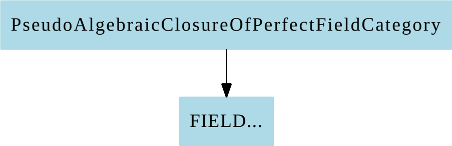
"InfinitlyClosePointCategory" [color=lightblue];
"InfinitlyClosePointCategory" -> "AssocationList(SetCategory,SetCategory)"

"AssocationList(SetCategory,SetCategory)" -> "AssocationList()"

"AssocationList()" [color=lightblue];

}
```

17.0.197 PseudoAlgebraicClosureOfPerfectFieldCategory (PACPERC)



⟨*PseudoAlgebraicClosureOfPerfectFieldCategory.input*⟩≡

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfPerfectFieldCategory.output
)spool PseudoAlgebraicClosureOfPerfectFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfPerfectFieldCategory
--R
--R PseudoAlgebraicClosureOfPerfectFieldCategory  is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfPerfectFieldCategory is PACPERC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACPERC
--R
--R------------------------------- Operations ---------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              conjugate : % -> %
--R extDegree : % -> PositiveInteger      factor : % -> Factored %
--R fullOutput : % -> OutputForm          gcd : List % -> %
--R gcd : (%,%) -> %                      ground? : % -> Boolean
--R hash : % -> SingleInteger             inv : % -> %
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      maxTower : List % -> %
--R one? : % -> Boolean                   previousTower : % -> %
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
```

```
--R sample : () -> %                       setTower! : % -> Void
--R sizeLess? : (%,%) -> Boolean           squareFree : % -> Factored %
--R squareFreePart : % -> %                unit? : % -> Boolean
--R unitCanonical : % -> %                 vectorise : (%,%) -> Vector %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R lift : (%,%) -> SparseUnivariatePolynomial %
--R lift : % -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PseudoAlgebraicClosureOfPerfectFieldCategory.help*⟩≡

```
====================================================================
PseudoAlgebraicClosureOfPerfectFieldCategory examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfPerfectFieldCategory
```

⇐ "Field" (FIELD) 16.0.185 on page 1347

Exports:

0
1
associates?
characteristic
coerce
conjugate
definingPolynomial
distinguishedRootsOf
divide
euclideanSize
expressIdealMember
exquo
extDegree
extendedEuclidean
factor
fullOutput
gcd
gcdPolynomial
ground?
hash
inv
latex
lcm
lift
maxTower
multiEuclidean
newElement
one?
previousTower
prime?
principalIdeal
?quo?
recip
reduce
?rem?
sample
setTower!
sizeLess?
squareFree
squareFreePart
subtractIfCan
unit?
unitCanonical
unitNormal
vectorise
zero?
?*?
?**?
?+?
?-?
-?
?/?
?=?
?^?
?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conjugate: % -> %
definingPolynomial: () -> SUP(%)
definingPolynomial: % -> SUP %
distinguishedRootsOf: (SparseUnivariatePolynomial %,%) -> List %
extDegree: % -> PI
fullOutput: % -> OutputForm
ground_? : % -> Boolean
```

```
 lift: % -> SUP(%)
 lift: (%,%) -> SUP(%)
 maxTower: List % -> %
 newElement: (SUP(%), %, Symbol) -> %
 newElement: (SUP(%), Symbol) -> %
 previousTower: % -> %
 reduce: SUP(%) -> %
 setTower_!: % -> Void
 vectorise: (%,%) -> Vector(%)
```

These exports come from (p1347) Field():

```
associates? : (%,%) -> Boolean
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
inv : % -> %
prime? : % -> Boolean
squareFree : % -> Factored %
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
```

These exports come from (p1307) EuclideanDomain():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                   SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1294) UniqueFactorizationDomain():

```
squareFreePart : % -> %
```

These exports come from (p1116) DivisionRing():

```
coerce : Fraction Integer -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
```

⟨*category PACPERC PseudoAlgebraicClosureOfPerfectFieldCategory*⟩≡

```
)abbrev category PACPERC PseudoAlgebraicClosureOfPerfectFieldCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description: This category exports the function for domains
++ which implement dynamic extension using the simple notion of tower
++ extensions. ++ A tower extension T  of the ground
++ field K is any sequence of field extension
++ (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1
++ and defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1 (or i=1,2,...,n2).
++ Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfPerfectFieldCategory() : Category == PUB where

 INT      ==> Integer
 K        ==> Fraction Integer
 NNI      ==> NonNegativeInteger
 SUP      ==> SparseUnivariatePolynomial
 BOOLEAN  ==> Boolean
 PI       ==> PositiveInteger
 FFFACTSE ==> FiniteFieldFactorizationWithSizeParseBySideEffect

 PUB ==> Field with

  definingPolynomial: () -> SUP(%)
  definingPolynomial: % -> SUP %

  lift: % -> SUP(%)
  lift: (%,%) -> SUP(%)
  reduce: SUP(%) -> %

  distinguishedRootsOf: (SparseUnivariatePolynomial %,%) -> List %
    ++ distinguishedRootsOf(p,a) returns a (distinguised) root for each
    ++ irreducible factor of the polynomial p (factored over the field defined
    ++ by the element a).

  ground_? : % -> Boolean
  maxTower: List % -> %
    ++ maxTower(l) returns the tower in the list having the maximal extension
    ++ degree over the ground field. It has no meaning if the towers are
    ++ not related.
  extDegree: % -> PI
    ++ extDegree(a) returns the extension degree of the extension tower
    ++ over which the element is defined.
  previousTower: % -> %
    ++ previousTower(a) returns the previous tower extension over which
    ++ the element a is defined.

  vectorise: (%,%) -> Vector(%)

  conjugate: % -> %
  newElement: (SUP(%), %, Symbol) -> %
  newElement: (SUP(%), Symbol) -> %
  setTower_!: % -> Void
  fullOutput: % -> OutputForm
```

⟨*PACPERC.dotabb*⟩≡
```
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"PACPERC" -> "FIELD"
```

⟨*PACPERC.dotfull*⟩≡
```
"PseudoAlgebraicClosureOfPerfectFieldCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"PseudoAlgebraicClosureOfPerfectFieldCategory" -> "Field()"
```

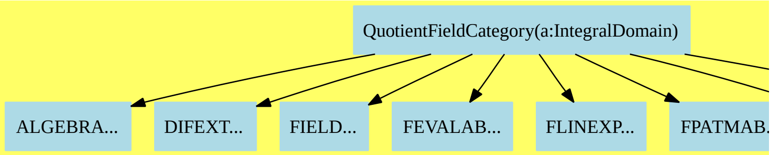
⟨*PACPERC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfPerfectFieldCategory" [color=lightblue];
"PseudoAlgebraicClosureOfPerfectFieldCategory" -> "FIELD..."

"FIELD..." [color=lightblue];

}
```

17.0.198 QuotientFieldCategory (QFCAT)



⟨*QuotientFieldCategory.input*⟩≡

```
)set break resume
)sys rm -f QuotientFieldCategory.output
)spool QuotientFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuotientFieldCategory
--R QuotientFieldCategory S: IntegralDomain  is a category constructor
--R Abbreviation for QuotientFieldCategory is QFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (S,S) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : (%,(S -> S)) -> %
--R D : % -> % if S has DIFRING           1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R ceiling : % -> S if S has INS         coerce : S -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R denom : % -> S                        denominator : % -> %
--R factor : % -> Factored %              floor : % -> S if S has INS
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             init : () -> % if S has STEP
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R map : ((S -> S),%) -> %               numer : % -> S
```

```
--R numerator : % -> %                    one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R random : () -> % if S has INS         recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> S
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R D : (%,(S -> S),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL
--R D : (%,Symbol) -> % if S has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if S has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if S has OINTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if S has CHARNZ or and(has($,CharacteristicNonZero),
--R coerce : Symbol -> % if S has RETRACT SYMBOL
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),ha
--R convert : % -> DoubleFloat if S has REAL
--R convert : % -> Float if S has REAL
--R convert : % -> InputForm if S has KONVERT INFORM
--R convert : % -> Pattern Float if S has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if S has KONVERT PATTERN INT
--R differentiate : (%,(S -> S)) -> %
--R differentiate : (%,(S -> S),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING
--R differentiate : % -> % if S has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,S) -> % if S has ELTAB(S,S)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,S) -> % if S has IEVALAB(SYMBOL,S)
--R eval : (%,List Symbol,List S) -> % if S has IEVALAB(SYMBOL,S)
--R eval : (%,List Equation S) -> % if S has EVALAB S
--R eval : (%,Equation S) -> % if S has EVALAB S
```

```
--R eval : (%,S,S) -> % if S has EVALAB S
--R eval : (%,List S,List S) -> % if S has EVALAB S
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R fractionPart : % -> % if S has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if S has OINTDOM
--R nextItem : % -> Union(%,"failed") if S has STEP
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchR
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMa
--R positive? : % -> Boolean if S has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix S
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT
--R retract : % -> Integer if S has RETRACT INT
--R retract : % -> Fraction Integer if S has RETRACT INT
--R retract : % -> Symbol if S has RETRACT SYMBOL
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed") if S has RETRACT SYMBOL
--R retractIfCan : % -> Union(S,"failed")
--R sign : % -> Integer if S has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUniv
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivari
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> S if S has EUCDOM
--R
--E 1

)spool
)lisp (bye)
```

⟨*QuotientFieldCategory.help*⟩≡

```
====================================================================
QuotientFieldCategory examples
====================================================================

See Also:
o )show QuotientFieldCategory
```

See:

⇐ “Algebra” (ALGEBRA) 11.0.152 on page 1045
⇐ “CharacteristicNonZero” (CHARNZ) 10.0.135 on page 922
⇐ “CharacteristicZero” (CHARZ) 10.0.136 on page 927
⇐ “ConvertibleTo” (KONVERT) 2.0.8 on page 25
⇐ “DifferentialExtension” (DIFEXT) 11.0.153 on page 1053
⇐ “EuclideanDomain” (EUCDOM) 15.0.181 on page 1307
⇐ “Field” (FIELD) 16.0.185 on page 1347
⇐ “FullyEvalableOver” (FEVALAB) 4.0.44 on page 178
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.154 on page 1060
⇐ “FullyPatternMatchable” (FPATMAB) 3.0.28 on page 105
⇐ “OrderedIntegralDomain” (OINTDOM) 13.0.174 on page 1253
⇐ “OrderedSet” (ORDSET) 4.0.56 on page 242
⇐ “Patternable” (PATAB) 2.0.15 on page 53
⇐ “PolynomialFactorizationExplicit” (PFECAT) 15.0.183 on page 1325
⇐ “RealConstant” (REAL) 3.0.32 on page 122
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “StepThrough” (STEP) 4.0.64 on page 283

Exports:

0
1
abs
associates?
ceiling
characteristic
charthRoot
coerce
conditionP
convert
D
denom
denominator
differentiate
divide
euclideanSize
eval
expressIdealMember
exquo
extendedEuclidean
factor
factorPolynomial
factorSquareFreePolynomial
floor
fractionPart
gcd
gcdPolynomial
hash
init
inv
latex
lcm
map
max
min
multiEuclidean
negative?
nextItem
numer
numerator
one?
patternMatch
positive?
prime?
principalIdeal
random
recip
reducedSystem
retract
retractIfCan
sample
sign
sizeLess?
solveLinearPolynomialEquation
squareFree
squareFreePart
squareFreePolynomial
subtractIfCan
unit?
unitNormal
unitCanonical
wholePart
zero?
?.?
?*?
?**?
?+?
?-?
-?
?/?
?=?
?ˆ?
?quo?
?rem?
?˜=?
?<?
?<=?
?>?
?>=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

- **nil**

These are directly exported but not implemented:

```
ceiling : % -> S if S has INS
denom : % -> S
floor : % -> S if S has INS
numer : % -> S
wholePart : % -> S if S has EUCDOM
?/? : (S,S) -> %
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
coerce : Symbol -> % if S has RETRACT SYMBOL
coerce : Fraction Integer -> %
convert : % -> InputForm if S has KONVERT INFORM
convert : % -> DoubleFloat if S has REAL
convert : % -> Float if S has REAL
convert : % -> Pattern Integer if S has KONVERT PATTERN INT
convert : % -> Pattern Float if S has KONVERT PATTERN FLOAT
denominator : % -> %
differentiate : (%,(S -> S)) -> %
fractionPart : % -> % if S has EUCDOM
init : () -> % if S has STEP
map : ((S -> S),%) -> %
nextItem : % -> Union(%,"failed") if S has STEP
numerator : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if S has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if S has PATMAB INT
random : () -> % if S has INS
reducedSystem : Matrix % -> Matrix S
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S)
retract : % -> Symbol if S has RETRACT SYMBOL
retract : % -> Integer if S has RETRACT INT
retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT
retractIfCan : % -> Union(Symbol,"failed") if S has RETRACT SYMBOL
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p1347) Field():

```
0 : () -> %
1 : () -> %
```

```
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1045) Algebra(S:IntegralDomain):

```
coerce : S -> %
?*? : (%,S) -> %
?*? : (S,%) -> %
```

These exports come from (p62) RetractableTo(S:IntegralDomain):

```
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

These exports come from (p178) FullyEvalableOver(S:IntegralDomain):

```
?.? : (%,S) -> % if S has ELTAB(S,S)
eval : (%,Equation S) -> % if S has EVALAB S
eval : (%,List Symbol,List S) -> % if S has IEVALAB(SYMBOL,S)
eval : (%,List Equation S) -> % if S has EVALAB S
eval : (%,S,S) -> % if S has EVALAB S
eval : (%,List S,List S) -> % if S has EVALAB S
eval : (%,Symbol,S) -> % if S has IEVALAB(SYMBOL,S)
```

These exports come from (p1053) DifferentialExtension(S:IntegralDomain):

```
D : (%,(S -> S)) -> %
D : (%,(S -> S),NonNegativeInteger) -> %
D : % -> % if S has DIFRING
D : (%,NonNegativeInteger) -> % if S has DIFRING
D : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,List Symbol) -> % if S has PDRING SYMBOL
D : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if S has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if S has DIFRING
differentiate : % -> % if S has DIFRING
differentiate : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,(S -> S),NonNegativeInteger) -> %
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(S:IntegralDomain):

```
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if S has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT
```

These exports come from (p62) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer if S has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if S has RETRACT INT
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p1253) OrderedIntegralDomain():

```
abs : % -> % if S has OINTDOM
negative? : % -> Boolean if S has OINTDOM
positive? : % -> Boolean if S has OINTDOM
sign : % -> Integer if S has OINTDOM
```

These exports come from (p922) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
  if S has CHARNZ
  or and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
```

These exports come from (p1325) PolynomialFactorizationExplicit():

```
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
solveLinearPolynomialEquation :
   (List SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
      Union(List SparseUnivariatePolynomial %,"failed")
       if S has PFECAT
 squareFreePolynomial :
   SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
       if S has PFECAT
```

⟨*category QFCAT QuotientFieldCategory*⟩≡

```
)abbrev category QFCAT QuotientFieldCategory
++ Author:
++ Date Created:
++ Date Last Updated: 5th March 1996
++ Basic Functions: + - * / numer denom
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ QuotientField(S) is the category of fractions of an Integral Domain S.

QuotientFieldCategory(S: IntegralDomain): Category ==
  Join(Field, Algebra S, RetractableTo S, FullyEvalableOver S,
         DifferentialExtension S, FullyLinearlyExplicitRingOver S,
           Patternable S, FullyPatternMatchable S) with
    _/     : (S, S) -> %
       ++ d1 / d2 returns the fraction d1 divided by d2.
    numer  : % -> S
       ++ numer(x) returns the numerator of the fraction x.
    denom  : % -> S
       ++ denom(x) returns the denominator of the fraction x.
    numerator : % -> %
       ++ numerator(x) is the numerator of the fraction x converted to %.
    denominator : % -> %
       ++ denominator(x) is the denominator of the fraction x converted to %.
    if S has StepThrough then StepThrough
    if S has RetractableTo Integer then
             RetractableTo Integer
             RetractableTo Fraction Integer
    if S has OrderedSet then OrderedSet
    if S has OrderedIntegralDomain then OrderedIntegralDomain
    if S has RealConstant then RealConstant
    if S has ConvertibleTo InputForm then ConvertibleTo InputForm
    if S has CharacteristicZero then CharacteristicZero
    if S has CharacteristicNonZero then CharacteristicNonZero
```

```
   if S has RetractableTo Symbol then RetractableTo Symbol
   if S has EuclideanDomain then
     wholePart: % -> S
       ++ wholePart(x) returns the whole part of the fraction x
       ++ i.e. the truncated quotient of the numerator by the denominator.
     fractionPart: % -> %
       ++ fractionPart(x) returns the fractional part of x.
       ++ x = wholePart(x) + fractionPart(x)
   if S has IntegerNumberSystem then
     random: () -> %
       ++ random() returns a random fraction.
     ceiling : % -> S
       ++ ceiling(x) returns the smallest integral element above x.
     floor: % -> S
       ++ floor(x) returns the largest integral element below x.
   if S has PolynomialFactorizationExplicit then
     PolynomialFactorizationExplicit

add
   import MatrixCommonDenominator(S, %)

   numerator(x) == numer(x)::%

   denominator(x) == denom(x) ::%

   if S has StepThrough then
      init() == init()$S / 1$S

      nextItem(n) ==
        m:= nextItem(numer(n))
        m case "failed" =>
          error "We seem to have a Fraction of a finite object"
        m / 1

   map(fn, x)                         == (fn numer x) / (fn denom x)

   reducedSystem(m:Matrix %):Matrix S == clearDenominator m

   characteristic()                   == characteristic()$S

   differentiate(x:%, deriv:S -> S) ==
       n := numer x
       d := denom x
       (deriv n * d - n * deriv d) / (d**2)

   if S has ConvertibleTo InputForm then
```

```
    convert(x:%):InputForm == (convert numer x) / (convert denom x)

  if S has RealConstant then
    convert(x:%):Float == (convert numer x) / (convert denom x)

    convert(x:%):DoubleFloat == (convert numer x) / (convert denom x)

  -- Note that being a Join(OrderedSet,IntegralDomain) is not the same
  -- as being an OrderedIntegralDomain.
  if S has OrderedIntegralDomain then
     if S has canonicalUnitNormal then
         x:% < y:% ==
           (numer x  * denom y) < (numer y * denom x)
       else
         x:% < y:% ==
           if denom(x) < 0 then (x,y):=(y,x)
           if denom(y) < 0 then (x,y):=(y,x)
           (numer x  * denom y) < (numer y * denom x)
  else if S has OrderedSet then
     x:% < y:% ==
       (numer x  * denom y) < (numer y * denom x)

  if (S has EuclideanDomain) then
    fractionPart x == x - (wholePart(x)::%)

  if S has RetractableTo Symbol then
    coerce(s:Symbol):%  == s::S::%

    retract(x:%):Symbol == retract(retract(x)@S)

    retractIfCan(x:%):Union(Symbol, "failed") ==
      (r := retractIfCan(x)@Union(S,"failed")) case "failed" =>"failed"
      retractIfCan(r::S)

  if (S has ConvertibleTo Pattern Integer) then
    convert(x:%):Pattern(Integer)==(convert numer x)/(convert denom x)

    if (S has PatternMatchable Integer) then
      patternMatch(x:%, p:Pattern Integer,
       l:PatternMatchResult(Integer, %)) ==
         patternMatch(x, p,
                  l)$PatternMatchQuotientFieldCategory(Integer, S, %)

  if (S has ConvertibleTo Pattern Float) then
    convert(x:%):Pattern(Float) == (convert numer x)/(convert denom x)
```

```
        if (S has PatternMatchable Float) then
          patternMatch(x:%, p:Pattern Float,
           l:PatternMatchResult(Float, %)) ==
             patternMatch(x, p,
                        l)$PatternMatchQuotientFieldCategory(Float, S, %)

      if S has RetractableTo Integer then
        coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

        if not(S is Integer) then
          retract(x:%):Integer == retract(retract(x)@S)

          retractIfCan(x:%):Union(Integer, "failed") ==
            (u := retractIfCan(x)@Union(S, "failed")) case "failed" =>
              "failed"
            retractIfCan(u::S)

      if S has IntegerNumberSystem then
        random():% ==
          while zero?(d:=random()$S) repeat d
          random()$S / d

      reducedSystem(m:Matrix %, v:Vector %):
        Record(mat:Matrix S, vec:Vector S) ==
          n := reducedSystem(horizConcat(v::Matrix(%), m))@Matrix(S)
          [subMatrix(n, minRowIndex n, maxRowIndex n, 1 + minColIndex n,
                                maxColIndex n), column(n, minColIndex n)]
```

⟨*QFCAT.dotabb*⟩≡

```
  "QFCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
  "QFCAT" -> "ALGEBRA"
  "QFCAT" -> "DIFEXT"
  "QFCAT" -> "FIELD"
  "QFCAT" -> "FEVALAB"
  "QFCAT" -> "FLINEXP"
  "QFCAT" -> "FPATMAB"
  "QFCAT" -> "PATAB"
  "QFCAT" -> "RETRACT"
```

⟨*QFCAT.dotfull*⟩≡

```
"QuotientFieldCategory(a:IntegralDomain)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
"QuotientFieldCategory(a:IntegralDomain)" -> "Field()"
"QuotientFieldCategory(a:IntegralDomain)" -> "Algebra(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" -> "RetractableTo(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyEvalableOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "DifferentialExtension(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyLinearlyExplicitRingOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "Patternable(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyPatternMatchable(IntegralDomain)"
```

⟨*QFCAT.dotpic*⟩≡

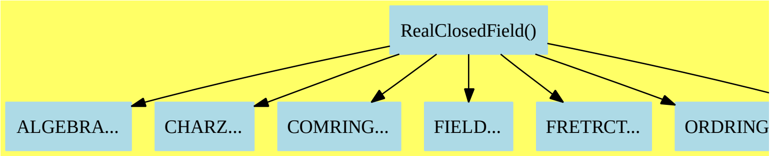
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QuotientFieldCategory(a:IntegralDomain)" [color=lightblue];
"QuotientFieldCategory(a:IntegralDomain)" -> "ALGEBRA..."
"QuotientFieldCategory(a:IntegralDomain)" -> "DIFEXT..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FIELD..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FEVALAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FLINEXP..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FPATMAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "PATAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "RETRACT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FIELD..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];

}
```

17.0.199 RealClosedField (RCFIELD)



⟨*RealClosedField.input*⟩≡
```
)set break resume
)sys rm -f RealClosedField.output
)spool RealClosedField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealClosedField
--R RealClosedField  is a category constructor
--R Abbreviation for RealClosedField is RCFIELD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RCFIELD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (%,Integer) -> %                ?*? : (Integer,%) -> %
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        abs : % -> %
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : Integer -> %                 coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Fraction Integer -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R factor : % -> Factored %              gcd : (%,%) -> %
--R gcd : List % -> %                     hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : (%,%) -> %                      lcm : List % -> %
```

```
--R max : (%,%) -> %                      min : (%,%) -> %
--R negative? : % -> Boolean              nthRoot : (%,Integer) -> %
--R one? : % -> Boolean                   positive? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R rename : (%,OutputForm) -> %          rename! : (%,OutputForm) -> %
--R retract : % -> Fraction Integer       sample : () -> %
--R sign : % -> Integer                   sizeLess? : (%,%) -> Boolean
--R sqrt : Integer -> %                   sqrt : Fraction Integer -> %
--R sqrt : % -> %                         squareFree : % -> Factored %
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R allRootsOf : Polynomial Integer -> List %
--R allRootsOf : Polynomial Fraction Integer -> List %
--R allRootsOf : Polynomial % -> List %
--R allRootsOf : SparseUnivariatePolynomial Integer -> List %
--R allRootsOf : SparseUnivariatePolynomial Fraction Integer -> List %
--R allRootsOf : SparseUnivariatePolynomial % -> List %
--R approximate : (%,%) -> Fraction Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R mainDefiningPolynomial : % -> Union(SparseUnivariatePolynomial %,"failed")
--R mainForm : % -> Union(OutputForm,"failed")
--R mainValue : % -> Union(SparseUnivariatePolynomial %,"failed")
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retract : % -> Fraction Integer if Fraction Integer has RETRACT FRAC INT
--R retract : % -> Integer if Fraction Integer has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if Fraction Integer has R
--R retractIfCan : % -> Union(Integer,"failed") if Fraction Integer has RETRACT I
--R rootOf : (SparseUnivariatePolynomial %,PositiveInteger) -> Union(%,"failed")
--R rootOf : (SparseUnivariatePolynomial %,PositiveInteger,OutputForm) -> Union(%
--R sqrt : (%,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*RealClosedField.help*⟩≡

```
====================================================================
RealClosedField examples
====================================================================

See Also:
o )show RealClosedField
```

See:

⇐ "Algebra" (ALGEBRA) 11.0.152 on page 1045
⇐ "CharacteristicZero" (CHARZ) 10.0.136 on page 927
⇐ "CommutativeRing" (COMRING) 10.0.137 on page 932
⇐ "Field" (FIELD) 16.0.185 on page 1347
⇐ "FullyRetractableTo" (FRETRCT) 3.0.27 on page 100
⇐ "OrderedRing" (ORDRING) 10.0.144 on page 971
⇐ "RadicalCategory" (RADCAT) 2.0.17 on page 59

Exports:

0	1	abs	allRootsOf
approximate	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	gcd	gcdPolynomial
hash	inv	latex	lcm
mainDefiningPolynomial	mainForm	mainValue	max
min	multiEuclidean	negative?	nthRoot
one?	positive?	prime?	principalIdeal
recip	rename	rename!	retract
retractIfCan	rootOf	sample	sign
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?/?	?<?
?<=?	?=?	?>?	?>=?
?^?	?~=?	?quo?	?rem?

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
allRootsOf : SparseUnivariatePolynomial % -> List %
approximate : (%,%) -> Fraction Integer
mainDefiningPolynomial :
   % -> Union(SparseUnivariatePolynomial %,"failed")
mainForm : % -> Union(OutputForm,"failed")
mainValue : % -> Union(SparseUnivariatePolynomial %,"failed")
rename : (%,OutputForm) -> %
rename! : (%,OutputForm) -> %
```

These are implemented by this category:

```
allRootsOf : Polynomial Integer -> List %
allRootsOf : Polynomial Fraction Integer -> List %
allRootsOf : Polynomial % -> List %
allRootsOf : SparseUnivariatePolynomial Integer -> List %
allRootsOf : SparseUnivariatePolynomial Fraction Integer -> List %
characteristic : () -> NonNegativeInteger
nthRoot : (%,Integer) -> %
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger) ->
     Union(%,"failed")
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger,OutputForm) ->
     Union(%,"failed")
sqrt : (%,NonNegativeInteger) -> %
sqrt : Integer -> %
sqrt : Fraction Integer -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p927) CharacteristicZero():

```
0 : () -> %
1 : () -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
```

```
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

These exports come from (p971) OrderedRing():

```
abs : % -> %
coerce : Integer -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
```

These exports come from (p1347) Field():

```
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
inv : % -> %
lcm : (%,%) -> %
lcm : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p100) FullyRetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retract : % -> Fraction Integer
  if Fraction Integer has RETRACT FRAC INT
retract : % -> Integer if Fraction Integer has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
  if Fraction Integer has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
  if Fraction Integer has RETRACT INT
```

These exports come from (p1045) Algebra(Integer):

```
?*? : (%,Integer) -> %
```

⟨*category RCFIELD RealClosedField*⟩≡

```
)abbrev category RCFIELD RealClosedField
++ Author: Renaud Rioboo
++ Date Created: may 1993
++ Date Last Updated: January 2004
++ Basic Functions: provides computations with generic real roots of
++                  polynomials
++ Related Constructors: SimpleOrderedAlgebraicExtension, RealClosure
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
```

```
++ \axiomType{RealClosedField} provides common acces
++ functions for all real closed fields.

RealClosedField : Category == PUB where

    E ==> OutputForm
    SUP ==> SparseUnivariatePolynomial
    OFIELD ==> Join(OrderedRing,Field)
    PME ==> SUP($)
    N ==> NonNegativeInteger
    PI ==> PositiveInteger
    RN ==> Fraction(Integer)
    Z  ==> Integer
    POLY ==> Polynomial
    PACK ==> SparseUnivariatePolynomialFunctions2

    PUB == Join(CharacteristicZero,
                OrderedRing,
                CommutativeRing,
                Field,
                FullyRetractableTo(Fraction(Integer)),
                Algebra Integer,
                Algebra(Fraction(Integer)),
                RadicalCategory) with

        mainForm :   $ -> Union(E,"failed")
             ++ \axiom{mainForm(x)} is the main algebraic quantity name of
             ++ \axiom{x}

        mainDefiningPolynomial :   $ -> Union(PME,"failed")
             ++ \axiom{mainDefiningPolynomial(x)} is the defining
             ++ polynomial for the main algebraic quantity of \axiom{x}

        mainValue :   $ -> Union(PME,"failed")
             ++ \axiom{mainValue(x)} is the expression of \axiom{x} in terms
             ++ of \axiom{SparseUnivariatePolynomial($)}

        rootOf:          (PME,PI,E)           -> Union($,"failed")
             ++ \axiom{rootOf(pol,n,name)} creates the nth root for the order
             ++ of \axiom{pol} and names it \axiom{name}

        rootOf:          (PME,PI)             -> Union($,"failed")
             ++ \axiom{rootOf(pol,n)} creates the nth root for the order
             ++ of \axiom{pol} and gives it unique name

        allRootsOf:       PME                -> List $
```

```
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:       (SUP(RN))          ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:       (SUP(Z))          ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:       (POLY($))         ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:       (POLY(RN))        ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:       (POLY(Z))         ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

sqrt:            ($,N)               ->     $
     ++ \axiom{sqrt(x,n)} is \axiom{x ** (1/n)}

sqrt:              $                 ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

sqrt:             RN                 ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

sqrt:              Z                 ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

rename! :        ($,E)               ->     $
     ++ \axiom{rename!(x,name)} changes the way \axiom{x} is printed

rename :         ($,E)               ->     $
     ++ \axiom{rename(x,name)} gives a new number that prints as name

approximate:       ($,$) -> RN
      ++ \axiom{approximate(n,p)} gives an approximation of \axiom{n}
      ++ that has precision \axiom{p}

add
```

```
sqrt(a:$):$ == sqrt(a,2)

sqrt(a:RN):$ == sqrt(a::$,2)

sqrt(a:Z):$ == sqrt(a::$,2)

characteristic() == 0

rootOf(pol,n,o) ==
  r := rootOf(pol,n)
  r case "failed" => "failed"
  rename!(r,o)

rootOf(pol,n) ==
  liste:List($):= allRootsOf(pol)
  # liste > n => "failed"
  liste.n


sqrt(x,n) ==
  n = 0 => 1
  n = 1 => x
  zero?(x) => 0
  one?(x) => 1
  if odd?(n)
  then
    r := rootOf(monomial(1,n) - (x :: PME), 1)
  else
    r := rootOf(monomial(1,n) - (x :: PME), 2)
  r case "failed" => error "no roots"
  n = 2 => rename(r,root(x::E)$E)
  rename(r,root(x :: E, n :: E)$E)

(x : $) ** (rn : RN) == sqrt(x**numer(rn),denom(rn)::N)

nthRoot(x, n) ==
  zero?(n) => x
  negative?(n) => inv(sqrt(x,(-n) :: N))
  sqrt(x,n :: N)

allRootsOf(p:SUP(RN)) == allRootsOf(map(z +-> z::$ ,p)$PACK(RN,$))

allRootsOf(p:SUP(Z)) == allRootsOf(map(z +-> z::$ ,p)$PACK(Z,$))

allRootsOf(p:POLY($)) == allRootsOf(univariate(p))
```

```
        allRootsOf(p:POLY(RN)) == allRootsOf(univariate(p))

        allRootsOf(p:POLY(Z)) == allRootsOf(univariate(p))
```

⟨RCFIELD.dotabb⟩≡

```
"RCFIELD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RCFIELD" -> "ALGEBRA"
"RCFIELD" -> "CHARZ"
"RCFIELD" -> "COMRING"
"RCFIELD" -> "FIELD"
"RCFIELD" -> "FRETRCT"
"RCFIELD" -> "ORDRING"
"RCFIELD" -> "RADCAT"
```

⟨RCFIELD.dotfull⟩≡

```
"RealClosedField()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RealClosedField()" -> "Algebra(Integer)"
"RealClosedField()" -> "Algebra(Fraction(Integer))"
"RealClosedField()" -> "CharacteristicZero()"
"RealClosedField()" -> "CommutativeRing()"
"RealClosedField()" -> "Field()"
"RealClosedField()" -> "FullyRetractableTo(Fraction(Integer))"
"RealClosedField()" -> "OrderedRing()"
"RealClosedField()" -> "RadicalCategory()"
```

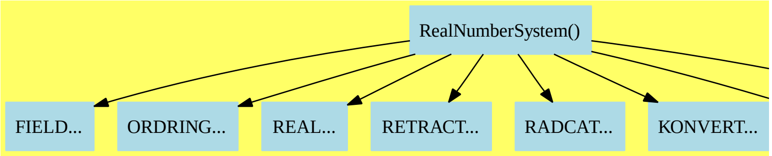
⟨*RCFIELD.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealClosedField()" [color=lightblue];
"RealClosedField()" -> "ALGEBRA..."
"RealClosedField()" -> "CHARZ..."
"RealClosedField()" -> "COMRING..."
"RealClosedField()" -> "FIELD..."
"RealClosedField()" -> "FRETRCT..."
"RealClosedField()" -> "ORDRING..."
"RealClosedField()" -> "RADCAT..."

"ALGEBRA..." [color=lightblue];
"CHARZ..." [color=lightblue];
"COMRING..." [color=lightblue];
"FIELD..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"ORDRING..." [color=lightblue];
"RADCAT..." [color=lightblue];

}
```

17.0.200 RealNumberSystem (RNS)



⟨*RealNumberSystem.input*⟩≡

```
  )set break resume
  )sys rm -f RealNumberSystem.output
  )spool RealNumberSystem.output
  )set message test on
  )set message auto off
  )clear all

  --S 1 of 1
  )show RealNumberSystem
  --R RealNumberSystem  is a category constructor
  --R Abbreviation for RealNumberSystem is RNS
  --R This constructor is exposed in this frame.
  --R Issue )edit bookvol10.2.pamphlet to see algebra source code for RNS
  --R
  --R------------------------------- Operations --------------------------------
  --R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
  --R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
  --R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
  --R ?**? : (%,Integer) -> %               ?**? : (%,PositiveInteger) -> %
  --R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
  --R -? : % -> %                           ?/? : (%,%) -> %
  --R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
  --R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
  --R ?>=? : (%,%) -> Boolean               1 : () -> %
  --R 0 : () -> %                           ?^? : (%,Integer) -> %
  --R ?^? : (%,PositiveInteger) -> %        abs : % -> %
  --R associates? : (%,%) -> Boolean        ceiling : % -> %
  --R coerce : Fraction Integer -> %        coerce : Integer -> %
  --R coerce : Fraction Integer -> %        coerce : % -> %
  --R coerce : Integer -> %                 coerce : % -> OutputForm
  --R convert : % -> Pattern Float          convert : % -> DoubleFloat
  --R convert : % -> Float                  factor : % -> Factored %
  --R floor : % -> %                        fractionPart : % -> %
  --R gcd : List % -> %                     gcd : (%,%) -> %
  --R hash : % -> SingleInteger             inv : % -> %
  --R latex : % -> String                   lcm : List % -> %
```

```
--R lcm : (%,%) -> %                       max : (%,%) -> %
--R min : (%,%) -> %                       negative? : % -> Boolean
--R norm : % -> %                          nthRoot : (%,Integer) -> %
--R one? : % -> Boolean                    positive? : % -> Boolean
--R prime? : % -> Boolean                  ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")         ?rem? : (%,%) -> %
--R retract : % -> Fraction Integer        retract : % -> Integer
--R round : % -> %                         sample : () -> %
--R sign : % -> Integer                    sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                          squareFree : % -> Factored %
--R squareFreePart : % -> %                truncate : % -> %
--R unit? : % -> Boolean                   unitCanonical : % -> %
--R wholePart : % -> Integer               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchR
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*RealNumberSystem.help*⟩≡

```
====================================================================
RealNumberSystem examples
====================================================================

See Also:
o )show RealNumberSystem
```

See:

⇒ "FloatingPointSystem" (FPS) 18.0.208 on page 1684
⇐ "CharacteristicZero" (CHARZ) 10.0.136 on page 927
⇐ "ConvertibleTo" (KONVERT) 2.0.8 on page 25
⇐ "Field" (FIELD) 16.0.185 on page 1347
⇐ "OrderedRing" (ORDRING) 10.0.144 on page 971
⇐ "PatternMatchable" (PATMAB) 4.0.58 on page 253
⇐ "RadicalCategory" (RADCAT) 2.0.17 on page 59
⇐ "RealConstant" (REAL) 3.0.32 on page 122
⇐ "RetractableTo" (RETRACT) 2.0.18 on page 62

Exports:

0
1
abs
associates?
ceiling
characteristic
coerce
convert
divide
euclideanSize
expressIdealMember
exquo
extendedEuclidean
factor
floor
fractionPart
gcd
gcdPolynomial
hash
inv
latex
lcm
max
min
multiEuclidean
negative?
norm
nthRoot
one?
patternMatch
positive?
prime?
principalIdeal
recip
retract
retractIfCan
round
sample
sign
sizeLess?
sqrt
squareFree
squareFreePart
subtractIfCan
truncate
unit?
unitCanonical
unitNormal
wholePart
zero?
?*?
?**?
?+?
?-?
-?
?/?
?<?
?<=?
?=?
?>?
?>=?
?^?
?quo?
?rem?
?~=?

These are directly exported but not implemented:

```
abs : % -> %
wholePart : % -> Integer
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
ceiling : % -> %
coerce : Fraction Integer -> %
convert : % -> Pattern Float
floor : % -> %
fractionPart : % -> %
norm : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
```

```
    PatternMatchResult(Float,%)
 round : % -> %
 truncate : % -> %
```

These exports come from (p1347) Field():

```
 0 : () -> %
 1 : () -> %
 associates? : (%,%) -> Boolean
 coerce : % -> %
 coerce : Integer -> %
 coerce : Integer -> %
 coerce : Fraction Integer -> %
 coerce : % -> OutputForm
 divide : (%,%) -> Record(quotient: %,remainder: %)
 euclideanSize : % -> NonNegativeInteger
 expressIdealMember : (List %,%) -> Union(List %,"failed")
 extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
 extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
 exquo : (%,%) -> Union(%,"failed")
 factor : % -> Factored %
 gcd : List % -> %
 gcd : (%,%) -> %
 gcdPolynomial :
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       SparseUnivariatePolynomial %
 hash : % -> SingleInteger
 inv : % -> %
 latex : % -> String
 lcm : List % -> %
 lcm : (%,%) -> %
 multiEuclidean : (List %,%) -> Union(List %,"failed")
 one? : % -> Boolean
 prime? : % -> Boolean
 principalIdeal : List % -> Record(coef: List %,generator: %)
 recip : % -> Union(%,"failed")
 sample : () -> %
 sizeLess? : (%,%) -> Boolean
 squareFree : % -> Factored %
 squareFreePart : % -> %
 subtractIfCan : (%,%) -> Union(%,"failed")
 unit? : % -> Boolean
 unitCanonical : % -> %
 unitNormal : % -> Record(unit: %,canonical: %,associate: %)
 zero? : % -> Boolean
 ?+? : (%,%) -> %
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
 ?*? : (Fraction Integer,%) -> %
```

```
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p971) OrderedRing():

```
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p122) RealConstant():

```
convert : % -> DoubleFloat
convert : % -> Float
```

These exports come from (p62) RetractableTo(Integer):

```
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

These exports come from (p62) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

These exports come from (p25) ConvertibleTo(Pattern(Float)):

These exports come from (p253) PatternMatchable(Float):

These exports come from (p927) CharacteristicZero():

⟨*category RNS RealNumberSystem*⟩≡

```
)abbrev category RNS RealNumberSystem
++ Author: Michael Monagan and Stephen M. Watt
++ Date Created:
++   January 1988
++ Change History:
++ Related Constructors:
++ Keywords: real numbers
++ Description:
++ The real number system category is intended as a model for the real
++ numbers.  The real numbers form an ordered normed field.  Note that
++ we have purposely not included \spadtype{DifferentialRing} or
++ the elementary functions (see \spadtype{TranscendentalFunctionCategory})
++ in the definition.

RealNumberSystem(): Category ==
  Join(Field, OrderedRing, RealConstant, RetractableTo Integer,
       RetractableTo Fraction Integer, RadicalCategory,
        ConvertibleTo Pattern Float, PatternMatchable Float,
          CharacteristicZero) with
    norm : % -> %
      ++ norm x returns the same as absolute value.
    ceiling : % -> %
      ++ ceiling x returns the small integer \spad{>= x}.
    floor: % -> %
      ++ floor x returns the largest integer \spad{<= x}.
    wholePart  : % -> Integer
      ++ wholePart x returns the integer part of x.
    fractionPart : % -> %
      ++ fractionPart x returns the fractional part of x.
    truncate: % -> %
      ++ truncate x returns the integer between x and 0 closest to x.
    round: % -> %
      ++ round x computes the integer closest to x.
    abs  : % -> %
```

```
     ++ abs x returns the absolute value of x.
add
  characteristic() == 0

  fractionPart x == x - truncate x

  truncate x == (negative? x => -floor(-x); floor x)

  round x == (negative? x => truncate(x-1/2::%); truncate(x+1/2::%))

  norm x == abs x

  coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

  convert(x:%):Pattern(Float)  == convert(x)@Float :: Pattern(Float)

  floor x ==
     x1 := (wholePart x) :: %
     x = x1 => x
     x < 0 => (x1 - 1)
     x1

  ceiling x ==
     x1 := (wholePart x)::%
     x = x1 => x
     x >= 0 => (x1 + 1)
     x1

  patternMatch(x, p, l) ==
    generic? p => addMatch(p, x, l)
    constant? p =>
      (r := retractIfCan(p)@Union(Float, "failed")) case Float =>
        convert(x)@Float = r::Float => l
        failed()
      failed()
    failed()
```

⟨*RNS.dotabb*⟩≡

```
"RNS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
"RNS" -> "FIELD"
"RNS" -> "ORDRING"
"RNS" -> "REAL"
"RNS" -> "RETRACT"
"RNS" -> "RADCAT"
"RNS" -> "KONVERT"
"RNS" -> "PATMAB"
"RNS" -> "CHARZ"
```

⟨*RNS.dotfull*⟩≡

```
"RealNumberSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
"RealNumberSystem()" -> "Field()"
"RealNumberSystem()" -> "OrderedRing()"
"RealNumberSystem()" -> "RealConstant()"
"RealNumberSystem()" -> "RetractableTo(Integer)"
"RealNumberSystem()" -> "RetractableTo(Fraction(Integer))"
"RealNumberSystem()" -> "RadicalCategory()"
"RealNumberSystem()" -> "ConvertibleTo(Pattern(Float))"
"RealNumberSystem()" -> "PatternMatchable(Float)"
"RealNumberSystem()" -> "CharacteristicZero()"
```

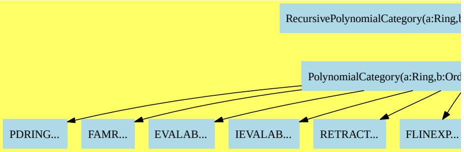
⟨*RNS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

17.0.201 RecursivePolynomialCategory (RPOLCAT)


RecursivePolynomialCategory(a:Ring,b
PolynomialCategory(a:Ring,b:Ord
PDRING...
FAMR...
EVALAB...
IEVALAB...
RETRACT...
FLINEXP...

⟨*RecursivePolynomialCategory.input*⟩≡

```
)set break resume
)sys rm -f RecursivePolynomialCategory.output
)spool RecursivePolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RecursivePolynomialCategory
--R RecursivePolynomialCategory(R: Ring,E: OrderedAbelianMonoidSup,V: OrderedSet)
--R Abbreviation for RecursivePolynomialCategory is RPOLCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RPOLCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,List V) -> %                   D : (%,V) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coefficients : % -> List R            coerce : V -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              deepestInitial : % -> %
--R deepestTail : % -> %                  degree : % -> E
--R differentiate : (%,List V) -> %       differentiate : (%,V) -> %
--R eval : (%,List V,List %) -> %         eval : (%,V,%) -> %
--R eval : (%,List V,List R) -> %         eval : (%,V,R) -> %
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             head : % -> %
--R headReduce : (%,%) -> %               headReduced? : (%,%) -> Boolean
--R infRittWu? : (%,%) -> Boolean         init : % -> %
```

```
--R initiallyReduce : (%,%) -> %          iteratedInitials : % -> List %
--R latex : % -> String                   lazyPquo : (%,%,V) -> %
--R lazyPquo : (%,%) -> %                 lazyPrem : (%,%,V) -> %
--R lazyPrem : (%,%) -> %                 leadingCoefficient : (%,V) -> %
--R leadingCoefficient : % -> R           leadingMonomial : % -> %
--R leastMonomial : % -> %                mainCoefficients : % -> List %
--R mainMonomial : % -> %                 mainMonomials : % -> List %
--R map : ((R -> R),%) -> %               mapExponents : ((E -> E),%) -> %
--R mdeg : % -> NonNegativeInteger        minimumDegree : % -> E
--R monic? : % -> Boolean                 monicModulo : (%,%) -> %
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R monomials : % -> List %               mvar : % -> V
--R normalized? : (%,%) -> Boolean        one? : % -> Boolean
--R pomopo! : (%,R,E,%) -> %              pquo : (%,%,V) -> %
--R pquo : (%,%) -> %                     prem : (%,%,V) -> %
--R prem : (%,%) -> %                     primitiveMonomials : % -> List %
--R quasiMonic? : % -> Boolean            recip : % -> Union(%,"failed")
--R reduced? : (%,List %) -> Boolean      reduced? : (%,%) -> Boolean
--R reductum : (%,V) -> %                 reductum : % -> %
--R retract : % -> V                      retract : % -> R
--R sample : () -> %                      supRittWu? : (%,%) -> Boolean
--R tail : % -> %                         variables : % -> List V
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List V,List NonNegativeInteger) -> %
--R D : (%,V,NonNegativeInteger) -> %
--R LazardQuotient : (%,%,NonNegativeInteger) -> % if R has INTDOM
--R LazardQuotient2 : (%,%,%,NonNegativeInteger) -> % if R has INTDOM
--R RittWuCompare : (%,%) -> Union(Boolean,"failed")
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,Polynomial
--R coefficient : (%,List V,List NonNegativeInteger) -> %
--R coefficient : (%,V,NonNegativeInteger) -> %
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT or R has ALGEBRA FRAC INT
```

```
--R coerce : % -> Polynomial R if V has KONVERT SYMBOL
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,Characteristic
--R content : (%,V) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> Polynomial R if V has KONVERT SYMBOL
--R convert : % -> String if R has RETRACT INT and V has KONVERT SYMBOL
--R convert : Polynomial R -> % if V has KONVERT SYMBOL
--R convert : Polynomial Integer -> % if not has(R,Algebra Fraction Integer) and
--R convert : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and V ha
--R convert : % -> InputForm if V has KONVERT INFORM and R has KONVERT INFORM
--R convert : % -> Pattern Integer if V has KONVERT PATTERN INT and R has KONVERT
--R convert : % -> Pattern Float if V has KONVERT PATTERN FLOAT and R has KONVERT
--R degree : (%,List V) -> List NonNegativeInteger
--R degree : (%,V) -> NonNegativeInteger
--R differentiate : (%,List V,List NonNegativeInteger) -> %
--R differentiate : (%,V,NonNegativeInteger) -> %
--R discriminant : (%,V) -> % if R has COMRING
--R exactQuotient : (%,%) -> % if R has INTDOM
--R exactQuotient : (%,R) -> % if R has INTDOM
--R exactQuotient! : (%,%) -> % if R has INTDOM
--R exactQuotient! : (%,R) -> % if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %) if R has
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcd : (R,%) -> R if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %) if R has INTD
--R halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %) if R has INTD
--R headReduced? : (%,List %) -> Boolean
--R initiallyReduced? : (%,List %) -> Boolean
--R initiallyReduced? : (%,%) -> Boolean
--R isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lastSubResultant : (%,%) -> % if R has INTDOM
--R lazyPremWithDefault : (%,%,V) -> Record(coef: %,gap: NonNegativeInteger,remai
--R lazyPremWithDefault : (%,%) -> Record(coef: %,gap: NonNegativeInteger,remaind
--R lazyPseudoDivide : (%,%,V) -> Record(coef: %,gap: NonNegativeInteger,quotient
--R lazyPseudoDivide : (%,%) -> Record(coef: %,gap: NonNegativeInteger,quotient:
--R lazyResidueClass : (%,%) -> Record(polnum: %,polden: %,power: NonNegativeInte
--R lcm : (%,%) -> % if R has GCDDOM
```

```
--R lcm : List % -> % if R has GCDDOM
--R mainContent : % -> % if R has GCDDOM
--R mainPrimitivePart : % -> % if R has GCDDOM
--R mainSquareFreePart : % -> % if R has GCDDOM
--R mainVariable : % -> Union(V,"failed")
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List V) -> List NonNegativeInteger
--R minimumDegree : (%,V) -> NonNegativeInteger
--R monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
--R monomial : (%,List V,List NonNegativeInteger) -> %
--R monomial : (%,V,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,V) -> %
--R multivariate : (SparseUnivariatePolynomial R,V) -> %
--R nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
--R normalized? : (%,List %) -> Boolean
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R primPartElseUnitCanonical : % -> % if R has INTDOM
--R primPartElseUnitCanonical! : % -> % if R has INTDOM
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R primitivePart! : % -> % if R has GCDDOM
--R pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%) -> % if R has INTDOM
--R resultant : (%,%,V) -> % if R has COMRING
--R retract : Polynomial R -> % if not has(R,Algebra Fraction Integer) and not has(R,Algebra
--R retract : Polynomial Integer -> % if not has(R,Algebra Fraction Integer) and R has ALGEB
--R retract : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and V has KONVERT S
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : Polynomial R -> Union(%,"failed") if not has(R,Algebra Fraction Integer)
--R retractIfCan : Polynomial Integer -> Union(%,"failed") if not has(R,Algebra Fraction Int
--R retractIfCan : Polynomial Fraction Integer -> Union(%,"failed") if R has ALGEBRA FRAC IN
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyn
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivari
--R subResultantChain : (%,%) -> List % if R has INTDOM
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List V) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,V) -> SparseUnivariatePolynomial %
--R
--E 1

)spool
)lisp (bye)
```

⟨*RecursivePolynomialCategory.help*⟩≡

```
====================================================================
RecursivePolynomialCategory examples
====================================================================

See Also:
o )show RecursivePolynomialCategory
```

See:

⇐ “PolynomialCategory” (POLYCAT) 16.0.189 on page 1385

Exports:

0
1
associates?
binomThmExpt
characteristic
charthRoot
coefficient
coefficients
coerce
conditionP
convert
D
deepestInitial
deepestTail
degree
differentiate
discriminant
eval
exactQuotient
exactQuotient!
exquo
extendedSubResultantGcd
factor
factorPolynomial
factorSquareFreePolynomial
gcd
gcdPolynomial
ground
ground?
halfExtendedSubResultantGcd1
halfExtendedSubResultantGcd2
hash
head
headReduce
headReduced?
infRittWu?
init
initiallyReduce
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?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?<?
?<=?
?>?
?>=?

Attributes Exported:

- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
exactQuotient! : (%,R) -> % if R has INTDOM
extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %)
  if R has INTDOM
halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %)
  if R has INTDOM
halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %)
  if R has INTDOM
lastSubResultant : (%,%) -> % if R has INTDOM
LazardQuotient : (%,%,NonNegativeInteger) -> %
  if R has INTDOM
LazardQuotient2 : (%,%,%,NonNegativeInteger) -> %
  if R has INTDOM
nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
resultant : (%,%) -> % if R has INTDOM
subResultantChain : (%,%) -> List % if R has INTDOM
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
coerce : % -> OutputForm
coerce : % -> Polynomial R if V has KONVERT SYMBOL
convert : % -> String
  if R has RETRACT INT
  and V has KONVERT SYMBOL
convert : % -> Polynomial R if V has KONVERT SYMBOL
convert : Polynomial R -> % if V has KONVERT SYMBOL
convert : Polynomial Integer -> %
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
convert : Polynomial Fraction Integer -> %
  if R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
deepestInitial : % -> %
deepestTail : % -> %
exactQuotient : (%,R) -> % if R has INTDOM
exactQuotient : (%,%) -> % if R has INTDOM
exactQuotient! : (%,%) -> % if R has INTDOM
gcd : (R,%) -> R if R has GCDDOM
head : % -> %
headReduce : (%,%) -> %
headReduced? : (%,List %) -> Boolean
headReduced? : (%,%) -> Boolean
infRittWu? : (%,%) -> Boolean
init : % -> %
initiallyReduce : (%,%) -> %
initiallyReduced? : (%,%) -> Boolean
initiallyReduced? : (%,List %) -> Boolean
iteratedInitials : % -> List %
lazyPremWithDefault : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPremWithDefault : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPquo : (%,%) -> %
lazyPquo : (%,%,V) -> %
lazyPrem : (%,%,V) -> %
lazyPrem : (%,%) -> %
lazyPseudoDivide : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyPseudoDivide : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyResidueClass : (%,%) ->
  Record(polnum: %,polden: %,power: NonNegativeInteger)
leadingCoefficient : (%,V) -> %
leastMonomial : % -> %
mainCoefficients : % -> List %
mainContent : % -> % if R has GCDDOM
mainMonomial : % -> %
mainMonomials : % -> List %
mainPrimitivePart : % -> % if R has GCDDOM
mainSquareFreePart : % -> % if R has GCDDOM
mdeg : % -> NonNegativeInteger
monic? : % -> Boolean
monicModulo : (%,%) -> %
mvar : % -> V
normalized? : (%,%) -> Boolean
```

```
normalized? : (%,List %) -> Boolean
pquo : (%,%) -> %
pquo : (%,%,V) -> %
prem : (%,%,V) -> %
prem : (%,%) -> %
primitivePart! : % -> % if R has GCDDOM
primPartElseUnitCanonical : % -> % if R has INTDOM
primPartElseUnitCanonical! : % -> % if R has INTDOM
pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
quasiMonic? : % -> Boolean
reduced? : (%,%) -> Boolean
reduced? : (%,List %) -> Boolean
reductum : (%,V) -> %
retract : Polynomial R -> %
  if not has(R,Algebra Fraction Integer)
  and not has(R,Algebra Integer)
  and V has KONVERT SYMBOL
  or not has(R,IntegerNumberSystem)
  and not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or not has(R,QuotientFieldCategory Integer)
  and R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retract : Polynomial Integer -> %
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retract : Polynomial Fraction Integer -> %
  if R has ALGEBRA FRAC INT and V has KONVERT SYMBOL
retractIfCan : Polynomial R -> Union(%,"failed")
  if not has(R,Algebra Fraction Integer)
  and not has(R,Algebra Integer)
  and V has KONVERT SYMBOL
  or not has(R,IntegerNumberSystem)
  and not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or not has(R,QuotientFieldCategory Integer)
  and R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retractIfCan : Polynomial Fraction Integer -> Union(%,"failed")
  if R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retractIfCan : Polynomial Integer -> Union(%,"failed")
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
RittWuCompare : (%,%) -> Union(Boolean,"failed")
supRittWu? : (%,%) -> Boolean
tail : % -> %
```

These exports come from (p1385) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
  if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
  if R has COMRING
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(has($,CharacteristicNonZero),
     has(R,PolynomialFactorizationExplicit))
  or R has CHARNZ
coefficient : (%,List V,List NonNegativeInteger) -> %
coefficient : (%,V,NonNegativeInteger) -> %
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT or R has ALGEBRA FRAC INT
coerce : V -> %
coerce : % -> % if R has INTDOM
coerce : Integer -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
differentiate : (%,List V) -> %
differentiate : (%,V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,V,R) -> %
eval : (%,List V,List R) -> %
eval : (%,V,%) -> %
eval : (%,List V,List %) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if V has PATMAB FLOAT and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
  if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
  if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
```

```
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

⟨*category RPOLCAT RecursivePolynomialCategory*⟩≡

```
)abbrev category RPOLCAT RecursivePolynomialCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/22/1994
++ Date Last Updated: 14/12/1998
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References:
```

```
++ Description:
++ A category for general multi-variate polynomials with coefficients
++ in a ring, variables in an ordered set, and exponents from an
++ ordered abelian monoid, with a \axiomOp{sup} operation.
++ When not constant, such a polynomial is viewed as a univariate polynomial
++ in its main variable w. r. t. to the total ordering on the elements in
++ the ordered set, so that some operations usually defined for univariate
++ polynomials make sense here.

RecursivePolynomialCategory(R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet):_
 Category ==
  PolynomialCategory(R, E, V) with
     mvar : $ -> V
         ++ \axiom{mvar(p)} returns an error if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its main variable w. r. t. to the
         ++ total ordering on the elements in \axiom{V}.
     mdeg  : $ -> NonNegativeInteger
         ++ \axiom{mdeg(p)} returns an error if \axiom{p} is \axiom{0},
         ++ otherwise, if \axiom{p} belongs to \axiom{R} returns \axiom{0},
         ++ otherwise, returns the degree of \axiom{p} in its main variable.
     init : $ -> $
         ++ \axiom{init(p)} returns an error if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its leading coefficient, where
         ++ \axiom{p} is viewed as a univariate polynomial in its main
         ++ variable.
     head  : $ -> $
         ++ \axiom{head(p)} returns \axiom{p} if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its leading term (monomial in the
         ++ AXIOM sense), where \axiom{p} is viewed as a univariate polynomial
         ++  in its main variable.
     tail  : $ -> $
         ++ \axiom{tail(p)} returns its reductum, where \axiom{p} is viewed
         ++ as a univariate polynomial in its main variable.
     deepestTail : $ -> $
         ++ \axiom{deepestTail(p)} returns \axiom{0} if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns tail(p), if \axiom{tail(p)} belongs
         ++ to  \axiom{R} or \axiom{mvar(tail(p)) < mvar(p)}, otherwise
         ++ returns \axiom{deepestTail(tail(p))}.
     iteratedInitials : $ -> List $
         ++ \axiom{iteratedInitials(p)} returns \axiom{[]} if \axiom{p}
         ++ belongs to \axiom{R},
         ++ otherwise returns the list of the iterated initials of \axiom{p}.
     deepestInitial : $ -> $
         ++ \axiom{deepestInitial(p)} returns an error if \axiom{p} belongs
         ++ to \axiom{R},
         ++ otherwise returns the last term of \axiom{iteratedInitials(p)}.
```

```
leadingCoefficient : ($,V) -> $
    ++ \axiom{leadingCoefficient(p,v)} returns the leading coefficient
    ++ of \axiom{p}, where \axiom{p} is viewed as A univariate
    ++ polynomial in \axiom{v}.
reductum  : ($,V) -> $
    ++ \axiom{reductum(p,v)} returns the reductum of \axiom{p}, where
    ++ \axiom{p} is viewed as a univariate polynomial in \axiom{v}.
monic? : $ -> Boolean
    ++ \axiom{monic?(p)} returns false if \axiom{p} belongs to \axiom{R},
    ++ otherwise returns true iff \axiom{p} is monic as a univariate
    ++ polynomial in its main variable.
quasiMonic? : $ -> Boolean
    ++ \axiom{quasiMonic?(p)} returns false if \axiom{p} belongs to
    ++ \axiom{R}, otherwise returns true iff the initial of \axiom{p}
    ++ lies in the base ring \axiom{R}.
mainMonomial : $ -> $
    ++ \axiom{mainMonomial(p)} returns an error if \axiom{p} is
    ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ \axiom{1}, otherwise, \axiom{mvar(p)} raised to the power
    ++ \axiom{mdeg(p)}.
leastMonomial : $ -> $
    ++ \axiom{leastMonomial(p)} returns an error if \axiom{p} is
    ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ \axiom{1}, otherwise, the monomial of \axiom{p} with lowest
    ++ degree, where \axiom{p} is viewed as a univariate polynomial in
    ++ its main variable.
mainCoefficients : $ -> List $
    ++ \axiom{mainCoefficients(p)} returns an error if \axiom{p} is
    ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ [p], otherwise returns the list of the coefficients of \axiom{p},
    ++ where \axiom{p} is viewed as a univariate polynomial in its main
    ++ variable.
mainMonomials : $ -> List $
    ++ \axiom{mainMonomials(p)} returns an error if \axiom{p} is
    ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ [1], otherwise returns the list of the monomials of \axiom{p},
    ++ where \axiom{p} is viewed as a univariate polynomial in its main
    ++ variable.
RittWuCompare : ($, $) -> Union(Boolean,"failed")
    ++ \axiom{RittWuCompare(a,b)} returns \axiom{"failed"} if \axiom{a}
    ++ and \axiom{b} have same rank w.r.t.
    ++ Ritt and Wu Wen Tsun ordering using the refinement of Lazard,
    ++ otherwise returns \axiom{infRittWu?(a,b)}.
infRittWu?  : ($, $) -> Boolean
    ++ \axiom{infRittWu?(a,b)} returns true if \axiom{a} is less than
    ++ \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
    ++ refinement of Lazard.
supRittWu? : ($, $) -> Boolean
    ++ \axiom{supRittWu?(a,b)} returns true if \axiom{a} is greater
    ++ than \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
    ++ refinement of Lazard.
reduced? : ($,$) -> Boolean
    ++ \axiom{reduced?(a,b)} returns true iff
    ++ \axiom{degree(a,mvar(b)) < mdeg(b)}.
reduced? : ($,List($)) -> Boolean
    ++ \axiom{reduced?(q,lp)} returns true iff \axiom{reduced?(q,p)}
    ++ holds for every \axiom{p} in \axiom{lp}.
headReduced? : ($,$) -> Boolean
    ++ \axiom{headReduced?(a,b)} returns true iff
    ++ \axiom{degree(head(a),mvar(b)) < mdeg(b)}.
headReduced? : ($,List($)) -> Boolean
    ++ \axiom{headReduced?(q,lp)} returns true iff
    ++ \axiom{headReduced?(q,p)} holds for every \axiom{p} in \axiom{lp}.
initiallyReduced? : ($,$) -> Boolean
    ++ \axiom{initiallyReduced?(a,b)} returns false iff there exists an
    ++ iterated initial of \axiom{a} which is not reduced w.r.t \axiom{b}.
initiallyReduced? :  ($,List($)) -> Boolean
    ++ \axiom{initiallyReduced?(q,lp)} returns true iff
    ++ \axiom{initiallyReduced?(q,p)} holds for every \axiom{p} in
    ++ \axiom{lp}.
normalized? : ($,$) -> Boolean
    ++ \axiom{normalized?(a,b)} returns true iff \axiom{a} and its
    ++ iterated initials have degree zero w.r.t. the main variable of
    ++ \axiom{b}
normalized? : ($,List($)) -> Boolean
    ++ \axiom{normalized?(q,lp)} returns true iff
    ++ \axiom{normalized?(q,p)} holds
    ++ for every \axiom{p} in \axiom{lp}.
prem : ($, $) -> $
    ++ \axiom{prem(a,b)} computes the pseudo-remainder of \axiom{a} by
    ++ \axiom{b}, both viewed as univariate polynomials in the main
    ++ variable of \axiom{b}.
pquo : ($, $) -> $
    ++ \axiom{pquo(a,b)} computes the pseudo-quotient of \axiom{a} by
    ++ \axiom{b}, both viewed as univariate polynomials in the main
    ++ variable of \axiom{b}.
prem : ($, $, V) -> $
    ++ \axiom{prem(a,b,v)} computes the pseudo-remainder of \axiom{a}
    ++ by \axiom{b}, both viewed as univariate polynomials in \axiom{v}.
pquo : ($, $, V) -> $
    ++ \axiom{pquo(a,b,v)} computes the pseudo-quotient of \axiom{a} by
    ++ \axiom{b}, both viewed as univariate polynomials in \axiom{v}.
```

```
lazyPrem : ($, $) ->  $
    ++ \axiom{lazyPrem(a,b)} returns the polynomial \axiom{r} reduced
    ++ w.r.t. \axiom{b} and such that \axiom{b} divides
    ++ \axiom{init(b)^e a - r} where \axiom{e}
    ++ is the number of steps of this pseudo-division.
lazyPquo : ($, $) ->  $
    ++ \axiom{lazyPquo(a,b)} returns the polynomial \axiom{q} such that
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}.
lazyPrem : ($, $, V) -> $
    ++ \axiom{lazyPrem(a,b,v)} returns the polynomial \axiom{r}
    ++ reduced w.r.t. \axiom{b} viewed as univariate polynomials in the
    ++ variable \axiom{v} such that \axiom{b} divides
    ++ \axiom{init(b)^e a - r} where \axiom{e} is the number of steps of
    ++ this pseudo-division.
lazyPquo : ($, $, V) ->  $
    ++ \axiom{lazyPquo(a,b,v)} returns the polynomial \axiom{q} such that
    ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]}.
lazyPremWithDefault : ($, $) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
    ++ \axiom{lazyPremWithDefault(a,b)} returns \axiom{[c,g,r]}
    ++ such that \axiom{r = lazyPrem(a,b)} and
    ++ \axiom{(c**g)*r = prem(a,b)}.
lazyPremWithDefault : ($, $, V) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
    ++ \axiom{lazyPremWithDefault(a,b,v)} returns \axiom{[c,g,r]}
    ++ such that \axiom{r = lazyPrem(a,b,v)} and
    ++ \axiom{(c**g)*r = prem(a,b,v)}.
lazyPseudoDivide : ($,$) -> _
  Record(coef:$, gap: NonNegativeInteger,quotient:$, remainder:$)
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}
    ++ such that \axiom{[c,g,r] = lazyPremWithDefault(a,b)} and
    ++ \axiom{q} is the pseudo-quotient computed in this lazy
    ++ pseudo-division.
lazyPseudoDivide : ($,$,V) -> _
  Record(coef:$, gap:NonNegativeInteger, quotient:$, remainder: $)
    ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]} such
    ++ that  \axiom{r = lazyPrem(a,b,v)}, \axiom{(c**g)*r = prem(a,b,v)}
    ++ and \axiom{q} is the pseudo-quotient computed in this lazy
    ++ pseudo-division.
pseudoDivide : ($, $) -> Record (quotient : $, remainder : $)
    ++ \axiom{pseudoDivide(a,b)} computes \axiom{[pquo(a,b),prem(a,b)]},
    ++ both polynomials viewed as univariate polynomials in the main
    ++ variable of \axiom{b}, if \axiom{b} is not a constant polynomial.
monicModulo : ($, $) -> $
    ++ \axiom{monicModulo(a,b)} computes \axiom{a mod b}, if \axiom{b} is
    ++ monic as univariate polynomial in its main variable.
```

```
lazyResidueClass : ($,$) -> _
 Record(polnum:$, polden:$, power:NonNegativeInteger)
    ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[p,q,n]} where
    ++ \axiom{p / q**n} represents the residue class of \axiom{a}
    ++ modulo \axiom{b} and \axiom{p} is reduced w.r.t. \axiom{b} and
    ++ \axiom{q} is \axiom{init(b)}.
headReduce: ($, $) ->  $
    ++ \axiom{headReduce(a,b)} returns a polynomial \axiom{r} such that
    ++ \axiom{headReduced?(r,b)} holds and there exists an integer
    ++ \axiom{e} such that \axiom{init(b)^e a - r} is zero modulo
    ++ \axiom{b}.
initiallyReduce: ($, $) ->  $
    ++ \axiom{initiallyReduce(a,b)} returns a polynomial \axiom{r} such
    ++ that \axiom{initiallyReduced?(r,b)} holds and there exists an
    ++ integer \axiom{e} such that \axiom{init(b)^e a - r} is zero
    ++ modulo \axiom{b}.

if (V has ConvertibleTo(Symbol))
then
  CoercibleTo(Polynomial R)
  ConvertibleTo(Polynomial R)
  if R has Algebra Fraction Integer
    then
      retractIfCan : Polynomial Fraction Integer -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
          ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial Fraction Integer -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not return
          ++ "failed", otherwise an error is produced.
      convert : Polynomial Fraction Integer -> $
          ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.
      retractIfCan : Polynomial Integer -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
          ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial Integer -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not return
          ++ "failed", otherwise an error is produced.
      convert : Polynomial Integer -> $
          ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}
      if not (R has QuotientFieldCategory(Integer))
        then
          retractIfCan : Polynomial R -> Union($,"failed")
            ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
            ++ of the current domain if all its variables belong to
          ++ \axiom{V}.
        retract : Polynomial R -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not
          ++ return "failed", otherwise an error is produced.
  if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
    then
      retractIfCan : Polynomial Integer -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
          ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial Integer -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not return
          ++ "failed", otherwise an error is produced.
      convert : Polynomial Integer -> $
          ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.
      if not (R has IntegerNumberSystem)
        then
          retractIfCan : Polynomial R -> Union($,"failed")
            ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
            ++ of the current domain if all its variables belong to
            ++ \axiom{V}.
          retract : Polynomial R -> $
            ++ \axiom{retract(p)} returns \axiom{p} as an element of the
            ++ current domain if \axiom{retractIfCan(p)} does not
            ++ return "failed", otherwise an error is produced.
  if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
    then
      retractIfCan : Polynomial R -> Union($,"failed")
        ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
        ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial R -> $
        ++ \axiom{retract(p)} returns \axiom{p} as an element of the
        ++ current domain if \axiom{retractIfCan(p)} does not return
        ++ "failed", otherwise an error is produced.
  convert : Polynomial R -> $
    ++ \axiom{convert(p)} returns \axiom{p} as an element of the current
    ++ domain if all its variables belong to \axiom{V}, otherwise an
    ++ error is produced.

  if R has RetractableTo(Integer)
  then
    ConvertibleTo(String)

if R has IntegralDomain
then
```

```
primPartElseUnitCanonical : $ -> $
    ++ \axiom{primPartElseUnitCanonical(p)} returns
    ++ \axiom{primitivePart(p)} if \axiom{R} is a gcd-domain,
    ++ otherwise \axiom{unitCanonical(p)}.
primPartElseUnitCanonical! : $ -> $
    ++ \axiom{primPartElseUnitCanonical!(p)} replaces  \axiom{p}
    ++ by \axiom{primPartElseUnitCanonical(p)}.
exactQuotient : ($,R) -> $
    ++ \axiom{exactQuotient(p,r)} computes the exact quotient of
    ++ \axiom{p} by \axiom{r}, which is assumed to be a divisor of
    ++ \axiom{p}. No error is returned if this exact quotient fails!
exactQuotient! : ($,R) -> $
    ++ \axiom{exactQuotient!(p,r)} replaces \axiom{p} by
    ++ \axiom{exactQuotient(p,r)}.
exactQuotient : ($,$) -> $
    ++ \axiom{exactQuotient(a,b)} computes the exact quotient of
    ++ \axiom{a} by \axiom{b}, which is assumed to be a divisor of
    ++ \axiom{a}. No error is returned if this exact quotient fails!
exactQuotient! : ($,$) -> $
    ++ \axiom{exactQuotient!(a,b)} replaces \axiom{a} by
    ++ \axiom{exactQuotient(a,b)}
subResultantGcd : ($, $) -> $
    ++ \axiom{subResultantGcd(a,b)} computes a gcd of \axiom{a} and
    ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
    ++ same main variable \axiom{v} and are viewed as univariate
    ++ polynomials in \axiom{v} with coefficients in the fraction
    ++ field of the polynomial ring generated by their other variables
    ++ over \axiom{R}.
extendedSubResultantGcd : ($, $) -> _
 Record (gcd : $, coef1 : $, coef2 : $)
    ++ \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[ca,cb,r]}
    ++ such that \axiom{r} is \axiom{subResultantGcd(a,b)} and we have
    ++ \axiom{ca * a + cb * cb = r} .
halfExtendedSubResultantGcd1: ($, $) -> Record (gcd : $, coef1 : $)
    ++ \axiom{halfExtendedSubResultantGcd1(a,b)} returns \axiom{[g,ca]}
    ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
    ++ otherwise produces an error.
halfExtendedSubResultantGcd2: ($, $) -> Record (gcd : $, coef2 : $)
    ++ \axiom{halfExtendedSubResultantGcd2(a,b)} returns \axiom{[g,cb]}
    ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
    ++ otherwise produces an error.
resultant  : ($, $) -> $
    ++ \axiom{resultant(a,b)} computes the resultant of \axiom{a} and
    ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
    ++ same main variable \axiom{v} and are viewed as univariate
    ++ polynomials in \axiom{v}.
```

```
  subResultantChain : ($, $) -> List $
      ++ \axiom{subResultantChain(a,b)}, where \axiom{a} and \axiom{b}
      ++ are not contant polynomials with the same main variable, returns
      ++ the subresultant chain of \axiom{a} and \axiom{b}.
  lastSubResultant: ($, $) -> $
      ++ \axiom{lastSubResultant(a,b)} returns the last non-zero
      ++ subresultant of \axiom{a} and \axiom{b} where \axiom{a} and
      ++ \axiom{b} are assumed to have the same main variable \axiom{v}
      ++ and are viewed as univariate polynomials in \axiom{v}.
  LazardQuotient: ($, $, NonNegativeInteger) -> $
      ++ \axiom{LazardQuotient(a,b,n)} returns \axiom{a**n exquo b**(n-1)}
      ++ assuming that this quotient does not fail.
  LazardQuotient2: ($, $, $, NonNegativeInteger) -> $
      ++ \axiom{LazardQuotient2(p,a,b,n)} returns
      ++ \axiom{(a**(n-1) * p) exquo b**(n-1)}
      ++ assuming that this quotient does not fail.
  next_subResultant2: ($, $, $, $) -> $
      ++ \axiom{nextsubResultant2(p,q,z,s)} is the multivariate version
      ++ of the operation
      ++ next_sousResultant2 from PseudoRemainderSequence from
      ++ the \axiomType{PseudoRemainderSequence} constructor.

if R has GcdDomain
then
  gcd : (R,$) -> R
      ++ \axiom{gcd(r,p)} returns the gcd of \axiom{r} and the content
      ++ of \axiom{p}.
  primitivePart! : $ -> $
      ++ \axiom{primitivePart!(p)} replaces \axiom{p}  by its primitive
      ++ part.
  mainContent : $ -> $
      ++ \axiom{mainContent(p)} returns the content of \axiom{p} viewed
      ++ as a univariate polynomial in its main variable and with
      ++ coefficients in the polynomial ring generated by its other
      ++ variables over \axiom{R}.
  mainPrimitivePart : $ -> $
      ++ \axiom{mainPrimitivePart(p)} returns the primitive part of
      ++ \axiom{p} viewed as a univariate polynomial in its main
      ++ variable and with coefficients in the polynomial ring generated
      ++ by its other variables over \axiom{R}.
  mainSquareFreePart : $ -> $
      ++ \axiom{mainSquareFreePart(p)} returns the square free part of
      ++ \axiom{p} viewed as a univariate polynomial in its main
      ++ variable and with coefficients in the polynomial ring
      ++ generated by its other variables over \axiom{R}.
```

```
 add
     O ==> OutputForm
     NNI ==> NonNegativeInteger
     INT ==> Integer

     exactQuo : (R,R) -> R

     coerce(p:$):O ==
       ground? (p) => (ground(p))::O
--       if one?((ip := init(p)))
       if (((ip := init(p))) = 1)
         then
           if zero?((tp := tail(p)))
             then
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O)
                 else
                   return(((mvar(p))::O **$O (dp::O)))
             else
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O +$O (tp::O))
                 else
                   return(((mvar(p))::O **$O (dp::O)) +$O (tp::O))
          else
           if zero?((tp := tail(p)))
             then
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((ip::O) *$O  (mvar(p))::O)
                 else
                   return((ip::O) *$O ((mvar(p))::O **$O (dp::O)))
             else
--               if one?(mdeg(p))
               if ((mdeg(p)) = 1)
                 then
                   return(((ip::O) *$O  (mvar(p))::O) +$O (tp::O))
       ((ip)::O *$O ((mvar(p))::O **$O ((mdeg(p)::O))) +$O (tail(p)::O))

     mvar p ==
       ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
       mainVariable(p)::V
```

```
mdeg p ==
  ground?(p) => 0$NNI
  degree(p,mainVariable(p)::V)

init p ==
  ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
  v := mainVariable(p)::V
  coefficient(p,v,degree(p,v))

leadingCoefficient (p,v) ==
  zero? (d := degree(p,v)) => p
  coefficient(p,v,d)

head p ==
  ground? p => p
  v := mainVariable(p)::V
  d := degree(p,v)
  monomial(coefficient(p,v,d),v,d)

reductum(p,v) ==
  zero? (d := degree(p,v)) => 0$$
  p - monomial(coefficient(p,v,d),v,d)

tail p ==
  ground? p => 0$$
  p - head(p)

deepestTail p ==
  ground? p => 0$$
  ground? tail(p) => tail(p)
  mvar(p) > mvar(tail(p)) => tail(p)
  deepestTail(tail(p))

iteratedInitials p ==
  ground? p => []
  p := init(p)
  cons(p,iteratedInitials(p))

localDeepestInitial (p : $) : $ ==
  ground? p => p
  localDeepestInitial init p

deepestInitial p ==
  ground? p => _
    error"Error in deepestInitial from RPOLCAT : #1 is constant."
    localDeepestInitial init p

  monic? p ==
    ground? p => false
    (recip(init(p))$$ case $)@Boolean

  quasiMonic?  p ==
    ground? p => false
    ground?(init(p))

  mainMonomial p ==
    zero? p => error"Error in mainMonomial from RPOLCAT : #1 is zero"
    ground? p => 1$$
    v := mainVariable(p)::V
    monomial(1$$,v,degree(p,v))

  leastMonomial p ==
    zero? p => error"Error in leastMonomial from RPOLCAT : #1 is zero"
    ground? p => 1$$
    v := mainVariable(p)::V
    monomial(1$$,v,minimumDegree(p,v))

  mainCoefficients p ==
    zero? p => error"Error in mainCoefficients from RPOLCAT : #1 is zero"
    ground? p => [p]
    v := mainVariable(p)::V
    coefficients(univariate(p,v)@SparseUnivariatePolynomial($))

  mainMonomials p ==
    zero? p => error"Error in mainMonomials from RPOLCAT : #1 is zero"
    ground? p => [1$$]
    v := mainVariable(p)::V
    lm := monomials(univariate(p,v)@SparseUnivariatePolynomial($))
    [monomial(1$$,v,degree(m)) for m in lm]

  RittWuCompare (a,b) ==
    (ground? b and  ground? a) => "failed"::Union(Boolean,"failed")
    ground? b => false::Union(Boolean,"failed")
    ground? a => true::Union(Boolean,"failed")
    mvar(a) < mvar(b) => true::Union(Boolean,"failed")
    mvar(a) > mvar(b) => false::Union(Boolean,"failed")
    mdeg(a) < mdeg(b) => true::Union(Boolean,"failed")
    mdeg(a) > mdeg(b) => false::Union(Boolean,"failed")
    lc := RittWuCompare(init(a),init(b))
    lc case Boolean => lc
    RittWuCompare(tail(a),tail(b))
```

```
infRittWu? (a,b) ==
  lc : Union(Boolean,"failed") := RittWuCompare(a,b)
  lc case Boolean => lc::Boolean
  false

supRittWu? (a,b) ==
  infRittWu? (b,a)

prem (a:$, b:$)  : $ ==
  cP := lazyPremWithDefault (a,b)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$)  : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

prem (a:$, b:$, v:V) : $ ==
  cP := lazyPremWithDefault (a,b,v)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$, v:V)  : $ ==
  cPS := lazyPseudoDivide (a,b,v)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

lazyPrem (a:$, b:$) : $ ==
  (not ground?(b)) and (monic?(b)) => monicModulo(a,b)
  (lazyPremWithDefault (a,b)).remainder

lazyPquo (a:$, b:$) : $ ==
  (lazyPseudoDivide (a,b)).quotient

lazyPrem (a:$, b:$, v:V) : $ ==
  zero? b => _
    error"Error in lazyPrem : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => 0$$
  (v = mvar(b)) => lazyPrem(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => 0$$
  dav : NNI  := degree(a,v)
  zero? dav => a
  test : INT := dav::INT - dbv
  lcbv : $ := leadingCoefficient(b,v)
  while not zero?(a) and not negative?(test) repeat
```

```
      lcav := leadingCoefficient(a,v)
      term := monomial(lcav,v,test::NNI)
      a := lcbv * a - term * b
      test := degree(a,v)::INT - dbv
    a

  lazyPquo (a:$, b:$, v:V) : $ ==
    (lazyPseudoDivide (a,b,v)).quotient

  headReduce (a:$,b:$) ==
    ground? b => error _
     "Error in headReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
    ground? a => a
    mvar(a) = mvar(b) => lazyPrem(a,b)
    while not reduced?((ha := head a),b) repeat
      lrc := lazyResidueClass(ha,b)
      if zero? tail(a)
        then
          a := lrc.polnum
        else
          a := lrc.polnum +  (lrc.polden)**(lrc.power) * tail(a)
    a

  initiallyReduce(a:$,b:$) ==
    ground? b => error _
"Error in initiallyReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
    ground? a => a
    v := mvar(b)
    mvar(a) = v => lazyPrem(a,b)
    ia := a
    ma := 1$$
    ta := 0$$
    while (not ground?(ia)) and (mvar(ia) >= mvar(b)) repeat
      if (mvar(ia) = mvar(b)) and (mdeg(ia) >= mdeg(b))
        then
          iamodb := lazyResidueClass(ia,b)
          ia := iamodb.polnum
          if not zero? ta
            then
              ta :=  (iamodb.polden)**(iamodb.power) * ta
      if zero? ia
        then
          ia := ta
          ma := 1$$
          ta := 0$$
        else
        if not ground?(ia)
          then
            ta := tail(ia) * ma + ta
            ma := mainMonomial(ia) * ma
            ia := init(ia)
  ia * ma + ta

lazyPremWithDefault (a,b) ==
  ground?(b) => error _
    "Error in lazyPremWithDefault from RPOLCAT : #2 is constant"
  ground?(a) => [1$$,0$NNI,a]
  xa := mvar a
  xb := mvar b
  xa < xb => [1$$,0$NNI,a]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  delta : INT := max(test + 1$INT, 0$INT)
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
    else
      while not zero?(a) and not negative?(test) repeat
        term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
        a := lcb * a - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
  [lcb, (delta::NNI), a]

lazyPremWithDefault (a,b,v) ==
  zero? b =>  error _
   "Error in lazyPremWithDefault : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,0$$]
  (v = mvar(b)) => lazyPremWithDefault(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
  lcbv : $ := leadingCoefficient(b,v)
```

```
    while not zero?(a) and not negative?(test) repeat
      lcav := leadingCoefficient(a,v)
      term := monomial(lcav,v,test::NNI)
      a := lcbv * a - term * b
      delta := delta - 1$INT
      test := degree(a,v)::INT - dbv
    [lcbv, (delta::NNI), a]

  pseudoDivide (a,b) ==
    cPS := lazyPseudoDivide (a,b)
    c := (cPS.coef) ** (cPS.gap)
    [c * cPS.quotient, c * cPS.remainder]

  lazyPseudoDivide (a,b) ==
    ground?(b) => error _
       "Error in lazyPseudoDivide from RPOLCAT : #2 is constant"
    ground?(a) => [1$$,0$NNI,0$$,a]
    xa := mvar a
    xb := mvar b
    xa < xb => [1$$,0$NNI,0$$, a]
    lcb : $ := init b
    db : NNI := mdeg b
    q := 0$$
    test : INT := degree(a,xb)::INT - db
    delta : INT := max(test + 1$INT, 0$INT)
    if xa = xb
      then
        b := tail b
        while not zero?(a) and not negative?(test) repeat
          term := monomial(init(a),xb,test::NNI)
          a := lcb * tail(a) - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
      else
        while not zero?(a) and not negative?(test) repeat
          term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
          a := lcb * a - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
    [lcb, (delta::NNI), q, a]

  lazyPseudoDivide (a,b,v) ==
    zero? b =>  error _
      "Error in lazyPseudoDivide : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,a,0$$]
  (v = mvar(b)) => lazyPseudoDivide(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,a,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,0$$, a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
  lcbv : $ := leadingCoefficient(b,v)
  q := 0$$
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
    term := monomial(lcav,v,test::NNI)
    a := lcbv * a - term * b
    q := lcbv * q + term
    delta := delta - 1$INT
    test := degree(a,v)::INT - dbv
  [lcbv, (delta::NNI), q, a]

monicModulo (a,b) ==
  ground?(b) => error"Error in monicModulo from RPOLCAT : #2 is constant"
  rec : Union($,"failed")
  rec := recip((ib := init(b)))$$
  (rec case "failed")@Boolean => error _
    "Error in monicModulo from RPOLCAT : #2 is not monic"
  ground? a => a
  ib * ((lazyPremWithDefault ((rec::$) * a,(rec::$) * b)).remainder)

lazyResidueClass(a,b) ==
  zero? b => [a,1$$,0$NNI]
  ground? b => [0$$,1$$,0$NNI]
  ground? a => [a,1$$,0$NNI]
  xa := mvar a
  xb := mvar b
  xa < xb => [a,1$$,0$NNI]
  monic?(b) => [monicModulo(a,b),1$$,0$NNI]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  pow : NNI := 0
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
```

```
          pow := pow + 1$NNI
          test := degree(a,xb)::INT - db
      else
        while not zero?(a) and not negative?(test) repeat
          term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
          a := lcb * a - term * b
          pow := pow + 1$NNI
          test := degree(a,xb)::INT - db
    [a,lcb,pow]

  reduced? (a:$,b:$) : Boolean ==
    degree(a,mvar(b)) < mdeg(b)

  reduced? (p:$, lq : List($)) : Boolean ==
    ground? p => true
    while (not empty? lq) and (reduced?(p, first lq)) repeat
      lq := rest lq
    empty? lq

  headReduced? (a:$,b:$) : Boolean ==
    reduced?(head(a),b)

  headReduced? (p:$, lq : List($)) : Boolean ==
    reduced?(head(p),lq)

  initiallyReduced? (a:$,b:$) : Boolean ==
    ground? b => error _
"Error in initiallyReduced? : ($,$) -> Bool. from RPOLCAT : #2 is constant"
    ground?(a) => true
    mvar(a) < mvar(b) => true
    (mvar(a) = mvar(b)) => reduced?(a,b)
    initiallyReduced?(init(a),b)

  initiallyReduced? (p:$, lq : List($)) : Boolean ==
    ground? p => true
    while (not empty? lq) and (initiallyReduced?(p, first lq)) repeat
      lq := rest lq
    empty? lq

  normalized?(a:$,b:$) : Boolean ==
    ground? b => error _
   "Error in  normalized? : ($,$) -> Boolean from TSETCAT : #2 is constant"
    ground? a => true
    mvar(a) < mvar(b) => true
    (mvar(a) = mvar(b)) => false
    normalized?(init(a),b)
```

```
     normalized? (p:$, lq : List($)) : Boolean ==
       while (not empty? lq) and (normalized?(p, first lq)) repeat
         lq := rest lq
       empty? lq

     if R has IntegralDomain
     then

       if R has EuclideanDomain
         then
           exactQuo(r:R,s:R):R ==
             r quo$R s
         else
           exactQuo(r:R,s:R):R ==
             (r exquo$R s)::R

       exactQuotient (p:$,r:R) ==
         (p exquo$$ r)::$

       exactQuotient (a:$,b:$) ==
         ground? b => exactQuotient(a,ground(b))
         (a exquo$$ b)::$

       exactQuotient! (a:$,b:$) ==
         ground? b => exactQuotient!(a,ground(b))
         a := (a exquo$$ b)::$

       if (R has GcdDomain) and not(R has Field)
       then

         primPartElseUnitCanonical p ==
           primitivePart p

         primitivePart! p ==
           zero? p => p
--           if one?(cp := content(p))
           if ((cp := content(p)) = 1)
             then
               p := unitCanonical p
             else
               p := unitCanonical exactQuotient!(p,cp)
           p

         primPartElseUnitCanonical! p ==
           primitivePart! p
```

```
          else
            primPartElseUnitCanonical p ==
              unitCanonical p

            primPartElseUnitCanonical! p ==
              p := unitCanonical p


        if R has GcdDomain
        then

          gcd(r:R,p:$):R ==
--            one? r => r
            (r = 1) => r
            zero? p => r
            ground? p => gcd(r,ground(p))$R
            gcd(gcd(r,init(p)),tail(p))

          mainContent p ==
            zero? p => p
            "gcd"/mainCoefficients(p)

          mainPrimitivePart p ==
            zero? p => p
            (unitNormal((p exquo$$ mainContent(p))::$)).canonical

          mainSquareFreePart p ==
            ground? p => p
            v := mainVariable(p)::V
            sfp : SparseUnivariatePolynomial($)
            sfp := squareFreePart(univariate(p,v)@SparseUnivariatePolynomial($))
            multivariate(sfp,v)

        if (V has ConvertibleTo(Symbol))
          then

            PR ==> Polynomial R
            PQ ==> Polynomial Fraction Integer
            PZ ==> Polynomial Integer
            IES ==> IndexedExponents(Symbol)
            Q ==> Fraction Integer
            Z ==> Integer

            convert(p:$) : PR ==
              ground? p => (ground(p)$$)::PR
```

```
  v : V := mvar(p)
  d : NNI := mdeg(p)
  convert(init(p))@PR *$PR _
               ((convert(v)@Symbol)::PR)**d +$PR convert(tail(p))@PR

coerce(p:$) : PR ==
  convert(p)@PR

localRetract : PR -> $
localRetractPQ : PQ -> $
localRetractPZ : PZ -> $
localRetractIfCan : PR -> Union($,"failed")
localRetractIfCanPQ : PQ -> Union($,"failed")
localRetractIfCanPZ : PZ -> Union($,"failed")

if V has Finite
  then

    sizeV : NNI := size()$V
    lv : List Symbol
    lv := _
      [convert(index(i::PositiveInteger)$V)@Symbol for i in 1..sizeV]

    localRetract(p : PR) : $ ==
      ground? p => (ground(p)$PR)::$
      mvp : Symbol := (mainVariable(p)$PR)::Symbol
      d : NNI
      imvp : PositiveInteger := _
                     (position(mvp,lv)$(List Symbol))::PositiveInteger
      vimvp : V := index(imvp)$V
      xvimvp,c : $
      newp := 0$$
      while (not zero? (d := degree(p,mvp))) repeat
        c := localRetract(coefficient(p,mvp,d)$PR)
        xvimvp := monomial(c,vimvp,d)$$
        newp := newp +$$ xvimvp
        p := p -$PR monomial(coefficient(p,mvp,d)$PR,mvp,d)$PR
      newp +$$ localRetract(p)

    if R has Algebra Fraction Integer
      then
        localRetractPQ(pq:PQ):$ ==
          ground? pq => ((ground(pq)$PQ)::R)::$
          mvp : Symbol := (mainVariable(pq)$PQ)::Symbol
          d : NNI
          imvp : PositiveInteger := _
```

```
                   (position(mvp,lv)$(List Symbol))::PositiveInteger
         vimvp : V := index(imvp)$V
         xvimvp,c : $
         newp := 0$$
         while (not zero? (d := degree(pq,mvp))) repeat
           c := localRetractPQ(coefficient(pq,mvp,d)$PQ)
           xvimvp := monomial(c,vimvp,d)$$
           newp := newp +$$ xvimvp
           pq := pq -$PQ monomial(coefficient(pq,mvp,d)$PQ,mvp,d)$PQ
         newp +$$ localRetractPQ(pq)

   if R has Algebra Integer
     then
       localRetractPZ(pz:PZ):$ ==
         ground? pz => ((ground(pz)$PZ)::R)::$
         mvp : Symbol := (mainVariable(pz)$PZ)::Symbol
         d : NNI
         imvp : PositiveInteger := _
                   (position(mvp,lv)$(List Symbol))::PositiveInteger
         vimvp : V := index(imvp)$V
         xvimvp,c : $
         newp := 0$$
         while (not zero? (d := degree(pz,mvp))) repeat
           c := localRetractPZ(coefficient(pz,mvp,d)$PZ)
           xvimvp := monomial(c,vimvp,d)$$
           newp := newp +$$ xvimvp
           pz := pz -$PZ monomial(coefficient(pz,mvp,d)$PZ,mvp,d)$PZ
         newp +$$ localRetractPZ(pz)

   retractable?(p:PR):Boolean ==
     lvp := variables(p)$PR
     while not empty? lvp and member?(first lvp,lv) repeat
       lvp := rest lvp
     empty? lvp

   retractablePQ?(p:PQ):Boolean ==
     lvp := variables(p)$PQ
     while not empty? lvp and member?(first lvp,lv) repeat
       lvp := rest lvp
     empty? lvp

   retractablePZ?(p:PZ):Boolean ==
     lvp := variables(p)$PZ
     while not empty? lvp and member?(first lvp,lv) repeat
       lvp := rest lvp
     empty? lvp
```

```
    localRetractIfCan(p : PR): Union($,"failed") ==
      not retractable?(p) => "failed"::Union($,"failed")
      localRetract(p)::Union($,"failed")

    localRetractIfCanPQ(p : PQ): Union($,"failed") ==
      not retractablePQ?(p) => "failed"::Union($,"failed")
      localRetractPQ(p)::Union($,"failed")

    localRetractIfCanPZ(p : PZ): Union($,"failed") ==
      not retractablePZ?(p) => "failed"::Union($,"failed")
      localRetractPZ(p)::Union($,"failed")

  if R has Algebra Fraction Integer
    then

      mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
      mpc2Q := MPolyCatFunctions2(Symbol,IES,IES,Q,R,PQ,PR)
      ZToR (z:Z):R == coerce(z)@R
      QToR (q:Q):R == coerce(q)@R
      PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z
      PQToPR (pq:PQ):PR == map(QToR,pq)$mpc2Q

      retract(pz:PZ) ==
        rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
        (rif case "failed") => error _
                           "failed in retract: POLY Z -> $ from RPOLCAT"
        rif::$

      convert(pz:PZ) ==
        retract(pz)@$

      retract(pq:PQ) ==
        rif : Union($,"failed") := retractIfCan(pq)@Union($,"failed")
        (rif case "failed") => error _
                           "failed in retract: POLY Z -> $ from RPOLCAT"
        rif::$

      convert(pq:PQ) ==
        retract(pq)@$

      if not (R has QuotientFieldCategory(Integer))
        then
          -- the only operation to implement is
          -- retractIfCan : PR -> Union($,"failed")
          -- when V does not have Finite
    if V has Finite
      then
        retractIfCan(pr:PR) ==
          localRetractIfCan(pr)@Union($,"failed")

        retractIfCan(pq:PQ) ==
          localRetractIfCanPQ(pq)@Union($,"failed")
      else
        retractIfCan(pq:PQ) ==
          pr : PR := PQToPR(pq)
          retractIfCan(pr)@Union($,"failed")

    retractIfCan(pz:PZ) ==
      pr : PR := PZToPR(pz)
      retractIfCan(pr)@Union($,"failed")

    retract(pr:PR) ==
      rif : Union($,"failed") := _
                           retractIfCan(pr)@Union($,"failed")
      (rif case "failed") => error _
                 "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pr:PR) ==
      retract(pr)@$

  else
    -- the only operation to implement is
    -- retractIfCan : PQ -> Union($,"failed")
    -- when V does not have Finite
    mpc2ZQ := MPolyCatFunctions2(Symbol,IES,IES,Z,Q,PZ,PQ)
    mpc2RQ := MPolyCatFunctions2(Symbol,IES,IES,R,Q,PR,PQ)
    ZToQ(z:Z):Q == coerce(z)@Q
    RToQ(r:R):Q == retract(r)@Q

    PZToPQ (pz:PZ):PQ == map(ZToQ,pz)$mpc2ZQ
    PRToPQ (pr:PR):PQ == map(RToQ,pr)$mpc2RQ

    retractIfCan(pz:PZ) ==
      pq : PQ := PZToPQ(pz)
      retractIfCan(pq)@Union($,"failed")

    if V has Finite
      then
        retractIfCan(pq:PQ) ==
              localRetractIfCanPQ(pq)@Union($,"failed")

            convert(pr:PR) ==
              lrif : Union($,"failed") := _
                              localRetractIfCan(pr)@Union($,"failed")
              (lrif case "failed") => error _
                              "failed in convert: PR->$ from RPOLCAT"
              lrif::$
          else
            convert(pr:PR) ==
              pq : PQ := PRToPQ(pr)
              retract(pq)@$

    if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
      then

        mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
        ZToR (z:Z):R == coerce(z)@R
        PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z

        retract(pz:PZ) ==
          rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
          (rif case "failed") => error _
                          "failed in retract: POLY Z -> $ from RPOLCAT"
          rif::$

        convert(pz:PZ) ==
          retract(pz)@$

        if not (R has IntegerNumberSystem)
          then
            -- the only operation to implement is
            -- retractIfCan : PR -> Union($,"failed")
            -- when V does not have Finite

            if V has Finite
              then
                retractIfCan(pr:PR) ==
                  localRetractIfCan(pr)@Union($,"failed")

                retractIfCan(pz:PZ) ==
                  localRetractIfCanPZ(pz)@Union($,"failed")
              else
                retractIfCan(pz:PZ) ==
                  pr : PR := PZToPR(pz)
                  retractIfCan(pr)@Union($,"failed")
        retract(pr:PR) ==
          rif : Union($,"failed"):=retractIfCan(pr)@Union($,"failed")
          (rif case "failed") => error _
                         "failed in retract: POLY Z -> $ from RPOLCAT"
          rif::$

        convert(pr:PR) ==
          retract(pr)@$

      else
        -- the only operation to implement is
        -- retractIfCan : PZ -> Union($,"failed")
        -- when V does not have Finite

        mpc2RZ := MPolyCatFunctions2(Symbol,IES,IES,R,Z,PR,PZ)
        RToZ(r:R):Z == retract(r)@Z
        PRToPZ (pr:PR):PZ == map(RToZ,pr)$mpc2RZ

        if V has Finite
          then
            convert(pr:PR) ==
              lrif : Union($,"failed") := _
                              localRetractIfCan(pr)@Union($,"failed")
              (lrif case "failed") => error _
                              "failed in convert: PR->$ from RPOLCAT"
              lrif::$
            retractIfCan(pz:PZ) ==
              localRetractIfCanPZ(pz)@Union($,"failed")
          else
            convert(pr:PR) ==
              pz : PZ := PRToPZ(pr)
              retract(pz)@$


  if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
    then
      -- the only operation to implement is
      -- retractIfCan : PR -> Union($,"failed")

      if V has Finite
        then
          retractIfCan(pr:PR) ==
            localRetractIfCan(pr)@Union($,"failed")

      retract(pr:PR) ==
                rif : Union($,"failed") := retractIfCan(pr)@Union($,"failed")
                (rif case "failed") => error _
                                "failed in retract: POLY Z -> $ from RPOLCAT"
                rif::$

              convert(pr:PR) ==
                retract(pr)@$

          if (R has RetractableTo(INT))
            then

              convert(pol:$):String ==
                ground?(pol) => convert(retract(ground(pol))@INT)@String
                ipol : $ := init(pol)
                vpol : V := mvar(pol)
                dpol : NNI := mdeg(pol)
                tpol: $  := tail(pol)
                sipol,svpol,sdpol,stpol : String
--                if one? ipol
                if (ipol = 1)
                  then
                    sipol := empty()$String
                  else
--                    if one?(-ipol)
                    if ((-ipol) = 1)
                      then
                        sipol := "-"
                      else
                        sipol := convert(ipol)@String
                        if not monomial?(ipol)
                          then
                            sipol := concat(["(",sipol,")*"])$String
                          else
                            sipol := concat(sipol,"*")$String
                svpol := string(convert(vpol)@Symbol)
--                if one? dpol
                if (dpol = 1)
                  then
                    sdpol :=  empty()$String
                  else
                    sdpol := _
                         concat("**",convert(convert(dpol)@INT)@String )$String
                if zero? tpol
                  then
                    stpol :=  empty()$String
                  else
                  if ground?(tpol)
                    then
                      n := retract(ground(tpol))@INT
                      if n > 0
                        then
                          stpol :=  concat(" +",convert(n)@String)$String
                        else
                          stpol := convert(n)@String
                    else
                      stpol := convert(tpol)@String
                      if _
                        not member?((stpol.1)::String,["+","-"])$(List String)
                        then
                          stpol :=  concat(" + ",stpol)$String
              concat([sipol,svpol,sdpol,stpol])$String
```

⟨*RPOLCAT.dotabb*⟩≡

```
"RPOLCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RPOLCAT" -> "POLYCAT"
```

⟨*RPOLCAT.dotfull*⟩≡

```
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

⟨*RPOLCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

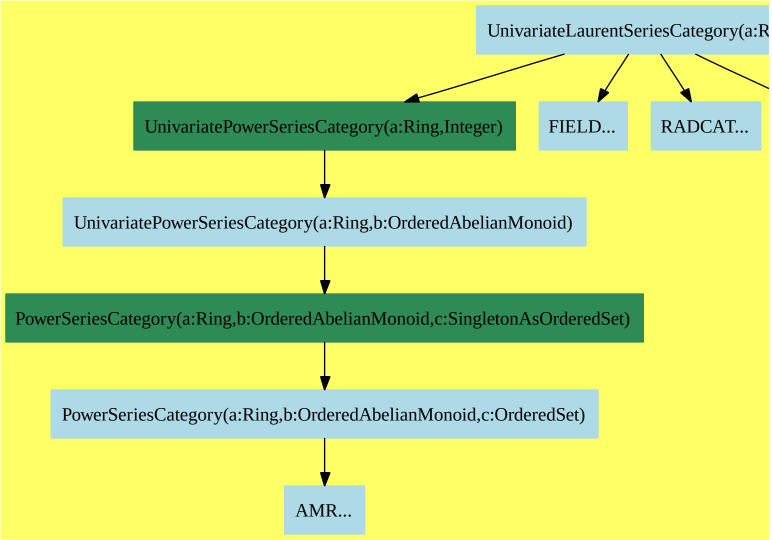
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "KONVERT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PATMAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "COMRING..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"COMRING..." [color=lightblue];

}
```

17.0.202 UnivariateLaurentSeriesCategory (ULSCAT)


UnivariateLaurentSeriesCategory(a:R
UnivariatePowerSeriesCategory(a:Ring,Integer)
FIELD...
RADCAT...
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AMR...

⟨*UnivariateLaurentSeriesCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariateLaurentSeriesCategory.output
)spool UnivariateLaurentSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeriesCategory
--R UnivariateLaurentSeriesCategory Coef: Ring  is a category constructor
--R Abbreviation for UnivariateLaurentSeriesCategory is ULSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ULSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficient : (%,Integer) -> Coef     coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
```

```
--R degree : % -> Integer                 ?.? : (%,Integer) -> Coef
--R extend : (%,Integer) -> %             hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Integer) -> %        monomial? : % -> Boolean
--R one? : % -> Boolean                   order : (%,Integer) -> Integer
--R order : % -> Integer                  pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      truncate : (%,Integer) -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Integer,Coef)
--R D : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Integer,C
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL an
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
```

```
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Integer,Coef)
--R differentiate : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Integer,
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef ha
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL a
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FI
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R integrate : (%,Symbol) -> % if Coef has ACFS INT and Coef has PRIMCAT and Coef has TRAN
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R ?rem? : (%,%) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDO
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariateLaurentSeriesCategory.help*⟩≡

```
====================================================================
UnivariateLaurentSeriesCategory examples
====================================================================

See Also:
o )show UnivariateLaurentSeriesCategory
```

See:

⇒ "UnivariateLaurentSeriesConstructorCategory" (ULSCCAT) 18.0.211 on page 1709

⇐ "Field" (FIELD) 16.0.185 on page 1347
⇐ "RadicalCategory" (RADCAT) 2.0.17 on page 59
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.0.35 on page 135
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.0.184 on page 1334

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyCoefficients	multiplyExponents
nthRoot	one?	order	pi
pole?	prime?	principalIdeal	rationalFunction
recip	reductum	sample	sec
sech	series	sin	sinh
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	tan	tanh	terms
truncate	unit?	unitCanonical	unitNormal
variable	variables	zero?	?*?
?**?	?+?	?-? * -?	
?=?	?^?	?~=?	?/?
?.?	?quo?	?rem?	

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
rationalFunction : (%,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
series : Stream Record(k: Integer,c: Coef) -> %
```

These exports come from (p1334) UnivariatePowerSeriesCategory(Coef,Integer) where Coef:Ring:

```
0 : () -> %
1 : () -> %
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
D : % -> % if Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
degree : % -> Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : % -> %
  if Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exquo : (%,%) -> Union(%,"failed")
  if Coef has INTDOM
extend : (%,Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (Coef,Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
```

```
order : (%,Integer) -> Integer
order : % -> Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?.? : (%,Integer) -> Coef
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Integer has SGROUP
```

These exports come from (p135) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
```

```
atanh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1347) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger
  if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
inv : % -> % if Coef has FIELD
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
```

```
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

⟨*category ULSCAT UnivariateLaurentSeriesCategory*⟩≡

```
)abbrev category ULSCAT UnivariateLaurentSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent
++ Examples:
++ References:
++ Description:
++ \spadtype{UnivariateLaurentSeriesCategory} is the category of
++ Laurent series in one variable.

UnivariateLaurentSeriesCategory(Coef): Category == Definition where
  Coef  : Ring
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  Term ==> Record(k:I,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,Integer) with

    series: Stream Term -> %
      ++ \spad{series(st)} creates a series from a stream of non-zero terms,
      ++ where a term is an exponent-coefficient pair.  The terms in the
      ++ stream should be ordered by increasing order of exponents.
    multiplyCoefficients: (I -> Coef,%) -> %
      ++ \spad{multiplyCoefficients(f,sum(n = n0..infinity,a[n] * x**n)) =
      ++ sum(n = 0..infinity,f(n) * a[n] * x**n)}.
      ++ This function is used when Puiseux series are represented by
      ++ a Laurent series and an exponent.
    if Coef has IntegralDomain then
      rationalFunction: (%,I) -> Fraction Polynomial Coef
      ++ \spad{rationalFunction(f,k)} returns a rational function
      ++ consisting of the sum of all terms of f of degree <= k.
    rationalFunction: (%,I,I) -> Fraction Polynomial Coef
      ++ \spad{rationalFunction(f,k1,k2)} returns a rational function
      ++ consisting of the sum of all terms of f of degree d with
      ++ \spad{k1 <= d <= k2}.

  if Coef has Algebra Fraction Integer then
    integrate: % -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 1.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.
    if Coef has integrate: (Coef,Symbol) -> Coef and _
       Coef has variables: Coef -> List Symbol then
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of the power
        ++ series \spad{f(x)} with respect to the variable \spad{y}.
    if Coef has TranscendentalFunctionCategory and _
       Coef has PrimitiveFunctionCategory and _
       Coef has AlgebraicallyClosedFunctionSpace Integer then
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of
        ++ the power series \spad{f(x)} with respect to the variable
        ++ \spad{y}.
    RadicalCategory
      --++ We provide rational powers when we can divide coefficients
      --++ by integers.
    TranscendentalFunctionCategory
      --++ We provide transcendental functions when we can divide
      --++ coefficients by integers.
  if Coef has Field then Field
      --++ Univariate Laurent series over a field form a field.
      --++ In fact, K((x)) is the quotient field of K[[x]].
```

⟨*ULSCAT.dotabb*⟩≡

```
"ULSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"ULSCAT" -> "UPSCAT"
"ULSCAT" -> "FIELD"
"ULSCAT" -> "TRANFUN"
"ULSCAT" -> "RADCAT"
```

⟨*ULSCAT.dotfull*⟩≡

```
"UnivariateLaurentSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "Field()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RadicalCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TranscendentalFunctionCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
```

⟨*ULSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

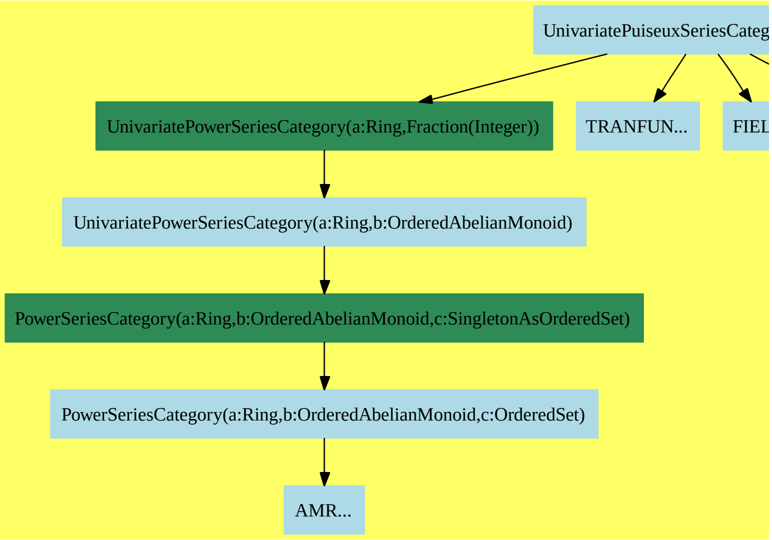
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

17.0.203 UnivariatePuiseuxSeriesCategory (UPXSCAT)


UnivariatePuiseuxSeriesCateg
UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))
TRANFUN...
FIEL
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AMR...

⟨*UnivariatePuiseuxSeriesCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesCategory.output
)spool UnivariatePuiseuxSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesCategory
--R UnivariatePuiseuxSeriesCategory Coef: Ring  is a category constructor
--R Abbreviation for UnivariatePuiseuxSeriesCategory is UPXSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPXSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Fraction Integer
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial? : % -> Boolean
--R one? : % -> Boolean                   order : % -> Fraction Integer
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fraction Integer,Coef) ->
--R D : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fraction Integer,Coe
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has *: (Fracti
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,
--R differentiate : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fr
--R differentiate : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRI
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coe
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef h
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if C
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R integrate : (%,Symbol) -> % if Coef has ACFS INT and Coef has PRIMCAT and Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIEL
--R ?quo? : (%,%) -> % if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariatePuiseuxSeriesCategory.help*⟩≡

```
====================================================================
UnivariatePuiseuxSeriesCategory examples
====================================================================

See Also:
o )show UnivariatePuiseuxSeriesCategory
```

See:

⇒ “UnivariatePuiseuxSeriesConstructorCategory” (UPXSCCA) 18.0.212 on page 1728

⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.0.35 on page 135
⇐ “Field” (FIELD) 16.0.185 on page 1347
⇐ “RadicalCategory” (RADCAT) 2.0.17 on page 59
⇐ “UnivariatePowerSeriesCategory” (UPSCAT) 15.0.184 on page 1334

Exports:

0
1
acos
acosh
acot
acoth
acsc
acsch
approximate
asec
asech
asin
asinh
associates?
atan
atanh
center
characteristic
charthRoot
coefficient
coerce
complete
cos
cosh
cot
coth
csc
csch
D
degree
differentiate
divide
euclideanSize
eval
exp
expressIdealMember
exquo
extend
extendedEuclidean
factor
gcd
gcdPolynomial
hash
integrate
inv
latex
lcm
leadingCoefficient
leadingMonomial
log
map
monomial
monomial?
multiEuclidean
multiplyExponents
nthRoot
one?
order
pi
pole?
prime?
principalIdeal
recip
reductum
sample
sec
sech
series
sin
sinh
sizeLess?
sqrt
squareFree
squareFreePart
subtractIfCan
tan
tanh
terms
truncate
unit?
unitCanonical
unitNormal
variable
variables
zero?
?*?
?**?
?+?
?-?
-?
?=?
?~=?
?/?
?^?
?.?
?quo?
?rem?

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyExponents : (%,Fraction Integer) -> %
series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
```

These exports come from (p1334) UnivariatePowerSeriesCategory(Coef,RN) where Coef:Ring and RN:Fraction(Integer):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
```

```
complete : % -> %
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
degree : % -> Fraction Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : % -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
```

```
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
```

These exports come from (p135) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
```

```
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1347) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
inv : % -> % if Coef has FIELD
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
```

```
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

⟨*category UPXSCAT UnivariatePuiseuxSeriesCategory*⟩≡

```
)abbrev category UPXSCAT UnivariatePuiseuxSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++ \spadtype{UnivariatePuiseuxSeriesCategory} is the category of Puiseux
++ series in one variable.

UnivariatePuiseuxSeriesCategory(Coef): Category == Definition where
  Coef : Ring
  NNI  ==> NonNegativeInteger
  RN   ==> Fraction Integer
  Term ==> Record(k:RN,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,RN) with

    series: (NNI,Stream Term) -> %
      ++ \spad{series(n,st)} creates a series from a common denomiator and
      ++ a stream of non-zero terms, where a term is an exponent-coefficient
      ++ pair.  The terms in the stream should be ordered by increasing order
      ++ of exponents and \spad{n} should be a common denominator for the
      ++ exponents in the stream of terms.
    multiplyExponents: (%,Fraction Integer) -> %
      ++ \spad{multiplyExponents(f,r)} multiplies all exponents of the power
      ++ series f by the positive rational number r.

    if Coef has Algebra Fraction Integer then
      integrate: % -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 1.
        ++ We may integrate a series when we can divide coefficients
        ++ by rational numbers.
      if Coef has integrate: (Coef,Symbol) -> Coef and _
         Coef has variables: Coef -> List Symbol then
        integrate: (%,Symbol) -> %
          ++ \spad{integrate(f(x),var)} returns an anti-derivative of the power
          ++ series \spad{f(x)} with respect to the variable \spad{var}.
      if Coef has TranscendentalFunctionCategory and _
         Coef has PrimitiveFunctionCategory and _
         Coef has AlgebraicallyClosedFunctionSpace Integer then
        integrate: (%,Symbol) -> %
          ++ \spad{integrate(f(x),y)} returns an anti-derivative of
          ++ the power series \spad{f(x)} with respect to the variable
          ++ \spad{y}.
      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.
      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.
    if Coef has Field then Field
        --++ Univariate Puiseux series over a field form a field.
```

⟨*UPXSCAT.dotabb*⟩≡

```
  "UPXSCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
  "UPXSCAT" -> "TRANFUN"
  "UPXSCAT" -> "FIELD"
  "UPXSCAT" -> "RADCAT"
  "UPXSCAT" -> "UPSCAT"
```

⟨*UPXSCAT.dotfull*⟩≡

```
  "UnivariatePuiseuxSeriesCategory(a:Ring)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
  "UnivariatePuiseuxSeriesCategory(a:Ring)" ->
      "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
  "UnivariatePuiseuxSeriesCategory(a:Ring)" ->
      "TranscendentalFunctionCategory()"
  "UnivariatePuiseuxSeriesCategory(a:Ring)" ->
      "Field()"
  "UnivariatePuiseuxSeriesCategory(a:Ring)" ->
      "RadicalCategory()"
```

⟨*UPXSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

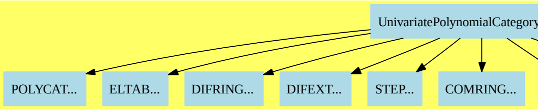
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
"RADCAT..." [color=lightblue];
"AMR..." [color=lightblue];
}
```

17.0.204 UnivariatePolynomialCategory (UPOLYC)



⟨*UnivariatePolynomialCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariatePolynomialCategory.output
)spool UnivariatePolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePolynomialCategory
--R UnivariatePolynomialCategory R: Ring  is a category constructor
--R Abbreviation for UnivariatePolynomialCategory is UPOLYC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPOLYC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> %
--R D : (%,NonNegativeInteger) -> %       1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      differentiate : % -> %
--R ?.? : (%,%) -> %                      ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             init : () -> % if R has STEP
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R monomial? : % -> Boolean              monomials : % -> List %
--R one? : % -> Boolean                   primitiveMonomials : % -> List %
--R pseudoRemainder : (%,%) -> %          recip : % -> Union(%,"failed")
--R reductum : % -> %                     retract : % -> R
--R sample : () -> %                      zero? : % -> Boolean
```

```
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,Polynomial
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT or R has ALGEBRA FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),ha
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R ha
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R ha
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
```

```
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeIn
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
```

```
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyn
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomi
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariatePolynomialCategory.help*⟩≡

```
====================================================================
UnivariatePolynomialCategory examples
====================================================================

See Also:
o )show UnivariatePolynomialCategory
```

See:

⇐ "Algebra" (ALGEBRA) 11.0.152 on page 1045
⇐ "CommutativeRing" (COMRING) 10.0.137 on page 932
⇐ "DifferentialExtension" (DIFEXT) 11.0.153 on page 1053
⇐ "DifferentialRing" (DIFRING) 10.0.138 on page 938
⇐ "Eltable" (ELTAB) 2.0.10 on page 32
⇐ "Field" (FIELD) 16.0.185 on page 1347
⇐ "GcdDomain" (GCDDOM) 13.0.173 on page 1246
⇐ "IntegralDomain" (INTDOM) 12.0.164 on page 1153
⇐ "PolynomialCategory" (POLYCAT) 16.0.189 on page 1385
⇐ "PolynomialFactorizationExplicit" (PFECAT) 15.0.183 on page 1325
⇐ "StepThrough" (STEP) 4.0.64 on page 283

Exports:

0
1
associates?
binomThmExpt
characteristic
charthRoot
coefficient
coefficients
coerce
composite
conditionP
content
convert
D
degree
differentiate
discriminant
divide
divideExponents
elt
euclideanSize
eval
expressIdealMember
exquo
extendedEuclidean
factor
factorPolynomial
factorSquareFreePolynomial
gcd
gcdPolynomial
ground
ground?
hash
init
integrate
isExpt
isPlus
isTimes
karatsubaDivide
latex
lcm
leadingCoefficient
leadingMonomial
mainVariable
makeSUP
map
mapExponents
max
min
minimumDegree
monicDivide
monomial
monomial?
monomials
multiEuclidean
multiplyExponents
multivariate
nextItem
numberOfMonomials
one?
order
patternMatch
pomopo!
prime?
primitiveMonomials
primitivePart
principalIdeal
pseudoDivide
pseudoQuotient
pseudoRemainder
recip
reducedSystem
reductum
resultant
retract
retractIfCan
sample
separate
shiftLeft
shiftRight
sizeLess?
solveLinearPolynomialEquation
squareFree
squareFreePart
squareFreePolynomial
subResultantGcd
subtractIfCan
totalDegree
unit?
unitCanonical
unitNormal
univariate
unmakeSUP
variables
vectorise
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?.?
?~=?
?/?
?<?
?<=?
?>?
?>=?
?quo?
?rem?

Attributes exported:

- if \$ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D)- > D$ which is commutative.
- if \$ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if R has Field then additiveValuation where **additiveValuation** implies

`euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.

- if $ has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
discriminant : % -> R if R has COMRING
divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
monicDivide : (%,%) -> Record(quotient: %,remainder: %)
multiplyExponents : (%,NonNegativeInteger) -> %
pseudoRemainder : (%,%) -> %
resultant : (%,%) -> R if R has COMRING
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
characteristic : () -> NonNegativeInteger
coefficient : (%,NonNegativeInteger) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : SingletonAsOrderedSet -> %
composite : (Fraction %,%) -> Union(Fraction %,"failed")
    if R has INTDOM
composite : (%,%) -> Union(%,"failed")
    if R has INTDOM
content : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
```

```
D : (%,List SingletonAsOrderedSet) -> %
D : (%,SingletonAsOrderedSet) -> %
D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
degree : % -> NonNegativeInteger
degree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
differentiate :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
differentiate :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
differentiate : (%,List SingletonAsOrderedSet) -> %
differentiate : (%,(R -> R),%) -> %
differentiate : (%,(R -> R)) -> %
differentiate : % -> %
differentiate : (%,SingletonAsOrderedSet) -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
elt : (Fraction %,Fraction %) -> Fraction %
    if R has INTDOM
elt : (Fraction %,R) -> R if R has FIELD
euclideanSize : % -> NonNegativeInteger
    if R has FIELD
eval : (%,List SingletonAsOrderedSet,List %) -> %
eval : (%,SingletonAsOrderedSet,%) -> %
eval : (%,List SingletonAsOrderedSet,List R) -> %
eval : (%,SingletonAsOrderedSet,R) -> %
eval : (%,List Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
exquo : (%,R) -> Union(%,"failed")
    if R has INTDOM
exquo : (%,%) -> Union(%,"failed")
    if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
```

```
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
init : () -> % if R has STEP
integrate : % -> % if R has ALGEBRA FRAC INT
karatsubaDivide :
  (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
makeSUP : % -> SparseUnivariatePolynomial R
map : ((R -> R),%) -> %
mapExponents :
  ((NonNegativeInteger -> NonNegativeInteger),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree :
  (%,SingletonAsOrderedSet) -> NonNegativeInteger
minimumDegree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial? : % -> Boolean
nextItem : % -> Union(%,"failed") if R has STEP
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
order : (%,%) -> NonNegativeInteger
    if R has INTDOM
pomopo! : (%,R,NonNegativeInteger,%) -> %
prime? : % -> Boolean if R has PFECAT
pseudoDivide :
  (%,%) -> Record(coef: R,quotient: %,remainder: %)
    if R has INTDOM
pseudoQuotient : (%,%) -> % if R has INTDOM
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
```

```
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(R,"failed")
sample : () -> %
separate : (%,%) -> Record(primePart: %,commonPart: %)
    if R has GCDDOM
shiftLeft : (%,NonNegativeInteger) -> %
shiftRight : (%,NonNegativeInteger) -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
       if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree :
  (%,List SingletonAsOrderedSet) -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
unmakeSUP : SparseUnivariatePolynomial R -> %
variables : % -> List SingletonAsOrderedSet
vectorise : (%,NonNegativeInteger) -> Vector R
zero? : % -> Boolean
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
```

```
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?.? : (%,Fraction %) -> Fraction %
    if R has INTDOM
```

These exports come from (p1385) PolynomialCategory(R,N,S)
where R:Ring, N:NonNegativeInteger, S:SingletonAsOrderedSet:

```
charthRoot : % -> Union(%,"failed")
  if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
coefficient :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if and(has($,CharacteristicNonZero),
        has(R,PolynomialFactorizationExplicit))
convert : % -> Pattern Integer
    if SingletonAsOrderedSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if SingletonAsOrderedSet has KONVERT PATTERN FLOAT
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if SingletonAsOrderedSet has KONVERT INFORM
    and R has KONVERT INFORM
degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
discriminant : (%,SingletonAsOrderedSet) -> %
    if R has COMRING
isExpt : % ->
  Union(
   Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),
  "failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
minimumDegree : % -> NonNegativeInteger
monicDivide :
  (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
monomial :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomials : % -> List %
multivariate :
  (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
multivariate :
  (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
```

```
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
    if SingletonAsOrderedSet has PATMAB INT
    and R has PATMAB INT
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
    if SingletonAsOrderedSet has PATMAB FLOAT
    and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,SingletonAsOrderedSet) -> %
    if R has COMRING
retract : % -> SingletonAsOrderedSet
retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
totalDegree : % -> NonNegativeInteger
univariate : % -> SparseUnivariatePolynomial R
univariate :
  (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p32) Eltable(R:Ring,R:Ring):

```
?.? : (%,R) -> R
```

These exports come from (p32) Eltable(R:UPOLYC,R:UPOLYC):

```
?.? : (%,%) -> %
```

These exports come from (p938) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p1053) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

```
 D : (%,List Symbol,List NonNegativeInteger) -> %
     if R has PDRING SYMBOL
 differentiate : (%,(R -> R),NonNegativeInteger) -> %
 differentiate : (%,Symbol) -> %
     if R has PDRING SYMBOL
 differentiate : (%,List Symbol) -> %
     if R has PDRING SYMBOL
 differentiate : (%,Symbol,NonNegativeInteger) -> %
     if R has PDRING SYMBOL
 differentiate : (%,List Symbol,List NonNegativeInteger) -> %
     if R has PDRING SYMBOL
```

These exports come from (p283) StepThrough()

These exports come from (p932) CommutativeRing()

These exports come from (p1153) IntegralDomain()

These exports come from (p1246) GcdDomain()

These exports come from (p1347) Field()

```
 expressIdealMember : (List %,%) -> Union(List %,"failed")
     if R has FIELD
 extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
     if R has FIELD
 extendedEuclidean :
   (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
     if R has FIELD
 multiEuclidean : (List %,%) -> Union(List %,"failed")
     if R has FIELD
 principalIdeal : List % -> Record(coef: List %,generator: %)
     if R has FIELD
 sizeLess? : (%,%) -> Boolean if R has FIELD
 ?quo? : (%,%) -> % if R has FIELD
 ?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p1045) Algebra(Fraction(Integer))

These exports come from (p1325) PolynomialFactorizationExplicit()

⟨*category UPOLYC UnivariatePolynomialCategory*⟩≡

```
)abbrev category UPOLYC UnivariatePolynomialCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of univariate polynomials over a ring R.
++ No particular model is assumed - implementations can be either
++ sparse or dense.

UnivariatePolynomialCategory(R:Ring): Category ==
 Join(PolynomialCategory(R, NonNegativeInteger, SingletonAsOrderedSet),
      Eltable(R, R), Eltable(%, %), DifferentialRing,
      DifferentialExtension R) with
    vectorise        : (%,NonNegativeInteger) -> Vector R
      ++ vectorise(p, n) returns \spad{[a0,...,a(n-1)]} where
      ++ \spad{p = a0 + a1*x + ... + a(n-1)*x**(n-1)} + higher order terms.
      ++ The degree of polynomial p can be different from \spad{n-1}.
    makeSUP: % -> SparseUnivariatePolynomial R
      ++ makeSUP(p) converts the polynomial p to be of type
      ++ SparseUnivariatePolynomial over the same coefficients.
    unmakeSUP: SparseUnivariatePolynomial R -> %
      ++ unmakeSUP(sup) converts sup of type
      ++ \spadtype{SparseUnivariatePolynomial(R)}
      ++ to be a member of the given type.
      ++ Note that converse of makeSUP.
    multiplyExponents: (%,NonNegativeInteger) -> %
      ++ multiplyExponents(p,n) returns a new polynomial resulting from
      ++ multiplying all exponents of the polynomial p by the non negative
      ++ integer n.
    divideExponents: (%,NonNegativeInteger) -> Union(%,"failed")
      ++ divideExponents(p,n) returns a new polynomial resulting from
      ++ dividing all exponents of the polynomial p by the non negative
      ++ integer n, or "failed" if some exponent is not exactly divisible
      ++ by n.
    monicDivide: (%,%) -> Record(quotient:%,remainder:%)
```

```
      ++ monicDivide(p,q) divide the polynomial p by the monic polynomial q,
      ++ returning the pair \spad{[quotient, remainder]}.
      ++ Error: if q isn't monic.
-- These three are for Karatsuba
    karatsubaDivide: (%,NonNegativeInteger) -> Record(quotient:%,remainder:%)
      ++ \spad{karatsubaDivide(p,n)} returns the same as
      ++ \spad{monicDivide(p,monomial(1,n))}
    shiftRight: (%,NonNegativeInteger) -> %
      ++ \spad{shiftRight(p,n)} returns
      ++ \spad{monicDivide(p,monomial(1,n)).quotient}
    shiftLeft: (%,NonNegativeInteger) -> %
      ++ \spad{shiftLeft(p,n)} returns \spad{p * monomial(1,n)}
    pseudoRemainder: (%,%) -> %
       ++ pseudoRemainder(p,q) = r, for polynomials p and q, returns the
       ++ remainder when
       ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
       ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
    differentiate: (%, R -> R, %) -> %
       ++ differentiate(p, d, x') extends the R-derivation d to an
       ++ extension D in \spad{R[x]} where Dx is given by x', and
       ++ returns \spad{Dp}.
    if R has StepThrough then StepThrough
    if R has CommutativeRing then
      discriminant: % -> R
        ++ discriminant(p) returns the discriminant of the polynomial p.
      resultant: (%,%) -> R
        ++ resultant(p,q) returns the resultant of the polynomials p and q.
    if R has IntegralDomain then
        Eltable(Fraction %, Fraction %)
        elt  : (Fraction %, Fraction %) -> Fraction %
             ++ elt(a,b) evaluates the fraction of univariate polynomials
             ++ \spad{a} with the distinguished variable replaced by b.
        order: (%, %) -> NonNegativeInteger
             ++ order(p, q) returns the largest n such that \spad{q**n}
             ++ divides polynomial p
             ++ i.e. the order of \spad{p(x)} at \spad{q(x)=0}.
        subResultantGcd: (%,%) -> %
           ++ subResultantGcd(p,q) computes the gcd of the polynomials p
           ++ and q using the SubResultant GCD algorithm.
        composite: (%, %) -> Union(%, "failed")
           ++ composite(p, q) returns h if \spad{p = h(q)}, and "failed"
           ++ no such h exists.
        composite: (Fraction %, %) -> Union(Fraction %, "failed")
           ++ composite(f, q) returns h if f = h(q), and "failed" is
           ++ no such h exists.
        pseudoQuotient: (%,%) -> %
```

```
          ++ pseudoQuotient(p,q) returns r, the quotient when
          ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
          ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
       pseudoDivide: (%, %) -> Record(coef:R, quotient: %, remainder:%)
          ++ pseudoDivide(p,q) returns \spad{[c, q, r]}, when
          ++ \spad{p' := p*lc(q)**(deg p - deg q + 1) = c * p}
          ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
   if R has GcdDomain then
       separate: (%, %) -> Record(primePart:%, commonPart: %)
          ++ separate(p, q) returns \spad{[a, b]} such that polynomial
          ++ \spad{p = a b} and \spad{a} is relatively prime to q.
   if R has Field then
       EuclideanDomain
       additiveValuation
         ++ euclideanSize(a*b) = euclideanSize(a) + euclideanSize(b)
       elt      : (Fraction %, R) -> R
         ++ elt(a,r) evaluates the fraction of univariate polynomials
         ++ \spad{a} with the distinguished variable replaced by the
         ++ constant r.
   if R has Algebra Fraction Integer then
     integrate: % -> %
       ++ integrate(p) integrates the univariate polynomial p with respect
       ++ to its distinguished variable.
 add
   pp,qq: SparseUnivariatePolynomial %

   variables(p) ==
     zero? p or zero?(degree p) => []
     [create()]

   degree(p:%,v:SingletonAsOrderedSet) == degree p

   totalDegree(p:%,lv:List SingletonAsOrderedSet) ==
      empty? lv => 0
      totalDegree p

   degree(p:%,lv:List SingletonAsOrderedSet) ==
      empty? lv => []
      [degree p]

   eval(p:%,lv: List SingletonAsOrderedSet,lq: List %):% ==
     empty? lv => p
     not empty? rest lv => _
       error "can only eval a univariate polynomial once"
     eval(p,first lv,first lq)$%
```

```
eval(p:%,v:SingletonAsOrderedSet,q:%):% == p(q)

eval(p:%,lv: List SingletonAsOrderedSet,lr: List R):% ==
  empty? lv => p
  not empty? rest lv => _
     error "can only eval a univariate polynomial once"
  eval(p,first lv,first lr)$%

eval(p:%,v:SingletonAsOrderedSet,r:R):% == p(r)::%

eval(p:%,le:List Equation %):% ==
  empty? le  => p
  not empty? rest le => _
     error "can only eval a univariate polynomial once"
  mainVariable(lhs first le) case "failed" => p
  p(rhs first le)

mainVariable(p:%) ==
  zero? degree p =>  "failed"
  create()$SingletonAsOrderedSet

minimumDegree(p:%,v:SingletonAsOrderedSet) == minimumDegree p

minimumDegree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => []
   [minimumDegree p]

monomial(p:%,v:SingletonAsOrderedSet,n:NonNegativeInteger) ==
   mapExponents(x1+->x1+n,p)

coerce(v:SingletonAsOrderedSet):% == monomial(1,1)

makeSUP p ==
  zero? p => 0
  monomial(leadingCoefficient p,degree p) + makeSUP reductum p

unmakeSUP sp ==
  zero? sp => 0
  monomial(leadingCoefficient sp,degree sp) + unmakeSUP reductum sp

karatsubaDivide(p:%,n:NonNegativeInteger) == monicDivide(p,monomial(1,n))

shiftRight(p:%,n:NonNegativeInteger) ==
   monicDivide(p,monomial(1,n)).quotient

shiftLeft(p:%,n:NonNegativeInteger) == p * monomial(1,n)
```

```
if R has PolynomialFactorizationExplicit then
   PFBRU ==> PolynomialFactorizationByRecursionUnivariate(R,%)
   pp,qq:SparseUnivariatePolynomial %
   lpp:List SparseUnivariatePolynomial %
   SupR ==> SparseUnivariatePolynomial R
   sp:SupR

   solveLinearPolynomialEquation(lpp,pp) ==
     solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBRU

   factorPolynomial(pp) ==
     factorByRecursion(pp)$PFBRU

   factorSquareFreePolynomial(pp) ==
     factorSquareFreeByRecursion(pp)$PFBRU

   import FactoredFunctions2(SupR,S)

   factor p ==
     zero? degree p  =>
       ansR:=factor leadingCoefficient p
       makeFR(unit(ansR)::%,
              [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
     map(unmakeSUP,factorPolynomial(makeSUP p)$R)

vectorise(p, n) ==
  m := minIndex(v := new(n, 0)$Vector(R))
  for i in minIndex v .. maxIndex v repeat
    qsetelt_!(v, i, coefficient(p, (i - m)::NonNegativeInteger))
  v

retract(p:%):R ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  error "Polynomial is not of degree 0"

retractIfCan(p:%):Union(R, "failed") ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  "failed"

if R has StepThrough then
   init() == init()$R::%

   nextItemInner: % -> Union(%,"failed")
```

```
     nextItemInner(n) ==
       zero? n => nextItem(0$R)::R::% -- assumed not to fail
       zero? degree n =>
         nn:=nextItem leadingCoefficient n
         nn case "failed" => "failed"
         nn::R::%
       n1:=reductum n
       n2:=nextItemInner n1 -- try stepping the reductum
       n2 case % => monomial(leadingCoefficient n,degree n) + n2
       1+degree n1 < degree n => -- there was a hole between lt n and n1
         monomial(leadingCoefficient n,degree n)+
           monomial(nextItem(init()$R)::R,1+degree n1)
       n3:=nextItem leadingCoefficient n
       n3 case "failed" => "failed"
       monomial(n3,degree n)

     nextItem(n) ==
       n1:=nextItemInner n
       n1 case "failed" => monomial(nextItem(init()$R)::R,1+degree(n))
       n1

  if R has GcdDomain then

    content(p:%,v:SingletonAsOrderedSet) == content(p)::%

    primeFactor: (%, %) -> %

    primeFactor(p, q) ==
      (p1 := (p exquo gcd(p, q))::%) = p => p
      primeFactor(p1, q)

    separate(p, q) ==
      a := primeFactor(p, q)
      [a, (p exquo a)::%]

  if R has CommutativeRing then
    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + x' * monomial(dg * lc, (dg - 1)::NonNegativeInteger)
               + monomial(deriv lc, dg)
        x := reductum x
      d + deriv(leadingCoefficient x)::%
  else
```

```
    ncdiff: (NonNegativeInteger, %) -> %
    -- computes d(x**n) given dx = x', non-commutative case
    ncdiff(n, x') ==
      zero? n => 0
      zero?(n1 := (n - 1)::NonNegativeInteger) => x'
      x' * monomial(1, n1) + monomial(1, 1) * ncdiff(n1, x')

    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + monomial(deriv lc, dg) + lc * ncdiff(dg, x')
        x := reductum x
      d + deriv(leadingCoefficient x)::%

  differentiate(x:%, deriv:R -> R) == differentiate(x, deriv, 1$%)$%

  differentiate(x:%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        d:=d+monomial(dg*leadingCoefficient x,(dg-1)::NonNegativeInteger)
        x := reductum x
      d

  differentiate(x:%,v:SingletonAsOrderedSet) == differentiate x

  if R has IntegralDomain then

    elt(g:Fraction %, f:Fraction %) == ((numer g) f) / ((denom g) f)

    pseudoQuotient(p, q) ==
      (n := degree(p)::Integer - degree q + 1) < 1 => 0
      ((leadingCoefficient(q)**(n::NonNegativeInteger) * p
        - pseudoRemainder(p, q)) exquo q)::%

    pseudoDivide(p, q) ==
      (n := degree(p)::Integer - degree q + 1) < 1 => [1, 0, p]
      prem := pseudoRemainder(p, q)
      lc   := leadingCoefficient(q)**(n::NonNegativeInteger)
      [lc,((lc*p - prem) exquo q)::%, prem]

    composite(f:Fraction %, q:%) ==
      (n := composite(numer f, q)) case "failed" => "failed"
      (d := composite(denom f, q)) case "failed" => "failed"
      n::% / d::%
  composite(p:%, q:%) ==
    ground? p => p
    cqr := pseudoDivide(p, q)
    ground?(cqr.remainder) and
      ((v := cqr.remainder exquo cqr.coef) case %) and
        ((u := composite(cqr.quotient, q)) case %) and
          ((w := (u::%) exquo cqr.coef) case %) =>
            v::% + monomial(1, 1) * w::%
    "failed"

  elt(p:%, f:Fraction %) ==
    zero? p => 0
    ans:Fraction(%) := (leadingCoefficient p)::%::Fraction(%)
    n := degree p
    while not zero?(p:=reductum p) repeat
      ans := ans * f ** (n - (n := degree p))::NonNegativeInteger +
                (leadingCoefficient p)::%::Fraction(%)
    zero? n => ans
    ans * f ** n

  order(p, q) ==
    zero? p => error "order: arguments must be nonzero"
    degree(q) < 1 => error "order: place must be non-trivial"
    ans:NonNegativeInteger := 0
    repeat
      (u  := p exquo q) case "failed" => return ans
      p   := u::%
      ans := ans + 1

if R has GcdDomain then
  squareFree(p:%) ==
    squareFree(p)$UnivariatePolynomialSquareFree(R, %)

  squareFreePart(p:%) ==
    squareFreePart(p)$UnivariatePolynomialSquareFree(R, %)

if R has PolynomialFactorizationExplicit then

  gcdPolynomial(pp,qq) ==
        zero? pp => unitCanonical qq  -- subResultantGcd can't handle 0
        zero? qq => unitCanonical pp
        unitCanonical(gcd(content (pp),content(qq))*
               primitivePart
                  subResultantGcd(primitivePart pp,primitivePart qq))

  squareFreePolynomial pp ==
        squareFree(pp)$UnivariatePolynomialSquareFree(%,
                                   SparseUnivariatePolynomial %)

  if R has Field then
    elt(f:Fraction %, r:R) == ((numer f) r) / ((denom f) r)

    euclideanSize x ==
          zero? x =>
            error "euclideanSize called on 0 in Univariate Polynomial"
          degree x

    divide(x,y) ==
          zero? y => error "division by 0 in Univariate Polynomials"
          quot:=0
          lc := inv leadingCoefficient y
          while not zero?(x) and (degree x >= degree y) repeat
             f:=lc*leadingCoefficient x
             n:=(degree x - degree y)::NonNegativeInteger
             quot:=quot+monomial(f,n)
             x:=x-monomial(f,n)*y
          [quot,x]

  if R has Algebra Fraction Integer then

    integrate p ==
      ans:% := 0
      while p ^= 0 repeat
        l := leadingCoefficient p
        d := 1 + degree p
        ans := ans + inv(d::Fraction(Integer)) * monomial(l, d)
        p := reductum p
      ans
```

⟨*UPOLYC.dotabb*⟩≡

```
"UPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UPOLYC" -> "POLYCAT"
"UPOLYC" -> "ELTAB"
"UPOLYC" -> "DIFRING"
"UPOLYC" -> "DIFEXT"
"UPOLYC" -> "STEP"
"UPOLYC" -> "COMRING"
"UPOLYC" -> "INTDOM"
"UPOLYC" -> "GCDDOM"
"UPOLYC" -> "FIELD"
"UPOLYC" -> "ALGEBRA"
"UPOLYC" -> "PFECAT"
```

⟨*UPOLYC.dotfull*⟩≡

```
"UnivariatePolynomialCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:Ring,b:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:Ring))"
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialExtension(a:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "StepThrough()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "CommutativeRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "IntegralDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "GcdDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Field()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Algebra(Fraction(Integer))"
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialFactorizationExplicit()"
```

⟨*UPOLYC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePolynomialCategory(a:Ring)" [color=lightblue];
"UnivariatePolynomialCategory(a:Ring)" -> "POLYCAT..."
"UnivariatePolynomialCategory(a:Ring)" -> "ELTAB..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFEXT..."
"UnivariatePolynomialCategory(a:Ring)" -> "STEP..."
"UnivariatePolynomialCategory(a:Ring)" -> "COMRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "INTDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "GCDDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "FIELD..."
"UnivariatePolynomialCategory(a:Ring)" -> "ALGEBRA..."
"UnivariatePolynomialCategory(a:Ring)" -> "PFECAT..."

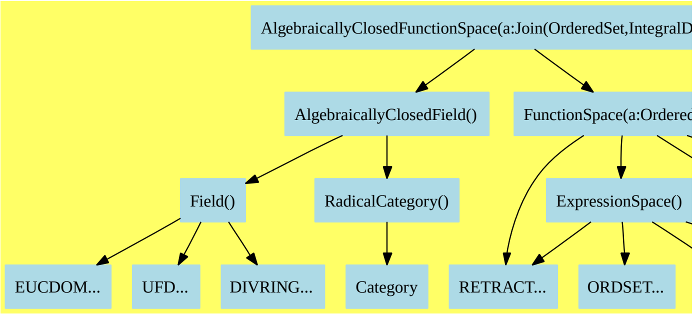
"POLYCAT..." [color=lightblue];
"ELTAB..." [color=lightblue];
"DIFRING..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"STEP..." [color=lightblue];
"COMRING..." [color=lightblue];
"INTDOM..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"ALGEBRA..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

Chapter 18

Category Layer 17

18.0.205 AlgebraicallyClosedFunctionSpace (ACFS)


AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralD
AlgebraicallyClosedField()
FunctionSpace(a:Orderec
Field()
RadicalCategory()
ExpressionSpace()
EUCDOM...
UFD...
DIVRING...
Category
RETRACT...
ORDSET...

⟨*AlgebraicallyClosedFunctionSpace.input*⟩≡

```
)set break resume
)sys rm -f AlgebraicallyClosedFunctionSpace.output
)spool AlgebraicallyClosedFunctionSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicallyClosedFunctionSpace
--R AlgebraicallyClosedFunctionSpace R: Join(OrderedSet,IntegralDomain)  is a category const
--R Abbreviation for AlgebraicallyClosedFunctionSpace is ACFS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ACFS
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        applyQuote : (Symbol,%) -> %
--R applyQuote : (Symbol,%,%) -> %        associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean    box : % -> %
--R box : List % -> %                     coerce : Kernel % -> %
--R coerce : Symbol -> %                  coerce : R -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R distribute : % -> %                   distribute : (%,%) -> %
--R elt : (BasicOperator,%) -> %          elt : (BasicOperator,%,%) -> %
--R eval : (%,Kernel %,%) -> %            eval : (%,List Equation %) -> %
--R eval : (%,Equation %) -> %            eval : (%,%,%) -> %
--R eval : (%,List %,List %) -> %         factor : % -> Factored %
--R freeOf? : (%,%) -> Boolean            freeOf? : (%,Symbol) -> Boolean
--R gcd : List % -> %                     gcd : (%,%) -> %
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             height : % -> NonNegativeInteger
--R inv : % -> %                          is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %       kernels : % -> List Kernel %
--R latex : % -> String                   lcm : List % -> %
--R lcm : (%,%) -> %                      map : ((% -> %),Kernel %) -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R nthRoot : (%,Integer) -> %            one? : % -> Boolean
--R paren : % -> %                        paren : List % -> %
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> Kernel %               retract : % -> Symbol
--R retract : % -> R                      rootOf : (%,Symbol) -> %
--R rootOf : % -> %                       rootOf : Polynomial % -> %
--R rootsOf : (%,Symbol) -> List %        rootsOf : % -> List %
--R rootsOf : Polynomial % -> List %      sample : () -> %
--R sizeLess? : (%,%) -> Boolean          sqrt : % -> %
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R subst : (%,Equation %) -> %           tower : % -> List Kernel %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R variables : % -> List Symbol          zero? : % -> Boolean
--R zeroOf : (%,Symbol) -> %              zeroOf : % -> %
```

```
--R zeroOf : Polynomial % -> %            zerosOf : (%,Symbol) -> List %
--R zerosOf : % -> List %                 zerosOf : Polynomial % -> List %
--R ?~=? : (%,%) -> Boolean
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (SparseMultivariatePolynomial(R,Kernel %),SparseMultivariatePolynomial(R,Kernel %)
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R D : (%,List Symbol) -> % if R has RING
--R D : (%,Symbol) -> % if R has RING
--R ?^? : (%,NonNegativeInteger) -> %
--R applyQuote : (Symbol,%,%,%) -> %
--R applyQuote : (Symbol,%,%,%,%) -> %
--R applyQuote : (Symbol,List %) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : SparseMultivariatePolynomial(R,Kernel %) -> % if R has RING
--R coerce : Fraction R -> % if R has INTDOM
--R coerce : Polynomial Fraction R -> % if R has INTDOM
--R coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
--R coerce : Fraction Polynomial R -> % if R has INTDOM
--R coerce : Polynomial R -> % if R has RING
--R commutator : (%,%) -> % if R has GROUP
--R conjugate : (%,%) -> % if R has GROUP
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R convert : Factored % -> % if R has INTDOM
--R convert : % -> InputForm if R has KONVERT INFORM
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
--R denominator : % -> % if R has INTDOM
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List Symbol) -> % if R has RING
--R differentiate : (%,Symbol) -> % if R has RING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R elt : (BasicOperator,%,%,%) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,List %) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,List Kernel %,List %) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
```

```
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,List Symbol) -> % if R has KONVERT INFORM
--R eval : % -> % if R has KONVERT INFORM
--R eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,List BasicOperator,List %,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> % if R has RI
--R eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> % if R h
--R eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R is? : (%,BasicOperator) -> Boolean
--R isExpt : % -> Union(Record(var: Kernel %,exponent: Integer),"failed") if R ha
--R isExpt : (%,BasicOperator) -> Union(Record(var: Kernel %,exponent: Integer),"
--R isExpt : (%,Symbol) -> Union(Record(var: Kernel %,exponent: Integer),"failed"
--R isMult : % -> Union(Record(coef: Integer,var: Kernel %),"failed") if R has AB
--R isPlus : % -> Union(List %,"failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %,exponent: Integer),"failed") if R has RING
--R isTimes : % -> Union(List %,"failed") if R has SGROUP
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
--R numerator : % -> % if R has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMa
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchR
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix Integer if and(has(R,Ring),has(R,LinearlyE
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R retract : % -> Fraction Integer if R has RETRACT INT and R has INTDOM or R ha
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Polynomial R if R has INTDOM
--R retract : % -> Polynomial R if R has RING
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT INT and R has INTD
--R retractIfCan : % -> Union(Kernel %,"failed")
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Polynomial R,"failed") if R has INTDOM
--R retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
--R rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R rootOf : SparseUnivariatePolynomial % -> %
--R rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R rootsOf : SparseUnivariatePolynomial % -> List %
--R subst : (%,List Equation %) -> %
--R subst : (%,List Kernel %,List %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial % if R has INTDOM
--R zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R zeroOf : SparseUnivariatePolynomial % -> %
--R zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R zerosOf : SparseUnivariatePolynomial % -> List %
--R
--E 1

)spool
)lisp (bye)
```

⟨*AlgebraicallyClosedFunctionSpace.help*⟩≡

```
====================================================================
AlgebraicallyClosedFunctionSpace examples
====================================================================

See Also:
o )show AlgebraicallyClosedFunctionSpace
```

See:

⇐ "AlgebraicallyClosedField" (ACF) 17.0.191 on page 1425
⇐ "FunctionSpace" (FS) 17.0.195 on page 1472

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
euclideanSize	eval	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	nthRoot
numer	numerator	odd?	one?
operator	operators	paren	patternMatch
prime?	principalIdeal	recip	reducedSystem
retract	retractIfCan	rootOf	rootsOf
sample	sizeLess?	sqrt	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	zeroOf	zerosOf
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **nil**

These are implemented by this category:

```
rootOf : % -> %
rootOf : (%,Symbol) -> %
rootsOf : % -> List %
rootsOf : (%,Symbol) -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zeroOf : % -> %
zeroOf : (%,Symbol) -> %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zerosOf : % -> List %
zerosOf : (%,Symbol) -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
```

These exports come from (p1425) AlgebraicallyClosedField():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
```

```
multiEuclidean : (List %,%) -> Union(List %,"failed")
nthRoot : (%,Integer) -> %
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
rootOf : SparseUnivariatePolynomial % -> %
rootOf : Polynomial % -> %
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : SparseUnivariatePolynomial % -> List %
rootsOf : Polynomial % -> List %
sample : () -> %
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
zeroOf : Polynomial % -> %
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1472) FunctionSpace(R)
where R:Join(OrderedSet, IntegralDomain)):

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
box : % -> %
box : List % -> %
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
  if R has RING
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
coerce : Polynomial R -> % if R has RING
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : Factored % -> % if R has INTDOM
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol) -> % if R has RING
definingPolynomial : % -> % if $ has RING
denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
distribute : % -> %
distribute : (%,%) -> %
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
eval : (%,List BasicOperator,List %,Symbol) -> %
  if R has KONVERT INFORM
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,%) -> Boolean
freeOf? : (%,Symbol) -> Boolean
ground : % -> R
ground? : % -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt :
  (%,BasicOperator) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isExpt :
  (%,Symbol) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % -> Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernel : (BasicOperator,List %) -> %
kernel : (BasicOperator,%) -> %
```

```
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
numerator : % -> % if R has RING
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
     PatternMatchResult(Integer,%)
       if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if R has PATMAB FLOAT
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
  or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
     if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
     or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
   if R has RING
reducedSystem : Matrix % -> Matrix R if R has RING
retract : % -> Kernel %
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Kernel %,"failed")
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
  if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial %
  if R has INTDOM
variables : % -> List Symbol
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (R,%) -> % if R has COMRING
?*? : (%,R) -> % if R has COMRING
?/? :
 (SparseMultivariatePolynomial(R,Kernel %),
  SparseMultivariatePolynomial(R,Kernel %)) -> %
    if R has INTDOM
```

⟨*category ACFS AlgebraicallyClosedFunctionSpace*⟩≡

```
)abbrev category ACFS AlgebraicallyClosedFunctionSpace
++ Author: Manuel Bronstein
++ Date Created: 31 October 1988
++ Date Last Updated: 7 October 1991
++ Keywords: algebraic, closure, field.
++ Description:
++ Model for algebraically closed function spaces.

AlgebraicallyClosedFunctionSpace(R:Join(OrderedSet, IntegralDomain)):
 Category == Join(AlgebraicallyClosedField, FunctionSpace R) with
   rootOf : $ -> $
     ++ rootOf(p) returns y such that \spad{p(y) = 0}.
     ++ Error: if p has more than one variable y.
   rootsOf: $ -> List $
     ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
     ++ Note that the returned symbols y1,...,yn are bound in the interpreter
     ++ to respective root values.
     ++ Error: if p has more than one variable y.
   rootOf : ($, Symbol) -> $
     ++ rootOf(p,y) returns y such that \spad{p(y) = 0}.
     ++ The object returned displays as \spad{'y}.
   rootsOf: ($, Symbol) -> List $
     ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
     ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
     ++ Note that the returned symbols y1,...,yn are bound in the interpreter
```

```
      ++ to respective root values.
    zeroOf : $ -> $
      ++ zeroOf(p) returns y such that \spad{p(y) = 0}.
      ++ The value y is expressed in terms of radicals if possible,and otherwise
      ++ as an implicit algebraic quantity.
      ++ Error: if p has more than one variable.
    zerosOf: $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++ Error: if p has more than one variable.
    zeroOf : ($, Symbol) -> $
      ++ zeroOf(p, y) returns y such that \spad{p(y) = 0}.
      ++ The value y is expressed in terms of radicals if possible,and otherwise
      ++ as an implicit algebraic quantity
      ++ which displays as \spad{'y}.
    zerosOf: ($, Symbol) -> List $
      ++ zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible, and otherwise
      ++ as implicit algebraic quantities
      ++ which display as \spad{'yi}.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
   add
     rootOf(p:$) ==
       empty?(l := variables p) => error "rootOf: constant expression"
       rootOf(p, first l)

     rootsOf(p:$) ==
       empty?(l := variables p) => error "rootsOf: constant expression"
       rootsOf(p, first l)

     zeroOf(p:$) ==
       empty?(l := variables p) => error "zeroOf: constant expression"
       zeroOf(p, first l)

     zerosOf(p:$) ==
       empty?(l := variables p) => error "zerosOf: constant expression"
       zerosOf(p, first l)

     zeroOf(p:$, x:Symbol) ==
       n := numer(f := univariate(p, kernel(x)$Kernel($)))
       degree denom f > 0 => error "zeroOf: variable appears in denom"
       degree n = 0 => error "zeroOf: constant expression"
       zeroOf(n, x)
    rootOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "roofOf: variable appears in denom"
      degree n = 0 => error "rootOf: constant expression"
      rootOf(n, x)

    zerosOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "zerosOf: variable appears in denom"
      degree n = 0 => empty()
      zerosOf(n, x)

    rootsOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "roofsOf: variable appears in denom"
      degree n = 0 => empty()
      rootsOf(n, x)

    rootsOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => rootsOf(r::$,y)
      rootsOf(p, y)$AlgebraicallyClosedField_&($)

    zerosOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => zerosOf(r::$,y)
      zerosOf(p, y)$AlgebraicallyClosedField_&($)

    zeroOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => zeroOf(r::$, y)
      zeroOf(p, y)$AlgebraicallyClosedField_&($)
```

⟨*ACFS.dotabb*⟩≡

```
"ACFS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"ACFS" -> "ACF"
"ACFS" -> "FS"
```

⟨*ACFS.dotfull*⟩≡

```
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"
```

⟨ACFS.dotpic⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
 [color=lightblue];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

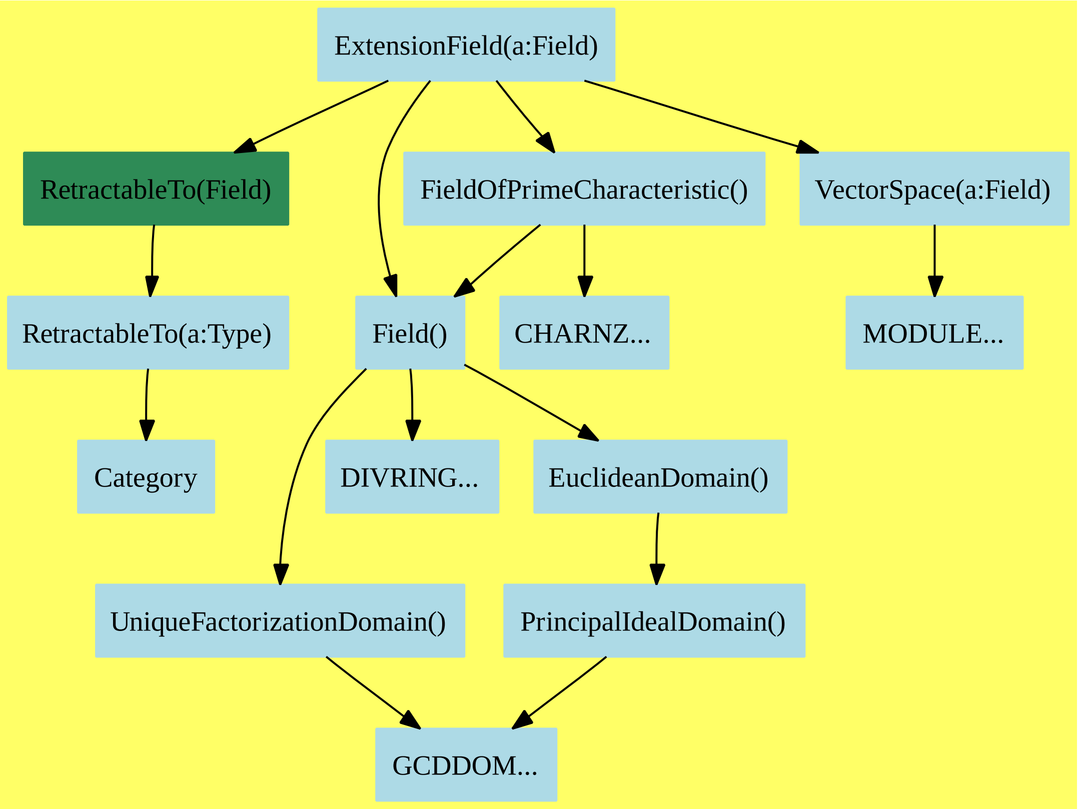
"Category" [color=lightblue];

"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "ORDSET..."
"ExpressionSpace()" -> "RETRACT..."
"ExpressionSpace()" -> "IEVALAB..."
"ExpressionSpace()" -> "EVALABLE..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"EVALABLE..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"ORDSET..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
}
```

18.0.206 ExtensionField (XF)


ExtensionField(a:Field)
RetractableTo(Field)
FieldOfPrimeCharacteristic()
VectorSpace(a:Field)
RetractableTo(a:Type)
Field()
CHARNZ...
MODULE...
Category
DIVRING...
EuclideanDomain()
UniqueFactorizationDomain()
PrincipalIdealDomain()
GCDDOM...

⟨*ExtensionField.input*⟩≡

```
)set break resume
)sys rm -f ExtensionField.output
)spool ExtensionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExtensionField
--R ExtensionField F: Field  is a category constructor
--R Abbreviation for ExtensionField is XF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (F,%) -> %                      ?*? : (%,F) -> %
```

```
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,F) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        coerce : F -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> F
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
--R Frobenius : % -> % if F has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if F has CHARNZ or F has FINITE
--R degree : % -> OnePointCompletion PositiveInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if F has CHARNZ or F has FINI
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R order : % -> OnePointCompletion PositiveInteger if F has CHARNZ or F has FINITE
--R primeFrobenius : % -> % if F has CHARNZ or F has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if F has CHARNZ or F has FINITE
```

```
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(F,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*ExtensionField.help*⟩≡

```
====================================================================
ExtensionField examples
====================================================================

See Also:
o )show ExtensionField
```

See:

⇒ “FiniteAlgebraicExtensionField” (FAXF) 19.0.213 on page 1741
⇐ “Field” (FIELD) 16.0.185 on page 1347
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “VectorSpace” (VSPACE) 11.0.158 on page 1089

Exports:

0
1
algebraic?
associates?
characteristic
charthRoot
coerce
degree
discreteLog
divide
euclideanSize
expressIdealMember
exquo
extendedEuclidean
extensionDegree
dimension
factor
Frobenius
gcd
gcdPolynomial
hash
inGroundField?
inv
latex
lcm
multiEuclidean
one?
order
prime?
primeFrbenius
principalIdeal
recip
retract
retractIfCan
sample
sizeLess?
squareFree
squareFreePart
subtractIfCan
transcendenceDegree
transcendent?
unit?
unitCanonical
unitNormal
zero?
?**?
?***?
?+?
?-?
-?
?/?
?=?
?^?
?quo?
?rem?
?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
inGroundField? : % -> Boolean
degree : % -> OnePointCompletion PositiveInteger
extensionDegree : () -> OnePointCompletion PositiveInteger
transcendenceDegree : () -> NonNegativeInteger
```

These are implemented by this category:

```
algebraic? : % -> Boolean
Frobenius : % -> % if F has FINITE
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
transcendent? : % -> Boolean
```

These exports come from (p1347) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
  Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
  Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p62) RetractableTo(F:Field):

```
coerce : F -> %
retract : % -> F
retractIfCan : % -> Union(F,"failed")
```

These exports come from (p1089) VectorSpace(F:Field):

```
dimension : () -> CardinalNumber
?/? : (%,F) -> %
```

These exports come from (p1457) FieldOfPrimeCharacteristic():

```
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if F has CHARNZ or F has FINITE
order : % -> OnePointCompletion PositiveInteger
    if F has CHARNZ or F has FINITE
primeFrobenius : % -> %
    if F has CHARNZ or F has FINITE
```

```
primeFrobenius : (%,NonNegativeInteger) -> %
    if F has CHARNZ or F has FINITE
```

⟨*category XF ExtensionField*⟩≡

```
)abbrev category XF ExtensionField
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*, extensionDegree, algebraic?, transcendent?
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field
++ References:
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ ExtensionField F is the category of fields which extend the field F

ExtensionField(F:Field) : Category  == _
  Join(Field,RetractableTo F,VectorSpace F) with
    if F has CharacteristicZero then CharacteristicZero
    if F has CharacteristicNonZero then FieldOfPrimeCharacteristic
    algebraic? : $ -> Boolean
      ++ algebraic?(a) tests whether an element \spad{a} is algebraic with
      ++ respect to the ground field F.
    transcendent? : $ -> Boolean
      ++ transcendent?(a) tests whether an element \spad{a} is transcendent
      ++ with respect to the ground field F.
    inGroundField?: $ -> Boolean
      ++ inGroundField?(a) tests whether an element \spad{a}
      ++ is already in the ground field F.
    degree : $ -> OnePointCompletion PositiveInteger
      ++ degree(a) returns the degree of minimal polynomial of an element
      ++ \spad{a} if \spad{a} is algebraic
      ++ with respect to the ground field F, and \spad{infinity} otherwise.
    extensionDegree : () -> OnePointCompletion PositiveInteger
      ++ extensionDegree() returns the degree of the field extension if the
      ++ extension is algebraic, and \spad{infinity} if it is not.
    transcendenceDegree : () -> NonNegativeInteger
      ++ transcendenceDegree() returns the transcendence degree of the
      ++ field extension, 0 if the extension is algebraic.
    -- perhaps more absolute degree functions
    if F has Finite then
      FieldOfPrimeCharacteristic
      Frobenius: $ -> $
```

```
          ++ Frobenius(a) returns \spad{a ** q} where q is the \spad{size()$F}.
        Frobenius:   ($,NonNegativeInteger) -> $
          ++ Frobenius(a,s) returns \spad{a**(q**s)} where q is the size()$F.
    add
      algebraic?(a) == not infinite? (degree(a)@OnePointCompletion_
        (PositiveInteger))$OnePointCompletion(PositiveInteger)
      transcendent? a == infinite?(degree(a)@OnePointCompletion _
        (PositiveInteger))$OnePointCompletion(PositiveInteger)
      if F has Finite then
        Frobenius(a) == a ** size()$F
        Frobenius(a,s) == a ** (size()$F ** s)
```

⟨*XF.dotabb*⟩≡

```
  "XF"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
  "XF" -> "FIELD"
  "XF" -> "RETRACT"
  "XF" -> "VSPACE"
  "XF" -> "FPC"
```

⟨*XF.dotfull*⟩≡

```
  "ExtensionField(a:Field)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
  "ExtensionField(a:Field)" -> "Field()"
  "ExtensionField(a:Field)" -> "RetractableTo(Field)"
  "ExtensionField(a:Field)" -> "VectorSpace(a:Field)"
  "ExtensionField(a:Field)" -> "FieldOfPrimeCharacteristic()"
```

⟨*XF.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

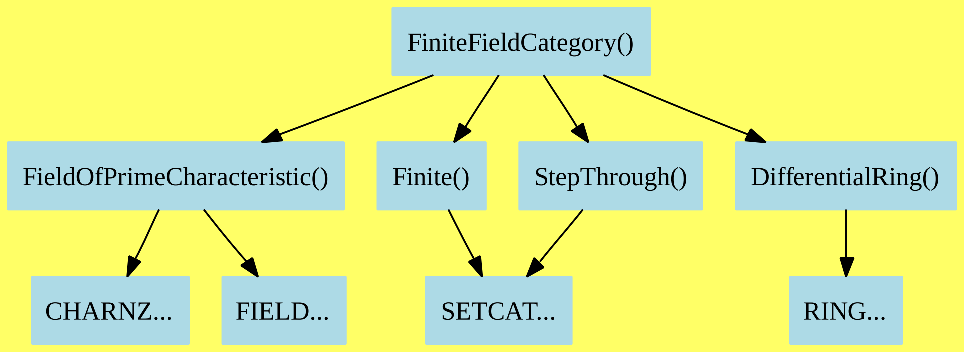
"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"Category" [color=lightblue];
}
```

18.0.207 FiniteFieldCategory (FFIELDC)


FiniteFieldCategory()
FieldOfPrimeCharacteristic()
Finite()
StepThrough()
DifferentialRing()
CHARNZ...
FIELD...
SETCAT...
RING...

⟨*FiniteFieldCategory.input*⟩≡

```
)set break resume
)sys rm -f FiniteFieldCategory.output
)spool FiniteFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCategory
--R FiniteFieldCategory  is a category constructor
--R Abbreviation for FiniteFieldCategory is FFIELDC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FFIELDC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        charthRoot : % -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R createPrimitiveElement : () -> %      differentiate : % -> %
--R factor : % -> Factored %              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
```

```
--R index : PositiveInteger -> %          init : () -> %
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R lookup : % -> PositiveInteger         one? : % -> Boolean
--R order : % -> PositiveInteger          prime? : % -> Boolean
--R primeFrobenius : % -> %               primitive? : % -> Boolean
--R primitiveElement : () -> %            ?quo? : (%,%) -> %
--R random : () -> %                      recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R size : () -> NonNegativeInteger       sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix % -> Union(Vector %,"failed")
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Intege
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteg
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteFieldCategory.help*⟩≡

```
====================================================================
FiniteFieldCategory examples
====================================================================

See Also:
o )show FiniteFieldCategory
```

See:

⇐ "DifferentialRing" (DIFRING) 10.0.138 on page 938
⇐ "FieldOfPrimeCharacteristic" (FPC) 17.0.193 on page 1457
⇐ "Finite" (FINITE) 4.0.46 on page 188
⇐ "StepThrough" (STEP) 4.0.64 on page 283

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	createPrimitiveElement	D
differentiate	discreteLog	divide
euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	factorsOfCyclicGroupSize
gcd	gcdPolynomial	hash
index	init	inv
latex	lcm	lookup
multiEuclidean	nextItem	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?quo?	?rem?
?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
primitiveElement : () -> %
representationType : () -> Union("prime",polynomial,normal,cyclic)
tableForDiscreteLogarithm : Integer ->
   Table(PositiveInteger,NonNegativeInteger)
```

These are implemented by this category:

```
charthRoot : % -> %
charthRoot : % -> Union(%,"failed")
conditionP : Matrix % -> Union(Vector %,"failed")
createPrimitiveElement : () -> %
differentiate : % -> %
discreteLog : % -> NonNegativeInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
init : () -> %
nextItem : % -> Union(%,"failed")
order : % -> OnePointCompletion PositiveInteger
order : % -> PositiveInteger
primitive? : % -> Boolean
```

These exports come from (p1457) FieldOfPrimeCharacteristic():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p188) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p283) StepThrough():

These exports come from (p938) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

⟨*category FFIELDC FiniteFieldCategory*⟩≡

```
)abbrev category FFIELDC FiniteFieldCategory
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 11 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*, extensionDegree, order, primitiveElement
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension, finite field
++  Galois field
++ References:
++  D.Lipson, Elements of Algebra and Algebraic Computing, The
++  Benjamin/Cummings Publishing Company, Inc.-Menlo Park, California, 1981.
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldCategory is the category of finite fields

FiniteFieldCategory() : Category ==_
  Join(FieldOfPrimeCharacteristic,Finite,StepThrough,DifferentialRing) with
--                 ,PolynomialFactorizationExplicit) with
    charthRoot: $ -> $
      ++ charthRoot(a) takes the characteristic'th root of a.
      ++ Note that such a root is alway defined in finite fields.
    conditionP: Matrix $ -> Union(Vector $,"failed")
      ++ conditionP(mat), given a matrix representing a homogeneous system
      ++ of equations, returns a vector whose characteristic'th powers
      ++ is a non-trivial solution, or "failed" if no such vector exists.
    -- the reason for implementing the following function is that we
    -- can implement the functions order, getGenerator and primitive? on
    -- category level without computing the, may be time intensive,
    -- factorization of size()-1 at every function call again.
    factorsOfCyclicGroupSize:_
      () -> List Record(factor:Integer,exponent:Integer)
      ++ factorsOfCyclicGroupSize() returns the factorization of size()-1
    -- the reason for implementing the function tableForDiscreteLogarithm
    -- is that we can implement the functions discreteLog and
    -- shanksDiscLogAlgorithm on category level
    -- computing the necessary exponentiation tables in the respective
    -- domains once and for all
    -- absoluteDegree : $ -> PositiveInteger
    --  ++ degree of minimal polynomial, if algebraic with respect
    --  ++ to the prime subfield
    tableForDiscreteLogarithm: Integer -> _
             Table(PositiveInteger,NonNegativeInteger)
      ++ tableForDiscreteLogarithm(a,n) returns a table of the discrete
      ++ logarithms of \spad{a**0} up to \spad{a**(n-1)} which, called with
      ++ key \spad{lookup(a**i)} returns i for i in \spad{0..n-1}.
      ++ Error: if not called for prime divisors of order of
      ++        multiplicative group.
    createPrimitiveElement: () -> $
      ++ createPrimitiveElement() computes a generator of the (cyclic)
      ++ multiplicative group of the field.
      -- RDJ: Are these next lines to be included?
      -- we run through the field and test, algorithms which construct
      -- elements of larger order were found to be too slow
    primitiveElement: () -> $
      ++ primitiveElement() returns a primitive element stored in a global
      ++ variable in the domain.
      ++ At first call, the primitive element is computed
      ++ by calling \spadfun{createPrimitiveElement}.
    primitive?: $ -> Boolean
      ++ primitive?(b) tests whether the element b is a generator of the
      ++ (cyclic) multiplicative group of the field, i.e. is a primitive
      ++ element.
      ++ Implementation Note that see ch.IX.1.3, th.2 in D. Lipson.
    discreteLog: $ -> NonNegativeInteger
      ++ discreteLog(a) computes the discrete logarithm of \spad{a}
      ++ with respect to \spad{primitiveElement()} of the field.
    order: $ -> PositiveInteger
      ++ order(b) computes the order of an element b in the multiplicative
      ++ group of the field.
      ++ Error: if b equals 0.
    representationType: () -> Union("prime","polynomial","normal","cyclic")
      ++ representationType() returns the type of the representation, one of:
      ++ \spad{prime}, \spad{polynomial}, \spad{normal}, or \spad{cyclic}.
  add
    I   ==> Integer
    PI  ==> PositiveInteger
    NNI ==> NonNegativeInteger
    SUP ==> SparseUnivariatePolynomial
    DLP ==> DiscreteLogarithmPackage
    -- exported functions

    differentiate x          == 0

    init() == 0

    nextItem(a) ==
      zero?(a:=index(lookup(a)+1)) => "failed"
      a

    order(e):OnePointCompletion(PositiveInteger) ==
      (order(e)@PI)::OnePointCompletion(PositiveInteger)

    conditionP(mat:Matrix $) ==
      l:=nullSpace mat
      empty? l or every?(zero?, first l) => "failed"
      map(charthRoot,first l)

    charthRoot(x:$):$ == x**(size() quo characteristic())

    charthRoot(x:%):Union($,"failed") ==
        (charthRoot(x)@$)::Union($,"failed")

    createPrimitiveElement() ==
      sm1  : PositiveInteger := (size()$$-1) pretend PositiveInteger
      start : Integer :=
        -- in the polynomial case, index from 1 to characteristic-1
        -- gives prime field elements
        representationType = "polynomial" => characteristic()::Integer
        1
      found : Boolean := false
      for i in start..  while not found repeat
        e : $ := index(i::PositiveInteger)
        found := (order(e) = sm1)
      e

    primitive? a ==
      -- add special implementation for prime field case
      zero?(a) => false
      explist := factorsOfCyclicGroupSize()
      q:=(size()-1)@Integer
      equalone : Boolean := false
      for exp in explist while not equalone repeat
--          equalone := one?(a**(q quo exp.factor))
        equalone := ((a**(q quo exp.factor)) = 1)
      not equalone

    order e ==
      e = 0 => error "order(0) is not defined "
      ord:Integer:= size()-1 -- order e divides ord
      a:Integer:= 0
      lof:=factorsOfCyclicGroupSize()
      for rec in lof repeat -- run through prime divisors
        a := ord quo (primeDivisor := rec.factor)
--        goon := one?(e**a)
        goon := ((e**a) = 1)
        -- run through exponents of the prime divisors
        for j in 0..(rec.exponent)-2 while goon repeat
          -- as long as we get (e**ord = 1) we
          -- continue dividing by primeDivisor
          ord := a
          a := ord quo primeDivisor
--          goon := one?(e**a)
          goon := ((e**a) = 1)
        if goon then ord := a
        -- as we do a top down search we have found the
        -- correct exponent of primeDivisor in order e
        -- and continue with next prime divisor
      ord pretend PositiveInteger

    discreteLog(b) ==
      zero?(b) => error "discreteLog: logarithm of zero"
      faclist:=factorsOfCyclicGroupSize()
      a:=b
      gen:=primitiveElement()
      -- in GF(2) its necessary to have discreteLog(1) = 1
      b = gen => 1
      disclog:Integer:=0
      mult:Integer:=1
      groupord := (size() - 1)@Integer
      exp:Integer:=groupord
      for f in faclist repeat
        fac:=f.factor
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          -- shanks discrete logarithm algorithm
          exptable:=tableForDiscreteLogarithm(fac)
          n:=#exptable
          c:=a**exp
          end:=(fac - 1) quo n
          found:=false
        disc1:Integer:=0
        for i in 0..end while not found repeat
          rho:= search(lookup(c),exptable)_
                $Table(PositiveInteger,NNI)
          rho case NNI =>
            found := true
            disc1:=((n * i + rho)@Integer) * mult
          c:=c* gen**((groupord quo fac) * (-n))
        not found => error "discreteLog: ?? discrete logarithm"
        -- end of shanks discrete logarithm algorithm
        mult := mult * fac
        disclog:=disclog+disc1
        a:=a * (gen ** (-disc1))
      disclog pretend NonNegativeInteger

    discreteLog(logbase,b) ==
      zero?(b) =>
        messagePrint("discreteLog: logarithm of zero")$OutputForm
        "failed"
      zero?(logbase) =>
        messagePrint("discreteLog: logarithm to base zero")$OutputForm
        "failed"
      b = logbase => 1
      not zero?((groupord:=order(logbase)@PI) rem order(b)@PI) =>
         messagePrint("discreteLog: second argument not in cyclic group_
generated by first argument")$OutputForm
         "failed"
      faclist:=factors factor groupord
      a:=b
      disclog:Integer:=0
      mult:Integer:=1
      exp:Integer:= groupord
      for f in faclist repeat
        fac:=f.factor
        primroot:= logbase ** (groupord quo fac)
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          rhoHelp:= shanksDiscLogAlgorithm(primroot,_
                a**exp,fac pretend NonNegativeInteger)$DLP($)
          rhoHelp case "failed" => return "failed"
          rho := (rhoHelp :: NNI) * mult
          disclog := disclog + rho
          mult := mult * fac
          a:=a * (logbase ** (-rho))
      disclog pretend NonNegativeInteger
      FP ==> SparseUnivariatePolynomial($)
      FRP ==> Factored FP
      f,g:FP

      squareFreePolynomial(f:FP):FRP ==
            squareFree(f)$UnivariatePolynomialSquareFree($,FP)

      factorPolynomial(f:FP):FRP == factor(f)$DistinctDegreeFactorize($,FP)

      factorSquareFreePolynomial(f:FP):FRP ==
          f = 0 => 0
          flist := distdfact(f,true)$DistinctDegreeFactorize($,FP)
          (flist.cont :: FP) *
              (*/[primeFactor(u.irr,u.pow) for u in flist.factors])

      gcdPolynomial(f:FP,g:FP):FP ==
           gcd(f,g)$EuclideanDomain_&(FP)
```

⟨*FFIELDC.dotabb*⟩≡

```
  "FFIELDC"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
  "FFIELDC" -> "FPC"
  "FFIELDC" -> "FINITE"
  "FFIELDC" -> "STEP"
  "FFIELDC" -> "DIFRING"
```

⟨*FFIELDC.dotfull*⟩≡

```
  "FiniteFieldCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
  "FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
  "FiniteFieldCategory()" -> "Finite()"
  "FiniteFieldCategory()" -> "StepThrough()"
  "FiniteFieldCategory()" -> "DifferentialRing()"
```

⟨*FFIELDC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteFieldCategory()" [color=lightblue];
"FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
"FiniteFieldCategory()" -> "Finite()"
"FiniteFieldCategory()" -> "StepThrough()"
"FiniteFieldCategory()" -> "DifferentialRing()"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "FIELD..."

"Finite()" [color=lightblue];
"Finite()" -> "SETCAT..."

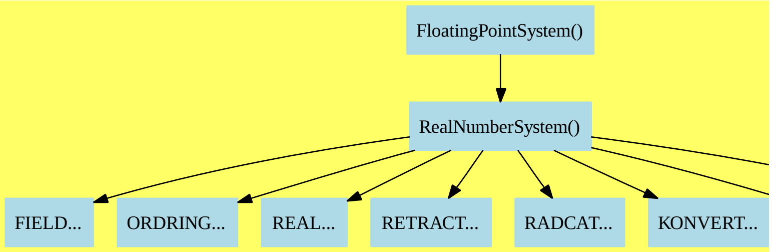
"StepThrough()" [color=lightblue];
"StepThrough()" -> "SETCAT..."

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "RING..."

"RING..." [color=lightblue];
"FIELD..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"SETCAT..." [color=lightblue];

}
```

18.0.208 FloatingPointSystem (FPS)



⟨*FloatingPointSystem.input*⟩≡

```
)set break resume
)sys rm -f FloatingPointSystem.output
)spool FloatingPointSystem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FloatingPointSystem
--R FloatingPointSystem  is a category constructor
--R Abbreviation for FloatingPointSystem is FPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        abs : % -> %
--R associates? : (%,%) -> Boolean        base : () -> PositiveInteger
--R bits : () -> PositiveInteger          ceiling : % -> %
--R coerce : Fraction Integer -> %        coerce : Integer -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R convert : % -> Pattern Float          convert : % -> DoubleFloat
--R convert : % -> Float                  digits : () -> PositiveInteger
```

```
--R exponent : % -> Integer               factor : % -> Factored %
--R float : (Integer,Integer) -> %        floor : % -> %
--R fractionPart : % -> %                 gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R mantissa : % -> Integer               max : (%,%) -> %
--R min : (%,%) -> %                      negative? : % -> Boolean
--R norm : % -> %                         nthRoot : (%,Integer) -> %
--R one? : % -> Boolean                   order : % -> Integer
--R positive? : % -> Boolean              precision : () -> PositiveInteger
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> Fraction Integer       retract : % -> Integer
--R round : % -> %                        sample : () -> %
--R sign : % -> Integer                   sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                         squareFree : % -> Factored %
--R squareFreePart : % -> %               truncate : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R wholePart : % -> Integer              zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R bits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R characteristic : () -> NonNegativeInteger
--R decreasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R digits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R float : (Integer,Integer,PositiveInteger) -> %
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R increasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R max : () -> % if not has($,arbitraryPrecision) and not has($,arbitraryExponent)
--R min : () -> % if not has($,arbitraryPrecision) and not has($,arbitraryExponent)
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R precision : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FloatingPointSystem.help*⟩≡

```
====================================================================
FloatingPointSystem examples
====================================================================

See Also:
o )show FloatingPointSystem
```

See:

⇐ “RealNumberSystem” (RNS) 17.0.200 on page 1543

Exports:

0
1
abs
associates?
base
bits
characteristic
ceiling
coerce
convert
decreasePrecision
digits
divide
euclideanSize
exponent
expressIdealMember
exquo
extendedEuclidean
factor
float
floor
fractionPart
gcd
gcdPolynomial
hash
increasePrecision
inv
latex
lcm
mantissa
max
min
multiEuclidean
negative?
norm
nthRoot
one?
order
patternMatch
positive?
precision
prime?
principalIdeal
recip
retract
retractIfCan
round
sample
sign
sizeLess?
sqrt
squareFree
squareFreePart
subtractIfCan
truncate
unit?
unitCanonical
unitNormal
wholePart
zero?
?*?
?**?
?+?
?-?
-?
?/?
?<?
?<=?
?=?
?>?
?>=?
?^?
?~=?
?quo?
?rem?

Attributes Exported:

- **approximate** means “is an approximation to the real numbers”.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative(“*”)** is true if it has an operation $" * "$: $(D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
base : () -> PositiveInteger
bits : () -> PositiveInteger
bits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
decreasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
digits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
exponent : % -> Integer
float : (Integer,Integer,PositiveInteger) -> %
increasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
mantissa : % -> Integer
max : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
min : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
order : % -> Integer
precision : () -> PositiveInteger
precision : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
```

These are implemented by this category:

```
digits : () -> PositiveInteger
float : (Integer,Integer) -> %
```

These exports come from (p1543) RealNumberSystem():

```
0 : () -> %
1 : () -> %
abs : % -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
ceiling : % -> %
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : Fraction Integer -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
convert : % -> Pattern Float
convert : % -> DoubleFloat
convert : % -> Float
divide : (%,%) -> Record(quotient: %,remainder: %)
```

```
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
floor : % -> %
fractionPart : % -> %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
max : (%,%) -> %
min : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
negative? : % -> Boolean
norm : % -> %
nthRoot : (%,Integer) -> %
one? : % -> Boolean
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
positive? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
retract : % -> Fraction Integer
retract : % -> Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Integer,"failed")
round : % -> %
sample : () -> %
sign : % -> Integer
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
truncate : % -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
wholePart : % -> Integer
```

```
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

⟨*category FPS FloatingPointSystem*⟩≡

```
)abbrev category FPS FloatingPointSystem
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: approximate, base, bits, digits, exponent, float,
++    mantissa, order, precision, round?
++ Related Constructors:
++ Keywords: float, floating point
++ Description:
++ This category is intended as a model for floating point systems.
++ A floating point system is a model for the real numbers.  In fact,
++ it is an approximation in the sense that not all real numbers are
++ exactly representable by floating point numbers.
++ A floating point system is characterized by the following:
++
++ 1: base of the exponent where the actual implemenations are
++ usually binary or decimal)\br
++ 2: precision of the mantissa (arbitrary or fixed)\br
++ 3: rounding error for operations
--++   4:  when, and what happens if exponent overflow/underflow occurs
```

```
++
++ Because a Float is an approximation to the real numbers, even though
++ it is defined to be a join of a Field and OrderedRing, some of
++ the attributes do not hold.  In particular associative("+")
++ does not hold.  Algorithms defined over a field need special
++ considerations when the field is a floating point system.

FloatingPointSystem(): Category == RealNumberSystem() with
   approximate
      ++ \spad{approximate} means "is an approximation to the real numbers".
   float: (Integer,Integer) -> %
      ++ float(a,e) returns \spad{a * base() ** e}.
   float: (Integer,Integer,PositiveInteger) -> %
      ++ float(a,e,b) returns \spad{a * b ** e}.
   order: % -> Integer
      ++ order x is the order of magnitude of x.
      ++ Note that \spad{base ** order x <= |x| < base ** (1 + order x)}.
   base: () -> PositiveInteger
      ++ base() returns the base of the
      ++\spadfunFrom{exponent}{FloatingPointSystem}.

   exponent: % -> Integer
      ++ exponent(x) returns the
      ++ \spadfunFrom{exponent}{FloatingPointSystem} part of x.

   mantissa: % -> Integer
      ++ mantissa(x) returns the mantissa part of x.
  -- round?: () -> B
  --    ++ round?() returns the rounding or chopping.

   bits: () -> PositiveInteger
      ++ bits() returns ceiling's precision in bits.
   digits: () -> PositiveInteger
      ++ digits() returns ceiling's precision in decimal digits.
   precision: () -> PositiveInteger
      ++ precision() returns the precision in digits base.

   if % has arbitraryPrecision then
      bits: PositiveInteger -> PositiveInteger
        ++ bits(n) set the \spadfunFrom{precision}{FloatingPointSystem}
        ++ to n bits.

      digits: PositiveInteger -> PositiveInteger
        ++ digits(d) set the \spadfunFrom{precision}{FloatingPointSystem}
        ++ to d digits.
        precision: PositiveInteger -> PositiveInteger
          ++ precision(n) set the precision in the base to n decimal digits.

        increasePrecision: Integer -> PositiveInteger
          ++ increasePrecision(n) increases the current
          ++ \spadfunFrom{precision}{FloatingPointSystem} by n decimal digits.

        decreasePrecision: Integer -> PositiveInteger
          ++ decreasePrecision(n) decreases the current
          ++ \spadfunFrom{precision}{FloatingPointSystem} precision
          ++ by n decimal digits.

     if not (% has arbitraryExponent) then
      --  overflow: (()->Exit) -> Void
      --    ++ overflow() returns the Exponent overflow of float
      --  underflow: (()->Exit) -> Void
      --    ++ underflow() returns the Exponent underflow of float
      --  maxExponent: () -> Integer
      --    ++ maxExponent() returns the max Exponent of float
        if not (% has arbitraryPrecision) then
           min: () -> %
            ++ min() returns the minimum floating point number.
           max: () -> %
            ++ max() returns the maximum floating point number.
   add
     float(ma, ex) == float(ma, ex, base())
     digits() == max(1,4004 * (bits()-1) quo 13301)::PositiveInteger
```

⟨*FPS.dotabb*⟩≡

```
  "FPS"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
  "FPS" -> "RNS"
```

⟨*FPS.dotfull*⟩≡

```
  "FloatingPointSystem()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
  "FloatingPointSystem()" -> "RealNumberSystem()"
```

⟨*FPS.dotpic*⟩≡

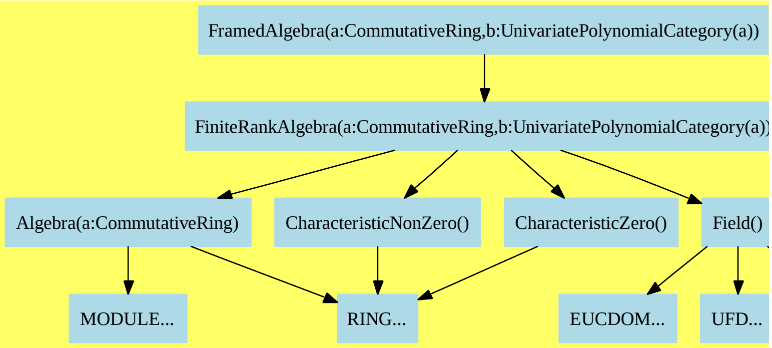
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FloatingPointSystem()" [color=lightblue];
"FloatingPointSystem()" -> "RealNumberSystem()"

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

18.0.209 FramedAlgebra (FRAMALG)


FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))
FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))
Algebra(a:CommutativeRing)
CharacteristicNonZero()
CharacteristicZero()
Field()
MODULE...
RING...
EUCDOM...
UFD...

⟨*FramedAlgebra.input*⟩≡

```
)set break resume
)sys rm -f FramedAlgebra.output
)spool FramedAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedAlgebra
--R FramedAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory t#1)  is a
--R Abbreviation for FramedAlgebra is FRAMALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRAMALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        basis : () -> Vector %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              convert : Vector R -> %
--R convert : % -> Vector R               coordinates : % -> Vector R
--R discriminant : () -> R                discriminant : Vector % -> R
--R hash : % -> SingleInteger             latex : % -> String
--R norm : % -> R                         one? : % -> Boolean
--R rank : () -> PositiveInteger          recip : % -> Union(%,"failed")
--R represents : Vector R -> %            sample : () -> %
```

```
--R trace : % -> R                        traceMatrix : () -> Matrix R
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UP
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R minimalPolynomial : % -> UP if R has FIELD
--R regularRepresentation : % -> Matrix R
--R regularRepresentation : (%,Vector %) -> Matrix R
--R represents : (Vector R,Vector %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R traceMatrix : Vector % -> Matrix R
--R
--E 1

)spool
)lisp (bye)
```

⟨*FramedAlgebra.help*⟩≡

```
====================================================================
FramedAlgebra examples
====================================================================

See Also:
o )show FramedAlgebra
```

See:

⇒ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇐ “FiniteRankAlgebra” (FINRALG) 17.0.194 on page 1465

Exports:

0	1	basis
characteristic	characteristicPolynomial	charthRoot
coerce	convert	coordinates
discriminant	hash	latex
minimalPolynomial	norm	one?
rank	recip	regularRepresentation
represents	sample	subtractIfCan
trace	traceMatrix	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
represents : Vector R -> %
```

These are implemented by this category:

```
convert : Vector R -> %
convert : % -> Vector R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : () -> R
regularRepresentation : % -> Matrix R
traceMatrix : () -> Matrix R
```

These exports come from (p1465) FiniteRankAlgebra(R, UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory R):

```
0 : () -> %
```

```
1 : () -> %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UP
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
one? : % -> Boolean
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category FRAMALG FramedAlgebra*⟩≡

```
)abbrev category FRAMALG FramedAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ A \spadtype{FramedAlgebra} is a \spadtype{FiniteRankAlgebra} together
++ with a fixed R-module basis.

FramedAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category == FiniteRankAlgebra(R, UP) with
      basis                  : () -> Vector %
        ++ basis() returns the fixed R-module basis.
      coordinates            : % -> Vector R
        ++ coordinates(a) returns the coordinates of \spad{a} with
        ++ respect to the fixed R-module basis.
      coordinates            : Vector % -> Matrix R
        ++ coordinates([v1,...,vm]) returns the coordinates of the
        ++ vi's with to the fixed basis.  The coordinates of vi are
        ++ contained in the ith row of the matrix returned by this
        ++ function.
      represents             : Vector R -> %
        ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
        ++ v1, ..., vn are the elements of the fixed basis.
      convert                : % -> Vector R
        ++ convert(a) returns the coordinates of \spad{a} with respect to the
        ++ fixed R-module basis.
      convert                : Vector R -> %
        ++ convert([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
        ++ v1, ..., vn are the elements of the fixed basis.
      traceMatrix            : () -> Matrix R
        ++ traceMatrix() is the n-by-n matrix ( \spad{Tr(vi * vj)} ), where
        ++ v1, ..., vn are the elements of the fixed basis.
      discriminant           : () -> R
        ++ discriminant() = determinant(traceMatrix()).
      regularRepresentation : % -> Matrix R
        ++ regularRepresentation(a) returns the matrix of the linear
        ++ map defined by left multiplication by \spad{a} with respect
        ++ to the fixed basis.
    --attributes
      --separable <=> discriminant() ^= 0
  add
   convert(x:%):Vector(R)  == coordinates(x)
   convert(v:Vector R):%   == represents(v)
   traceMatrix()           == traceMatrix basis()
   discriminant()          == discriminant basis()
   regularRepresentation x == regularRepresentation(x, basis())
   coordinates x           == coordinates(x, basis())
   represents x            == represents(x, basis())
```

```
coordinates(v:Vector %) ==
  m := new(#v, rank(), 0)$Matrix(R)
  for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates qelt(v, i))
  m

regularRepresentation x ==
  m := new(n := rank(), n, 0)$Matrix(R)
  b := basis()
  for i in minIndex b .. maxIndex b for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates(x * qelt(b, i)))
  m

characteristicPolynomial x ==
   mat00 := (regularRepresentation x)
   mat0 := map(y+->y::UP,mat00)$MatrixCategoryFunctions2(R, Vector R,
               Vector R, Matrix R, UP, Vector UP,Vector UP, Matrix UP)
   mat1 : Matrix UP := scalarMatrix(rank(),monomial(1,1)$UP)
   determinant(mat1 - mat0)

if R has Field then
 -- depends on the ordering of results from nullSpace, also see FFP
   minimalPolynomial(x:%):UP ==
     y:%:=1
     n:=rank()
     m:Matrix R:=zero(n,n+1)
     for i in 1..n+1 repeat
       setColumn_!(m,i,coordinates(y))
       y:=y*x
     v:=first nullSpace(m)
     +/[monomial(v.(i+1),i) for i in 0..#v-1]
```

⟨*FRAMALG.dotabb*⟩≡

```
"FRAMALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FRAMALG" -> "FINRALG"
```

⟨*FRAMALG.dotfull*⟩≡

```
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

⟨*FRAMALG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

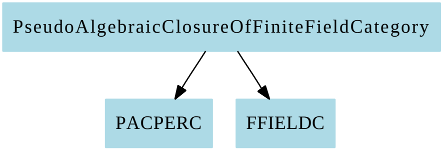
"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

18.0.210 PseudoAlgebraicClosureOfFiniteFieldCategory (PACFFC)



⟨*PseudoAlgebraicClosureOfFiniteFieldCategory.input*⟩≡
```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfFiniteFieldCategory.output
)spool PseudoAlgebraicClosureOfFiniteFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfFiniteFieldCategory
--R
--R PseudoAlgebraicClosureOfFiniteFieldCategory  is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfFiniteFieldCategory is PACFFC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACFFC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R D : % -> %                           D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       charthRoot : % -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R conjugate : % -> %                   createPrimitiveElement : () -> %
--R differentiate : % -> %               extDegree : % -> PositiveInteger
--R factor : % -> Factored %             fullOutput : % -> OutputForm
--R gcd : List % -> %                    gcd : (%,%) -> %
--R ground? : % -> Boolean               hash : % -> SingleInteger
--R index : PositiveInteger -> %         init : () -> %
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
```

```
--R lookup : % -> PositiveInteger         maxTower : List % -> %
--R one? : % -> Boolean                   order : % -> PositiveInteger
--R previousTower : % -> %                prime? : % -> Boolean
--R primeFrobenius : % -> %               primitive? : % -> Boolean
--R primitiveElement : () -> %            ?quo? : (%,%) -> %
--R random : () -> %                      recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R setTower! : % -> Void                 size : () -> NonNegativeInteger
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                vectorise : (%,%) -> Vector %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix % -> Union(Vector %,"failed")
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Intege
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R lift : % -> SparseUnivariatePolynomial %
--R lift : (%,%) -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R nextItem : % -> Union(%,"failed")
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteg
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PseudoAlgebraicClosureOfFiniteFieldCategory.help*⟩≡

```
====================================================================
PseudoAlgebraicClosureOfFiniteFieldCategory examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfFiniteFieldCategory
```

⇐ "FieldOfPrimeCharacteristic" (FPC) 17.0.193 on page 1457
⇐ "PseudoAlgebraicClosureOfPerfectFieldCategory" (PACPERC) 17.0.197 on page 1509

Exports:

0
1
associates?
characteristic
charthRoot
conditionP
coerce
conjugate
createPrimitiveElement
D
definingPolynomial
differentiate
discreteLog
distinguishedRootsOf
divide
euclideanSize
extendedEuclidean
expressIdealMember
exquo
extDegree
factor
factorsOfCyclicGroupSize
fullOutput
gcd
gcdPolynomial
ground?
hash
index
init
inv
latex
lcm
lift
lookup
maxTower
multiEuclidean
newElement
nextItem
one?
order
previousTower
prime?
primeFrobenius
primitive?
primitiveElement
principalIdeal
?quo?
random
recip
reduce
?rem?
representationType
sample
setTower!
size
sizeLess?
squareFree
squareFreePart
subtractIfCan
tableForDiscreteLogarithm
unit?
unitCanonical
unitNormal
vectorise
zero?
?*?
?**?
?+?
?-?
-?
?/?
?=?
?^?
?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1509) PseudoAlgebraicClosureOfPerfectFieldCategory():

```
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
1 : () -> %
0 : () -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?~=? : (%,%) -> Boolean
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Fraction Integer -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
definingPolynomial : () -> SUP %
definingPolynomial : % -> SUP %
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PI
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomi
ground? : % -> Boolean
```

```
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
lift : % -> SUP %
lift : (%,%) -> SUP %
maxTower : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
newElement : (SUP %,Symbol) -> %
newElement : (SUP %,%,Symbol) -> %
one? : % -> Boolean
previousTower : % -> %
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
?quo? : (%,%) -> %
recip : % -> Union(%,"failed")
reduce : SUP % -> %
?rem? : (%,%) -> %
sample : () -> %
setTower! : % -> Void
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
vectorise : (%,%) -> Vector %
zero? : % -> Boolean
```

These exports come from (p1671) FiniteFieldCategory():

```
charthRoot : % -> %
charthRoot : % -> Union(%,"failed")
conditionP : Matrix % -> Union(Vector %,"failed")
createPrimitiveElement : () -> %
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : % -> %
differentiate : (%,NonNegativeInteger) -> %
discreteLog : % -> NonNegativeInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
factorsOfCyclicGroupSize : () ->
    List Record(factor: Integer,exponent: Integer)
index : PositiveInteger -> %
init : () -> %
lookup : % -> PositiveInteger
nextItem : % -> Union(%,"failed")
order : % -> OnePointCompletion PositiveInteger
order : % -> PositiveInteger
primeFrobenius : (%,NonNegativeInteger) -> %
primeFrobenius : % -> %
primitive? : % -> Boolean
primitiveElement : () -> %
random : () -> %
representationType : () -> Union("prime",polynomial,normal,cyclic)
size : () -> NonNegativeInteger
tableForDiscreteLogarithm : Integer ->
    Table(PositiveInteger,NonNegativeInteger)
```

⟨*category PACFFC PseudoAlgebraicClosureOfFiniteFieldCategory*⟩≡

```
)abbrev category  PACFFC PseudoAlgebraicClosureOfFiniteFieldCategory
-- PseudoAlgebraicClosureOfFiniteFieldCategory
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated:
++ References:
++ Description: This category exports the function for the domain
++ PseudoAlgebraicClosureOfFiniteField which implement dynamic extension
++ using the simple notion of tower extensions.
++ A tower extension T  of the ground
++ field K is any sequence of field extension (T : K_0, K_1, ..., K_i...,K_n)
++ where K_0 = K and for i =1,2,...,n, K_i is an extension
++ of K_{i-1} of degree > 1 and defined by an irreducible polynomial
++ p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfFiniteFieldCategory:Category ==
  Join(FiniteFieldCategory, PseudoAlgebraicClosureOfPerfectFieldCategory)
```

⟨*PACFFC.dotabb*⟩≡

```
"PACFFC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACFFC"];
"PACFFC" -> "PACPERC"
```

⟨*PACFFC.dotfull*⟩≡
```
"PseudoAlgebraicClosureOfFiniteFieldCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACFFC"];
"PseudoAlgebraicClosureOfFiniteFieldCategory" ->
  "PseudoAlgebraicClosureOfPerfectFieldCategory()"
"PseudoAlgebraicClosureOfFiniteFieldCategory" ->
  "FiniteFieldCategory()"
```

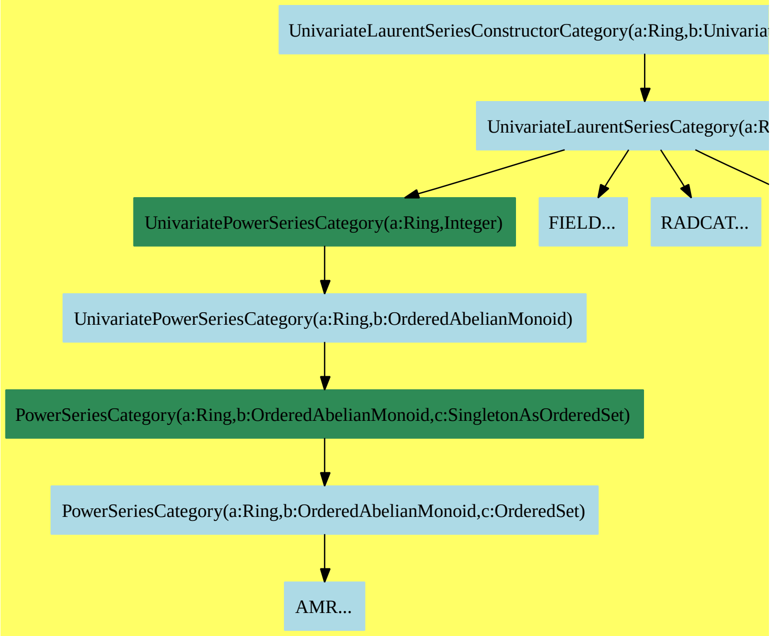
⟨*PACFFC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfFiniteFieldCategory" [color=lightblue];
"PseudoAlgebraicClosureOfFiniteFieldCategory" -> "PACPERC"
"PseudoAlgebraicClosureOfFiniteFieldCategory" -> "FFIELDC"

"PACPERC" [color=lightblue];
"FFIELDC" [color=lightblue];

}
```

18.0.211 UnivariateLaurentSeriesConstructorCategory (ULSCCAT)


UnivariateLaurentSeriesConstructorCategory(a:Ring,b:Univaria
UnivariateLaurentSeriesCategory(a:R
UnivariatePowerSeriesCategory(a:Ring,Integer)
FIELD...
RADCAT...
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AMR...

⟨*UnivariateLaurentSeriesConstructorCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariateLaurentSeriesConstructorCategory.output
)spool UnivariateLaurentSeriesConstructorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeriesConstructorCategory
--R UnivariateLaurentSeriesConstructorCategory(Coef: Ring,UTS: UnivariateTaylorSeriesCategor
--R Abbreviation for UnivariateLaurentSeriesConstructorCategory is ULSCCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ULSCCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficient : (%,Integer) -> Coef     coerce : UTS -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Integer
--R ?.? : (%,Integer) -> Coef             extend : (%,Integer) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R laurent : (Integer,UTS) -> %          leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Integer) -> %        monomial? : % -> Boolean
--R one? : % -> Boolean                   order : (%,Integer) -> Integer
--R order : % -> Integer                  pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R removeZeroes : (Integer,%) -> %       removeZeroes : % -> %
--R retract : % -> UTS                    sample : () -> %
--R taylor : % -> UTS                     taylorRep : % -> UTS
--R truncate : (%,Integer) -> %           variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (UTS,%) -> % if Coef has FIELD
--R ?*? : (%,UTS) -> % if Coef has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (UTS,UTS) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?<=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?>? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?>=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R D : (%,Symbol) -> % if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,F
--R D : (%,List Symbol) -> % if and(has(UTS,PartialDifferentialRing Symbol),has(C
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(UTS,PartialDifferentialRing
--R D : (%,List Symbol,List NonNegativeInteger) -> % if and(has(UTS,PartialDiffer
--R D : % -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *: (
--R D : (%,NonNegativeInteger) -> % if and(has(UTS,DifferentialRing),has(Coef,Fie
--R D : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R D : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R acos : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> UTS if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(OR(has(UTS,CharacteristicNonZero),and(has($,C
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Symbol -> % if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(and(has($,CharacteristicNonZero
--R convert : % -> Pattern Integer if and(has(UTS,ConvertibleTo Pattern Integer),has(Coef,Fi
--R convert : % -> Pattern Float if and(has(UTS,ConvertibleTo Pattern Float),has(Coef,Field)
--R convert : % -> InputForm if and(has(UTS,ConvertibleTo InputForm),has(Coef,Field))
--R convert : % -> Float if and(has(UTS,RealConstant),has(Coef,Field))
--R convert : % -> DoubleFloat if and(has(UTS,RealConstant),has(Coef,Field))
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> UTS if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,
--R differentiate : (%,List Symbol) -> % if and(has(UTS,PartialDifferentialRing Symbol),has(
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(UTS,PartialDifferentialRin
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if and(has(UTS,PartialDiffe
--R differentiate : % -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *:
--R differentiate : (%,NonNegativeInteger) -> % if and(has(UTS,DifferentialRing),has(Coef,Fi
--R differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UTS) -> % if and(has(UTS,Eltable(UTS,UTS)),has(Coef,Field))
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
```

```
--R eval : (%,List UTS,List UTS) -> % if and(has(UTS,Evalable UTS),has(Coef,Field
--R eval : (%,UTS,UTS) -> % if and(has(UTS,Evalable UTS),has(Coef,Field))
--R eval : (%,Equation UTS) -> % if and(has(UTS,Evalable UTS),has(Coef,Field))
--R eval : (%,List Equation UTS) -> % if and(has(UTS,Evalable UTS),has(Coef,Field
--R eval : (%,List Symbol,List UTS) -> % if and(has(UTS,InnerEvalable(Symbol,UTS)
--R eval : (%,Symbol,UTS) -> % if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef h
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if C
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R floor : % -> UTS if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R fractionPart : % -> % if and(has(UTS,EuclideanDomain),has(Coef,Field))
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R init : () -> % if and(has(UTS,StepThrough),has(Coef,Field))
--R integrate : (%,Symbol) -> % if Coef has ACFS INT and Coef has PRIMCAT and Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((UTS -> UTS),%) -> % if Coef has FIELD
--R max : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
--R min : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if and(has(UTS,OrderedIntegralDomain),has(Coef,Field
--R nextItem : % -> Union(%,"failed") if and(has(UTS,StepThrough),has(Coef,Field)
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> UTS if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMat
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchR
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if and(has(UTS,OrderedIntegralDomain),has(Coef,Field
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIEL
```

```
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if and(has(UTS,LinearlyExplicitRingOver Integ
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS) if Coef h
--R reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Symbol if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
--R retract : % -> Fraction Integer if and(has(UTS,RetractableTo Integer),has(Coef,Field))
--R retract : % -> Integer if and(has(UTS,RetractableTo Integer),has(Coef,Field))
--R retractIfCan : % -> Union(Symbol,"failed") if and(has(UTS,RetractableTo Symbol),has(Coef
--R retractIfCan : % -> Union(Fraction Integer,"failed") if and(has(UTS,RetractableTo Intege
--R retractIfCan : % -> Union(Integer,"failed") if and(has(UTS,RetractableTo Integer),has(Co
--R retractIfCan : % -> Union(UTS,"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyn
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomi
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylorIfCan : % -> Union(UTS,"failed")
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> UTS if and(has(UTS,EuclideanDomain),has(Coef,Field))
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariateLaurentSeriesConstructorCategory.help*⟩≡

```
====================================================================
UnivariateLaurentSeriesConstructorCategory examples
====================================================================

See Also:
o )show UnivariateLaurentSeriesConstructorCategory
```

See:

⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “UnivariateLaurentSeriesCategory” (ULSCAT) 17.0.202 on page 1595

Exports:

0
1
abs
acos
acosh
acot
acoth
acsc
acsch
approximate
asec
asech
asin
asinh
associates?
atan
atanh
ceiling
center
characteristic
charthRoot
coefficient
coerce
complete
conditionP
convert
cos
cosh
cot
coth
csc
csch
D
degree
denom
denominator
differentiate
divide
euclideanSize
eval
exp
expressIdealMember
exquo
extend
extendedEuclidean
factor
factorPolynomial
factorSquareFreePolynomial
floor
fractionPart
gcd
gcdPolynomial
hash
init
integrate
inv
latex
laurent
lcm
leadingCoefficient
leadingMonomial
log
map
max
min
monomial
monomial?
multiEuclidean
multiplyCoefficients
multiplyExponents
negative?
nextItem
nthRoot
numer
numerator
one?
order
patternMatch
pi
pole?
positive?
prime?
principalIdeal
random
rationalFunction
recip
reducedSystem
reductum
removeZeroes
retract
retractIfCan
sample
sec
sech
series
sign
sin
sinh
sizeLess?
solveLinearPolynomialEquation
sqrt
squareFree
squareFreePart
squareFreePolynomial
subtractIfCan
tan
tanh
taylor
taylorIfCan
taylorRep
terms
truncate
unit?
unitCanonical
unitNormal
variable
variables
wholePart
zero?
?*?
?**?
?+?
?-?
-?
?=?
?ˆ?
?˜=?
?/?
?<?
?<=?
?>?
?>=?
?.?
?quo?
?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has Field then nil

These are directly exported but not implemented:

```
coerce : UTS -> %
degree : % -> Integer
laurent : (Integer,UTS) -> %
removeZeroes : % -> %
removeZeroes : (Integer,%) -> %
taylor : % -> UTS
taylorIfCan : % -> Union(UTS,"failed")
taylorRep : % -> UTS
```

These are implemented by this category:

```
retract : % -> UTS
retractIfCan : % -> Union(UTS,"failed")
zero? : % -> Boolean
```

These exports come from (p1595) UnivariateLaurentSeriesCategory(Coef:Ring)

```
0 : () -> %
1 : () -> %
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
```

```
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
associates? : (%,%) -> Boolean if Coef has INTDOM
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(OR(has(UTS,CharacteristicNonZero),
            and(has($,CharacteristicNonZero),
                has(UTS,PolynomialFactorizationExplicit))),
         has(Coef,Field))
  or Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
```

```
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Integer) -> %
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
```

```
       if Coef has FIELD
 hash : % -> SingleInteger
 integrate : (%,Symbol) -> %
   if Coef has ACFS INT
   and Coef has PRIMCAT
   and Coef has TRANFUN
   and Coef has ALGEBRA FRAC INT
   or Coef has variables: Coef -> List Symbol
   and Coef has integrate: (Coef,Symbol) -> Coef
   and Coef has ALGEBRA FRAC INT
 integrate : % -> % if Coef has ALGEBRA FRAC INT
 inv : % -> % if Coef has FIELD
 latex : % -> String
 lcm : (%,%) -> % if Coef has FIELD
 lcm : List % -> % if Coef has FIELD
 leadingCoefficient : % -> Coef
 leadingMonomial : % -> %
 log : % -> % if Coef has ALGEBRA FRAC INT
 map : ((Coef -> Coef),%) -> %
 monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
 monomial : (%,SingletonAsOrderedSet,Integer) -> %
 monomial : (Coef,Integer) -> %
 monomial? : % -> Boolean
 multiEuclidean : (List %,%) -> Union(List %,"failed")
   if Coef has FIELD
 multiplyCoefficients : ((Integer -> Coef),%) -> %
 multiplyExponents : (%,PositiveInteger) -> %
 nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
 one? : % -> Boolean
 order : (%,Integer) -> Integer
 order : % -> Integer
 pi : () -> % if Coef has ALGEBRA FRAC INT
 pole? : % -> Boolean
 prime? : % -> Boolean if Coef has FIELD
 principalIdeal : List % -> Record(coef: List %,generator: %)
   if Coef has FIELD
 rationalFunction : (%,Integer) -> Fraction Polynomial Coef
   if Coef has INTDOM
 rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
   if Coef has INTDOM
 recip : % -> Union(%,"failed")
 reductum : % -> %
 sample : () -> %
 sec : % -> % if Coef has ALGEBRA FRAC INT
 sech : % -> % if Coef has ALGEBRA FRAC INT
 series : Stream Record(k: Integer,c: Coef) -> %
 sin : % -> % if Coef has ALGEBRA FRAC INT
 sinh : % -> % if Coef has ALGEBRA FRAC INT
 sizeLess? : (%,%) -> Boolean if Coef has FIELD
 squareFree : % -> Factored % if Coef has FIELD
```

```
squareFreePart : % -> % if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?.? : (%,Integer) -> Coef
?**? : (%,Integer) -> % if Coef has FIELD
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> % if Coef has FIELD
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Coef has FIELD
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
?.? : (%,%) -> % if Integer has SGROUP
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p1516) QuotientFieldCategory(UTS)
where UTS:UnivariateLaurentSeriesCategory(Coef:Ring)

```
abs : % -> %
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
ceiling : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
conditionP : Matrix % -> Union(Vector %,"failed")
```

```
  if and(and(has($,CharacteristicNonZero),
             has(UTS,PolynomialFactorizationExplicit)),
         has(Coef,Field))
coerce : Symbol -> %
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
convert : % -> Pattern Integer
  if and(has(UTS,ConvertibleTo Pattern Integer),has(Coef,Field))
convert : % -> Pattern Float
  if and(has(UTS,ConvertibleTo Pattern Float),has(Coef,Field)\hfill\\
convert : % -> InputForm
  if and(has(UTS,ConvertibleTo InputForm),has(Coef,Field))
convert : % -> Float
  if and(has(UTS,RealConstant),has(Coef,Field))
convert : % -> DoubleFloat
  if and(has(UTS,RealConstant),has(Coef,Field))
D : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
D : (%,(UTS -> UTS)) -> % if Coef has FIELD
denom : % -> UTS if Coef has FIELD
denominator : % -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
eval : (%,Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List Symbol,List UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
eval : (%,List Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,UTS,UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List UTS,List UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,Symbol,UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
floor : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
fractionPart : % -> %
  if and(has(UTS,EuclideanDomain),has(Coef,Field))
init : () -> % if and(has(UTS,StepThrough),has(Coef,Field))
map : ((UTS -> UTS),%) -> % if Coef has FIELD
max : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
```

```
min : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
negative? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
nextItem : % -> Union(%,"failed")
  if and(has(UTS,StepThrough),has(Coef,Field))
numer : % -> UTS if Coef has FIELD
numerator : % -> % if Coef has FIELD
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
     if and(has(UTS,PatternMatchable Integer),has(Coef,Field))
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
     if and(has(UTS,PatternMatchable Float),has(Coef,Field))
positive? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
random : () -> %
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS)
   if Coef has FIELD
reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
retract : % -> Symbol
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retract : % -> Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retract : % -> Fraction Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Symbol,"failed")
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
sign : % -> Integer
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
      if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
```

```
      if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
wholePart : % -> UTS
  if and(has(UTS,EuclideanDomain),has(Coef,Field))
?*? : (UTS,%) -> % if Coef has FIELD
?*? : (%,UTS) -> % if Coef has FIELD
?<? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?/? : (UTS,UTS) -> % if Coef has FIELD
?.? : (%,UTS) -> % if and(has(UTS,Eltable(UTS,UTS)),has(Coef,Field))
?<=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?>? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?>=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
```

⟨*category ULSCCAT UnivariateLaurentSeriesConstructorCategory*⟩≡

```
)abbrev category ULSCCAT UnivariateLaurentSeriesConstructorCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 February 1990
++ Date Last Updated: 10 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent, Taylor
++ Examples:
++ References:
++ Description:
++ This is a category of univariate Laurent series constructed from
++ univariate Taylor series.  A Laurent series is represented by a pair
++ \spad{[n,f(x)]}, where n is an arbitrary integer and \spad{f(x)}
++ is a Taylor series.  This pair represents the Laurent series
++ \spad{x**n * f(x)}.

UnivariateLaurentSeriesConstructorCategory(Coef,UTS):_
 Category == Definition where
  Coef: Ring
  UTS : UnivariateTaylorSeriesCategory Coef
  I ==> Integer

  Definition ==> Join(UnivariateLaurentSeriesCategory(Coef),_
                      RetractableTo UTS) with
    laurent: (I,UTS) -> %
      ++ \spad{laurent(n,f(x))} returns \spad{x**n * f(x)}.
    degree: % -> I
      ++ \spad{degree(f(x))} returns the degree of the lowest order term of
      ++ \spad{f(x)}, which may have zero as a coefficient.
    taylorRep: % -> UTS
      ++ \spad{taylorRep(f(x))} returns \spad{g(x)}, where
      ++ \spad{f = x**n * g(x)} is represented by \spad{[n,g(x)]}.
 removeZeroes: % -> %
   ++ \spad{removeZeroes(f(x))} removes leading zeroes from the
   ++ representation of the Laurent series \spad{f(x)}.
   ++ A Laurent series is represented by (1) an exponent and
   ++ (2) a Taylor series which may have leading zero coefficients.
   ++ When the Taylor series has a leading zero coefficient, the
   ++ 'leading zero' is removed from the Laurent series as follows:
   ++ the series is rewritten by increasing the exponent by 1 and
   ++ dividing the Taylor series by its variable.
   ++ Note that \spad{removeZeroes(f)} removes all leading zeroes from f
 removeZeroes: (I,%) -> %
   ++ \spad{removeZeroes(n,f(x))} removes up to n leading zeroes from
   ++ the Laurent series \spad{f(x)}.
   ++ A Laurent series is represented by (1) an exponent and
   ++ (2) a Taylor series which may have leading zero coefficients.
   ++ When the Taylor series has a leading zero coefficient, the
   ++ 'leading zero' is removed from the Laurent series as follows:
   ++ the series is rewritten by increasing the exponent by 1 and
   ++ dividing the Taylor series by its variable.
 coerce: UTS -> %
   ++ \spad{coerce(f(x))} converts the Taylor series \spad{f(x)} to a
   ++ Laurent series.
 taylor: % -> UTS
   ++ taylor(f(x)) converts the Laurent series f(x) to a Taylor series,
   ++ if possible.  Error: if this is not possible.
 taylorIfCan: % -> Union(UTS,"failed")
   ++ \spad{taylorIfCan(f(x))} converts the Laurent series \spad{f(x)}
   ++ to a Taylor series, if possible. If this is not possible,
   ++ "failed" is returned.
 if Coef has Field then QuotientFieldCategory(UTS)
   --++ the quotient field of univariate Taylor series over a field is
   --++ the field of Laurent series

add

 zero? x == zero? taylorRep x

 retract(x:%):UTS == taylor x

 retractIfCan(x:%):Union(UTS,"failed") == taylorIfCan x
```

⟨*ULSCCAT.dotabb*⟩≡

```
"ULSCCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"ULSCCAT" -> "ULSCAT"
```

⟨*ULSCCAT.dotfull*⟩≡

```
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCatego
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCatego
  -> "UnivariateLaurentSeriesCategory(a:Ring)"
```

⟨*ULSCCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
 [color=lightblue];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
  -> "UnivariateLaurentSeriesCategory(a:Ring)"

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

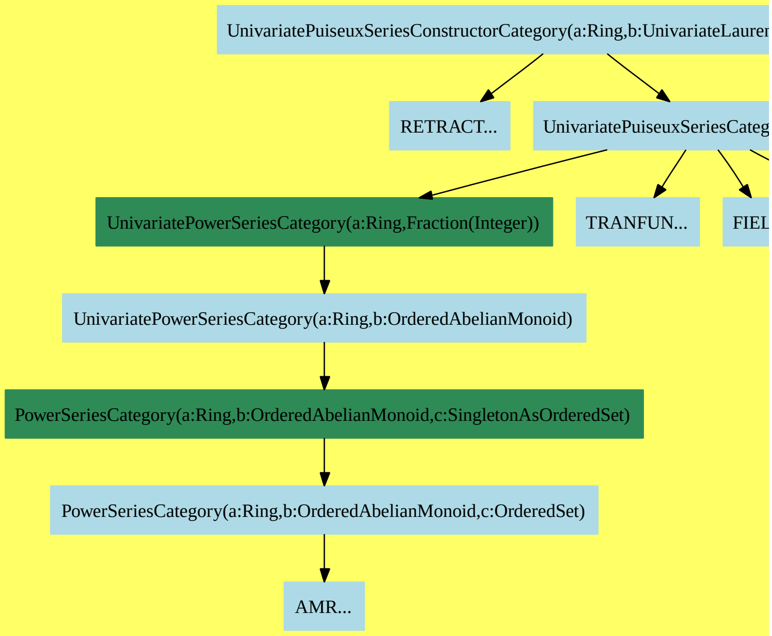
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

18.0.212 UnivariatePuiseuxSeriesConstructorCategory (UPXSCCA)


UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLauren
RETRACT...
UnivariatePuiseuxSeriesCateg
UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))
TRANFUN...
FIEL
UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)
PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)
AMR...

⟨*UnivariatePuiseuxSeriesConstructorCategory.input*⟩≡

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesConstructorCategory.output
)spool UnivariatePuiseuxSeriesConstructorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesConstructorCategory
--R UnivariatePuiseuxSeriesConstructorCategory(Coef: Ring,ULS: UnivariateLaurentS
--R Abbreviation for UnivariatePuiseuxSeriesConstructorCategory is UPXSCCA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPXSCCA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coerce : ULS -> %                     coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> Fraction Integer        hash : % -> SingleInteger
--R latex : % -> String                   laurent : % -> ULS
--R laurentRep : % -> ULS                 leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R order : % -> Fraction Integer         pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> ULS                    sample : () -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fraction Integer,Coef) ->
--R D : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fraction Integer,Coe
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has *: (Fracti
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRING SYMBOL and Coef has
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
```

```
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,
--R differentiate : (%,Symbol) -> % if Coef has PDRING SYMBOL and Coef has *: (Fr
--R differentiate : (%,List Symbol) -> % if Coef has PDRING SYMBOL and Coef has *
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has PDRI
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coe
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef h
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if C
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R integrate : (%,Symbol) -> % if Coef has ACFS INT and Coef has PRIMCAT and Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurentIfCan : % -> Union(ULS,"failed")
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R puiseux : (Fraction Integer,ULS) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retractIfCan : % -> Union(ULS,"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

⟨*UnivariatePuiseuxSeriesConstructorCategory.help*⟩≡

```
====================================================================
UnivariatePuiseuxSeriesConstructorCategory examples
====================================================================

See Also:
o )show UnivariatePuiseuxSeriesConstructorCategory
```

See:

⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62
⇐ “UnivariatePuiseuxSeriesCategory” (UPXSCAT) 17.0.203 on page 1608

Exports:

0
1
acos
acosh
acot
acoth
acsc
acsch
approximate
asec
asech
asin
asinh
associates?
atan
atanh
center
characteristic
charthRoot
coefficient
coerce
complete
cos
cosh
cot
coth
csc
csch
D
degree
differentiate
divide
euclideanSize
eval
exp
expressIdealMember
exquo
extend
extendedEuclidean
factor
gcd
gcdPolynomial
hash
integrate
inv
latex
laurent
laurentIfCan
laurentRep
lcm
leadingCoefficient
leadingMonomial
log
map
monomial
monomial?
multiEuclidean
multiplyExponents
nthRoot
one?
order
pi
pole?
prime?
principalIdeal
puiseux
rationalPower
recip
reductum
retract
retractIfCan
sample
sec
sech
series
sin
sinh
sizeLess?
sqrt
squareFree
squareFreePart
subtractIfCan
tan
tanh
terms
truncate
unit?
unitCanonical
unitNormal
variable
variables
zero?
?.?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?quo?
?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalsClosed where
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commututive("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
coerce : ULS -> %
degree : % -> Fraction Integer
laurent : % -> ULS
laurentIfCan : % -> Union(ULS,"failed")
laurentRep : % -> ULS
puiseux : (Fraction Integer,ULS) -> %
rationalPower : % -> Fraction Integer
```

These are implemented by this category:

```
retract : % -> ULS
retractIfCan : % -> Union(ULS,"failed")
zero? : % -> Boolean
```

These exports come from (p1608) UnivariatePuiseuxSeriesCategory(Coef:Ring):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
```

```
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
```

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
hash : % -> SingleInteger
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
inv : % -> % if Coef has FIELD
latex : % -> String
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
log : % -> % if Coef has ALGEBRA FRAC INT
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial? : % -> Boolean
multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
multiplyExponents : (%,PositiveInteger) -> %
multiplyExponents : (%,Fraction Integer) -> %
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
pi : () -> % if Coef has ALGEBRA FRAC INT
pole? : % -> Boolean
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
```

```
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
series :
  (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sizeLess? : (%,%) -> Boolean if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
```

```
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

⟨*category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory*⟩≡

```
)abbrev category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 February 1990
++ Date Last Updated: 22 March 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux, Laurent
++ Examples:
++ References:
++ Description:
++ This is a category of univariate Puiseux series constructed
++ from univariate Laurent series.  A Puiseux series is represented
++ by a pair \spad{[r,f(x)]}, where r is a positive rational number and
++ \spad{f(x)} is a Laurent series.  This pair represents the Puiseux
++ series \spad{f(x^r)}.

UnivariatePuiseuxSeriesConstructorCategory(Coef,ULS):_
 Category == Definition where
  Coef : Ring
  ULS  : UnivariateLaurentSeriesCategory Coef
  I  ==> Integer
  RN ==> Fraction Integer

  Definition ==> Join(UnivariatePuiseuxSeriesCategory(Coef),_
                     RetractableTo ULS) with
    puiseux: (RN,ULS) -> %
      ++ \spad{puiseux(r,f(x))} returns \spad{f(x^r)}.
    rationalPower: % -> RN
      ++ \spad{rationalPower(f(x))} returns r where the Puiseux series
      ++ \spad{f(x) = g(x^r)}.
    laurentRep  : % -> ULS
      ++ \spad{laurentRep(f(x))} returns \spad{g(x)} where the Puiseux series
      ++ \spad{f(x) = g(x^r)} is represented by \spad{[r,g(x)]}.
    degree: % -> RN
      ++ \spad{degree(f(x))} returns the degree of the leading term of the
      ++ Puiseux series \spad{f(x)}, which may have zero as a coefficient.
    coerce: ULS -> %
      ++ \spad{coerce(f(x))} converts the Laurent series \spad{f(x)} to a
      ++ Puiseux series.
    laurent: % -> ULS
```

```
      ++ \spad{laurent(f(x))} converts the Puiseux series \spad{f(x)} to a
      ++ Laurent series if possible. Error: if this is not possible.
    laurentIfCan: % -> Union(ULS,"failed")
      ++ \spad{laurentIfCan(f(x))} converts the Puiseux series \spad{f(x)}
      ++ to a Laurent series if possible.
      ++ If this is not possible, "failed" is returned.

   add

     zero? x == zero? laurentRep x

     retract(x:%):ULS == laurent x

     retractIfCan(x:%):Union(ULS,"failed") == laurentIfCan x
```

⟨*UPXSCCA.dotabb*⟩≡

```
  "UPXSCCA"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
  "UPXSCCA" -> "RETRACT"
  "UPXSCCA" -> "UPXSCAT"
```

⟨*UPXSCCA.dotfull*⟩≡

```
  "UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
   [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
  "UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
    -> "RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
  "UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
    -> "UnivariatePuiseuxSeriesCategory(a:Ring)"
```

⟨*UPXSCCA.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "RETRACT..."
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "UnivariatePuiseuxSeriesCategory(a:Ring)"

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

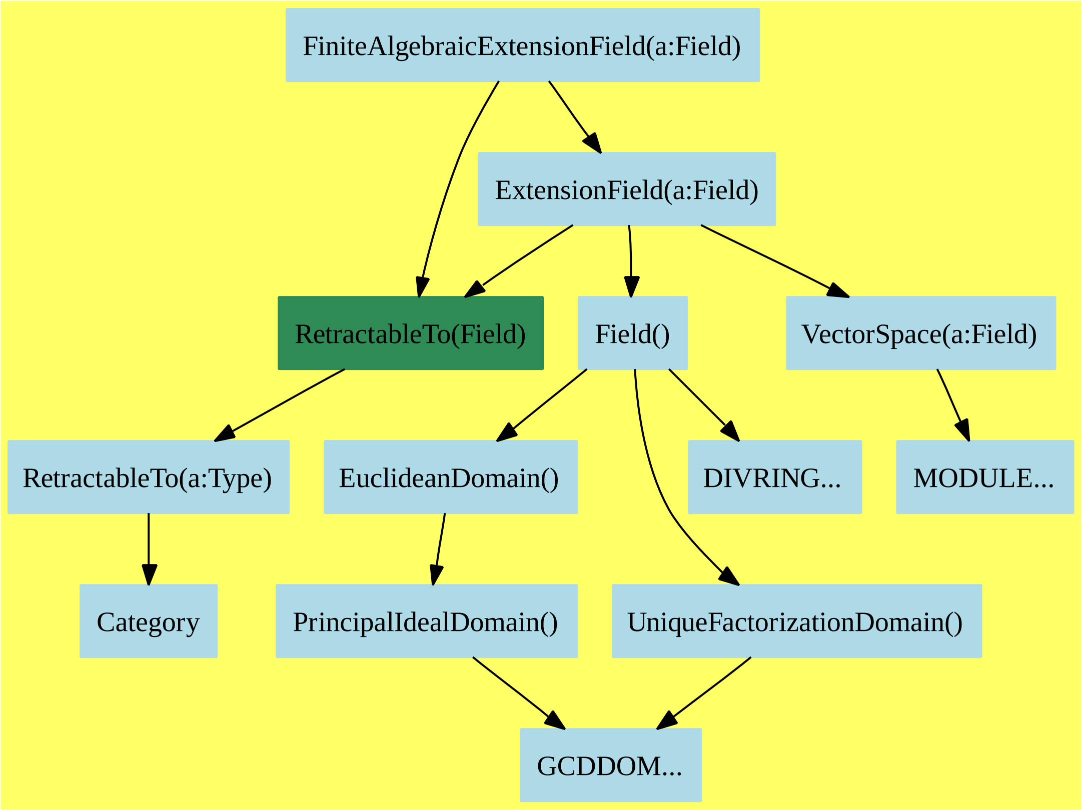
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"RETRACT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
"RADCAT..." [color=lightblue];
"AMR..." [color=lightblue];
}
```

Chapter 19

Category Layer 18

19.0.213 FiniteAlgebraicExtensionField (FAXF)


FiniteAlgebraicExtensionField(a:Field)
ExtensionField(a:Field)
RetractableTo(Field)
Field()
VectorSpace(a:Field)
RetractableTo(a:Type)
EuclideanDomain()
DIVRING...
MODULE...
Category
PrincipalIdealDomain()
UniqueFactorizationDomain()
GCDDOM...

⟨*FiniteAlgebraicExtensionField.input*⟩≡

```
)set break resume
)sys rm -f FiniteAlgebraicExtensionField.output
)spool FiniteAlgebraicExtensionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteAlgebraicExtensionField
--R FiniteAlgebraicExtensionField F: Field  is a category constructor
--R Abbreviation for FiniteAlgebraicExtensionField is FAXF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAXF
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (F,%) -> %                      ?*? : (%,F) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,F) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : % -> % if F has FINITE
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean             associates? : (%,%) -> Boolean
--R basis : () -> Vector %                coerce : F -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R coordinates : % -> Vector F           degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R norm : % -> F                         one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R represents : Vector F -> %            retract : % -> F
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R trace : % -> F                        transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
```

```
--R D : (%,NonNegativeInteger) -> % if F has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
--R Frobenius : % -> % if F has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if F has CHARNZ or F has FINITE
--R charthRoot : % -> % if F has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if F has FINITE
--R coordinates : Vector % -> Matrix F
--R createNormalElement : () -> % if F has FINITE
--R createPrimitiveElement : () -> % if F has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial F
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if F has FINITE
--R differentiate : % -> % if F has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if F has CHARNZ or F has FINI
--R discreteLog : % -> NonNegativeInteger if F has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if F has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R generator : () -> % if F has FINITE
--R index : PositiveInteger -> % if F has FINITE
--R init : () -> % if F has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial F) -> % if F has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial F,"failed") if F has FIN
--R linearAssociatedLog : % -> SparseUnivariatePolynomial F if F has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial F if F has FINITE
--R lookup : % -> PositiveInteger if F has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if F has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial F
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if F has FINITE
--R norm : (%,PositiveInteger) -> % if F has FINITE
--R normal? : % -> Boolean if F has FINITE
--R normalElement : () -> % if F has FINITE
--R order : % -> OnePointCompletion PositiveInteger if F has CHARNZ or F has FINITE
--R order : % -> PositiveInteger if F has FINITE
--R primeFrobenius : % -> % if F has CHARNZ or F has FINITE
```

```
--R primeFrobenius : (%,NonNegativeInteger) -> % if F has CHARNZ or F has FINITE
--R primitive? : % -> Boolean if F has FINITE
--R primitiveElement : () -> % if F has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if F has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if F has F
--R retractIfCan : % -> Union(F,"failed")
--R size : () -> NonNegativeInteger if F has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteg
--R trace : (%,PositiveInteger) -> % if F has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FiniteAlgebraicExtensionField.help*⟩≡

```
====================================================================
FiniteAlgebraicExtensionField examples
====================================================================

See Also:
o )show FiniteAlgebraicExtensionField
```

See:

⇐ “ExtensionField” (XF) 18.0.206 on page 1662
⇐ “RetractableTo” (RETRACT) 2.0.18 on page 62

Exports:

0
1
algebraic?
associates?
basis
characteristic
charthRoot
coerce
conditionP
coordinates
createNormalElement
createPrimitiveElement
D
definingPolynomial
degree
differentiate
dimension
discreteLog
divide
euclideanSize
expressIdealMember
exquo
extendedEuclidean
extensionDegree
factor
factorsOfCyclicGroupSize
Frobenius
gcd
gcdPolynomial
generator
hash
index
inGroundField?
init
inv
latex
lcm
linearAssociatedExp
linearAssociatedLog
linearAssociatedOrder
lookup
minimalPolynomial
multiEuclidean
nextItem
norm
normal?
normalElement
one?
order
prime?
primeFrobenius
primitive?
primitiveElement
principalIdeal
random
recip
representationType
represents
retract
retractIfCan
sample
size
sizeLess?
squareFree
squareFreePart
subtractIfCan
tableForDiscreteLogarithm
trace
transcendenceDegree
transcendent?
unit?
unitCanonical
unitNormal
zero?
?*?
?**?
?+?
?-?
-?
?/?
?=?
?^?
?quo?
?rem?
?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
basis : PositiveInteger -> Vector %
coordinates : % -> Vector F
definingPolynomial : () -> SparseUnivariatePolynomial F
generator : () -> % if F has FINITE
minimalPolynomial : (%,PositiveInteger) ->
   SparseUnivariatePolynomial %
     if F has FINITE
normalElement : () -> % if F has FINITE
```

These are implemented by this category:

```
algebraic? : % -> Boolean
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
coordinates : Vector % -> Matrix F
createNormalElement : () -> % if F has FINITE
degree : % -> PositiveInteger
dimension : () -> CardinalNumber
extensionDegree : () -> PositiveInteger
linearAssociatedExp : (%,SparseUnivariatePolynomial F) -> %
    if F has FINITE
linearAssociatedLog : (%,%) ->
   Union(SparseUnivariatePolynomial F,"failed")
     if F has FINITE
linearAssociatedLog : % -> SparseUnivariatePolynomial F
    if F has FINITE
linearAssociatedOrder : % -> SparseUnivariatePolynomial F
    if F has FINITE
minimalPolynomial : % -> SparseUnivariatePolynomial F
norm : % -> F
norm : (%,PositiveInteger) -> % if F has FINITE
normal? : % -> Boolean if F has FINITE
represents : Vector F -> %
size : () -> NonNegativeInteger if F has FINITE
trace : % -> F
trace : (%,PositiveInteger) -> % if F has FINITE
```

```
transcendenceDegree : () -> NonNegativeInteger
transcendent? : % -> Boolean
```

These exports come from (p1662) ExtensionField(F:Field):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : F -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
discreteLog : (%,%) ->
   Union(NonNegativeInteger,"failed")
     if F has CHARNZ or F has FINITE
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
   Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
Frobenius : % -> % if F has FINITE
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
inGroundField? : % -> Boolean
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
     if F has CHARNZ or F has FINITE
prime? : % -> Boolean
primeFrobenius : % -> %
    if F has CHARNZ or F has FINITE
primeFrobenius : (%,NonNegativeInteger) -> %
    if F has CHARNZ or F has FINITE
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
```

```
retract : % -> F
retractIfCan : % -> Union(F,"failed")
sample : () -> %
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
?/? : (%,F) -> %
```

These exports come from (p62) RetractableTo(F:Field):

These exports come from (p1671) FiniteFieldCategory():

```
charthRoot : % -> % if F has FINITE
conditionP : Matrix % -> Union(Vector %,"failed")
    if F has FINITE
createPrimitiveElement : () -> % if F has FINITE
D : % -> % if F has FINITE
D : (%,NonNegativeInteger) -> % if F has FINITE
differentiate : % -> % if F has FINITE
differentiate : (%,NonNegativeInteger) -> %
    if F has FINITE
```

```
discreteLog : % -> NonNegativeInteger if F has FINITE
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
     if F has FINITE
index : PositiveInteger -> % if F has FINITE
init : () -> % if F has FINITE
lookup : % -> PositiveInteger if F has FINITE
nextItem : % -> Union(%,"failed") if F has FINITE
order : % -> PositiveInteger if F has FINITE
primitive? : % -> Boolean if F has FINITE
primitiveElement : () -> % if F has FINITE
random : () -> % if F has FINITE
representationType : () ->
   Union("prime",polynomial,normal,cyclic)
     if F has FINITE
tableForDiscreteLogarithm : Integer ->
   Table(PositiveInteger,NonNegativeInteger)
     if F has FINITE
```

⟨*category FAXF FiniteAlgebraicExtensionField*⟩≡

```
)abbrev category FAXF FiniteAlgebraicExtensionField
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 11 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*, extensionDegree,
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension, finite extension
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983,
++  ISBN 0 521 30240 4 J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteAlgebraicExtensionField F is the category of fields
++ which are finite algebraic extensions of the field F.
++ If F is finite then any finite algebraic extension of F
++ is finite, too. Let K be a finite algebraic extension of the
++ finite field F. The exponentiation of elements of K
++ defines a Z-module structure on the multiplicative group of K.
++ The additive group of K becomes a module over the ring of
++ polynomials over F via the operation
++ \spadfun{linearAssociatedExp}(a:K,f:SparseUnivariatePolynomial F)
++ which is linear over F, i.e. for elements a from K,
++ c,d from F and f,g univariate polynomials over F
++ we have \spadfun{linearAssociatedExp}(a,cf+dg) equals c times
++ \spadfun{linearAssociatedExp}(a,f) plus d times
++ \spadfun{linearAssociatedExp}(a,g).
++ Therefore \spadfun{linearAssociatedExp} is defined completely by
++ its action on  monomials from F[X]:
++ \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is defined to be
++ \spadfun{Frobenius}(a,k) which is a**(q**k) where q=size()\$F.
++ The operations order and discreteLog associated with the multiplicative
++ exponentiation have additive analogues associated to the operation
++ \spadfun{linearAssociatedExp}. These are the functions
++ \spadfun{linearAssociatedOrder} and \spadfun{linearAssociatedLog},
++ respectively.

FiniteAlgebraicExtensionField(F : Field) : Category == _
  Join(ExtensionField F, RetractableTo F) with
  -- should be unified with algebras
  -- Join(ExtensionField F, FramedAlgebra F, RetractableTo F) with
    basis : () -> Vector $
      ++ basis() returns a fixed basis of \$ as \spad{F}-vectorspace.
    basis : PositiveInteger -> Vector $
      ++ basis(n) returns a fixed basis of a subfield of \$ as
      ++ \spad{F}-vectorspace.
    coordinates : $ -> Vector F
      ++ coordinates(a) returns the coordinates of \spad{a} with respect
      ++ to the fixed \spad{F}-vectorspace basis.
    coordinates : Vector $ -> Matrix F
      ++ coordinates([v1,...,vm]) returns the coordinates of the
      ++ vi's with to the fixed basis.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
      ++ function.
    represents:  Vector F -> $
      ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
      ++ v1,...,vn are the elements of the fixed basis.
    minimalPolynomial: $ -> SparseUnivariatePolynomial F
      ++ minimalPolynomial(a) returns the minimal polynomial of an
      ++ element \spad{a} over the ground field F.
    definingPolynomial: () -> SparseUnivariatePolynomial F
      ++ definingPolynomial() returns the polynomial used to define
      ++ the field extension.
    extensionDegree : () ->  PositiveInteger
      ++ extensionDegree() returns the degree of field extension.
    degree : $ -> PositiveInteger
      ++ degree(a) returns the degree of the minimal polynomial of an
      ++ element \spad{a} over the ground field F.
    norm: $  -> F
      ++ norm(a) computes the norm of \spad{a} with respect to the
      ++ field considered as an algebra with 1 over the ground field F.
    trace: $ -> F
  ++ trace(a) computes the trace of \spad{a} with respect to
  ++ the field considered as an algebra with 1 over the ground field F.
if F has Finite then
  FiniteFieldCategory
  minimalPolynomial: ($,PositiveInteger) -> SparseUnivariatePolynomial $
    ++ minimalPolynomial(x,n) computes the minimal polynomial of x over
    ++ the field of extension degree n over the ground field F.
  norm: ($,PositiveInteger)  -> $
    ++ norm(a,d) computes the norm of \spad{a} with respect to the field
    ++ of extension degree d over the ground field of size.
    ++ Error: if d does not divide the extension degree of \spad{a}.
    ++ Note that norm(a,d) = reduce(*,[a**(q**(d*i)) for i in 0..n/d])
  trace: ($,PositiveInteger)   -> $
    ++ trace(a,d) computes the trace of \spad{a} with respect to the
    ++ field of extension degree d over the ground field of size q.
    ++ Error: if d does not divide the extension degree of \spad{a}.
    ++ Note that
    ++ \spad{trace(a,d)=reduce(+,[a**(q**(d*i)) for i in 0..n/d])}.
  createNormalElement: () -> $
    ++ createNormalElement() computes a normal element over the ground
    ++ field F, that is,
    ++ \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ Reference: Such an element exists Lidl/Niederreiter: Theorem 2.35.
  normalElement: () -> $
    ++ normalElement() returns a element, normal over the ground field F,
    ++ i.e. \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ At the first call, the element is computed by
    ++ \spadfunFrom{createNormalElement}{FiniteAlgebraicExtensionField}
    ++ then cached in a global variable.
    ++ On subsequent calls, the element is retrieved by referencing the
    ++ global variable.
  normal?: $ -> Boolean
    ++ normal?(a) tests whether the element \spad{a} is normal over the
    ++ ground field F, i.e.
    ++ \spad{a**(q**i), 0 <= i <= extensionDegree()-1} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ Implementation according to Lidl/Niederreiter: Theorem 2.39.
  generator: () -> $
    ++ generator() returns a root of the defining polynomial.
    ++ This element generates the field as an algebra over the ground
    ++ field.
  linearAssociatedExp:($,SparseUnivariatePolynomial F) -> $
    ++ linearAssociatedExp(a,f) is linear over F, i.e.
    ++ for elements a from \$, c,d form F and
```

```
        ++ f,g univariate polynomials over F we have
        ++ \spadfun{linearAssociatedExp}(a,cf+dg) equals c times
        ++ \spadfun{linearAssociatedExp}(a,f) plus d times
        ++ \spadfun{linearAssociatedExp}(a,g). Therefore
        ++ \spadfun{linearAssociatedExp} is defined completely by its
        ++ action on monomials from F[X]:
        ++ \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is
        ++ defined to be \spadfun{Frobenius}(a,k) which is a**(q**k),
        ++ where q=size()\$F.
      linearAssociatedOrder: $ -> SparseUnivariatePolynomial F
        ++ linearAssociatedOrder(a) retruns the monic polynomial g of
        ++ least degree, such that \spadfun{linearAssociatedExp}(a,g) is 0.
      linearAssociatedLog: $ -> SparseUnivariatePolynomial F
        ++ linearAssociatedLog(a) returns a polynomial g, such that
        ++ \spadfun{linearAssociatedExp}(normalElement(),g) equals a.
      linearAssociatedLog: ($,$) -> _
        Union(SparseUnivariatePolynomial F,"failed")
        ++ linearAssociatedLog(b,a) returns a polynomial g, such
        ++ that the \spadfun{linearAssociatedExp}(b,g) equals a.
        ++ If there is no such polynomial g, then
        ++ \spadfun{linearAssociatedLog} fails.
  add
    I   ==> Integer
    PI  ==> PositiveInteger
    NNI ==> NonNegativeInteger
    SUP ==> SparseUnivariatePolynomial
    DLP ==> DiscreteLogarithmPackage

    represents(v) ==
      a:$:=0
      b:=basis()
      for i in 1..extensionDegree()@PI repeat
        a:=a+(v.i)*(b.i)
      a

    transcendenceDegree() == 0$NNI

    dimension() == (#basis()) ::NonNegativeInteger::CardinalNumber

    coordinates(v:Vector $) ==
      m := new(#v, extensionDegree(), 0)$Matrix(F)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates qelt(v, i))
      m

    algebraic? a == true
    transcendent? a == false

-- This definition is a duplicate and has been removed
--    extensionDegree():OnePointCompletion(PositiveInteger) ==
--      (#basis()) :: PositiveInteger::OnePointCompletion(PositiveInteger)

    extensionDegree() == (#basis()) :: PositiveInteger

-- These definitions are duplicates and have been removed
--    degree(a):OnePointCompletion(PositiveInteger) ==
--      degree(a)@PI::OnePointCompletion(PositiveInteger)

    -- degree a == degree(minimalPolynomial a)$SUP(F) :: PI

    trace a ==
      b := basis()
      abs : F := 0
      for i in 1..#b repeat
        abs := abs + coordinates(a*b.i).i
      abs

    norm a ==
      b := basis()
      m := new(#b,#b, 0)$Matrix(F)
      for i in 1..#b repeat
        setRow_!(m,i, coordinates(a*b.i))
      determinant(m)

    if F has Finite then
      linearAssociatedExp(x,f) ==
        erg:$:=0
        y:=x
        for i in 0..degree(f) repeat
          erg:=erg + coefficient(f,i) * y
          y:=Frobenius(y)
        erg

      linearAssociatedLog(b,x) ==
        x=0 => 0
        l:List List F:=[entries coordinates b]
        a:$:=b
        extdeg:NNI:=extensionDegree()@PI
        for i in 2..extdeg repeat
          a:=Frobenius(a)
          l:=concat(l,entries coordinates a)$(List List F)
```

```
  l:=concat(l,entries coordinates x)$(List List F)
  m1:=rowEchelon transpose matrix(l)$(Matrix F)
  v:=zero(extdeg)$(Vector F)
  rown:I:=1
  for i in 1..extdeg repeat
    if qelt(m1,rown,i) = 1$F then
      v.i:=qelt(m1,rown,extdeg+1)
      rown:=rown+1
  p:=+/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]
  p=0 =>
   messagePrint("linearAssociatedLog: second argument not in_
                group generated by first argument")$OutputForm
   "failed"
  p

linearAssociatedLog(x) == linearAssociatedLog(normalElement(),x) ::
                        SparseUnivariatePolynomial(F)

linearAssociatedOrder(x) ==
  x=0 => 0
  l:List List F:=[entries coordinates x]
  a:$:=x
  for i in 1..extensionDegree()@PI repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  v:=first nullSpace transpose matrix(l)$(Matrix F)
  +/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]

charthRoot(x):Union($,"failed") ==
  (charthRoot(x)@$)::Union($,"failed")
-- norm(e) == norm(e,1) pretend F
-- trace(e) == trace(e,1) pretend F

minimalPolynomial(a,n) ==
  extensionDegree()@PI rem n ^= 0 =>
    error "minimalPolynomial: 2. argument must divide extension degree"
  f:SUP $:=monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u:$:=Frobenius(a,n)
  while not(u = a) repeat
    f:=f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u:=Frobenius(u,n)
  f

norm(e,s) ==
  qr := divide(extensionDegree(), s)
  zero?(qr.remainder) =>
```

```
    pow := (size()-1) quo (size()$F ** s - 1)
    e ** (pow::NonNegativeInteger)
  error "norm: second argument must divide degree of extension"

trace(e,s) ==
  qr:=divide(extensionDegree(),s)
  q:=size()$F
  zero?(qr.remainder) =>
    a:$:=0
    for i in 0..qr.quotient-1 repeat
      a:=a + e**(q**(s*i))
    a
  error "trace: second argument must divide degree of extension"

size() == size()$F ** extensionDegree()

createNormalElement() ==
  characteristic() = size() => 1
  res : $
  for i in 1.. repeat
    res := index(i :: PI)
    not inGroundField? res =>
      normal? res => return res
  -- theorem: there exists a normal element, this theorem is
  -- unknown to the compiler
  res

normal?(x:$) ==
  p:SUP $:=(monomial(1,extensionDegree()) - monomial(1,0))@(SUP $)
  f:SUP $:= +/[monomial(Frobenius(x,i),i)$(SUP $) _
             for i in 0..extensionDegree()-1]
  gcd(p,f) = 1 => true
  false

degree a ==
  y:$:=Frobenius a
  deg:PI:=1
  while y^=a repeat
    y := Frobenius(y)
    deg:=deg+1
  deg
```

⟨*FAXF.dotabb*⟩≡

```
"FAXF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FAXF" -> "XF"
"FAXF" -> "RETRACT"
```

⟨*FAXF.dotfull*⟩≡

```
"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(a:Field)"
```

⟨*FAXF.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(Field)"

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

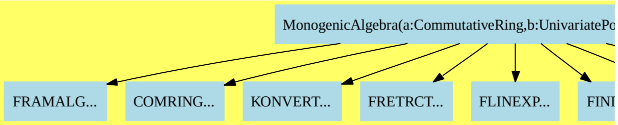
"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"Category" [color=lightblue];
}
```

19.0.214 MonogenicAlgebra (MONOGEN)



⟨*MonogenicAlgebra.input*⟩≡

```
)set break resume
)sys rm -f MonogenicAlgebra.output
)spool MonogenicAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonogenicAlgebra
--R MonogenicAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory t#1)  is
--R Abbreviation for MonogenicAlgebra is MONOGEN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONOGEN
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       basis : () -> Vector %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             convert : UP -> %
--R convert : % -> UP                    convert : Vector R -> %
--R convert : % -> Vector R              coordinates : % -> Vector R
--R definingPolynomial : () -> UP        discriminant : () -> R
--R discriminant : Vector % -> R         generator : () -> %
--R hash : % -> SingleInteger            inv : % -> % if R has FIELD
--R latex : % -> String                  lift : % -> UP
--R norm : % -> R                        one? : % -> Boolean
--R rank : () -> PositiveInteger         recip : % -> Union(%,"failed")
--R reduce : UP -> %                     represents : Vector R -> %
--R retract : % -> R                     sample : () -> %
--R trace : % -> R                       traceMatrix : () -> Matrix R
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has FIELD
```

```
--R ?*? : (Fraction Integer,%) -> % if R has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if R has FIELD
--R D : (%,(R -> R)) -> % if R has FIELD
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing Sy
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing Symbol),has(
--R D : (%,List Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
--R D : (%,Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
--R D : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FF
--R D : % -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R ?^? : (%,Integer) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has FIELD
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UP
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R charthRoot : % -> % if R has FFIELDC
--R coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
--R coerce : % -> % if R has FIELD
--R conditionP : Matrix % -> Union(Vector %,"failed") if R has FFIELDC
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R createPrimitiveElement : () -> % if R has FFIELDC
--R derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
--R differentiate : (%,(R -> R)) -> % if R has FIELD
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if and(has(R,PartialDiffere
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing
--R differentiate : (%,List Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,
--R differentiate : (%,Symbol) -> % if and(has(R,PartialDifferentialRing Symbol),has(R,Fiel
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Field))
--R differentiate : % -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if R has
```

```
--R gcd : (%,%) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R index : PositiveInteger -> % if R has FINITE
--R init : () -> % if R has FFIELDC
--R lcm : (%,%) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R lookup : % -> PositiveInteger if R has FINITE
--R minimalPolynomial : % -> UP if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R nextItem : % -> Union(%,"failed") if R has FFIELDC
--R order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R prime? : % -> Boolean if R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R ?quo? : (%,%) -> % if R has FIELD
--R random : () -> % if R has FINITE
--R reduce : Fraction UP -> Union(%,"failed") if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R regularRepresentation : % -> Matrix R
--R regularRepresentation : (%,Vector %) -> Matrix R
--R ?rem? : (%,%) -> % if R has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if R has F
--R represents : (Vector R,Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC IN
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R squareFree : % -> Factored % if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteg
--R traceMatrix : Vector % -> Matrix R
--R unit? : % -> Boolean if R has FIELD
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)
```

⟨*MonogenicAlgebra.help*⟩≡

```
====================================================================
MonogenicAlgebra examples
====================================================================

See Also:
o )show MonogenicAlgebra
```

See:

⇒ "FunctionFieldCategory" (FFCAT) 20.0.217 on page 1804
⇐ "CommutativeRing" (COMRING) 10.0.137 on page 932
⇐ "ConvertibleTo" (KONVERT) 2.0.8 on page 25
⇐ "FramedAlgebra" (FRAMALG) 18.0.209 on page 1694
⇐ "FullyRetractableTo" (FRETRCT) 3.0.27 on page 100
⇐ "LinearlyExplicitRingOver" (LINEXP) 10.0.142 on page 961

Exports:

0
1
associates?
basis
characteristic
characteristicPolynomial
charthRoot
coerce
conditionP
convert
coordinates
createPrimitiveElement
D
definingPolynomial
derivationCoordinates
differentiate
discreteLog
discriminant
divide
euclideanSize
expressIdealMember
exquo
extendedEuclidean
factor
factorsOfCyclicGroupSize
generator
gcd
gcdPolynomial
hash
index
init
inv
latex
lcm
lift
lookup
minimalPolynomial
multiEuclidean
nextItem
norm
one?
order
prime?
primeFrobenius
primitive?
primitiveElement
principalIdeal
random
rank
recip
reduce
reducedSystem
regularRepresentation
represents
representationType
retract
retractIfCan
sample
size
sizeLess?
squareFree
squareFreePart
subtractIfCan
tableForDiscreteLogarithm
trace
traceMatrix
unit?
unitCanonical
unitNormal
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?quo?
?rem?

Attributes Exported:

- if $ has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Field then canonicalClosed where **canonicalsClosed** is true if

`unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

- if $ has Field then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
definingPolynomial : () -> UP
lift : % -> UP
reduce : UP -> %
```

These are implemented by this category:

```
basis : () -> Vector %
characteristicPolynomial : % -> UP
convert : % -> UP
convert : UP -> %
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R
    if R has FIELD
differentiate : (%,(R -> R)) -> % if R has FIELD
generator : () -> %
norm : % -> R
random : () -> % if R has FINITE
recip : % -> Union(%,"failed")
reduce : Fraction UP -> Union(%,"failed") if R has FIELD
retract : % -> R
retractIfCan : % -> Union(R,"failed")
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p1694) FramedAlgebra(R,UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory(a)

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
```

```
convert : Vector R -> %
convert : % -> Vector R
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : Vector % -> R
discriminant : () -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
one? : % -> Boolean
rank : () -> PositiveInteger
regularRepresentation : (%,Vector %) -> Matrix R
regularRepresentation : % -> Matrix R
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
traceMatrix : () -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p100) FullyRetractableTo(R) where R:CommutativeRing

```
coerce : Fraction Integer -> %
    if R has FIELD or R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
```

```
    if R has RETRACT INT
```

These exports come from (p1060) FullyLinearlyExplicitRingOver(R)
where R:CommutativeRing

```
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

These exports come from (p188) Finite()

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
```

These exports come from (p1347) Field()

```
associates? : (%,%) -> Boolean if R has FIELD
coerce : % -> % if R has FIELD
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
euclideanSize : % -> NonNegativeInteger if R has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has FIELD
exquo : (%,%) -> Union(%,"failed") if R has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has FIELD
factor : % -> Factored % if R has FIELD
gcd : (%,%) -> % if R has FIELD
gcd : List % -> % if R has FIELD
gcdPolynomial :
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       SparseUnivariatePolynomial % if R has FIELD
inv : % -> % if R has FIELD
lcm : (%,%) -> % if R has FIELD
lcm : List % -> % if R has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has FIELD
prime? : % -> Boolean if R has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has FIELD
sizeLess? : (%,%) -> Boolean if R has FIELD
```

```
squareFree : % -> Factored % if R has FIELD
squareFreePart : % -> % if R has FIELD
unit? : % -> Boolean if R has FIELD
unitCanonical : % -> % if R has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has FIELD
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?quo? : (%,%) -> % if R has FIELD
?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p1053) DifferentialExtension(R)
where R:CommutativeRing

```
D : (%,(R -> R)) -> % if R has FIELD
D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
D : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
    if R has FIELD
differentiate : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
```

These exports come from (p1671) FiniteFieldCategory():

```
charthRoot : % -> % if R has FFIELDC
conditionP : Matrix % -> Union(Vector %,"failed")
    if R has FFIELDC
createPrimitiveElement : () -> % if R has FFIELDC
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if R has FFIELDC
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
     if R has FFIELDC
init : () -> % if R has FFIELDC
nextItem : % -> Union(%,"failed") if R has FFIELDC
order : % -> OnePointCompletion PositiveInteger
    if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
primeFrobenius : % -> % if R has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
representationType : () ->
   Union("prime",polynomial,normal,cyclic)
     if R has FFIELDC
tableForDiscreteLogarithm :
   Integer -> Table(PositiveInteger,NonNegativeInteger)
     if R has FFIELDC
```

⟨*category MONOGEN MonogenicAlgebra*⟩≡

```
)abbrev category MONOGEN MonogenicAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A \spadtype{MonogenicAlgebra} is an algebra of finite rank which
++ can be generated by a single element.

MonogenicAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category ==
    Join(FramedAlgebra(R, UP), CommutativeRing, ConvertibleTo UP,
              FullyRetractableTo R, FullyLinearlyExplicitRingOver R) with
      generator         : () -> %
        ++ generator() returns the generator for this domain.
    definingPolynomial: () -> UP
      ++ definingPolynomial() returns the minimal polynomial which
      ++ \spad{generator()} satisfies.
    reduce           : UP -> %
      ++ reduce(up) converts the univariate polynomial up to an algebra
      ++ element, reducing by the \spad{definingPolynomial()} if necessary.
    convert          : UP -> %
      ++ convert(up) converts the univariate polynomial up to an algebra
      ++ element, reducing by the \spad{definingPolynomial()} if necessary.
    lift             : % -> UP
      ++ lift(z) returns a minimal degree univariate polynomial up such that
      ++ \spad{z=reduce up}.
    if R has Finite then Finite
    if R has Field then
      Field
      DifferentialExtension R
      reduce               : Fraction UP -> Union(%, "failed")
        ++ reduce(frac) converts the fraction frac to an algebra element.
      derivationCoordinates: (Vector %, R -> R) -> Matrix R
        ++ derivationCoordinates(b, ’) returns M such that \spad{b’ = M b}.
    if R has FiniteFieldCategory then FiniteFieldCategory
add
 convert(x:%):UP == lift x
 convert(p:UP):% == reduce p
 generator()     == reduce monomial(1, 1)$UP
 norm x          == resultant(definingPolynomial(), lift x)
 retract(x:%):R  == retract lift x
 retractIfCan(x:%):Union(R, "failed") == retractIfCan lift x

 basis() ==
   [reduce monomial(1,i)$UP for i in 0..(rank()-1)::NonNegativeInteger]

 characteristicPolynomial(x:%):UP ==
   characteristicPolynomial(x)$CharacteristicPolynomialInMonogenicalAlgebra(R,U

 if R has Finite then
   size()   == size()$R ** rank()
   random() == represents [random()$R for i in 1..rank()]$Vector(R)

 if R has Field then
   reduce(x:Fraction UP) == reduce(numer x) exquo reduce(denom x)

   differentiate(x:%, d:R -> R) ==
     p := definingPolynomial()
     yprime := - reduce(map(d, p)) / reduce(differentiate p)
     reduce(map(d, lift x)) + yprime * reduce differentiate lift x
    derivationCoordinates(b, d) ==
      coordinates(map(x +-> differentiate(x, d), b), b)

    recip x ==
      (bc := extendedEuclidean(lift x, definingPolynomial(), 1))
                                        case "failed" => "failed"
      reduce(bc.coef1)
```

⟨*MONOGEN.dotabb*⟩≡

```
"MONOGEN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MONOGEN" -> "FRAMALG"
"MONOGEN" -> "COMRING"
"MONOGEN" -> "KONVERT"
"MONOGEN" -> "FRETRCT"
"MONOGEN" -> "FLINEXP"
"MONOGEN" -> "FINITE"
"MONOGEN" -> "FIELD"
"MONOGEN" -> "DIFEXT"
"MONOGEN" -> "FFIELDC"
```

⟨*MONOGEN.dotfull*⟩≡

```
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CommutativeRing()"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyRetractableTo(a:CommutativeRing)"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyLinearlyExplicitRingOver(a:CommutativeRing)"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

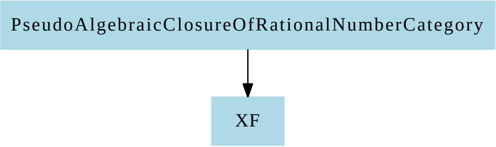
⟨*MONOGEN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FINITE..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FIELD..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "DIFEXT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FFIELDC..."

"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FINITE..." [color=lightblue];
"FIELD..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FFIELDC..." [color=lightblue];
}
```

19.0.215 PseudoAlgebraicClosureOfRationalNumberCategory (PACRATC)



⟨*PseudoAlgebraicClosureOfRationalNumberCategory.input*⟩≡

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfRationalNumberCategory.output
)spool PseudoAlgebraicClosureOfRationalNumberCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfRationalNumberCategory
--R
--R PseudoAlgebraicClosureOfRationalNumberCategory  is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfRationalNumberCategory is PACRATC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACRATC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %      ?*? : (Fraction Integer,%) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,Fraction Integer) -> %      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %       algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean       coerce : Fraction Integer -> %
--R coerce : Fraction Integer -> %       coerce : Integer -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R conjugate : % -> %                   dimension : () -> CardinalNumber
--R extDegree : % -> PositiveInteger     factor : % -> Factored %
--R fullOutput : % -> OutputForm         gcd : List % -> %
--R gcd : (%,%) -> %                     ground? : % -> Boolean
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
```

```
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R maxTower : List % -> %                one? : % -> Boolean
--R previousTower : % -> %                prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Fraction Integer
--R retract : % -> Fraction Integer       retract : % -> Integer
--R sample : () -> %                      setTower! : % -> Void
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R vectorise : (%,%) -> Vector %         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if Fraction Integer has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Fraction Integer has CHARNZ or Fractio
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction Integer
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R lift : (%,%) -> SparseUnivariatePolynomial %
--R lift : % -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R order : % -> OnePointCompletion PositiveInteger if Fraction Integer has CHARN
--R primeFrobenius : % -> % if Fraction Integer has CHARNZ or Fraction Integer ha
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction Integer has CHARNZ o
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PseudoAlgebraicClosureOfRationalNumberCategory.help*⟩≡

```
====================================================================
PseudoAlgebraicClosureOfRationalNumberCategory examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfRationalNumberCategory
```

See:

⇒ "PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory" (PACEXTC) 20.0.218 on page 1829
⇐ "ExtensionField" (XF) 18.0.206 on page 1662

Exports:

0
1
algebraic?
associates?
characteristic
charthRoot
coerce
conjugate
definingPolynomial
degree
dimension
discreteLog
distinguishedRootsOf
divide
euclideanSize
expressIdealMember
exquo
extDegree
extendedEuclidean
extensionDegree
factor
Frobenius
fullOutput
gcd
gcdPolynomial
ground?
hash
inGroundField?
inv
latex
lcm
lift
maxTower
multiEuclidean
newElement
one?
order
previousTower
prime?
primeFrobenius
principalIdeal
?quo?
recip
reduce
?rem?
retract
retractIfCan
sample
setTower!
sizeLess?
squareFree
squareFreePart
subtractIfCan
transcendenceDegree
transcendent?
unit?
unitCanonical
unitNormal
vectorise
zero?
?*?
?**?
?+?
?-?
-?
?/?
?=?
?~=?
?^?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1509) PseudoAlgebraicClosureOfPerfectFieldCategory():

```
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
1 : () -> %
0 : () -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : % -> %
coerce : % -> OutputForm
conjugate : % -> %
definingPolynomial : % -> SUP %
definingPolynomial : () -> SUP %
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PI
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
ground? : % -> Boolean
hash : % -> SingleInteger
inv : % -> %
```

```
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
lift : (%,%) -> SUP %
lift : % -> SUP %
maxTower : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
newElement : (SUP %,Symbol) -> %
newElement : (SUP %,%,Symbol) -> %
one? : % -> Boolean
previousTower : % -> %
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
?quo? : (%,%) -> %
recip : % -> Union(%,"failed")
reduce : SUP % -> %
?rem? : (%,%) -> %
sample : () -> %
setTower! : % -> Void
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
vectorise : (%,%) -> Vector %
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p927) CharacteristicZero():

These exports come from (p62) RetractableTo(Integer):

```
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

These exports come from (p62) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
```

These exports come from (p1662) ExtensionField(Fraction(Integer)):

```
?/? : (%,Fraction Integer) -> %
algebraic? : % -> Boolean
charthRoot : % -> Union(%,"failed")
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
degree : % -> OnePointCompletion PositiveInteger
dimension : () -> CardinalNumber
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
extensionDegree : () -> OnePointCompletion PositiveInteger
Frobenius : % -> % if Fraction Integer has FINITE
Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
inGroundField? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
primeFrobenius : % -> %
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
transcendent? : % -> Boolean
primeFrobenius : (%,NonNegativeInteger) -> %
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
retractIfCan : % -> Union(Fraction Integer,"failed")
transcendenceDegree : () -> NonNegativeInteger
```

⟨*category PACRATC PseudoAlgebraicClosureOfRationalNumberCategory*⟩≡

```
)abbrev category  PACRATC PseudoAlgebraicClosureOfRationalNumberCategory
++ Authors: Gaetan Hache
++ Date Created: feb 1997
++ Date Last Updated:
++ Description: This category exports the function for the domain
++ PseudoAlgebraicClosureOfRationalNumber
++ which implement dynamic extension using the simple notion of tower
++ extensions. A tower extension T  of the ground
++ field K is any sequence of field extension (T : K_0, K_1, ..., K_i...,K_n)
++ where K_0 = K and for i =1,2,...,n, K_i is an extension
++ of K_{i-1} of degree > 1 and defined by an irreducible polynomial
++ p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfRationalNumberCategory:Category ==
  Join(PseudoAlgebraicClosureOfPerfectFieldCategory,
        CharacteristicZero,RetractableTo(Integer),
         RetractableTo(Fraction(Integer)),ExtensionField(Fraction(Integer)))
```

⟨*PACRATC.dotabb*⟩≡
```
"PACRATC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];
"PACRATC" -> "XF"
```

⟨*PACRATC.dotfull*⟩≡
```
"PseudoAlgebraicClosureOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];
"PseudoAlgebraicClosureOfRationalNumberCategory" -> "ExtensionField(F:Field)"
```

⟨*PACRATC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfRationalNumberCategory" [color=lightblue];
"PseudoAlgebraicClosureOfRationalNumberCategory" -> "XF"

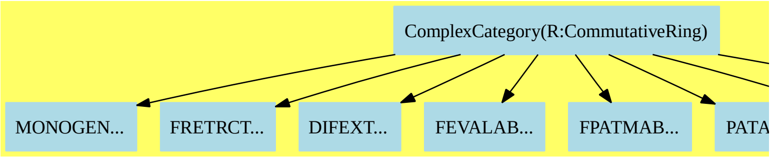
"XF" [color=lightblue];

}
```

Chapter 20

Category Layer 19

20.0.216 ComplexCategory (COMPCAT)



ComplexCategory(R:CommutativeRing)
MONOGEN...
FRETRCT...
DIFEXT...
FEVALAB...
FPATMAB...
PATA

⟨*ComplexCategory.input*⟩≡

```
)set break resume
)sys rm -f ComplexCategory.output
)spool ComplexCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ComplexCategory
--R ComplexCategory R: CommutativeRing  is a category constructor
--R Abbreviation for ComplexCategory is COMPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R D : (%,(R -> R)) -> %                 1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R abs : % -> % if R has RNS             basis : () -> Vector %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              complex : (R,R) -> %
--R conjugate : % -> %                    convert : Vector R -> %
--R convert : % -> Vector R               coordinates : % -> Vector R
--R discriminant : () -> R                discriminant : Vector % -> R
--R generator : () -> %                   hash : % -> SingleInteger
--R imag : % -> R                         imaginary : () -> %
--R inv : % -> % if R has FIELD           latex : % -> String
--R map : ((R -> R),%) -> %               norm : % -> R
--R one? : % -> Boolean                   pi : () -> % if R has TRANFUN
--R rank : () -> PositiveInteger          real : % -> R
--R recip : % -> Union(%,"failed")        represents : Vector R -> %
--R retract : % -> R                      sample : () -> %
--R trace : % -> R                        traceMatrix : () -> Matrix R
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has FIELD
--R ?*? : (Fraction Integer,%) -> % if R has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,%) -> % if R has TRANFUN
--R ?**? : (%,Fraction Integer) -> % if R has RADCAT and R has TRANFUN
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or a
--R D : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing))
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R ?^? : (%,Integer) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if R has TRANFUN
--R acosh : % -> % if R has TRANFUN
--R acot : % -> % if R has TRANFUN
--R acoth : % -> % if R has TRANFUN
--R acsc : % -> % if R has TRANFUN
--R acsch : % -> % if R has TRANFUN
--R argument : % -> R if R has TRANFUN
```

```
--R asec : % -> % if R has TRANFUN
--R asech : % -> % if R has TRANFUN
--R asin : % -> % if R has TRANFUN
--R asinh : % -> % if R has TRANFUN
--R associates? : (%,%) -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
--R atan : % -> % if R has TRANFUN
--R atanh : % -> % if R has TRANFUN
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> SparseUnivariatePolynomial R
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),AND(has(R,Eucli
--R charthRoot : % -> % if R has FFIELDC
--R coerce : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
--R coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
--R conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),AN
--R convert : % -> InputForm if R has KONVERT INFORM
--R convert : % -> Complex DoubleFloat if R has REAL
--R convert : % -> Complex Float if R has REAL
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R convert : SparseUnivariatePolynomial R -> %
--R convert : % -> SparseUnivariatePolynomial R
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R cos : % -> % if R has TRANFUN
--R cosh : % -> % if R has TRANFUN
--R cot : % -> % if R has TRANFUN
--R coth : % -> % if R has TRANFUN
--R createPrimitiveElement : () -> % if R has FFIELDC
--R csc : % -> % if R has TRANFUN
--R csch : % -> % if R has TRANFUN
--R definingPolynomial : () -> SparseUnivariatePolynomial R
--R derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
--R differentiate : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing))
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has EUCDOM
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R euclideanSize : % -> NonNegativeInteger if R has EUCDOM
```

```
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List Equation R) -> % if R has EVALAB R
--R eval : (%,Equation R) -> % if R has EVALAB R
--R eval : (%,R,R) -> % if R has EVALAB R
--R eval : (%,List R,List R) -> % if R has EVALAB R
--R exp : % -> % if R has TRANFUN
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has EUCDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM or R has EUCDOM and R has 
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has 
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R
--R factor : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariateP
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseU
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Intege
--R gcd : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R gcd : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R index : PositiveInteger -> % if R has FINITE
--R init : () -> % if R has FFIELDC
--R lcm : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R lcm : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R lift : % -> SparseUnivariatePolynomial R
--R log : % -> % if R has TRANFUN
--R lookup : % -> PositiveInteger if R has FINITE
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimalPolynomial : % -> SparseUnivariatePolynomial R if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has EUCDOM
--R nextItem : % -> Union(%,"failed") if R has FFIELDC
--R nthRoot : (%,Integer) -> % if R has RADCAT and R has TRANFUN
--R order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchR
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMa
--R polarCoordinates : % -> Record(r: R,phi: R) if R has RNS and R has TRANFUN
--R prime? : % -> Boolean if R has EUCDOM and R has PFECAT or R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has EUCDOM
--R ?quo? : (%,%) -> % if R has EUCDOM
--R random : () -> % if R has FINITE
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R reduce : Fraction SparseUnivariatePolynomial R -> Union(%,"failed") if R has FIELD
--R reduce : SparseUnivariatePolynomial R -> %
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R regularRepresentation : % -> Matrix R
--R regularRepresentation : (%,Vector %) -> Matrix R
--R ?rem? : (%,%) -> % if R has EUCDOM
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
--R represents : (Vector R,Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R sec : % -> % if R has TRANFUN
--R sech : % -> % if R has TRANFUN
--R sin : % -> % if R has TRANFUN
--R sinh : % -> % if R has TRANFUN
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has EUCDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePoly
--R sqrt : % -> % if R has RADCAT and R has TRANFUN
--R squareFree : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
--R squareFreePart : % -> % if R has EUCDOM and R has PFECAT or R has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomi
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R ha
--R tan : % -> % if R has TRANFUN
--R tanh : % -> % if R has TRANFUN
--R traceMatrix : Vector % -> Matrix R
--R unit? : % -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
--R unitCanonical : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM or R has EU
--R
--E 1

)spool
)lisp (bye)
```

⟨*ComplexCategory.help*⟩≡
```
====================================================================
ComplexCategory examples
====================================================================

See Also:
o )show ComplexCategory
```

See:

⇐ “CommutativeRing” (COMRING) 10.0.137 on page 932
⇐ “DifferentialExtension” (DIFEXT) 11.0.153 on page 1053
⇐ “FullyEvalableOver” (FEVALAB) 4.0.44 on page 178
⇐ “FullyPatternMatchable” (FPATMAB) 3.0.28 on page 105
⇐ “FullyRetractableTo” (FRETRCT) 3.0.27 on page 100
⇐ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758
⇐ “Patternable” (PATAB) 2.0.15 on page 53
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.154 on page 1060

Exports:

0
1
abs
acos
acosh
acot
acoth
acsc
acsch
argument
asec
asech
asin
asinh
associates?
atan
atanh
basis
characteristic
characteristicPolynomial
charthRoot
coerce
complex
conditionP
conjugate
convert
coordinates
cos
cosh
cot
coth
createPrimitiveElement
csc
csch
D
definingPolynomial
derivationCoordinates
differentiate
discreteLog
discriminant
divide
euclideanSize
eval
exp
expressIdealMember
exquo
extendedEuclidean
factor
factorPolynomial
factorSquareFreePolynomial
factorsOfCyclicGroupSize
gcd
gcdPolynomial
generator
hash
imag
imaginary
index
init
inv
latex
lcm
lift
log
lookup
map
max
min
minimalPolynomial
multiEuclidean
nextItem
norm
nthRoot
one?
order
patternMatch
pi
polarCoordinates
prime?
primeFrobenius
primitive?
primitiveElement
principalIdeal
random
rank
rational
rational?
rationalIfCan
real
recip
reduce
reducedSystem
regularRepresentation
represents
representationType
retract
retractIfCan
sample
sec
sech
sin
sinh
size
sizeLess?
solveLinearPolynomialEquation
sqrt
squareFree
squareFreePart
squareFreePolynomial
subtractIfCan
tableForDiscreteLogarithm
tan
tanh
trace
traceMatrix
unit?
unitCanonical
unitNormal
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?<?
?<=?
?>?
?>=?
?.?
?quo?
?rem?

Attributes Exported:

- if #1 has multiplicativeValuation then multiplicativeValuation where **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- if #1 has additiveValuation then additiveValuation where **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain or #1 has both EuclideanDomain and PolynomialFactorizationExplicit then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **complex** means that this domain has $\sqrt{-1}$
- **nil**

These are directly exported but not implemented:

```
abs : % -> % if R has RNS
argument : % -> R if R has TRANFUN
complex : (R,R) -> %
conjugate : % -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
imag : % -> R
imaginary : () -> %
norm : % -> R
polarCoordinates : % -> Record(r: R,phi: R) if R has RNS and R has TRANFUN
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
real : % -> R
```

These are implemented by this category:

```
acos : % -> % if R has TRANFUN
acosh : % -> % if R has TRANFUN
asin : % -> % if R has TRANFUN
asinh : % -> % if R has TRANFUN
atan : % -> % if R has TRANFUN
atanh : % -> % if R has TRANFUN
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> SparseUnivariatePolynomial R
coerce : % -> OutputForm
convert : % -> Complex DoubleFloat if R has REAL
convert : % -> Complex Float if R has REAL
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
coordinates : % -> Vector R
coordinates : (%,Vector %) -> Vector R
cos : % -> % if R has TRANFUN
cosh : % -> % if R has TRANFUN
definingPolynomial : () -> SparseUnivariatePolynomial R
differentiate : (%,(R -> R)) -> %
discriminant : () -> R
divide : (%,%) -> Record(quotient: %,remainder: %) if R has EUCDOM
euclideanSize : % -> NonNegativeInteger if R has EUCDOM
exp : % -> % if R has TRANFUN
exquo : (%,%) -> Union(%,"failed") if R has INTDOM or R has EUCDOM and R has PFECAT
factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if
factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolyno
inv : % -> % if R has FIELD
lift : % -> SparseUnivariatePolynomial R
log : % -> % if R has TRANFUN
map : ((R -> R),%) -> %
minimalPolynomial : % -> SparseUnivariatePolynomial R if R has FIELD
patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integ
patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
pi : () -> % if R has TRANFUN
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
reduce : SparseUnivariatePolynomial R -> %
reducedSystem : Matrix % -> Matrix R
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
sin : % -> % if R has TRANFUN
sinh : % -> % if R has TRANFUN
solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomia
tan : % -> % if R has TRANFUN
tanh : % -> % if R has TRANFUN
trace : % -> R
```

```
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM or R has EUCDOM and R has P
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?**? : (%,Fraction Integer) -> % if R has RADCAT and R has TRANFUN
?rem? : (%,%) -> % if R has EUCDOM
?quo? : (%,%) -> % if R has EUCDOM
```

These exports come from (p1758) MonogenicAlgebra(R,S)
where R:CommutativeRing and S:SparseUnivariatePolynomial(R):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
basis : () -> Vector %
charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanDomain),has(
charthRoot : % -> % if R has FFIELDC
coerce : R -> %
coerce : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
coerce : Integer -> %
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),AND(has(R,Euclide
convert : SparseUnivariatePolynomial R -> %
convert : Vector R -> %
convert : % -> Vector R
convert : % -> SparseUnivariatePolynomial R
coordinates : Vector % -> Matrix R
coordinates : (Vector %,Vector %) -> Matrix R
createPrimitiveElement : () -> % if R has FFIELDC
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,DifferentialRing)
D : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(ha
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol) -> % if R has PDRING SYMBOL
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
differentiate : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRI
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,Diffe
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discriminant : Vector % -> R
expressIdealMember : (List %,%) -> Union(List %,"failed") if R has EUCDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has EUCDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has EUCDOM
factor : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if R has FFI
gcd : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcd : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivari
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> % if R has FINITE
init : () -> % if R has FFIELDC
latex : % -> String
lcm : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lcm : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lookup : % -> PositiveInteger if R has FINITE
multiEuclidean : (List %,%) -> Union(List %,"failed") if R has EUCDOM
nextItem : % -> Union(%,"failed") if R has FFIELDC
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
prime? : % -> Boolean if R has EUCDOM and R has PFECAT or R has FIELD
primeFrobenius : % -> % if R has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
principalIdeal : List % -> Record(coef: List %,generator: %) if R has EUCDOM
random : () -> % if R has FINITE
reduce : Fraction SparseUnivariatePolynomial R -> Union(%,"failed") if R has FIELD
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R
regularRepresentation : % -> Matrix R
regularRepresentation : (%,Vector %) -> Matrix R
representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger if R has FINITE
sizeLess? : (%,%) -> Boolean if R has EUCDOM
subtractIfCan : (%,%) -> Union(%,"failed")
squareFree : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
squareFreePart : % -> % if R has EUCDOM and R has PFECAT or R has FIELD
tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has F
traceMatrix : () -> Matrix R
traceMatrix : Vector % -> Matrix R
unit? : % -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
unitCanonical : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
zero? : % -> Boolean
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p178) FullyEvalableOver(R:CommutativeRing)

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p932) CommutativeRing():

```
?~=? : (%,%) -> Boolean
```

These exports come from (p242) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p59) RadicalCategory():

```
nthRoot : (%,Integer) -> % if R has RADCAT and R has TRANFUN
sqrt : % -> % if R has RADCAT and R has TRANFUN
```

These exports come from (p135) TranscendentalFunctionCategory():

```
acot : % -> % if R has TRANFUN
acoth : % -> % if R has TRANFUN
```

```
acsc : % -> % if R has TRANFUN
acsch : % -> % if R has TRANFUN
asec : % -> % if R has TRANFUN
asech : % -> % if R has TRANFUN
cot : % -> % if R has TRANFUN
coth : % -> % if R has TRANFUN
csc : % -> % if R has TRANFUN
csch : % -> % if R has TRANFUN
sec : % -> % if R has TRANFUN
sech : % -> % if R has TRANFUN
?**? : (%,%) -> % if R has TRANFUN
```

These exports come from (p1325) PolynomialFactorizationExplicit():

```
squareFreePolynomial : SparseUnivariatePolynomial % ->
  Factored SparseUnivariatePolynomial % if R has EUCDOM and R has PFECAT
```

⟨*category COMPCAT ComplexCategory*⟩≡

```
)abbrev category COMPCAT ComplexCategory
++ Author:
++ Date Created:
++ Date Last Updated: 18 March 1994
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: complex, gaussian
++ References:
++ Description:
++ This category represents the extension of a ring by a square root of -1.

ComplexCategory(R:CommutativeRing): Category ==
  Join(MonogenicAlgebra(R, SparseUnivariatePolynomial R), FullyRetractableTo R,
   DifferentialExtension R, FullyEvalableOver R, FullyPatternMatchable(R),
    Patternable(R), FullyLinearlyExplicitRingOver R, CommutativeRing) with
     complex                        ++ indicates that % has sqrt(-1)
     imaginary:   () -> %           ++ imaginary() = sqrt(-1) = %i.
     conjugate:   % -> %            ++ conjugate(x + %i y) returns x - %i y.
     complex  :   (R, R) -> %       ++ complex(x,y) constructs x + %i*y.
     imag     :   % -> R            ++ imag(x) returns imaginary part of x.
     real     :   % -> R            ++ real(x) returns real part of x.
     norm     :   % -> R            ++ norm(x) returns x * conjugate(x)
     if R has OrderedSet then OrderedSet
     if R has IntegralDomain then
       IntegralDomain
       _exquo : (%,R) -> Union(%,"failed")
         ++ exquo(x, r) returns the exact quotient of x by r, or
         ++ "failed" if r does not divide x exactly.
    if R has EuclideanDomain then EuclideanDomain
    if R has multiplicativeValuation then multiplicativeValuation
    if R has additiveValuation then additiveValuation
    if R has Field then        -- this is a lie; we must know that
      Field                    -- x**2+1 is irreducible in R
    if R has ConvertibleTo InputForm then ConvertibleTo InputForm
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has RealConstant then
      ConvertibleTo Complex DoubleFloat
      ConvertibleTo Complex Float
    if R has RealNumberSystem then
      abs: % -> %
        ++ abs(x) returns the absolute value of x = sqrt(norm(x)).
    if R has TranscendentalFunctionCategory then
      TranscendentalFunctionCategory
      argument: % -> R
        ++ argument(x) returns the angle made by (0,1) and (0,x).
      if R has RadicalCategory then RadicalCategory
      if R has RealNumberSystem then
        polarCoordinates: % -> Record(r:R, phi:R)
          ++ polarCoordinates(x) returns (r, phi) such that
          ++ x = r * exp(%i * phi).
    if R has IntegerNumberSystem then
      rational?    : % -> Boolean
        ++ rational?(x) tests if x is a rational number.
      rational     : % -> Fraction Integer
        ++ rational(x) returns x as a rational number.
        ++ Error: if x is not a rational number.
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(x) returns x as a rational number, or
        ++ "failed" if x is not a rational number.
    if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
       PolynomialFactorizationExplicit
add
      import MatrixCategoryFunctions2(%, Vector %, Vector %, Matrix %,
                                      R, Vector R, Vector R, Matrix R)
      SUP ==> SparseUnivariatePolynomial
      characteristicPolynomial x ==
         v := monomial(1,1)$SUP(R)
         v**2 - trace(x)*v**1 + norm(x)*v**0
      if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
         SupR ==> SparseUnivariatePolynomial R
         Sup ==> SparseUnivariatePolynomial %
         import FactoredFunctionUtilities Sup
         import UnivariatePolynomialCategoryFunctions2(R,SupR,%,Sup)
```

```
   import UnivariatePolynomialCategoryFunctions2(%,Sup,R,SupR)
   pp,qq:Sup
   if R has IntegerNumberSystem then
      myNextPrime: (%,NonNegativeInteger) -> %
      myNextPrime(x,n ) == -- prime is actually in R, and = 3(mod 4)
         xr:=real(x)-4::R
         while not prime? xr repeat
            xr:=xr-4::R
         complex(xr,0)
      --!TT:=InnerModularGcd(%,Sup,32719 :: %,myNextPrime)
      --!gcdPolynomial(pp,qq) == modularGcd(pp,qq)$TT
      solveLinearPolynomialEquation(lp:List Sup,p:Sup) ==
        solveLinearPolynomialEquation(lp,p)$ComplexIntegerSolveLinearPolyn
   normPolynomial: Sup -> SupR
   normPolynomial pp ==
       map(z+->retract(z@%)::R,pp * map(conjugate,pp))
   factorPolynomial pp ==
       refine(squareFree pp,factorSquareFreePolynomial)
   factorSquareFreePolynomial pp ==
       pnorm:=normPolynomial pp
       k:R:=0
       while degree gcd(pnorm,differentiate pnorm)>0 repeat
          k:=k+1
          pnorm:=normPolynomial
                   elt(pp,monomial(1,1)-monomial(complex(0,k),0))
       fR:=factorSquareFreePolynomial pnorm
       numberOfFactors fR = 1 =>
           makeFR(1,[["irred",pp,1]])
       lF:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
                      fctr:Sup, xpnt:Integer):=[]
       for u in factorList fR repeat
           p1:=map((z:R):%+->z::%,u.fctr)
           if not zero? k then
              p1:=elt(p1,monomial(1,1)+monomial(complex(0,k),0))
           p2:=gcd(p1,pp)
           lF:=cons(["irred",p2,1],lF)
           pp:=(pp exquo p2)::Sup
       makeFR(pp,lF)
rank()           == 2
discriminant()   == -4 :: R
norm x           == real(x)**2 + imag(x)**2
trace x          == 2 * real x
imaginary()      == complex(0, 1)
conjugate x      == complex(real x, - imag x)
characteristic() == characteristic()$R
map(fn, x)       == complex(fn real x, fn imag x)
```

```
      x = y            == real(x) = real(y) and imag(x) = imag(y)
      x + y            == complex(real x + real y, imag x + imag y)
      - x              == complex(- real x, - imag x)
      r:R * x:%        == complex(r * real x, r * imag x)
      coordinates(x:%) == [real x, imag x]
      n:Integer * x:%  == complex(n * real x, n * imag x)
      differentiate(x:%, d:R -> R) == complex(d real x, d imag x)

      definingPolynomial() ==
        monomial(1,2)$(SUP R) + monomial(1,0)$(SUP R)

      reduce(pol:SUP R) ==
        part:= (monicDivide(pol,definingPolynomial())).remainder
        complex(coefficient(part,0),coefficient(part,1))

      lift(x) == monomial(real x,0)$(SUP R)+monomial(imag x,1)$(SUP R)

      minimalPolynomial x ==
        zero? imag x =>
          monomial(1, 1)$(SUP R) - monomial(real x, 0)$(SUP R)
        monomial(1, 2)$(SUP R) - monomial(trace x, 1)$(SUP R)
          + monomial(norm x, 0)$(SUP R)

      coordinates(x:%, v:Vector %):Vector(R) ==
        ra := real(a := v(minIndex v))
        rb := real(b := v(maxIndex v))
        (#v ^= 2) or
          ((d := recip(ra * (ib := imag b) - (ia := imag a) * rb))
            case "failed") =>error "coordinates: vector is not a basis"
        rx := real x
        ix := imag x
        [d::R * (rx * ib - ix * rb), d::R * (ra * ix - ia * rx)]

      coerce(x:%):OutputForm ==
        re := (r := real x)::OutputForm
        ie := (i := imag x)::OutputForm
        zero? i => re
        outi := "%i"::Symbol::OutputForm
        ip :=
--            one? i => outi
          (i = 1) => outi
--            one?(-i) => -outi
          ((-i) = 1) => -outi
          ie * outi
        zero? r => ip
        re + ip
```

```
retract(x:%):R ==
  not zero?(imag x) =>
    error "Imaginary part is nonzero. Cannot retract."
  real x

retractIfCan(x:%):Union(R, "failed") ==
  not zero?(imag x) => "failed"
  real x

x:% * y:% ==
  complex(real x * real y - imag x * imag y,
           imag x * real y + imag y * real x)

reducedSystem(m:Matrix %):Matrix R ==
  vertConcat(map(real, m), map(imag, m))

reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  rh := reducedSystem(v::Matrix %)@Matrix(R)
  [reducedSystem(m)@Matrix(R), column(rh, minColIndex rh)]

if R has RealNumberSystem then
  abs(x:%):%        == (sqrt norm x)::%

if R has RealConstant then
  convert(x:%):Complex(DoubleFloat) ==
   complex(convert(real x)@DoubleFloat,convert(imag x)@DoubleFloat)

  convert(x:%):Complex(Float) ==
    complex(convert(real x)@Float, convert(imag x)@Float)

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    convert([convert("complex"::Symbol), convert real x,
             convert imag x]$List(InputForm))@InputForm

if R has ConvertibleTo Pattern Integer then
  convert(x:%):Pattern Integer ==
     convert(x)$ComplexPattern(Integer, R, %)
if R has ConvertibleTo Pattern Float then
  convert(x:%):Pattern Float ==
     convert(x)$ComplexPattern(Float, R, %)

if R has PatternMatchable Integer then
  patternMatch(x:%, p:Pattern Integer,
        l:PatternMatchResult(Integer, %)) ==
         patternMatch(x, p, l)$ComplexPatternMatch(Integer, R, %)

    if R has PatternMatchable Float then
      patternMatch(x:%, p:Pattern Float,
        l:PatternMatchResult(Float, %)) ==
         patternMatch(x, p, l)$ComplexPatternMatch(Float, R, %)


    if R has OrderedSet then
      x < y ==
        real x = real y => imag x < imag y
        real x < real y

    if R has IntegerNumberSystem then
      rational? x == zero? imag x

      rational x ==
        zero? imag x => rational real x
        error "Not a rational number"

      rationalIfCan x ==
        zero? imag x => rational real x
        "failed"

    if R has Field then
      inv x ==
        zero? imag x => (inv real x)::%
        r := norm x
        complex(real(x) / r, - imag(x) / r)

    if R has IntegralDomain then
      _exquo(x:%, r:R) ==
--          one? r => x
        (r = 1) => x
        (r1 := real(x) exquo r) case "failed" => "failed"
        (r2 := imag(x) exquo r) case "failed" => "failed"
        complex(r1, r2)

      _exquo(x:%, y:%) ==
        zero? imag y => x exquo real y
        x * conjugate(y) exquo norm(y)

      recip(x:%) == 1 exquo x

      if R has OrderedRing then
    unitNormal x ==
      zero? x => [1,x,1]
      (u := recip x) case % => [x, 1, u]
      zero? real x =>
        c := unitNormal imag x
        [complex(0, c.unit), (c.associate * imag x)::%,
                                       complex(0, - c.associate)]
      c := unitNormal real x
      x := c.associate * x
      imag x < 0 =>
        x := complex(- imag x, real x)
        [- c.unit * imaginary(), x, c.associate * imaginary()]
      [c.unit ::%, x, c.associate ::%]
  else
    unitNormal x ==
      zero? x => [1,x,1]
      (u := recip x) case % => [x, 1, u]
      zero? real x =>
        c := unitNormal imag x
        [complex(0, c.unit), (c.associate * imag x)::%,
                                       complex(0, - c.associate)]
      c := unitNormal real x
      x := c.associate * x
      [c.unit ::%, x, c.associate ::%]

if R has EuclideanDomain then
   if R has additiveValuation then
       euclideanSize x == max(euclideanSize real x,
                              euclideanSize imag x)
   else
       euclideanSize x == euclideanSize(real(x)**2 + imag(x)**2)$R
   if R has IntegerNumberSystem then
     x rem y ==
       zero? imag y =>
         yr:=real y
         complex(symmetricRemainder(real(x), yr),
                 symmetricRemainder(imag(x), yr))
       divide(x, y).remainder
     x quo y ==
       zero? imag y =>
         yr:= real y
         xr:= real x
         xi:= imag x
         complex((xr-symmetricRemainder(xr,yr)) quo yr,
                 (xi-symmetricRemainder(xi,yr)) quo yr)
       divide(x, y).quotient
     else
       x rem y ==
         zero? imag y =>
           yr:=real y
           complex(real(x) rem yr,imag(x) rem yr)
         divide(x, y).remainder
       x quo y ==
         zero? imag y => complex(real x quo real y,imag x quo real y)
         divide(x, y).quotient

     divide(x, y) ==
       r := norm y
       y1 := conjugate y
       xx := x * y1
       x1 := real(xx) rem r
       a  := x1
       if x1^=0 and sizeLess?(r, 2 * x1) then
         a := x1 - r
         if sizeLess?(x1, a) then a := x1 + r
       x2 := imag(xx) rem r
       b  := x2
       if x2^=0 and sizeLess?(r, 2 * x2) then
         b := x2 - r
         if sizeLess?(x2, b) then b := x2 + r
       y1 := (complex(a, b) exquo y1)::%
       [((x - y1) exquo y)::%, y1]

  if R has TranscendentalFunctionCategory then
    half := recip(2::R)::R

    if R has RealNumberSystem then
      atan2loc(y: R, x: R): R ==
          pi1 := pi()$R
          pi2 := pi1 * half
          x = 0 => if y >= 0 then pi2 else -pi2

          -- Atan in (-pi/2,pi/2]
          theta := atan(y * recip(x)::R)
          while theta <= -pi2 repeat theta := theta + pi1
          while theta >   pi2 repeat theta := theta - pi1

          x >= 0 => theta      -- I or IV

          if y >= 0 then
              theta + pi1      -- II
      else
          theta - pi1      -- III

  argument x == atan2loc(imag x, real x)

else
  -- Not ordered so dictate two quadrants
  argument x ==
    zero? real x => pi()$R * half
    atan(imag(x) * recip(real x)::R)

pi()  == pi()$R :: %

if R is DoubleFloat then
   stoc ==> S_-TO_-C$Lisp
   ctos ==> C_-TO_-S$Lisp

   exp   x == ctos EXP(stoc x)$Lisp
   log   x == ctos LOG(stoc x)$Lisp

   sin   x == ctos SIN(stoc x)$Lisp
   cos   x == ctos COS(stoc x)$Lisp
   tan   x == ctos TAN(stoc x)$Lisp
   asin  x == ctos ASIN(stoc x)$Lisp
   acos  x == ctos ACOS(stoc x)$Lisp
   atan  x == ctos ATAN(stoc x)$Lisp

   sinh  x == ctos SINH(stoc x)$Lisp
   cosh  x == ctos COSH(stoc x)$Lisp
   tanh  x == ctos TANH(stoc x)$Lisp
   asinh x == ctos ASINH(stoc x)$Lisp
   acosh x == ctos ACOSH(stoc x)$Lisp
   atanh x == ctos ATANH(stoc x)$Lisp

else
  atan x ==
    ix := imaginary()*x
    - imaginary() * half * (log(1 + ix) - log(1 - ix))

  log x ==
    complex(log(norm x) * half, argument x)

  exp x ==
    e := exp real x
    complex(e * cos imag x, e * sin imag x)
  cos x ==
    e := exp(imaginary() * x)
    half * (e + recip(e)::%)

  sin x ==
    e := exp(imaginary() * x)
    - imaginary() * half * (e - recip(e)::%)

if R has RealNumberSystem then
  polarCoordinates x ==
    [sqrt norm x, (negative?(t := argument x) => t + 2 * pi(); t)]

  x:% ** q:Fraction(Integer) ==
    zero? q =>
      zero? x => error "0 ** 0 is undefined"
      1
    zero? x => 0
    rx := real x
    zero? imag x and positive? rx => (rx ** q)::%
    zero? imag x and denom q = 2 => complex(0, (-rx)**q)
    ax := sqrt(norm x) ** q
    tx := q::R * argument x
    complex(ax * cos tx, ax * sin tx)

else if R has RadicalCategory then
  x:% ** q:Fraction(Integer) ==
    zero? q =>
      zero? x => error "0 ** 0 is undefined"
      1
    r := real x
    zero?(i := imag x) => (r ** q)::%
    t := numer(q) * recip(denom(q)::R)::R * argument x
    e:R :=
      zero? r => i ** q
      norm(x) ** (q / (2::Fraction(Integer)))
    complex(e * cos t, e * sin t)
```

⟨*COMPCAT.dotabb*⟩≡
```
"COMPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"COMPCAT" -> "COMRING"
"COMPCAT" -> "DIFEXT"
"COMPCAT" -> "FEVALAB"
"COMPCAT" -> "FPATMAB"
"COMPCAT" -> "FRETRCT"
"COMPCAT" -> "MONOGEN"
"COMPCAT" -> "PATAB"
"COMPCAT" -> "FLINEXP"
"COMPCAT" -> "ORDSET"
```

⟨*COMPCAT.dotfull*⟩≡
```
"ComplexCategory(R:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"ComplexCategory(R:CommutativeRing)" ->
  "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyRetractableTo(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "DifferentialExtension(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyEvalableOver(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyPatternMatchable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "Patternable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "CommutativeRing()"
"ComplexCategory(R:CommutativeRing)" ->
  "OrderedSet()"
```

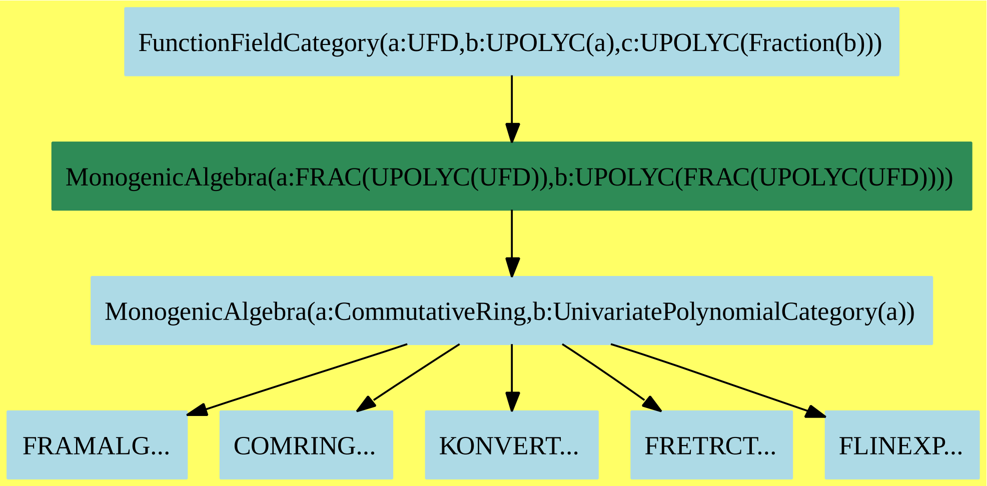
⟨*COMPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ComplexCategory(R:CommutativeRing)" [color=lightblue];
"ComplexCategory(R:CommutativeRing)" -> "MONOGEN..."
"ComplexCategory(R:CommutativeRing)" -> "FRETRCT..."
"ComplexCategory(R:CommutativeRing)" -> "DIFEXT..."
"ComplexCategory(R:CommutativeRing)" -> "FEVALAB..."
"ComplexCategory(R:CommutativeRing)" -> "FPATMAB..."
"ComplexCategory(R:CommutativeRing)" -> "PATAB..."
"ComplexCategory(R:CommutativeRing)" -> "FLINEXP..."
"ComplexCategory(R:CommutativeRing)" -> "COMRING..."
"ComplexCategory(R:CommutativeRing)" -> "ORDSET..."

"MONOGEN..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"COMRING..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
}
```

20.0.217 FunctionFieldCategory (FFCAT)


FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))
MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))
MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))
FRAMALG...
COMRING...
KONVERT...
FRETRCT...
FLINEXP...

⟨FunctionFieldCategory.input⟩≡

```
)set break resume
)sys rm -f FunctionFieldCategory.output
)spool FunctionFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionFieldCategory
--R FunctionFieldCategory(F: UniqueFactorizationDomain,UP: UnivariatePolynomialCa
--R Abbreviation for FunctionFieldCategory is FFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction UP,%) -> %            ?*? : (%,Fraction UP) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        basis : () -> Vector %
--R branchPoint? : UP -> Boolean          branchPoint? : F -> Boolean
--R coerce : Fraction UP -> %             coerce : Integer -> %
--R coerce : % -> OutputForm              convert : UPUP -> %
--R convert : % -> UPUP                   convert : Vector Fraction UP -> %
```

```
--R convert : % -> Vector Fraction UP     definingPolynomial : () -> UPUP
--R discriminant : () -> Fraction UP      elt : (%,F,F) -> F
--R generator : () -> %                   genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger              integral? : (%,UP) -> Boolean
--R integral? : (%,F) -> Boolean          integral? : % -> Boolean
--R integralBasis : () -> Vector %        latex : % -> String
--R lift : % -> UPUP                      norm : % -> Fraction UP
--R one? : % -> Boolean                   primitivePart : % -> %
--R ramified? : UP -> Boolean             ramified? : F -> Boolean
--R rank : () -> PositiveInteger          rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%,"failed")        reduce : UPUP -> %
--R represents : (Vector UP,UP) -> %      retract : % -> Fraction UP
--R sample : () -> %                      singular? : UP -> Boolean
--R singular? : F -> Boolean              trace : % -> Fraction UP
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
--R ?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if Fraction UP has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Fraction UP has FIELD
--R D : % -> % if and(has(Fraction UP,Field),has(Fraction UP,DifferentialRing)) or and(has(
--R D : (%,NonNegativeInteger) -> % if and(has(Fraction UP,Field),has(Fraction UP,Differenti
--R D : (%,Symbol) -> % if and(has(Fraction UP,Field),has(Fraction UP,PartialDifferentialRin
--R D : (%,List Symbol) -> % if and(has(Fraction UP,Field),has(Fraction UP,PartialDifferenti
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(Fraction UP,Field),has(Fraction UP,Par
--R D : (%,List Symbol,List NonNegativeInteger) -> % if and(has(Fraction UP,Field),has(Fract
--R D : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R ?^? : (%,Integer) -> % if Fraction UP has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (%,(UP -> UP)) -> Record(num: %,den: UP,derivden: UP,gd: UP)
--R associates? : (%,%) -> Boolean if Fraction UP has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(%,"failed") if Fraction UP has CHARNZ
--R charthRoot : % -> % if Fraction UP has FFIELDC
--R coerce : % -> % if Fraction UP has FIELD
--R coerce : Fraction Integer -> % if Fraction UP has FIELD or Fraction UP has RETRACT FRAC
--R complementaryBasis : Vector % -> Vector %
--R conditionP : Matrix % -> Union(Vector %,"failed") if Fraction UP has FFIELDC
--R coordinates : Vector % -> Matrix Fraction UP
--R coordinates : % -> Vector Fraction UP
--R coordinates : (Vector %,Vector %) -> Matrix Fraction UP
```

```
--R coordinates : (%,Vector %) -> Vector Fraction UP
--R createPrimitiveElement : () -> % if Fraction UP has FFIELDC
--R derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP)) -> Matrix Fra
--R differentiate : % -> % if and(has(Fraction UP,Field),has(Fraction UP,Differen
--R differentiate : (%,NonNegativeInteger) -> % if and(has(Fraction UP,Field),has
--R differentiate : (%,Symbol) -> % if and(has(Fraction UP,Field),has(Fraction UP
--R differentiate : (%,List Symbol) -> % if and(has(Fraction UP,Field),has(Fracti
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(Fraction UP,Fie
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if and(has(Fract
--R differentiate : (%,(UP -> UP)) -> %
--R differentiate : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIEL
--R differentiate : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if F
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction UP has
--R discreteLog : % -> NonNegativeInteger if Fraction UP has FFIELDC
--R discriminant : Vector % -> Fraction UP
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Fraction UP has FIELD
--R elliptic : () -> Union(UP,"failed")
--R euclideanSize : % -> NonNegativeInteger if Fraction UP has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Fraction UP has
--R exquo : (%,%) -> Union(%,"failed") if Fraction UP has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Fracti
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if F
--R factor : % -> Factored % if Fraction UP has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Intege
--R gcd : (%,%) -> % if Fraction UP has FIELD
--R gcd : List % -> % if Fraction UP has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R hyperelliptic : () -> Union(UP,"failed")
--R index : PositiveInteger -> % if Fraction UP has FINITE
--R init : () -> % if Fraction UP has FFIELDC
--R integralAtInfinity? : % -> Boolean
--R integralBasisAtInfinity : () -> Vector %
--R integralCoordinates : % -> Record(num: Vector UP,den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix UP,den: UP)
--R integralMatrix : () -> Matrix Fraction UP
--R integralMatrixAtInfinity : () -> Matrix Fraction UP
--R integralRepresents : (Vector UP,UP) -> %
--R inv : % -> % if Fraction UP has FIELD
--R inverseIntegralMatrix : () -> Matrix Fraction UP
--R inverseIntegralMatrixAtInfinity : () -> Matrix Fraction UP
--R lcm : (%,%) -> % if Fraction UP has FIELD
--R lcm : List % -> % if Fraction UP has FIELD
--R lookup : % -> PositiveInteger if Fraction UP has FINITE
--R minimalPolynomial : % -> UPUP if Fraction UP has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Fraction UP has FIEL
--R nextItem : % -> Union(%,"failed") if Fraction UP has FFIELDC
```

```
--R nonSingularModel : Symbol -> List Polynomial F if F has FIELD
--R normalizeAtInfinity : Vector % -> Vector %
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion PositiveInteger if Fraction UP has FFIELDC
--R order : % -> PositiveInteger if Fraction UP has FFIELDC
--R prime? : % -> Boolean if Fraction UP has FIELD
--R primeFrobenius : % -> % if Fraction UP has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction UP has FFIELDC
--R primitive? : % -> Boolean if Fraction UP has FFIELDC
--R primitiveElement : () -> % if Fraction UP has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Fraction UP has FIELD
--R ?quo? : (%,%) -> % if Fraction UP has FIELD
--R ramifiedAtInfinity? : () -> Boolean
--R random : () -> % if Fraction UP has FINITE
--R rationalPoints : () -> List List F if F has FINITE
--R reduce : Fraction UPUP -> Union(%,"failed") if Fraction UP has FIELD
--R reduceBasisAtInfinity : Vector % -> Vector %
--R reducedSystem : Matrix % -> Matrix Fraction UP
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction UP,vec: Vector Fracti
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if Fraction UP has LINEXP INT
--R regularRepresentation : % -> Matrix Fraction UP
--R regularRepresentation : (%,Vector %) -> Matrix Fraction UP
--R ?rem? : (%,%) -> % if Fraction UP has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction UP has FI
--R represents : Vector Fraction UP -> %
--R represents : (Vector Fraction UP,Vector %) -> %
--R retract : % -> Fraction Integer if Fraction UP has RETRACT FRAC INT
--R retract : % -> Integer if Fraction UP has RETRACT INT
--R retractIfCan : % -> Union(Fraction UP,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if Fraction UP has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if Fraction UP has RETRACT INT
--R singularAtInfinity? : () -> Boolean
--R size : () -> NonNegativeInteger if Fraction UP has FINITE
--R sizeLess? : (%,%) -> Boolean if Fraction UP has FIELD
--R squareFree : % -> Factored % if Fraction UP has FIELD
--R squareFreePart : % -> % if Fraction UP has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Frac
--R traceMatrix : () -> Matrix Fraction UP
--R traceMatrix : Vector % -> Matrix Fraction UP
--R unit? : % -> Boolean if Fraction UP has FIELD
--R unitCanonical : % -> % if Fraction UP has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction UP has FIELD
--R yCoordinates : % -> Record(num: Vector UP,den: UP)
--R
--E 1

)spool
)lisp (bye)
```

⟨*FunctionFieldCategory.help*⟩≡

```
====================================================================
FunctionFieldCategory examples
====================================================================

See Also:
o )show FunctionFieldCategory
```

See:

⇐ “MonogenicAlgebra” (MONOGEN) 19.0.214 on page 1758

Exports:

0
1
absolutelyIrreducible?
algSplitSimple
associates?
basis
branchPoint?
branchPointAtInfinity?
characteristic
characteristicPolynomial
charthRoot
coerce
complementaryBasis
conditionP
convert
coordinates
createPrimitiveElement
D
definingPolynomial
derivationCoordinates
differentiate
discreteLog
discriminant
divide
elliptic
elt
euclideanSize
expressIdealMember
exquo
extendedEuclidean
factor
factorsOfCyclicGroupSize
gcd
gcdPolynomial
generator
genus
hash
hyperelliptic
index
init
integral?
integralAtInfinity?
integralBasis
integralBasisAtInfinity
integralCoordinates
integralDerivationMatrix
integralMatrix
integralMatrixAtInfinity
integralRepresents
inv
inverseIntegralMatrix
inverseIntegralMatrixAtInfinity
latex
lcm
lift
lookup
minimalPolynomial
multiEuclidean
nextItem
nonSingularModel
norm
normalizeAtInfinity
numberOfComponents
one?
order
prime?
primeFrobenius
primitive?
primitiveElement
primitivePart
principalIdeal
ramified?
ramifiedAtInfinity?
rank
random
rationalPoints
rationalPoint?
recip
reduce
reduceBasisAtInfinity
reducedSystem
regularRepresentation
representationType
represents
retract
retractIfCan
sample
singular?
singularAtInfinity?
size
sizeLess?
squareFree
squareFreePart
subtractIfCan
tableForDiscreteLogarithm
trace
traceMatrix
unit?
unitCanonical
unitNormal
yCoordinates
zero?
?*?
?**?
?+?
?-?
-?
?=?
?^?
?~=?
?/?
?quo?
?rem?

Attributes Exported:

- if $ has Fraction(UPOLYC(UFD)) and Field then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
branchPointAtInfinity? : () -> Boolean
branchPoint? : UP -> Boolean
branchPoint? : F -> Boolean
integralBasis : () -> Vector %
integralBasisAtInfinity : () -> Vector %
ramifiedAtInfinity? : () -> Boolean
ramified? : UP -> Boolean
ramified? : F -> Boolean
singularAtInfinity? : () -> Boolean
singular? : F -> Boolean
singular? : UP -> Boolean
```

These are implemented by this category:

```
absolutelyIrreducible? : () -> Boolean
algSplitSimple : (%,(UP -> UP)) ->
     Record(num: %,den: UP,derivden: UP,gd: UP)
complementaryBasis : Vector % -> Vector %
differentiate : (%,(UP -> UP)) -> %
elliptic : () -> Union(UP,"failed")
elt : (%,F,F) -> F
genus : () -> NonNegativeInteger
```

```
hyperelliptic : () -> Union(UP,"failed")
integral? : % -> Boolean
integral? : (%,F) -> Boolean
integral? : (%,UP) -> Boolean
integralAtInfinity? : % -> Boolean
normalizeAtInfinity : Vector % -> Vector %
numberOfComponents : () -> NonNegativeInteger
primitivePart : % -> %
rationalPoint? : (F,F) -> Boolean
rationalPoints : () -> List List F if F has FINITE
reduceBasisAtInfinity : Vector % -> Vector %
represents : (Vector UP,UP) -> %
yCoordinates : % -> Record(num: Vector UP,den: UP)
```

These exports come from (p1758) MonogenicAlgebra(RF, UPUP)
where RF:Fraction UP, UP:UnivariatePolynomialCategory F
F:UniqueFactorizationDomain, and
UPUP:UnivariatePolynomialCategory Fraction UP

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
    if Fraction UP has FIELD
basis : () -> Vector %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UPUP
charthRoot : % -> Union(%,"failed")
    if Fraction UP has CHARNZ
charthRoot : % -> %
    if Fraction UP has FFIELDC
coerce : % -> %
    if Fraction UP has FIELD
coerce : Fraction Integer -> %
    if Fraction UP has FIELD
    or Fraction UP has RETRACT FRAC INT
coerce : Fraction UP -> %
coerce : Integer -> %
coerce : % -> OutputForm
conditionP : Matrix % -> Union(Vector %,"failed")
    if Fraction UP has FFIELDC
convert : UPUP -> %
convert : % -> UPUP
convert : Vector Fraction UP -> %
convert : % -> Vector Fraction UP
coordinates : Vector % -> Matrix Fraction UP
coordinates : % -> Vector Fraction UP
coordinates : (Vector %,Vector %) -> Matrix Fraction UP
coordinates : (%,Vector %) -> Vector Fraction UP
createPrimitiveElement : () -> %
    if Fraction UP has FFIELDC
```

```
D : % -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
          has(Fraction UP,Field),
          has(Fraction UP,PartialDifferentialRing Symbol))
      or
        and(
          has(Fraction UP,PartialDifferentialRing Symbol),
          has(Fraction UP,Field))
D : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
    if Fraction UP has FIELD
definingPolynomial : () -> UPUP
derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP))
  -> Matrix Fraction UP
    if Fraction UP has FIELD
differentiate : % -> %
    if
        and(
          has(Fraction UP,Field),
          has(Fraction UP,DifferentialRing))
      or
        and(
          has(Fraction UP,DifferentialRing),
          has(Fraction UP,Field))
      or Fraction UP has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if
        and(
          has(Fraction UP,Field),
          has(Fraction UP,DifferentialRing))
      or
        and(
          has(Fraction UP,DifferentialRing),
          has(Fraction UP,Field))
      or Fraction UP has FFIELDC
differentiate : (%,Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
    or
     and(
       has(Fraction UP,PartialDifferentialRing Symbol),
       has(Fraction UP,Field))
differentiate : (%,List Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
      or
       and(
        has(Fraction UP,PartialDifferentialRing Symbol),
        has(Fraction UP,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
      or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
differentiate :
  (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
    if Fraction UP has FIELD
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if Fraction UP has FFIELDC
discreteLog : % -> NonNegativeInteger
    if Fraction UP has FFIELDC
discriminant : Vector % -> Fraction UP
discriminant : () -> Fraction UP
divide : (%,%) -> Record(quotient: %,remainder: %)
    if Fraction UP has FIELD
euclideanSize : % -> NonNegativeInteger
    if Fraction UP has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
exquo : (%,%) -> Union(%,"failed")
    if Fraction UP has FIELD
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
    if Fraction UP has FIELD
extendedEuclidean : (%,%,%) ->
   Union(Record(coef1: %,coef2: %),"failed")
    if Fraction UP has FIELD
factor : % -> Factored %
    if Fraction UP has FIELD
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
    if Fraction UP has FFIELDC
```

```
gcd : (%,%) -> %
    if Fraction UP has FIELD
gcd : List % -> %
    if Fraction UP has FIELD
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if Fraction UP has FIELD
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> %
    if Fraction UP has FINITE
init : () -> %
    if Fraction UP has FFIELDC
integralCoordinates : % ->
   Record(num: Vector UP,den: UP)
integralDerivationMatrix : (UP -> UP) ->
   Record(num: Matrix UP,den: UP)
integralMatrix : () -> Matrix Fraction UP
integralMatrixAtInfinity : () -> Matrix Fraction UP
integralRepresents : (Vector UP,UP) -> %
inv : % -> %
    if Fraction UP has FIELD
inverseIntegralMatrix : () -> Matrix Fraction UP
inverseIntegralMatrixAtInfinity : () ->
   Matrix Fraction UP
latex : % -> String
lcm : (%,%) -> %
    if Fraction UP has FIELD
lcm : List % -> %
    if Fraction UP has FIELD
lift : % -> UPUP
lookup : % -> PositiveInteger
    if Fraction UP has FINITE
minimalPolynomial : % -> UPUP
    if Fraction UP has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
nextItem : % -> Union(%,"failed")
    if Fraction UP has FFIELDC
nonSingularModel : Symbol -> List Polynomial F
    if F has FIELD
norm : % -> Fraction UP
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
    if Fraction UP has FFIELDC
order : % -> PositiveInteger
    if Fraction UP has FFIELDC
prime? : % -> Boolean
    if Fraction UP has FIELD
```

```
primeFrobenius : % -> %
    if Fraction UP has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> %
    if Fraction UP has FFIELDC
primitive? : % -> Boolean
    if Fraction UP has FFIELDC
primitiveElement : () -> %
    if Fraction UP has FFIELDC
principalIdeal : List % ->
  Record(coef: List %,generator: %)
    if Fraction UP has FIELD
rank : () -> PositiveInteger
random : () -> %
    if Fraction UP has FINITE
recip : % -> Union(%,"failed")
reduce : UPUP -> %
reduce : Fraction UPUP -> Union(%,"failed")
    if Fraction UP has FIELD
reducedSystem : Matrix % -> Matrix Fraction UP
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Fraction UP,vec: Vector Fraction UP)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if Fraction UP has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if Fraction UP has LINEXP INT
regularRepresentation : % -> Matrix Fraction UP
regularRepresentation : (%,Vector %) -> Matrix Fraction UP
representationType : () ->
  Union("prime",polynomial,normal,cyclic)
    if Fraction UP has FFIELDC
represents : Vector Fraction UP -> %
represents : (Vector Fraction UP,Vector %) -> %
retract : % -> Fraction Integer
    if Fraction UP has RETRACT FRAC INT
retract : % -> Integer
    if Fraction UP has RETRACT INT
retract : % -> Fraction UP
retractIfCan : % -> Union(Fraction UP,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
    if Fraction UP has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if Fraction UP has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger
    if Fraction UP has FINITE
sizeLess? : (%,%) -> Boolean
    if Fraction UP has FIELD
squareFree : % -> Factored %
    if Fraction UP has FIELD
```

```
squareFreePart : % -> %
    if Fraction UP has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tableForDiscreteLogarithm : Integer ->
  Table(PositiveInteger,NonNegativeInteger)
    if Fraction UP has FFIELDC
trace : % -> Fraction UP
traceMatrix : () -> Matrix Fraction UP
traceMatrix : Vector % -> Matrix Fraction UP
unit? : % -> Boolean
    if Fraction UP has FIELD
unitCanonical : % -> %
    if Fraction UP has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Fraction UP has FIELD
zero? : % -> Boolean
?*? : (Fraction UP,%) -> %
?*? : (%,Fraction UP) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
?*? : (NonNegativeInteger,%) -> %
?**? : (%,Integer) -> % if Fraction UP has FIELD
?**? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Fraction UP has FIELD
?^? : (%,Integer) -> % if Fraction UP has FIELD
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> % if Fraction UP has FIELD
?rem? : (%,%) -> % if Fraction UP has FIELD
```

⟨*category FFCAT FunctionFieldCategory*⟩≡

```
)abbrev category FFCAT FunctionFieldCategory
++ Function field of a curve
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 19 Mai 1993
++ Keywords: algebraic, curve, function, field.
++ Description:
++ This category is a model for the function field of a
++ plane algebraic curve.
```

```
FunctionFieldCategory(F, UP, UPUP): Category == Definition where
  F   : UniqueFactorizationDomain
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP

  Z   ==> Integer
  Q   ==> Fraction F
  P   ==> Polynomial F
  RF  ==> Fraction UP
  QF  ==> Fraction UPUP
  SY  ==> Symbol
  REC ==> Record(num:$, den:UP, derivden:UP, gd:UP)

  Definition ==> MonogenicAlgebra(RF, UPUP) with
    numberOfComponents     : () -> NonNegativeInteger
      ++ numberOfComponents() returns the number of absolutely irreducible
      ++ components.
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
      ++X numberOfComponents()$R
    genus                  : () -> NonNegativeInteger
      ++ genus() returns the genus of one absolutely irreducible component
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
      ++X genus()$R
    absolutelyIrreducible? : () -> Boolean
      ++ absolutelyIrreducible?() tests if the curve absolutely irreducible?
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
      ++X absolutelyIrreducible?()$R2
    rationalPoint?         : (F, F) -> Boolean
      ++ rationalPoint?(a, b) tests if \spad{(x=a,y=b)} is on the curve.
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
      ++X rationalPoint?(0,0)$R
      ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
      ++X rationalPoint?(0,0)$R2
```

```
branchPointAtInfinity? : () -> Boolean
  ++ branchPointAtInfinity?() tests if there is a branch point
  ++ at infinity.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X branchPointAtInfinity?()$R
  ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
  ++X branchPointAtInfinity?()$R
branchPoint?           : F -> Boolean
  ++ branchPoint?(a) tests whether \spad{x = a} is a branch point.
branchPoint?           : UP -> Boolean
  ++ branchPoint?(p) tests whether \spad{p(x) = 0} is a branch point.
singularAtInfinity?    : () -> Boolean
  ++ singularAtInfinity?() tests if there is a singularity at infinity.
singular?              : F -> Boolean
  ++ singular?(a) tests whether \spad{x = a} is singular.
singular?              : UP -> Boolean
  ++ singular?(p) tests whether \spad{p(x) = 0} is singular.
ramifiedAtInfinity?    : () -> Boolean
  ++ ramifiedAtInfinity?() tests if infinity is ramified.
ramified?              : F -> Boolean
  ++ ramified?(a) tests whether \spad{x = a} is ramified.
ramified?              : UP -> Boolean
  ++ ramified?(p) tests whether \spad{p(x) = 0} is ramified.
integralBasis          : () -> Vector $
  ++ integralBasis() returns the integral basis for the curve.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasis()$R
integralBasisAtInfinity: () -> Vector $
  ++ integralBasisAtInfinity() returns the local integral basis
  ++ at infinity
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasisAtInfinity()$R
integralAtInfinity?    : $  -> Boolean
  ++ integralAtInfinity?() tests if f is locally integral at infinity.
integral?              : $  -> Boolean
  ++ integral?() tests if f is integral over \spad{k[x]}.
complementaryBasis     : Vector $ -> Vector $
```

```
  ++ complementaryBasis(b1,...,bn) returns the complementary basis
  ++ \spad{(b1',...,bn')} of \spad{(b1,...,bn)}.
normalizeAtInfinity    : Vector $ -> Vector $
  ++ normalizeAtInfinity(v) makes v normal at infinity.
reduceBasisAtInfinity  : Vector $ -> Vector $
  ++ reduceBasisAtInfinity(b1,...,bn) returns \spad{(x**i * bj)}
  ++ for all i,j such that \spad{x**i*bj} is locally integral
  ++ at infinity.
integralMatrix         : () -> Matrix RF
  ++ integralMatrix() returns M such that
  ++ \spad{(w1,...,wn) = M (1, y, ..., y**(n-1))},
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralMatrix()$R
inverseIntegralMatrix  : () -> Matrix RF
  ++ inverseIntegralMatrix() returns M such that
  ++ \spad{M (w1,...,wn) = (1, y, ..., y**(n-1))}
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X inverseIntegralMatrix()$R
integralMatrixAtInfinity       : () -> Matrix RF
  ++ integralMatrixAtInfinity() returns M such that
  ++ \spad{(v1,...,vn) = M (1, y, ..., y**(n-1))}
  ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
  ++ returned by \spad{infIntBasis()}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralMatrixAtInfinity()$R
inverseIntegralMatrixAtInfinity: () -> Matrix RF
  ++ inverseIntegralMatrixAtInfinity() returns M such
  ++ that \spad{M (v1,...,vn) = (1, y, ..., y**(n-1))}
  ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
  ++ returned by \spad{infIntBasis()}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
```

```
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X inverseIntegralMatrixAtInfinity()$R
  yCoordinates          : $ -> Record(num:Vector(UP), den:UP)
    ++ yCoordinates(f) returns \spad{[[A1,...,An], D]} such that
    ++ \spad{f = (A1 + A2 y +...+ An y**(n-1)) / D}.
  represents            : (Vector UP, UP) -> $
    ++ represents([A0,...,A(n-1)],D) returns
    ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.
  integralCoordinates    : $ -> Record(num:Vector(UP), den:UP)
    ++ integralCoordinates(f) returns \spad{[[A1,...,An], D]} such that
    ++ \spad{f = (A1 w1 +...+ An wn) / D}  where \spad{(w1,...,wn)} is the
    ++ integral basis returned by \spad{integralBasis()}.
  integralRepresents     : (Vector UP, UP) -> $
    ++ integralRepresents([A1,...,An], D) returns
    ++ \spad{(A1 w1+...+An wn)/D}
    ++ where \spad{(w1,...,wn)} is the integral
    ++ basis of \spad{integralBasis()}.
  integralDerivationMatrix:(UP -> UP) -> Record(num:Matrix(UP),den:UP)
    ++ integralDerivationMatrix(d) extends the derivation d from UP to $
    ++ and returns (M, Q) such that the i^th row of M divided by Q form
    ++ the coordinates of \spad{d(wi)} with respect to \spad{(w1,...,wn)}
    ++ where \spad{(w1,...,wn)} is the integral basis returned
    ++ by integralBasis().
  integral?             : ($,  F) -> Boolean
    ++ integral?(f, a) tests whether f is locally integral at \spad{x = a}.
  integral?             : ($, UP) -> Boolean
    ++ integral?(f, p) tests whether f is locally integral at
    ++ \spad{p(x) = 0}
  differentiate         : ($, UP -> UP) -> $
    ++ differentiate(x, d) extends the derivation d from UP to $ and
    ++ applies it to x.
  represents            : (Vector UP, UP) -> $
    ++ represents([A0,...,A(n-1)],D) returns
    ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.
  primitivePart         : $ -> $
    ++ primitivePart(f) removes the content of the denominator and
    ++ the common content of the numerator of f.
  elt                   : ($, F, F) -> F
    ++ elt(f,a,b) or f(a, b) returns the value of f
    ++ at the point \spad{(x = a, y = b)}
    ++ if it is not singular.
  elliptic              : () -> Union(UP, "failed")
    ++ elliptic() returns \spad{p(x)} if the curve is the elliptic
    ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.
  hyperelliptic         : () -> Union(UP, "failed")
    ++ hyperelliptic() returns \spad{p(x)} if the curve is the
```

```
      ++ hyperelliptic
      ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.
    algSplitSimple         : ($, UP -> UP) -> REC
      ++ algSplitSimple(f, D) returns \spad{[h,d,d',g]} such that
      ++ \spad{f=h/d},
      ++ \spad{h} is integral at all the normal places w.r.t. \spad{D},
      ++ \spad{d' = Dd}, \spad{g = gcd(d, discriminant())} and \spad{D}
      ++ is the derivation to use. \spad{f} must have at most simple finite
      ++ poles.
    if F has Field then
      nonSingularModel: SY -> List Polynomial F
        ++ nonSingularModel(u) returns the equations in u1,...,un of
        ++ an affine non-singular model for the curve.
    if F has Finite then
      rationalPoints: () -> List List F
        ++ rationalPoints() returns the list of all the affine
        ++rational points.
   add
    import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
    import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)

    repOrder: (Matrix RF, Z) -> Z
    Q2RF    : Q  -> RF
    infOrder: RF -> Z
    infValue: RF -> Fraction F
    intvalue: (Vector UP, F, F) -> F
    rfmonom : Z  -> RF
    kmin    : (Matrix RF,Vector Q) -> Union(Record(pos:Z,km:Z),"failed")

    Q2RF q                 == numer(q)::UP / denom(q)::UP
    infOrder f             == (degree denom f)::Z - (degree numer f)::Z
    integral? f            == ground?(integralCoordinates(f).den)
    integral?(f:$, a:F)    == (integralCoordinates(f).den)(a) ^= 0
--    absolutelyIrreducible? == one? numberOfComponents()
    absolutelyIrreducible? == numberOfComponents() = 1
    yCoordinates f         == splitDenominator coordinates f

    hyperelliptic() ==
      degree(f := definingPolynomial()) ^= 2 => "failed"
      (u:=retractIfCan(reductum f)@Union(RF,"failed"))
        case "failed" => "failed"
      (v:=retractIfCan(-(u::RF) / leadingCoefficient f)@Union(UP, "failed"))
        case "failed" => "failed"
      odd? degree(p := v::UP) => p
      "failed"
    algSplitSimple(f, derivation) ==
      cd := splitDenominator lift f
      dd := (cd.den exquo (g := gcd(cd.den, derivation(cd.den))))::UP
      [reduce(inv(g::RF) * cd.num), dd, derivation dd,
                                  gcd(dd, retract(discriminant())@UP)]

    elliptic() ==
      (u := hyperelliptic()) case "failed" => "failed"
      degree(p := u::UP) = 3 => p
      "failed"

    rationalPoint?(x, y)   ==
      zero?((definingPolynomial() (y::UP::RF)) (x::UP::RF))

    if F has Field then
      import PolyGroebner(F)
      import MatrixCommonDenominator(UP, RF)

      UP2P  : (UP,   P)    -> P
      UPUP2P: (UPUP, P, P) -> P

      UP2P(p, x) ==
        (map((s:F):P +-> s::P, p)_
          $UnivariatePolynomialCategoryFunctions2(F, UP,
                                   P, SparseUnivariatePolynomial P)) x

      UPUP2P(p, x, y) ==
        (map((s:RF):P +-> UP2P(retract(s)@UP, x),p)_
          $UnivariatePolynomialCategoryFunctions2(RF, UPUP,
                                   P, SparseUnivariatePolynomial P)) y

      nonSingularModel u ==
        d    := commonDenominator(coordinates(w := integralBasis()))::RF
        vars := [concat(string u, string i)::SY for i in 1..(n := #w)]
        x    := "%%dummy1"::SY
        y    := "%%dummy2"::SY
        select_!(s+->zero?(degree(s, x)) and zero?(degree(s, y)),
                 lexGroebner([v::P - UPUP2P(lift(d * w.i), x::P, y::P)
                    for v in vars for i in 1..n], concat([x, y], vars)))

    if F has Finite then
      ispoint: (UPUP, F, F) -> List F

-- must use the 'elt function explicitely or the compiler takes 45 mins
-- on that function    MB 5/90
-- still takes ages : I split the expression up. JHD 6/Aug/90
```

```
    ispoint(p, x, y) ==
      jhd:RF:=p(y::UP::RF)
      zero?(jhd (x::UP::RF)) => [x, y]
      empty()

    rationalPoints() ==
      p := definingPolynomial()
      concat [[pt for y in 1..size()$F | not empty?(pt :=
        ispoint(p, index(x::PositiveInteger)$F,
                   index(y::PositiveInteger)$F))]$List(List F)
                              for x in 1..size()$F]$List(List(List F))

  intvalue(v, x, y) ==
    singular? x => error "Point is singular"
    mini := minIndex(w := integralBasis())
    rec := yCoordinates(+/[qelt(v, i)::RF * qelt(w, i)
                         for i in mini .. maxIndex w])
    n   := +/[(qelt(rec.num, i) x) *
              (y ** ((i - mini)::NonNegativeInteger))
                         for i in mini .. maxIndex w]
    zero?(d := (rec.den) x) =>
      zero? n => error "0/0 -- cannot compute value yet"
      error "Shouldn't happen"
    (n exquo d)::F

  elt(f, x, y) ==
    rec := integralCoordinates f
    n   := intvalue(rec.num, x, y)
    zero?(d := (rec.den) x) =>
      zero? n => error "0/0 -- cannot compute value yet"
      error "Function has a pole at the given point"
    (n exquo d)::F

  primitivePart f ==
    cd := yCoordinates f
    d  := gcd([content qelt(cd.num, i)
               for i in minIndex(cd.num) .. maxIndex(cd.num)]$List(F))
                 * primitivePart(cd.den)
    represents [qelt(cd.num, i) / d
             for i in minIndex(cd.num) .. maxIndex(cd.num)]$Vector(RF)

  reduceBasisAtInfinity b ==
    x := monomial(1, 1)$UP ::RF
    concat([[f for j in 0.. while
              integralAtInfinity?(f := x**j * qelt(b, i))]$Vector($)
                    for i in minIndex b .. maxIndex b]$List(Vector $))
```

```
complementaryBasis b ==
  m := inverse(traceMatrix b)::Matrix(RF)
  [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

integralAtInfinity? f ==
  not any?(s +-> infOrder(s) < 0,
     coordinates(f) * inverseIntegralMatrixAtInfinity())$Vector(RF)

numberOfComponents() ==
  count(integralAtInfinity?, integralBasis())$Vector($)

represents(v:Vector UP, d:UP) ==
  represents
    [qelt(v, i) / d for i in minIndex v .. maxIndex v]$Vector(RF)

genus() ==
  ds := discriminant()
  d  := degree(retract(ds)@UP) + infOrder(ds * determinant(
         integralMatrixAtInfinity() * inverseIntegralMatrix()) ** 2)
  dd := (((d exquo 2)::Z - rank()) exquo numberOfComponents())::Z
  (dd + 1)::NonNegativeInteger

repOrder(m, i) ==
  nostart:Boolean := true
  ans:Z := 0
  r := row(m, i)
  for j in minIndex r .. maxIndex r | qelt(r, j) ^= 0 repeat
    ans :=
      nostart => (nostart := false; infOrder qelt(r, j))
      min(ans, infOrder qelt(r,j))
  nostart => error "Null row"
  ans

infValue f ==
  zero? f => 0
  (n := infOrder f) > 0 => 0
  zero? n =>
    (leadingCoefficient numer f) / (leadingCoefficient denom f)
  error "f not locally integral at infinity"

rfmonom n ==
  n < 0 => inv(monomial(1, (-n)::NonNegativeInteger)$UP :: RF)
  monomial(1, n::NonNegativeInteger)$UP :: RF

kmin(m, v) ==
      nostart:Boolean := true
      k:Z := 0
      ii  := minRowIndex m - (i0  := minIndex v)
      for i in minIndex v .. maxIndex v | qelt(v, i) ^= 0 repeat
        nk := repOrder(m, i + ii)
        if nostart then (nostart := false; k := nk; i0 := i)
        else
          if nk < k then (k := nk; i0 := i)
      nostart => "failed"
      [i0, k]

    normalizeAtInfinity w ==
      ans   := copy w
      infm  := inverseIntegralMatrixAtInfinity()
      mhat  := zero(rank(), rank())$Matrix(RF)
      ii    := minIndex w - minRowIndex mhat
      repeat
        m := coordinates(ans) * infm
        r := [rfmonom repOrder(m, i)
                     for i in minRowIndex m .. maxRowIndex m]$Vector(RF)
        for i in minRowIndex m .. maxRowIndex m repeat
          for j in minColIndex m .. maxColIndex m repeat
            qsetelt_!(mhat, i, j, qelt(r, i + ii) * qelt(m, i, j))
        sol := first nullSpace transpose map(infValue,
                mhat)$MatrixCategoryFunctions2(RF, Vector RF, Vector RF,
                             Matrix RF, Q, Vector Q, Vector Q, Matrix Q)
        (pr := kmin(m, sol)) case "failed" => return ans
        qsetelt_!(ans, pr.pos,
         +/[Q2RF(qelt(sol, i)) * rfmonom(repOrder(m, i - ii) - pr.km)
                  * qelt(ans, i) for i in minIndex sol .. maxIndex sol])

    integral?(f:$, p:UP) ==
      (r:=retractIfCan(p)@Union(F,"failed")) case F => integral?(f,r::F)
      (integralCoordinates(f).den exquo p) case "failed"

    differentiate(f:$, d:UP -> UP) ==
      differentiate(f, x +-> differentiate(x, d)$RF)
```

⟨*FFCAT.dotabb*⟩≡

```
  "FFCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
  "FFCAT" -> "MONOGEN"
```

⟨*FFCAT.dotfull*⟩≡
```
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
```

⟨*FFCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

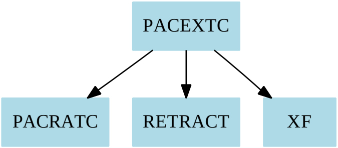
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."

"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
}
```

20.0.218 PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory (PACEXTC)



⟨*PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.input*⟩≡

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.output
)spool PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
--R PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory  is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory is PACEXTC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACEXTC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,Fraction Integer) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : Fraction Integer -> %        coerce : Integer -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R conjugate : % -> %                    dimension : () -> CardinalNumber
--R extDegree : % -> PositiveInteger      factor : % -> Factored %
--R fullOutput : % -> OutputForm          gcd : List % -> %
--R gcd : (%,%) -> %                      ground? : % -> Boolean
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
```

```
--R lcm : List % -> %                     lcm : (%,%) -> %
--R maxTower : List % -> %                one? : % -> Boolean
--R previousTower : % -> %                prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Fraction Integer
--R retract : % -> Fraction Integer       retract : % -> Integer
--R sample : () -> %                      setTower! : % -> Void
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R vectorise : (%,%) -> Vector %         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
--R ?*? : (PseudoAlgebraicClosureOfRationalNumber,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
--R Frobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fr
--R Frobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNum
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if PseudoAlgebraicClosureOfRationalNumber
--R coerce : PseudoAlgebraicClosureOfRationalNumber -> %
--R coerce : PseudoAlgebraicClosureOfRationalNumber -> %
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PseudoAlgebraicC
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -
--R lift : (%,%) -> SparseUnivariatePolynomial %
--R lift : % -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R order : % -> OnePointCompletion PositiveInteger if PseudoAlgebraicClosureOfRa
--R primeFrobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ
--R primeFrobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRatio
--R principalIdeal : List % -> Record(coef: List %,generator: %)
```

```
--R reduce : SparseUnivariatePolynomial % -> %
--R retract : % -> PseudoAlgebraicClosureOfRationalNumber
--R retract : % -> PseudoAlgebraicClosureOfRationalNumber
--R retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
--R retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

⟨*PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.help*⟩≡

```
====================================================================
PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
```

See:

⇐ "PseudoAlgebraicClosureOfRationalNumberCategory" (PACRATC) 19.0.215 on page 1771

Exports:

0
1
-?
?**?
?*?
?+?
?-?
?/?
?=?
?^?
?~=?
?quo?
?rem?
algebraic?
associates?
characteristic
charthRoot
coerce
conjugate
definingPolynomial
degree
dimension
discreteLog
distinguishedRootsOf
divide
euclideanSize
expressIdealMember
exquo
extDegree
extendedEuclidean
extensionDegree
factor
Frobenius
fullOutput
gcd
gcdPolynomial
ground?
hash
inGroundField?
inv
latex
lcm
lift
maxTower
multiEuclidean
newElement
one?
order
previousTower
prime?
primeFrobenius
principalIdeal
recip
reduce
retract
retractIfCan
sample
setTower!
sizeLess?
squareFree
squareFreePart
subtractIfCan
transcendenceDegree
transcendent?
unit?
unitCanonical
unitNormal
vectorise
zero?

Attributes Exported:

- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

These exports come from (p1771) PseudoAlgebraicClosureOfRationalNumberCategory

```
-? : % -> %
0 : () -> %
1 : () -> %
?**? : (%,Integer) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?/? : (%,%) -> %
?/? : (%,Fraction Integer) -> %
?=? : (%,%) -> Boolean
?^? : (%,Integer) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
?~=? : (%,%) -> Boolean
Frobenius : % -> % if Fraction Integer has FINITE
Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
algebraic? : % -> Boolean
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
coerce : % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : Integer -> %
conjugate : % -> %
definingPolynomial : % -> SparseUnivariatePolynomial %
definingPolynomial : () -> SparseUnivariatePolynomial %
degree : % -> OnePointCompletion PositiveInteger
dimension : () -> CardinalNumber
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
```

```
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PositiveInteger
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extensionDegree : () -> OnePointCompletion PositiveInteger
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
             SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
ground? : % -> Boolean
hash : % -> SingleInteger
inGroundField? : % -> Boolean
inv : % -> %
latex : % -> String
lcm : (%,%) -> %
lcm : List % -> %
lift : % -> SparseUnivariatePolynomial %
lift : (%,%) -> SparseUnivariatePolynomial %
maxTower : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
newElement : (SparseUnivariatePolynomial %,Symbol) -> %
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
previousTower : % -> %
prime? : % -> Boolean
primeFrobenius : % -> %
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
primeFrobenius : (%,NonNegativeInteger) -> %
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
reduce : SparseUnivariatePolynomial % -> %
retract : % -> Fraction Integer
retract : % -> Fraction Integer
retract : % -> Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Integer,"failed")
sample : () -> %
setTower! : % -> Void
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
transcendenceDegree : () -> NonNegativeInteger
transcendent? : % -> Boolean
```

```
 unit? : % -> Boolean
 unitCanonical : % -> %
 unitNormal : % -> Record(unit: %,canonical: %,associate: %)
 vectorise : (%,%) -> Vector %
 zero? : % -> Boolean

These exports come from \refto{RetractableTo}
\begin{verbatim}
 coerce : PseudoAlgebraicClosureOfRationalNumber -> %
 coerce : PseudoAlgebraicClosureOfRationalNumber -> %
 retract : % -> PseudoAlgebraicClosureOfRationalNumber
 retract : % -> PseudoAlgebraicClosureOfRationalNumber
 retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
 retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
```

These exports come from (p1662) ExtensionField

```
 ?*? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
 ?*? : (PseudoAlgebraicClosureOfRationalNumber,%) -> %
 ?/? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
 charthRoot : % -> Union(%,"failed") if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgeb
 discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PseudoAlgebraicClosureOfRationalNumber has
 Frobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fraction Integer has FINITE
 Frobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fractio
 order : % -> OnePointCompletion PositiveInteger if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or
 primeFrobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgebraicClosureO
 primeFrobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or Ps
```

⟨*category PACEXTC PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory*⟩≡

```
 )abbrev category PACEXTC PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
 ++ Authors: Gaetan Hache
 ++ Date Created: jan 1998
 ++ Date Last Updated: May 2010 by Tim Daly
 ++ Description:
 ++ This category exports the function for the domain
 ++ PseudoAlgebraicClosureOfAlgExtOfRationalNumber which implement dynamic
 ++ extension using the simple notion of tower extensions. A tower extension
 ++ T of the ground field K is any sequence of field extension
 ++ (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K and for i =1,2,...,n,
 ++ K_i is an extension of K_{i-1} of degree > 1 and defined by an
 ++ irreducible polynomial p(Z) in K_{i-1}.
 ++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1) and
 ++ (T_2: K_02, K_12,...,K_i2,...,K_n2)
 ++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
 ++ that is if K_i1 = K_i2 for i=1,2,...,n1 (or i=1,2,...,n2).
 ++ Any algebraic operations defined for several elements
 ++ are only defined if all of the concerned elements are comming from
 ++ a set of related tour extensions.
 PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory:Category == Impl where
```

```
Q    ==> PseudoAlgebraicClosureOfRationalNumber
Impl ==>
  Join(PseudoAlgebraicClosureOfRationalNumberCategory,_
       RetractableTo(Q),ExtensionField(Q))
```

⟨*PACEXTC.dotabb*⟩≡

```
"PACEXTC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
"PACEXTC" -> "PACRATC"
"PACEXTC" -> "RETRACT"
"PACEXTC" -> "XF"
```

⟨*PACEXTC.dotfull*⟩≡

```
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
   -> "PseudoAlgebraicClosureOfRationalNumberCategory"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"

"PseudoAlgebraicClosureOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];

"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
 ->  "RetractableTo(a:Type)"

"RetractableTo(a:Type)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];

"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)" ->
 ->  "ExtensionField(a:Field)"

"ExtensionField(a:Field)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
```

⟨*PACEXTC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
   -> "PseudoAlgebraicClosureOfRationalNumberCategory"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"

"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
  -> "RETRACT..."

"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)" ->
 ->  "XF..."

"XF..." [color=lightblue];
"RETRACT..." [color=lightblue];

}
```

Chapter 21

The bootstrap code

21.1 ABELGRP.lsp BOOTSTRAP

ABELGRP depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELGRP.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianGroup;AL| (QUOTE NIL))

(DEFUN |AbelianGroup| NIL
  (LET (#:G82664)
    (COND
      (|AbelianGroup;AL|)
      (T (SETQ |AbelianGroup;AL| (|AbelianGroup;|))))))

(DEFUN |AbelianGroup;| NIL
  (PROG (#1=#:G82662)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|CancellationAbelianMonoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|-| (|$| |$|)) T)
                ((|-| (|$| |$| |$|)) T)
                ((|*| (|$| (|Integer|) |$|)) T)))
              NIL
              (QUOTE ((|Integer|)))
              NIL))
           |AbelianGroup|)
         (SETELT #1# 0 (QUOTE (|AbelianGroup|)))))))

(MAKEPROP (QUOTE |AbelianGroup|) (QUOTE NILADIC) T)
```

21.2 ABELGRP-.lsp BOOTSTRAP

ABELGRP- depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELGRP-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELGRP-;-;3S;1| (|x| |y| |$|)
  (SPADCALL |x| (SPADCALL |y| (QREFELT |$| 7)) (QREFELT |$| 8)))

(DEFUN |ABELGRP-;subtractIfCan;2SU;2| (|x| |y| |$|)
  (CONS 0 (SPADCALL |x| |y| (QREFELT |$| 10))))

(DEFUN |ABELGRP-;*;Nni2S;3| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 14)))

(DEFUN |ABELGRP-;*;I2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 17))
    ((|<| 0 |n|) (SPADCALL |n| |x| (QREFELT |$| 20)))
    ((QUOTE T)
      (SPADCALL (|-| |n|) (SPADCALL |x| (QREFELT |$| 7)) (QREFELT |$| 20)))))

(DEFUN |AbelianGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianGroup&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 21
              (CONS (|dispatchFunction| |ABELGRP-;*;I2S;4|) |$|))))
        |$|))))
```

```
(MAKEPROP
  (QUOTE |AbelianGroup&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |-|)
        (5 . |+|)
        |ABELGRP-;-;3S;1|
        (11 . |-|)
        (|Union| |$| (QUOTE "failed"))
        |ABELGRP-;subtractIfCan;2SU;2|
        (|Integer|)
        (17 . |*|)
        (|NonNegativeInteger|)
        |ABELGRP-;*;Nni2S;3|
        (23 . |Zero|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (27 . |double|)
        (33 . |*|)))
    (QUOTE #(|subtractIfCan| 39 |-| 45 |*| 51))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 21
            (QUOTE (1 6 0 0 7 2 6 0 0 0 8 2 6 0 0 0 10 2 6 0 13 0 14 0 6 0 17
                    2 19 6 18 6 20 2 0 0 13 0 21 2 0 11 0 0 12 2 0 0 0 0 9 2
                    0 0 13 0 21 2 0 0 15 0 16))))))
    (QUOTE |lookupComplete|)))
```

21.3 ABELMON.lsp BOOTSTRAP

ABELMON which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON**. We break this chain with **ABELMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELMON.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianMonoid;AL| (QUOTE NIL))

(DEFUN |AbelianMonoid| NIL
  (LET (#:G82597)
    (COND
      (|AbelianMonoid;AL|)
      (T (SETQ |AbelianMonoid;AL| (|AbelianMonoid;|))))))

(DEFUN |AbelianMonoid;| NIL
  (PROG (#1=#:G82595)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianSemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|Zero| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|zero?| ((|Boolean|) |$|)) T)
                ((|*| (|$| (|NonNegativeInteger|) |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |AbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|AbelianMonoid|)))))))

(MAKEPROP (QUOTE |AbelianMonoid|) (QUOTE NILADIC) T)
```

21.4 ABELMON-.lsp BOOTSTRAP

ABELMON- which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON-**. We break this chain with **ABELMON-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELMON-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELMON-;zero?;SB;1| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ABELMON-;*;Pi2S;2| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 12)))

(DEFUN |ABELMON-;sample;S;3| (|$|)
  (|spadConstant| |$| 7))

(DEFUN |ABELMON-;*;Nni2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 7))
    ((QUOTE T) (SPADCALL |n| |x| (QREFELT |$| 17)))))

(DEFUN |AbelianMonoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianMonoid&|))
        (LETT |dv$| (LIST (QUOTE |AbelianMonoid&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 19) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 18
              (CONS (|dispatchFunction| |ABELMON-;*;Nni2S;4|) |$|)))) |$|))))

(MAKEPROP
  (QUOTE |AbelianMonoid&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |=|)
        |ABELMON-;zero?;SB;1|
        (|NonNegativeInteger|)
        (10 . |*|)
        (|PositiveInteger|)
        |ABELMON-;*;Pi2S;2|
        |ABELMON-;sample;S;3|
        (|RepeatedDoubling| 6)
        (16 . |double|)
        (22 . |*|)))
    (QUOTE #(|zero?| 28 |sample| 33 |*| 37))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 18
            (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 11 0 12 2 16 6 13 6 17 2 0 0 11
                    0 18 1 0 8 0 10 0 0 0 15 2 0 0 11 0 18 2 0 0 13 0 14))))))
   (QUOTE |lookupComplete|)))
```

21.5 ABELSG.lsp BOOTSTRAP

ABELSG needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG**. We break this chain with **ABELSG.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELSG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianSemiGroup;AL| (QUOTE NIL))

(DEFUN |AbelianSemiGroup| NIL
  (LET (#:G82568)
    (COND
      (|AbelianSemiGroup;AL|)
      (T (SETQ |AbelianSemiGroup;AL| (|AbelianSemiGroup;|))))))

(DEFUN |AbelianSemiGroup;| NIL
  (PROG (#1=#:G82566)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SetCategory|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|+| (|$| |$| |$|)) T)
                ((|*| (|$| (|PositiveInteger|) |$|)) T)))
              NIL
              (QUOTE ((|PositiveInteger|)))
              NIL))
           |AbelianSemiGroup|)
        (SETELT #1# 0 (QUOTE (|AbelianSemiGroup|)))))))

(MAKEPROP (QUOTE |AbelianSemiGroup|) (QUOTE NILADIC) T)
```

21.6 ABELSG-.lsp BOOTSTRAP

ABELSG- needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG-**. We break this chain with **ABELSG-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELSG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELSG-;*;Pi2S;1| (|n| |x| |$|) (SPADCALL |n| |x| (QREFELT |$| 9)))

(DEFUN |AbelianSemiGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianSemiGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianSemiGroup&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 11) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 10
              (CONS (|dispatchFunction| |ABELSG-;*;Pi2S;1|) |$|))))
        |$|))))

(MAKEPROP
  (QUOTE |AbelianSemiGroup&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (0 . |double|)
    (6 . |*|)))
(QUOTE #(|*| 12))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 10
        (QUOTE (2 8 6 7 6 9 2 0 0 7 0 10 2 0 0 7 0 10))))))
(QUOTE |lookupComplete|)))
```

21.7 ALAGG.lsp BOOTSTRAP

ALAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ALAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ALAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ALAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AssociationListAggregate;CAT| (QUOTE NIL))

(SETQ |AssociationListAggregate;AL| (QUOTE NIL))

(DEFUN |AssociationListAggregate|
 (|&REST| #1=#:G88404 |&AUX| #2=#:G88402)
  (DSETQ #2# #1#)
  (LET (#3=#:G88403)
   (COND
    ((SETQ #3# (|assoc| (|devaluateList| #2#) |AssociationListAggregate;AL|))
      (CDR #3#))
    (T
      (SETQ |AssociationListAggregate;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |AssociationListAggregate;|) #2#)))
        |AssociationListAggregate;AL|)) #3#))))

(DEFUN |AssociationListAggregate;| (|t#1| |t#2|)
 (PROG (#1=#:G88401)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR
        (QUOTE (#2=#:G88400))
        (LIST (QUOTE (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)))))
       (COND
        (|AssociationListAggregate;CAT|)
```

```
          ((QUOTE T)
           (LETT |AssociationListAggregate;CAT|
            (|Join|
             (|TableAggregate| (QUOTE |t#1|) (QUOTE |t#2|))
             (|ListAggregate| (QUOTE #2#))
             (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
               (((|assoc|
                  ((|Union|
                    (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)) "failed")
                   |t#1| |$|))
                  T)))
              NIL (QUOTE NIL) NIL))
            . #3=(|AssociationListAggregate|))))))
         . #3#)
      (SETELT #1# 0
       (LIST
        (QUOTE |AssociationListAggregate|)
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)))))))
```

21.8 CABMON.lsp BOOTSTRAP

CABMON which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON**. We break this chain with **CABMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CABMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **CABMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CABMON.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |CancellationAbelianMonoid;AL| (QUOTE NIL))

(DEFUN |CancellationAbelianMonoid| NIL
  (LET (#:G82646)
    (COND
      (|CancellationAbelianMonoid;AL|)
      (T
        (SETQ
          |CancellationAbelianMonoid;AL|
          (|CancellationAbelianMonoid;|))))))

(DEFUN |CancellationAbelianMonoid;| NIL
  (PROG (#1=#:G82644)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianMonoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
               (((|subtractIfCan| ((|Union| |$| "failed") |$| |$|)) T)))
              NIL
             (QUOTE NIL)
             NIL))
           |CancellationAbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|CancellationAbelianMonoid|)))))))

(MAKEPROP (QUOTE |CancellationAbelianMonoid|) (QUOTE NILADIC) T)
```

21.9 CLAGG.lsp BOOTSTRAP

CLAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CLAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Collection;CAT| (QUOTE NIL))

(SETQ |Collection;AL| (QUOTE NIL))

(DEFUN |Collection| (#1=#:G82618)
 (LET (#2=#:G82619)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |Collection;AL|)) (CDR #2#))
   (T
    (SETQ |Collection;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|Collection;| #1#)))
      |Collection;AL|))
     #2#))))

(DEFUN |Collection;| (|t#1|)
 (PROG (#1=#:G82617)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|Collection;CAT|)
       ((QUOTE T)
        (LETT |Collection;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|construct| (|$| (|List| |t#1|))) T)
            ((|find| ((|Union| |t#1| "failed")
                       (|Mapping| (|Boolean|) |t#1|) |$|))
              T)
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|remove| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|select| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1| |t#1|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))
            ((|remove| (|$| |t#1| |$|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))
            ((|removeDuplicates| (|$| |$|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
           (QUOTE (((|ConvertibleTo| (|InputForm|))
                    (|has| |t#1| (|ConvertibleTo| (|InputForm|))))))
           (QUOTE ((|List| |t#1|))) NIL))
         . #2=(|Collection|)))))
      . #2#)
    (SETELT #1# 0 (LIST (QUOTE |Collection|) (|devaluate| |t#1|)))))))
```

21.10 CLAGG-.lsp BOOTSTRAP

CLAGG- depends on **CLAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CLAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |CLAGG-;#;ANni;1| (|c| |$|) (LENGTH (SPADCALL |c| (QREFELT |$| 9))))

(DEFUN |CLAGG-;count;MANni;2| (|f| |c| |$|)
 (PROG (|x| #1=#:G82637 #2=#:G82634 #3=#:G82632 #4=#:G82633)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;count;MANni;2|)
     (SEQ
      (LETT |x| NIL |CLAGG-;count;MANni;2|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;count;MANni;2|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;count;MANni;2|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (COND
         ((SPADCALL |x| |f|)
          (PROGN
           (LETT #2# 1 |CLAGG-;count;MANni;2|)
           (COND
            (#4# (LETT #3# (|+| #3# #2#) |CLAGG-;count;MANni;2|))
            ((QUOTE T)
             (PROGN
              (LETT #3# #2# |CLAGG-;count;MANni;2|)
              (LETT #4# (QUOTE T) |CLAGG-;count;MANni;2|)))))))))
      (LETT #1# (CDR #1#) |CLAGG-;count;MANni;2|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) 0)))))))
```

```
(DEFUN |CLAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82642 #2=#:G82640 #3=#:G82638 #4=#:G82639)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |CLAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |CLAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# (QUOTE T)) ((QUOTE T) #2#))
            |CLAGG-;any?;MAB;3|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |CLAGG-;any?;MAB;3|)
            (LETT #4# (QUOTE T) |CLAGG-;any?;MAB;3|)))))))
      (LETT #1# (CDR #1#) |CLAGG-;any?;MAB;3|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |CLAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82647 #2=#:G82645 #3=#:G82643 #4=#:G82644)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |CLAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;every?;MAB;4|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;every?;MAB;4|) NIL))
        (GO G191)))
      (SEQ
        (EXIT
         (PROGN
          (LETT #2# (SPADCALL |x| |f|) |CLAGG-;every?;MAB;4|)
          (COND
           (#4#
            (LETT #3#
             (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
             |CLAGG-;every?;MAB;4|))
           ((QUOTE T)
            (PROGN
             (LETT #3# #2# |CLAGG-;every?;MAB;4|)
             (LETT #4# (QUOTE T) |CLAGG-;every?;MAB;4|)))))))
       (LETT #1# (CDR #1#) |CLAGG-;every?;MAB;4|)
       (GO G190)
       G191
       (EXIT NIL))
      (COND (#4# #3#) ((QUOTE T) (QUOTE T)))))))

(DEFUN |CLAGG-;find;MAU;5| (|f| |c| |$|)
 (SPADCALL |f| (SPADCALL |c| (QREFELT |$| 9)) (QREFELT |$| 18)))

(DEFUN |CLAGG-;reduce;MAS;6| (|f| |x| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 21)))

(DEFUN |CLAGG-;reduce;MA2S;7| (|f| |x| |s| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s| (QREFELT |$| 23)))

(DEFUN |CLAGG-;remove;M2A;8| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 25))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;select;M2A;9| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 28))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;remove;S2A;10| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |CLAGG-;remove;S2A;10!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 31)))

(DEFUN |CLAGG-;remove;S2A;10!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 30)))
```

```
(DEFUN |CLAGG-;reduce;MA3S;11| (|f| |x| |s1| |s2| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s1| |s2| (QREFELT |$| 33)))

(DEFUN |CLAGG-;removeDuplicates;2A;12| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 35))
  (QREFELT |$| 26)))

(DEFUN |Collection&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|Collection&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |Collection&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 37) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 3)
      (PROGN
       (QSETREFV |$| 11 (CONS (|dispatchFunction| |CLAGG-;#;ANni;1|) |$|))
       (QSETREFV |$| 13 (CONS (|dispatchFunction| |CLAGG-;count;MANni;2|) |$|))
       (QSETREFV |$| 15 (CONS (|dispatchFunction| |CLAGG-;any?;MAB;3|) |$|))
       (QSETREFV |$| 16 (CONS (|dispatchFunction| |CLAGG-;every?;MAB;4|) |$|))
       (QSETREFV |$| 19 (CONS (|dispatchFunction| |CLAGG-;find;MAU;5|) |$|))
       (QSETREFV |$| 22 (CONS (|dispatchFunction| |CLAGG-;reduce;MAS;6|) |$|))
       (QSETREFV |$| 24 (CONS (|dispatchFunction| |CLAGG-;reduce;MA2S;7|) |$|))
       (QSETREFV |$| 27 (CONS (|dispatchFunction| |CLAGG-;remove;M2A;8|) |$|))
       (QSETREFV |$| 29 (CONS (|dispatchFunction| |CLAGG-;select;M2A;9|) |$|))
       (COND
        ((|testBitVector| |pv$| 2)
         (PROGN
          (QSETREFV |$| 32
           (CONS (|dispatchFunction| |CLAGG-;remove;S2A;10|) |$|))
          (QSETREFV |$| 34
           (CONS (|dispatchFunction| |CLAGG-;reduce;MA3S;11|) |$|))
          (QSETREFV |$| 36
           (CONS (|dispatchFunction| |CLAGG-;removeDuplicates;2A;12|)
                 |$|))))))))
    |$|))))

(MAKEPROP
 (QUOTE |Collection&|)
 (QUOTE |infovec|)
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 7)
   (0 . |parts|) (|NonNegativeInteger|) (5 . |#|) (|Mapping| 14 7)
   (10 . |count|) (|Boolean|) (16 . |any?|) (22 . |every?|)
   (|Union| 7 (QUOTE "failed")) (28 . |find|) (34 . |find|)
   (|Mapping| 7 7 7) (40 . |reduce|) (46 . |reduce|) (52 . |reduce|)
   (59 . |reduce|) (66 . |remove|) (72 . |construct|) (77 . |remove|)
   (83 . |select|) (89 . |select|) (95 . |=|) (101 . |remove|)
   (107 . |remove|) (113 . |reduce|) (121 . |reduce|)
   (129 . |removeDuplicates|) (134 . |removeDuplicates|)))
   (QUOTE #(|select| 139 |removeDuplicates| 145 |remove| 150 |reduce|
   162 |find| 183 |every?| 189 |count| 195 |any?| 201 |#| 207))
   (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
     (QUOTE #())
     (CONS
      (QUOTE #())
      (|makeByteWordVec2| 36
       (QUOTE (1 6 8 0 9 1 0 10 0 11 2 0 10 12 0 13 2 0 14 12 0 15 2 0 14
        12 0 16 2 8 17 12 0 18 2 0 17 12 0 19 2 8 7 20 0 21 2 0 7 20 0 22
        3 8 7 20 0 7 23 3 0 7 20 0 7 24 2 8 0 12 0 25 1 6 0 8 26 2 0 0 12
        0 27 2 8 0 12 0 28 2 0 0 12 0 29 2 7 14 0 0 30 2 6 0 12 0 31 2 0 0
        7 0 32 4 8 7 20 0 7 7 33 4 0 7 20 0 7 7 34 1 8 0 0 35 1 0 0 0 36 2
        0 0 12 0 29 1 0 0 0 36 2 0 0 7 0 32 2 0 0 12 0 27 4 0 7 20 0 7 7 34
        3 0 7 20 0 7 24 2 0 7 20 0 22 2 0 17 12 0 19 2 0 14 12 0 16 2 0 10
        12 0 13 2 0 14 12 0 15 1 0 10 0 11))))))
    (QUOTE |lookupComplete|)))
```

21.11 COMRING.lsp BOOTSTRAP

COMRING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **COMRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **COMRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*COMRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |CommutativeRing;AL| (QUOTE NIL))

(DEFUN |CommutativeRing| NIL
  (LET (#:G82892)
    (COND
      (|CommutativeRing;AL|)
      (T (SETQ |CommutativeRing;AL| (|CommutativeRing;|))))))

(DEFUN |CommutativeRing;| NIL
  (PROG (#1=#:G82890)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE (((|commutative| "*") T)))
              (QUOTE NIL)
              NIL))
           |CommutativeRing|)
        (SETELT #1# 0 (QUOTE (|CommutativeRing|)))))))

(MAKEPROP (QUOTE |CommutativeRing|) (QUOTE NILADIC) T)
```

21.12 DIFRING.lsp BOOTSTRAP

DIFRING needs **INT** which needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIFRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |DifferentialRing;AL| (QUOTE NIL))

(DEFUN |DifferentialRing| NIL
  (LET (#:G84565)
    (COND
      (|DifferentialRing;AL|)
      (T (SETQ |DifferentialRing;AL| (|DifferentialRing;|))))))

(DEFUN |DifferentialRing;| NIL
  (PROG (#1=#:G84563)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
                (((|differentiate| (|$| |$|)) T)
                 ((D (|$| |$|)) T)
                 ((|differentiate| (|$| |$| (|NonNegativeInteger|))) T)
                 ((D (|$| |$| (|NonNegativeInteger|))) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|)))
              NIL))
          |DifferentialRing|)
        (SETELT #1# 0 (QUOTE (|DifferentialRing|)))))))

(MAKEPROP (QUOTE |DifferentialRing|) (QUOTE NILADIC) T)
```

21.13 DIFRING-.lsp BOOTSTRAP

DIFRING- needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIFRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |DIFRING-;D;2S;1| (|r| |$|)
  (SPADCALL |r| (QREFELT |$| 7)))

(DEFUN |DIFRING-;differentiate;SNniS;2| (|r| |n| |$|)
  (PROG (|i|)
    (RETURN
      (SEQ
        (SEQ
          (LETT |i| 1 |DIFRING-;differentiate;SNniS;2|)
          G190
          (COND ((QSGREATERP |i| |n|) (GO G191)))
          (SEQ
            (EXIT
              (LETT |r|
                (SPADCALL |r| (QREFELT |$| 7))
                |DIFRING-;differentiate;SNniS;2|)))
          (LETT |i| (QSADD1 |i|) |DIFRING-;differentiate;SNniS;2|)
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT |r|)))))

(DEFUN |DIFRING-;D;SNniS;3| (|r| |n| |$|)
  (SPADCALL |r| |n| (QREFELT |$| 11)))

(DEFUN |DifferentialRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DifferentialRing&|))
        (LETT |dv$| (LIST (QUOTE |DifferentialRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 13) . #1#)
        (QSETREFV |$| 0 |dv$|)
```

```
          (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
          (|stuffDomainSlots| |$|)
          (QSETREFV |$| 6 |#1|)
          |$|))))

(MAKEPROP
  (QUOTE |DifferentialRing&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |differentiate|)
        |DIFRING-;D;2S;1|
        (|NonNegativeInteger|)
        |DIFRING-;differentiate;SNniS;2|
        (5 . |differentiate|)
        |DIFRING-;D;SNniS;3|))
    (QUOTE #(|differentiate| 11 D 17))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 12
            (QUOTE
             (1 6 0 0 7 2 6 0 0 9 11 2 0 0 0 9 10 2 0 0 0 9 12 1 0 0 0 8))))))
    (QUOTE |lookupComplete|)))
```

21.14 DIVRING.lsp BOOTSTRAP

DIVRING depends on **QFCAT** which eventually depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIVRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |DivisionRing;AL| (QUOTE NIL))

(DEFUN |DivisionRing| NIL
  (LET (#:G84035)
    (COND
      (|DivisionRing;AL|)
      (T (SETQ |DivisionRing;AL| (|DivisionRing;|))))))

(DEFUN |DivisionRing;| NIL
  (PROG (#1=#:G84033)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G84032))
              (LIST (QUOTE (|Fraction| (|Integer|)))))
            (|Join|
              (|EntireRing|)
              (|Algebra| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|**| (|$| |$| (|Integer|))) T)
                  ((|^| (|$| |$| (|Integer|))) T)
                  ((|inv| (|$| |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL)))
          |DivisionRing|)
        (SETELT #1# 0 (QUOTE (|DivisionRing|)))))))

(MAKEPROP (QUOTE |DivisionRing|) (QUOTE NILADIC) T)
```

21.15 DIVRING-.lsp BOOTSTRAP

DIVRING- depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIVRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |DIVRING-;^;SIS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |DIVRING-;**;SIS;2| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((SPADCALL |x| (QREFELT |$| 12))
      (COND
        ((|<| |n| 0) (|error| "division by zero"))
        ((QUOTE T) |x|)))
    ((|<| |n| 0)
      (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (|-| |n|) (QREFELT |$| 17)))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 17)))))

(DEFUN |DIVRING-;*;F2S;3| (|q| |x| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |q| (QREFELT |$| 20))
      (SPADCALL
        (SPADCALL (SPADCALL |q| (QREFELT |$| 21)) (QREFELT |$| 22))
        (QREFELT |$| 14))
      (QREFELT |$| 23))
    |x|
    (QREFELT |$| 24)))

(DEFUN |DivisionRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DivisionRing&|))
        (LETT |dv$| (LIST (QUOTE |DivisionRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 27) . #1#)
        (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    |$|))))

(MAKEPROP
 (QUOTE |DivisionRing&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE
   #(NIL NIL NIL NIL NIL NIL
    (|local| |#1|)
    (|Integer|)
    (0 . |**|)
    |DIVRING-;^;SIS;1|
    (6 . |One|)
    (|Boolean|)
    (10 . |zero?|)
    (15 . |Zero|)
    (19 . |inv|)
    (|PositiveInteger|)
    (|RepeatedSquaring| 6)
    (24 . |expt|)
    |DIVRING-;**;SIS;2|
    (|Fraction| 7)
    (30 . |numer|)
    (35 . |denom|)
    (40 . |coerce|)
    (45 . |*|)
    (51 . |*|)
    |DIVRING-;*;F2S;3|
    (|NonNegativeInteger|)))
  (QUOTE #(|^| 57 |**| 63 |*| 69))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
      (|makeByteWordVec2| 25
       (QUOTE
        (2 6 0 0 7 8 0 6 0 10 1 6 11 0 12 0 6 0 13 1 6 0 0 14 2 16 6
         6 15 17 1 19 7 0 20 1 19 7 0 21 1 6 0 7 22 2 6 0 7 0 23 2 6
         0 0 0 24 2 0 0 0 7 9 2 0 0 0 7 18 2 0 0 19 0 25))))))
  (QUOTE |lookupComplete|)))
```

21.16 ES.lsp BOOTSTRAP

ES depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ES.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |ExpressionSpace;AL| (QUOTE NIL))

(DEFUN |ExpressionSpace| NIL
 (LET (#:G82344)
  (COND
   (|ExpressionSpace;AL|)
   (T (SETQ |ExpressionSpace;AL| (|ExpressionSpace;|))))))

(DEFUN |ExpressionSpace;| NIL
 (PROG (#1=#:G82342)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (#2=#:G82340 #3=#:G82341))
       (LIST (QUOTE (|Kernel| |$|)) (QUOTE (|Kernel| |$|))))
      (|Join|
       (|OrderedSet|)
       (|RetractableTo| (QUOTE #2#))
       (|InnerEvalable| (QUOTE #3#) (QUOTE |$|))
       (|Evalable| (QUOTE |$|))
       (|mkCategory|
        (QUOTE |domain|)
        (QUOTE (
         ((|elt| (|$| (|BasicOperator|) |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$| |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) (|List| |$|))) T)
         ((|subst| (|$| |$| (|Equation| |$|))) T)
         ((|subst| (|$| |$| (|List| (|Equation| |$|)))) T)
         ((|subst| (|$| |$| (|List| (|Kernel| |$|)) (|List| |$|))) T)
         ((|box| (|$| |$|)) T)
 ((|box| (|$| (|List| |$|))) T)
 ((|paren| (|$| |$|)) T)
 ((|paren| (|$| (|List| |$|))) T)
 ((|distribute| (|$| |$|)) T)
 ((|distribute| (|$| |$| |$|)) T)
 ((|height| ((|NonNegativeInteger|) |$|)) T)
 ((|mainKernel| ((|Union| (|Kernel| |$|) "failed") |$|)) T)
 ((|kernels| ((|List| (|Kernel| |$|)) |$|)) T)
 ((|tower| ((|List| (|Kernel| |$|)) |$|)) T)
 ((|operators| ((|List| (|BasicOperator|)) |$|)) T)
 ((|operator| ((|BasicOperator|) (|BasicOperator|))) T)
 ((|belong?| ((|Boolean|) (|BasicOperator|))) T)
 ((|is?| ((|Boolean|) |$| (|BasicOperator|))) T)
 ((|is?| ((|Boolean|) |$| (|Symbol|))) T)
 ((|kernel| (|$| (|BasicOperator|) |$|)) T)
 ((|kernel| (|$| (|BasicOperator|) (|List| |$|))) T)
 ((|map| (|$| (|Mapping| |$| |$|) (|Kernel| |$|))) T)
 ((|freeOf?| ((|Boolean|) |$| |$|)) T)
 ((|freeOf?| ((|Boolean|) |$| (|Symbol|))) T)
 ((|eval| (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| |$|))))
   T)
 ((|eval|
  (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| (|List| |$|)))))
  T)
 ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| (|List| |$|)))) T)
 ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| |$|))) T)
 ((|eval|
  (|$| |$| (|List| (|BasicOperator|)) (|List| (|Mapping| |$| |$|))))
  T)
 ((|eval|
  (|$| |$| (|List| (|BasicOperator|))
    (|List| (|Mapping| |$| (|List| |$|)))))
  T)
 ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| (|List| |$|)))) T)
 ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| |$|))) T)
 ((|minPoly|
  ((|SparseUnivariatePolynomial| |$|) (|Kernel| |$|)))
  (|has| |$| (|Ring|)))
 ((|definingPolynomial| (|$| |$|)) (|has| |$| (|Ring|)))
 ((|even?|
  ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|))))
 ((|odd?|
  ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|)))))
NIL
(QUOTE (
 (|Boolean|)
```

```
          (|SparseUnivariatePolynomial| |$|)
          (|Kernel| |$|)
          (|BasicOperator|)
          (|List| (|BasicOperator|))
          (|List| (|Mapping| |$| (|List| |$|)))
          (|List| (|Mapping| |$| |$|))
          (|Symbol|)
          (|List| (|Symbol|))
          (|List| |$|)
          (|List| (|Kernel| |$|))
          (|NonNegativeInteger|)
          (|List| (|Equation| |$|))
          (|Equation| |$|)))
         NIL)))
      |ExpressionSpace|)
     (SETELT #1# 0 (QUOTE (|ExpressionSpace|)))))))

(MAKEPROP (QUOTE |ExpressionSpace|) (QUOTE NILADIC) T)
```

21.17 ES-.lsp BOOTSTRAP

ES- depends on **ES**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ES-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ES-;box;2S;1| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 16)))

(DEFUN |ES-;paren;2S;2| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 18)))

(DEFUN |ES-;belong?;BoB;3| (|op| |$|)
 (COND
  ((SPADCALL |op| (QREFELT |$| 13) (QREFELT |$| 21)) (QUOTE T))
  ((QUOTE T) (SPADCALL |op| (QREFELT |$| 14) (QREFELT |$| 21)))))

(DEFUN |ES-;listk| (|f| |$|)
 (SPADCALL (|ES-;allKernels| |f| |$|) (QREFELT |$| 25)))

(DEFUN |ES-;tower;SL;5| (|f| |$|)
 (SPADCALL (|ES-;listk| |f| |$|) (QREFELT |$| 26)))

(DEFUN |ES-;allk| (|l| |$|)
 (PROG (#1=#:G82361 |f| #2=#:G82362)
  (RETURN
   (SEQ
    (SPADCALL
     (ELT |$| 30)
     (PROGN
      (LETT #1# NIL |ES-;allk|)
      (SEQ
       (LETT |f| NIL |ES-;allk|)
       (LETT #2# |l| |ES-;allk|)
       G190
       (COND
        ((OR (ATOM #2#)
             (PROGN (LETT |f| (CAR #2#) |ES-;allk|) NIL))
         (GO G191)))
       (SEQ (EXIT (LETT #1# (CONS (|ES-;allKernels| |f| |$|) #1#) |ES-;allk|)))
```

```
        (LETT #2# (CDR #2#) |ES-;allk|) (GO G190) G191 (EXIT (NREVERSE0 #1#))))
      (SPADCALL NIL (QREFELT |$| 29))
      (QREFELT |$| 33))))))

(DEFUN |ES-;operators;SL;7| (|f| |$|)
 (PROG (#1=#:G82365 |k| #2=#:G82366)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |ES-;operators;SL;7|)
     (SEQ
      (LETT |k| NIL |ES-;operators;SL;7|)
      (LETT #2# (|ES-;listk| |f| |$|) |ES-;operators;SL;7|)
      G190
      (COND
       ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;operators;SL;7|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (LETT #1#
         (CONS (SPADCALL |k| (QREFELT |$| 35)) #1#) |ES-;operators;SL;7|)))
      (LETT #2# (CDR #2#) |ES-;operators;SL;7|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#))))))))

(DEFUN |ES-;height;SNni;8| (|f| |$|)
 (PROG (#1=#:G82371 |k| #2=#:G82372)
  (RETURN
   (SEQ
    (SPADCALL
     (ELT |$| 41)
     (PROGN
      (LETT #1# NIL |ES-;height;SNni;8|)
      (SEQ
       (LETT |k| NIL |ES-;height;SNni;8|)
       (LETT #2# (SPADCALL |f| (QREFELT |$| 38)) |ES-;height;SNni;8|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;height;SNni;8|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |k| (QREFELT |$| 40)) #1#) |ES-;height;SNni;8|)))
       (LETT #2# (CDR #2#) |ES-;height;SNni;8|)
```

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        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      0
     (QREFELT |$| 44))))))

(DEFUN |ES-;freeOf?;SSB;9| (|x| |s| |$|)
 (PROG (#1=#:G82377 |k| #2=#:G82378)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |s|
       (PROGN
        (LETT #1# NIL |ES-;freeOf?;SSB;9|)
        (SEQ
         (LETT |k| NIL |ES-;freeOf?;SSB;9|)
         (LETT #2# (|ES-;listk| |x| |$|) |ES-;freeOf?;SSB;9|)
         G190
         (COND
          ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;freeOf?;SSB;9|) NIL))
            (GO G191)))
         (SEQ
          (EXIT
           (LETT #1#
            (CONS (SPADCALL |k| (QREFELT |$| 46)) #1#) |ES-;freeOf?;SSB;9|)))
         (LETT #2# (CDR #2#) |ES-;freeOf?;SSB;9|)
         (GO G190)
         G191
         (EXIT (NREVERSE0 #1#))))
        (QREFELT |$| 48))
       (QUOTE NIL))
      ((QUOTE T) (QUOTE T)))))))

(DEFUN |ES-;distribute;2S;10| (|x| |$|)
 (PROG (#1=#:G82381 |k| #2=#:G82382)
  (RETURN
   (SEQ
    (|ES-;unwrap|
     (PROGN (LETT #1# NIL |ES-;distribute;2S;10|)
      (SEQ
       (LETT |k| NIL |ES-;distribute;2S;10|)
       (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;2S;10|)
       G190
       (COND
        ((OR
          (ATOM #2#)
          (PROGN (LETT |k| (CAR #2#) |ES-;distribute;2S;10|) NIL))
           (GO G191)))
       (SEQ
        (EXIT
         (COND
          ((SPADCALL |k| (QREFELT |$| 13) (QREFELT |$| 50))
            (LETT #1# (CONS |k| #1#) |ES-;distribute;2S;10|)))))
       (LETT #2# (CDR #2#) |ES-;distribute;2S;10|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |x|
     |$|)))))

(DEFUN |ES-;box;LS;11| (|l| |$|)
 (SPADCALL (QREFELT |$| 14) |l| (QREFELT |$| 52)))

(DEFUN |ES-;paren;LS;12| (|l| |$|)
 (SPADCALL (QREFELT |$| 13) |l| (QREFELT |$| 52)))

(DEFUN |ES-;freeOf?;2SB;13| (|x| |k| |$|)
 (COND
  ((SPADCALL
    (SPADCALL |k| (QREFELT |$| 56))
    (|ES-;listk| |x| |$|)
    (QREFELT |$| 57))
     (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |ES-;kernel;Bo2S;14| (|op| |arg| |$|)
 (SPADCALL |op| (LIST |arg|) (QREFELT |$| 59)))

(DEFUN |ES-;elt;Bo2S;15| (|op| |x| |$|)
 (SPADCALL |op| (LIST |x|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo3S;16| (|op| |x| |y| |$|)
 (SPADCALL |op| (LIST |x| |y|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo4S;17| (|op| |x| |y| |z| |$|)
 (SPADCALL |op| (LIST |x| |y| |z|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo5S;18| (|op| |x| |y| |z| |t| |$|)
 (SPADCALL |op| (LIST |x| |y| |z| |t|) (QREFELT |$| 52)))

(DEFUN |ES-;eval;SSMS;19| (|x| |s| |f| |$|)
 (SPADCALL |x| (LIST |s|) (LIST |f|) (QREFELT |$| 67)))
```

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(DEFUN |ES-;eval;SBoMS;20| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST (SPADCALL |s| (QREFELT |$| 69)))
  (LIST |f|)
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SSMS;21!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SBoMS;22| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SBoMS;22!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 75)))

(DEFUN |ES-;eval;SBoMS;22!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;subst;SES;23| (|x| |e| |$|)
 (SPADCALL |x| (LIST |e|) (QREFELT |$| 78)))

(DEFUN |ES-;eval;SLLS;24| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82403 |f| #2=#:G82404)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;24|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;24|)
       (LETT #2# |lf| |ES-;eval;SLLS;24|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;24|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (CONS (FUNCTION |ES-;eval;SLLS;24!0|) (VECTOR |f| |$|)) #1#)
          |ES-;eval;SLLS;24|)))
       (LETT #2# (CDR #2#) |ES-;eval;SLLS;24|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 75))))))

(DEFUN |ES-;eval;SLLS;24!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;25| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82407 |f| #2=#:G82408)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;25|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;25|)
       (LETT #2# |lf| |ES-;eval;SLLS;25|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;25|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (CONS (FUNCTION |ES-;eval;SLLS;25!0|) (VECTOR |f| |$|)) #1#)
          |ES-;eval;SLLS;25|)))
       (LETT #2# (CDR #2#) |ES-;eval;SLLS;25|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 67))))))

(DEFUN |ES-;eval;SLLS;25!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;26| (|x| |ls| |lf| |$|)
  (PROG (#1=#:G82412 |s| #2=#:G82413)
   (RETURN
    (SEQ
     (SPADCALL
      |x|
      (PROGN
       (LETT #1# NIL |ES-;eval;SLLS;26|)
       (SEQ
        (LETT |s| NIL |ES-;eval;SLLS;26|)
        (LETT #2# |ls| |ES-;eval;SLLS;26|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |s| (CAR #2#) |ES-;eval;SLLS;26|) NIL))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |s| (QREFELT |$| 69)) #1#) |ES-;eval;SLLS;26|)))
        (LETT #2# (CDR #2#) |ES-;eval;SLLS;26|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      |lf|
      (QREFELT |$| 67))))))

(DEFUN |ES-;map;MKS;27| (|fn| |k| |$|)
 (PROG (#1=#:G82428 |x| #2=#:G82429 |l|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL
       (LETT |l|
        (PROGN
         (LETT #1# NIL |ES-;map;MKS;27|)
         (SEQ
          (LETT |x| NIL |ES-;map;MKS;27|)
          (LETT #2# (SPADCALL |k| (QREFELT |$| 85)) |ES-;map;MKS;27|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |x| (CAR #2#) |ES-;map;MKS;27|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1# (CONS (SPADCALL |x| |fn|) #1#) |ES-;map;MKS;27|)))
          (LETT #2# (CDR #2#) |ES-;map;MKS;27|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        |ES-;map;MKS;27|)
        (SPADCALL |k| (QREFELT |$| 85)) (QREFELT |$| 86))
       (SPADCALL |k| (QREFELT |$| 87)))
     ((QUOTE T)
       (SPADCALL (SPADCALL |k| (QREFELT |$| 35)) |l| (QREFELT |$| 52))))))))

(DEFUN |ES-;operator;2Bo;28| (|op| |$|)
 (COND
  ((SPADCALL |op| (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 13))
  ((SPADCALL |op| (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 14))
  ((QUOTE T) (|error| "Unknown operator"))))

(DEFUN |ES-;mainKernel;SU;29| (|x| |$|)
 (PROG (|l| |kk| #1=#:G82445 |n| |k|)
  (RETURN
   (SEQ
    (COND
     ((NULL (LETT |l| (SPADCALL |x| (QREFELT |$| 38)) |ES-;mainKernel;SU;29|))
       (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |n|
        (SPADCALL
         (LETT |k| (|SPADfirst| |l|) |ES-;mainKernel;SU;29|) (QREFELT |$| 40))
        |ES-;mainKernel;SU;29|)
       (SEQ
        (LETT |kk| NIL |ES-;mainKernel;SU;29|)
        (LETT #1# (CDR |l|) |ES-;mainKernel;SU;29|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |kk| (CAR #1#) |ES-;mainKernel;SU;29|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((|<| |n| (SPADCALL |kk| (QREFELT |$| 40)))
            (SEQ
             (LETT |n| (SPADCALL |kk| (QREFELT |$| 40)) |ES-;mainKernel;SU;29|)
             (EXIT (LETT |k| |kk| |ES-;mainKernel;SU;29|)))))))
        (LETT #1# (CDR #1#) |ES-;mainKernel;SU;29|) (GO G190) G191 (EXIT NIL))
        (EXIT (CONS 0 |k|)))))))))

  (DEFUN |ES-;allKernels| (|f| |$|)
   (PROG (|l| |k| #1=#:G82458 |u| |s0| |n| |arg| |t| |s|)
    (RETURN
     (SEQ
      (LETT |s|
       (SPADCALL
        (LETT |l|
         (SPADCALL |f| (QREFELT |$| 38))
         |ES-;allKernels|)
        (QREFELT |$| 29))
       |ES-;allKernels|)
      (SEQ
       (LETT |k| NIL |ES-;allKernels|)
       (LETT #1# |l| |ES-;allKernels|)
       G190
       (COND
        ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;allKernels|) NIL))
         (GO G191)))
       (SEQ
        (LETT |t|
         (SEQ
          (LETT |u|
           (SPADCALL
            (SPADCALL |k| (QREFELT |$| 35))
             "%dummyVar"
             (QREFELT |$| 94))
            |ES-;allKernels|)
          (EXIT
           (COND
            ((QEQCAR |u| 0)
             (SEQ
              (LETT |arg| (SPADCALL |k| (QREFELT |$| 85)) |ES-;allKernels|)
              (LETT |s0|
               (SPADCALL
                (SPADCALL
                 (SPADCALL |arg| (QREFELT |$| 95))
                 (QREFELT |$| 56))
                (|ES-;allKernels| (|SPADfirst| |arg|) |$|)
                (QREFELT |$| 96))
               |ES-;allKernels|)
              (LETT |arg| (CDR (CDR |arg|)) |ES-;allKernels|)
              (LETT |n| (QCDR |u|) |ES-;allKernels|)
              (COND ((|<| 1 |n|) (LETT |arg| (CDR |arg|) |ES-;allKernels|)))
              (EXIT (SPADCALL |s0| (|ES-;allk| |arg| |$|) (QREFELT |$| 30)))))
           ((QUOTE T) (|ES-;allk| (SPADCALL |k| (QREFELT |$| 85)) |$|)))))
         |ES-;allKernels|)
        (EXIT
         (LETT |s| (SPADCALL |s| |t| (QREFELT |$| 30)) |ES-;allKernels|)))
       (LETT #1# (CDR #1#) |ES-;allKernels|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |s|)))))

(DEFUN |ES-;kernel;BoLS;31| (|op| |args| |$|)
 (COND
  ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
  ((QUOTE T) (|ES-;okkernel| |op| |args| |$|))))

(DEFUN |ES-;okkernel| (|op| |l| |$|)
 (PROG (#1=#:G82465 |f| #2=#:G82466)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL |op| |l|
      (|+| 1
       (SPADCALL
        (ELT |$| 41)
        (PROGN
         (LETT #1# NIL |ES-;okkernel|)
         (SEQ
          (LETT |f| NIL |ES-;okkernel|)
          (LETT #2# |l| |ES-;okkernel|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;okkernel|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1#
             (CONS (SPADCALL |f| (QREFELT |$| 99)) #1#) |ES-;okkernel|)))
          (LETT #2# (CDR #2#) |ES-;okkernel|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        0
        (QREFELT |$| 44)))
      (QREFELT |$| 100))
     (QREFELT |$| 87))))))
```

```
(DEFUN |ES-;elt;BoLS;33| (|op| |args| |$|)
 (PROG (|u| #1=#:G82482 |v|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |u| (SPADCALL |op| (QREFELT |$| 102)) |ES-;elt;BoLS;33|)
         (EXIT
          (COND
           ((QEQCAR |u| 0)
            (COND
             ((NULL (EQL (LENGTH |args|) (QCDR |u|)))
              (PROGN
               (LETT #1#
                (|error| "Wrong number of arguments")
                |ES-;elt;BoLS;33|)
                (GO #1#))))))))
        (LETT |v| (SPADCALL |op| |args| (QREFELT |$| 105)) |ES-;elt;BoLS;33|)
        (EXIT
         (COND
          ((QEQCAR |v| 0) (QCDR |v|))
          ((QUOTE T) (|ES-;okkernel| |op| |args| |$|))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |ES-;retract;SK;34| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retract;SK;34|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
          |f|
          (QREFELT |$| 108))))
        (|error| "not a kernel"))
      ((QUOTE T) (QCDR |k|))))))))
```

```
(DEFUN |ES-;retractIfCan;SU;35| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retractIfCan;SU;35|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
          |f|
          (QREFELT |$| 108))))
        (CONS 1 "failed"))
      ((QUOTE T) |k|)))))))

(DEFUN |ES-;is?;SSB;36| (|f| |s| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SSB;36|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |s| (QREFELT |$| 112)))))))))

(DEFUN |ES-;is?;SBoB;37| (|f| |op| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SBoB;37|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |op| (QREFELT |$| 50)))))))))

(DEFUN |ES-;unwrap| (|l| |x| |$|)
 (PROG (|k| #1=#:G82507)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| NIL |ES-;unwrap|)
     (LETT #1# (NREVERSE |l|) |ES-;unwrap|)
     G190
     (COND
        ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;unwrap|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT |x|
          (SPADCALL |x| |k|
           (|SPADfirst| (SPADCALL |k| (QREFELT |$| 85)))
           (QREFELT |$| 115))
          |ES-;unwrap|)))
       (LETT #1# (CDR #1#) |ES-;unwrap|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |x|)))))

(DEFUN |ES-;distribute;3S;39| (|x| |y| |$|)
 (PROG (|ky| #1=#:G82512 |k| #2=#:G82513)
  (RETURN
   (SEQ
    (LETT |ky| (SPADCALL |y| (QREFELT |$| 56)) |ES-;distribute;3S;39|)
    (EXIT
     (|ES-;unwrap|
      (PROGN
       (LETT #1# NIL |ES-;distribute;3S;39|)
       (SEQ
        (LETT |k| NIL |ES-;distribute;3S;39|)
        (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;3S;39|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |k| (CAR #2#) |ES-;distribute;3S;39|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((COND
             ((SPADCALL |k|
               (SPADCALL "%paren" (QREFELT |$| 9))
                (QREFELT |$| 112))
               (SPADCALL |ky|
                (|ES-;listk| (SPADCALL |k| (QREFELT |$| 87)) |$|)
                (QREFELT |$| 57)))
             ((QUOTE T) (QUOTE NIL)))
              (LETT #1# (CONS |k| #1#) |ES-;distribute;3S;39|)))))
        (LETT #2# (CDR #2#) |ES-;distribute;3S;39|)
```

```
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        |x|
        |$|))))))

(DEFUN |ES-;eval;SLS;40| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;eval;SLS;40|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 117)))))))

(DEFUN |ES-;subst;SLS;41| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;subst;SLS;41|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 119)))))))

(DEFUN |ES-;mkKerLists| (|leq| |$|)
 (PROG (|eq| #1=#:G82530 |k| |lk| |lv|)
  (RETURN
   (SEQ
    (LETT |lk| NIL |ES-;mkKerLists|)
    (LETT |lv| NIL |ES-;mkKerLists|)
    (SEQ
     (LETT |eq| NIL |ES-;mkKerLists|)
     (LETT #1# |leq| |ES-;mkKerLists|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |eq| (CAR #1#) |ES-;mkKerLists|) NIL))
        (GO G191)))
     (SEQ
      (LETT |k|
       (SPADCALL (SPADCALL |eq| (QREFELT |$| 122)) (QREFELT |$| 111))
       |ES-;mkKerLists|)
      (EXIT
       (COND
        ((QEQCAR |k| 1) (|error| "left hand side must be a single kernel"))
        ((NULL (SPADCALL (QCDR |k|) |lk| (QREFELT |$| 57)))
         (SEQ
          (LETT |lk| (CONS (QCDR |k|) |lk|) |ES-;mkKerLists|)
          (EXIT
           (LETT |lv|
            (CONS (SPADCALL |eq| (QREFELT |$| 123)) |lv|)
           |ES-;mkKerLists|)))))))
      (LETT #1# (CDR #1#) |ES-;mkKerLists|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (CONS |lk| |lv|))))))

(DEFUN |ES-;even?;SB;43| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 125) |$|))

(DEFUN |ES-;odd?;SB;44| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 127) |$|))

(DEFUN |ES-;intpred?| (|x| |pred?| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |x| (QREFELT |$| 130)) |ES-;intpred?|)
    (EXIT
     (COND
      ((QEQCAR |u| 0) (SPADCALL (QCDR |u|) |pred?|))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |ExpressionSpace&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ExpressionSpace&|))
    (LETT |dv$| (LIST (QUOTE |ExpressionSpace&|) |DV$1|) . #1#)
    (LETT |$| (GETREFV 131) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#1| (QUOTE (|RetractableTo| (|Integer|))))
        (|HasCategory| |#1| (QUOTE (|Ring|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 13
     (SPADCALL (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (QSETREFV |$| 14
     (SPADCALL (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 126 (CONS (|dispatchFunction| |ES-;even?;SB;43|) |$|))
       (QSETREFV |$| 128 (CONS (|dispatchFunction| |ES-;odd?;SB;44|) |$|)))))
    |$|))))

(MAKEPROP
 (QUOTE |ExpressionSpace&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL NIL (|local| |#1|) (|String|) (|Symbol|)
   (0 . |coerce|) (|BasicOperator|) (|CommonOperators|) (5 . |operator|)
   (QUOTE |oppren|) (QUOTE |opbox|) (|List| |$|) (10 . |box|)
   |ES-;box;2S;1| (15 . |paren|) |ES-;paren;2S;2| (|Boolean|)
   (20 . |=|) |ES-;belong?;BoB;3| (|List| 34) (|Set| 34)
   (26 . |parts|) (31 . |sort!|) (|List| 55) |ES-;tower;SL;5|
   (36 . |brace|) (41 . |union|) (|Mapping| 24 24 24) (|List| 24)
   (47 . |reduce|) (|Kernel| 6) (54 . |operator|) (|List| 10)
   |ES-;operators;SL;7| (59 . |kernels|) (|NonNegativeInteger|)
   (64 . |height|) (69 . |max|) (|Mapping| 39 39 39) (|List| 39)
   (75 . |reduce|) |ES-;height;SNni;8| (82 . |name|) (|List| 8)
   (87 . |member?|) |ES-;freeOf?;SSB;9| (93 . |is?|)
   |ES-;distribute;2S;10| (99 . |elt|) |ES-;box;LS;11|
   |ES-;paren;LS;12| (|Kernel| |$|) (105 . |retract|)
   (110 . |member?|) |ES-;freeOf?;2SB;13| (116 . |kernel|)
   |ES-;kernel;Bo2S;14| |ES-;elt;Bo2S;15| |ES-;elt;Bo3S;16|
   |ES-;elt;Bo4S;17| |ES-;elt;Bo5S;18| (|Mapping| |$| 15)
   (|List| 65) (122 . |eval|) |ES-;eval;SSMS;19| (129 . |name|)
   |ES-;eval;SBoMS;20| (|List| 6) (134 . |first|)
   (|Mapping| |$| |$|) |ES-;eval;SSMS;21| (139 . |eval|)
   |ES-;eval;SBoMS;22| (|List| 79) (146 . |subst|) (|Equation| |$|)
   |ES-;subst;SES;23| (|List| 73) |ES-;eval;SLLS;24|
   |ES-;eval;SLLS;25| |ES-;eval;SLLS;26| (152 . |argument|)
   (157 . |=|) (163 . |coerce|) |ES-;map;MKS;27| (168 . |is?|)
   |ES-;operator;2Bo;28| (|Union| 55 (QUOTE "failed"))
   |ES-;mainKernel;SU;29| (|Union| (|None|) (QUOTE "failed"))
   (174 . |property|) (180 . |second|) (185 . |remove!|)
   (191 . |belong?|) |ES-;kernel;BoLS;31| (196 . |height|)
   (201 . |kernel|) (|Union| 39 (QUOTE "failed")) (208 . |arity|)
   (|Union| 6 (QUOTE "failed")) (|BasicOperatorFunctions1| 6)
   (213 . |evaluate|) |ES-;elt;BoLS;33| (219 . |mainKernel|)
   (224 . |=|) |ES-;retract;SK;34| |ES-;retractIfCan;SU;35|
   (230 . |retractIfCan|) (235 . |is?|) |ES-;is?;SSB;36|
   |ES-;is?;SBoB;37| (241 . |eval|) |ES-;distribute;3S;39|
   (248 . |eval|) |ES-;eval;SLS;40| (255 . |subst|)
   |ES-;subst;SLS;41| (|Equation| 6) (262 . |lhs|) (267 . |rhs|)
   (|Integer|) (272 . |even?|) (277 . |even?|) (282 . |odd?|)
 (287 . |odd?|) (|Union| 124 (QUOTE "failed")) (292 . |retractIfCan|)))
(QUOTE #(
 |tower| 297 |subst| 302 |retractIfCan| 314 |retract| 319 |paren| 324
 |operators| 334 |operator| 339 |odd?| 344 |map| 349 |mainKernel| 355
 |kernel| 360 |is?| 372 |height| 384 |freeOf?| 389 |even?| 401 |eval| 406
 |elt| 461 |distribute| 497 |box| 508 |belong?| 518))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 130 (QUOTE
    (1 8 0 7 9 1 11 10 8 12 1 6 0 15 16 1 6 0 15 18 2 10 20 0 0 21 1 24
     23 0 25 1 23 0 0 26 1 24 0 23 29 2 24 0 0 0 30 3 32 24 31 0 24 33
     1 34 10 0 35 1 6 27 0 38 1 34 39 0 40 2 39 0 0 0 41 3 43 39 42 0
     39 44 1 34 8 0 46 2 47 20 8 0 48 2 34 20 0 10 50 2 6 0 10 15 52 1
     6 55 0 56 2 23 20 34 0 57 2 6 0 10 15 59 3 6 0 0 47 66 67 1 10 8
     0 69 1 71 6 0 72 3 6 0 0 36 66 75 2 6 0 0 77 78 1 34 71 0 85 2 71
     20 0 0 86 1 6 0 55 87 2 10 20 0 8 89 2 10 93 0 7 94 1 71 6 0 95
     2 24 0 34 0 96 1 6 20 10 97 1 6 39 0 99 3 34 0 10 71 39 100 1 10
     101 0 102 2 104 103 10 71 105 1 6 91 0 107 2 6 20 0 0 108 1 6 91
     0 111 2 34 20 0 8 112 3 6 0 0 55 0 115 3 6 0 0 27 15 117 3 6 0 0
     27 15 119 1 121 6 0 122 1 121 6 0 123 1 124 20 0 125 1 0 20 0 126
     1 124 20 0 127 1 0 20 0 128 1 6 129 0 130 1 0 27 0 28 2 0 0 0 77
     120 2 0 0 0 79 80 1 0 91 0 110 1 0 55 0 109 1 0 0 0 19 1 0 0 15
     54 1 0 36 0 37 1 0 10 10 90 1 0 20 0 128 2 0 0 73 55 88 1 0 91 0
     92 2 0 0 10 15 98 2 0 0 10 0 60 2 0 20 0 8 113 2 0 20 0 10 114 1
     0 39 0 45 2 0 20 0 8 49 2 0 20 0 0 58 1 0 20 0 126 3 0 0 0 10 73
     76 3 0 0 0 36 66 84 3 0 0 0 10 65 70 3 0 0 0 36 81 82 3 0 0 0 8
     65 68 3 0 0 0 8 73 74 3 0 0 0 47 81 83 2 0 0 0 77 118 2 0 0 10
     15 106 5 0 0 10 0 0 0 0 64 3 0 0 10 0 0 62 4 0 0 10 0 0 0 63 2
     0 0 10 0 61 2 0 0 0 0 116 1 0 0 0 51 1 0 0 15 53 1 0 0 0 17 1 0
     20 10 22))))))
(QUOTE |lookupComplete|)))
```

21.18 EUCDOM.lsp BOOTSTRAP

EUCDOM depends on **INT** which depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.18.1 The Lisp Implementation

EUCDOM;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

⟨*EUCDOM;VersionCheck*⟩≡

```
(|/VERSIONCHECK| 2)
```

The Domain Cache Variable

We create a variable which is formed by concatenating the string "**;AL**" to the domain name forming, in this case, "**EuclideanDomain;AL**". The variable has the initial value at load time of a list of one element, **NIL**. This list is a data structure that will be modified to hold an executable function. This function is created the first time the domain is used which it replaces the **NIL**.

⟨*EuclideanDomain;AL*⟩≡

```
(SETQ |EuclideanDomain;AL| (QUOTE NIL))
```

The Domain Function

When you call a domain the code is pretty simple at the top level. This code will check to see if this domain has ever been used. It does this by checking the value of the cached domain variable (which is the domain name **EuclideanDomain** concatenated with the string "**;AL**" to form the cache variable name which is **EuclideanDomain;AL**).

If this value is NIL we have never executed this function before. If it is not NIL we have executed this function before and we need only return the cached function which was stored in the cache variable.

If this is the first time this function is called, the cache variable is NIL and we execute the other branch of the conditional. This calls a function which

1. creates a procedure
2. returns the procedure as a value.

This procedure replaces the cached variable **EuclideanDomain;AL** value so it will be non-NIL the second time this domain is used. Thus the work of building the domain only happens once.

If this function has never been called before we call the

⟨*EuclideanDomain*⟩≡

```
(DEFUN |EuclideanDomain| NIL
  (LET (#:G83585)
    (COND
      (|EuclideanDomain;AL|)
      (T (SETQ |EuclideanDomain;AL| (|EuclideanDomain;|))))))
```

The First Call Domain Function

⟨*EuclideanDomain;*⟩≡

```
(DEFUN |EuclideanDomain;| NIL
  (PROG (#1=#:G83583)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|PrincipalIdealDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|sizeLess?| ((|Boolean|) |$| |$|)) T)
                ((|euclideanSize| ((|NonNegativeInteger|) |$|)) T)
                ((|divide|
                  ((|Record|
                    (|:| |quotient| |$|)
                    (|:| |remainder| |$|))
                  |$| |$|)) T)
                ((|quo| (|$| |$| |$|)) T)
                ((|rem| (|$| |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Record|
                    (|:| |coef1| |$|)
                    (|:| |coef2| |$|)
                    (|:| |generator| |$|))
                  |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Union|
                      (|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
                      "failed")
                    |$| |$| |$|)) T)
                ((|multiEuclidean|
                  ((|Union|
                      (|List| |$|)
                      "failed")
                   (|List| |$|) |$|)) T)))
              NIL
              (QUOTE ((|List| |$|) (|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |EuclideanDomain|)
        (SETELT #1# 0 (QUOTE (|EuclideanDomain|)))))))
```

EUCDOM;MAKEPROP

⟨*EUCDOM;MAKEPROP*⟩≡

```
(MAKEPROP (QUOTE |EuclideanDomain|) (QUOTE NILADIC) T)
```

⟨*EUCDOM.lsp BOOTSTRAP*⟩≡
  ⟨*EUCDOM;VersionCheck*⟩
  ⟨*EuclideanDomain;AL*⟩
  ⟨*EuclideanDomain*⟩
  ⟨*EuclideanDomain;*⟩
  ⟨*EUCDOM;MAKEPROP*⟩

21.19 EUCDOM-.lsp BOOTSTRAP

EUCDOM- depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.19.1 The Lisp Implementation

EUCDOM-;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

⟨*EUCDOM-;VersionCheck*⟩≡

```
(|/VERSIONCHECK| 2)
```

EUCDOM-;sizeLess?;2SB;1

⟨EUCDOM-;sizeLess?;2SB;1⟩≡

```
(DEFUN |EUCDOM-;sizeLess?;2SB;1| (|x| |y| $)
 (COND
  ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
  ((SPADCALL |x| (QREFELT $ 8)) (QUOTE T))
  ((QUOTE T)
   (< (SPADCALL |x| (QREFELT $ 10)) (SPADCALL |y| (QREFELT $ 10))))))
```

EUCDOM-;quo;3S;2

⟨EUCDOM-;quo;3S;2⟩≡

```
(DEFUN |EUCDOM-;quo;3S;2| (|x| |y| $)
 (QCAR (SPADCALL |x| |y| (QREFELT $ 13))))
```

EUCDOM-;rem;3S;3

⟨EUCDOM-;rem;3S;3⟩≡

```
(DEFUN |EUCDOM-;rem;3S;3| (|x| |y| $)
 (QCDR (SPADCALL |x| |y| (QREFELT $ 13))))
```

EUCDOM-;exquo;2SU;4

⟨EUCDOM-;exquo;2SU;4⟩≡

```
(DEFUN |EUCDOM-;exquo;2SU;4| (|x| |y| $)
 (PROG (|qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT $ 8)) (CONS 0 (|spadConstant| $ 16)))
     ((SPADCALL |y| (QREFELT $ 8)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |qr|
        (SPADCALL |x| |y| (QREFELT $ 13))
        |EUCDOM-;exquo;2SU;4|)
       (EXIT
        (COND
         ((SPADCALL (QCDR |qr|) (QREFELT $ 8)) (CONS 0 (QCAR |qr|)))
         ((QUOTE T) (CONS 1 "failed")))))))))))
```

EUCDOM-;gcd;3S;5

⟨*EUCDOM-;gcd;3S;5*⟩≡

```
(DEFUN |EUCDOM-;gcd;3S;5| (|x| |y| $)
 (PROG (|#G13| |#G14|)
  (RETURN
   (SEQ
    (LETT |x| (SPADCALL |x| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (SEQ G190
     (COND
      ((NULL
       (COND
        ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (PROGN
       (LETT |#G13| |y| |EUCDOM-;gcd;3S;5|)
       (LETT |#G14| (SPADCALL |x| |y| (QREFELT $ 20)) |EUCDOM-;gcd;3S;5|)
       (LETT |x| |#G13| |EUCDOM-;gcd;3S;5|)
       (LETT |y| |#G14| |EUCDOM-;gcd;3S;5|))
      (EXIT
       (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)))
     NIL
     (GO G190)
    G191
   (EXIT NIL))
  (EXIT |x|)))))
```

EUCDOM-;unitNormalizeIdealElt

⟨*EUCDOM-;unitNormalizeIdealElt*⟩≡

```
(DEFUN |EUCDOM-;unitNormalizeIdealElt| (|s| $)
 (PROG (|#G16| |u| |c| |a|)
  (RETURN
   (SEQ
    (PROGN
     (LETT |#G16|
       (SPADCALL (QVELT |s| 2) (QREFELT $ 23))
       |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |u| (QVELT |#G16| 0) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |c| (QVELT |#G16| 1) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |a| (QVELT |#G16| 2) |EUCDOM-;unitNormalizeIdealElt|)
     |#G16|)
    (EXIT
     (COND
      ((SPADCALL |a| (|spadConstant| $ 24) (QREFELT $ 25)) |s|)
      ((QUOTE T)
       (VECTOR
        (SPADCALL |a| (QVELT |s| 0) (QREFELT $ 26))
        (SPADCALL |a| (QVELT |s| 1) (QREFELT $ 26))
        |c|))))))))
```

EUCDOM-;extendedEuclidean;2SR;7

⟨*EUCDOM-;extendedEuclidean;2SR;7*⟩≡

```
(DEFUN |EUCDOM-;extendedEuclidean;2SR;7| (|x| |y| $)
 (PROG (|s3| |s2| |qr| |s1|)
  (RETURN
   (SEQ
    (LETT |s1|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 24) (|spadConstant| $ 16) |x|)
       $)
     |EUCDOM-;extendedEuclidean;2SR;7|)
    (LETT |s2|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 16) (|spadConstant| $ 24) |y|)
      $)
      |EUCDOM-;extendedEuclidean;2SR;7|)
    (EXIT
     (COND
      ((SPADCALL |y| (QREFELT $ 8)) |s1|)
      ((SPADCALL |x| (QREFELT $ 8)) |s2|)
      ((QUOTE T)
       (SEQ
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL (QVELT |s2| 2) (QREFELT $ 8)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |qr|
           (SPADCALL (QVELT |s1| 2) (QVELT |s2| 2) (QREFELT $ 13))
           |EUCDOM-;extendedEuclidean;2SR;7|)
          (LETT |s3|
           (VECTOR
            (SPADCALL (QVELT |s1| 0)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 0) (QREFELT $ 26))
             (QREFELT $ 27))
            (SPADCALL (QVELT |s1| 1)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 1) (QREFELT $ 26))
             (QREFELT $ 27))
            (QCDR |qr|))
           |EUCDOM-;extendedEuclidean;2SR;7|)
          (LETT |s1| |s2| |EUCDOM-;extendedEuclidean;2SR;7|)
    (EXIT
     (LETT |s2|
      (|EUCDOM-;unitNormalizeIdealElt| |s3| $)
      |EUCDOM-;extendedEuclidean;2SR;7|)))
   NIL
   (GO G190)
   G191
   (EXIT NIL))
  (COND
   ((NULL (SPADCALL (QVELT |s1| 0) (QREFELT $ 8)))
     (COND
      ((NULL (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 28)))
       (SEQ
        (LETT |qr|
         (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 13))
         |EUCDOM-;extendedEuclidean;2SR;7|)
        (QSETVELT |s1| 0 (QCDR |qr|))
        (QSETVELT |s1| 1
         (SPADCALL (QVELT |s1| 1)
          (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26)) (QREFELT $ 29)))
        (EXIT
         (LETT |s1|
          (|EUCDOM-;unitNormalizeIdealElt| |s1| $)
          |EUCDOM-;extendedEuclidean;2SR;7|)))))))
  (EXIT |s1|)))))))))
```

EUCDOM-;extendedEuclidean;3SU;8

⟨EUCDOM-;extendedEuclidean;3SU;8⟩≡

```
(DEFUN |EUCDOM-;extendedEuclidean;3SU;8| (|x| |y| |z| $)
 (PROG (|s| |w| |qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (QREFELT $ 8))
       (CONS 0 (CONS (|spadConstant| $ 16) (|spadConstant| $ 16))))
     ((QUOTE T)
       (SEQ
        (LETT |s|
         (SPADCALL |x| |y| (QREFELT $ 32))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (LETT |w|
         (SPADCALL |z| (QVELT |s| 2) (QREFELT $ 33))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (EXIT
         (COND
          ((QEQCAR |w| 1) (CONS 1 "failed"))
          ((SPADCALL |y| (QREFELT $ 8))
            (CONS 0
             (CONS (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26)))))
          ((QUOTE T)
           (SEQ
            (LETT |qr|
             (SPADCALL
              (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              |y|
              (QREFELT $ 13))
             |EUCDOM-;extendedEuclidean;3SU;8|)
            (EXIT
             (CONS 0
              (CONS (QCDR |qr|)
               (SPADCALL
                (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26))
                (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26))
                (QREFELT $ 29)))))))))))))))))
```

EUCDOM-;principalIdeal;LR;9

⟨*EUCDOM-;principalIdeal;LR;9*⟩≡

```
(DEFUN |EUCDOM-;principalIdeal;LR;9| (|l| $)
 (PROG (|uca| |v| |u| #0=#:G1497 |vv| #1=#:G1498)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |l| NIL (QREFELT $ 38))
       (|error| "empty list passed to principalIdeal"))
     ((SPADCALL (CDR |l|) NIL (QREFELT $ 38))
       (SEQ
        (LETT |uca|
         (SPADCALL (|SPADfirst| |l|) (QREFELT $ 23))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |uca| 0)) (QVELT |uca| 1)))))
     ((SPADCALL (CDR (CDR |l|)) NIL (QREFELT $ 38))
       (SEQ
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|)
          (SPADCALL |l| (QREFELT $ 39)) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |u| 0) (QVELT |u| 1)) (QVELT |u| 2)))))
     ((QUOTE T)
       (SEQ
        (LETT |v|
         (SPADCALL (CDR |l|) (QREFELT $ 42))
         |EUCDOM-;principalIdeal;LR;9|)
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|) (QCDR |v|) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT
         (CONS
          (CONS (QVELT |u| 0)
           (PROGN
            (LETT #0# NIL |EUCDOM-;principalIdeal;LR;9|)
            (SEQ
             (LETT |vv| NIL |EUCDOM-;principalIdeal;LR;9|)
             (LETT #1# (QCAR |v|) |EUCDOM-;principalIdeal;LR;9|)
             G190
             (COND
              ((OR (ATOM #1#)
                   (PROGN
                    (LETT |vv| (CAR #1#) |EUCDOM-;principalIdeal;LR;9|) NIL))
                (GO G191)))
             (SEQ
    (EXIT
     (LETT #0#
      (CONS (SPADCALL (QVELT |u| 1) |vv| (QREFELT $ 26))
         #0#)
      |EUCDOM-;principalIdeal;LR;9|)))
   (LETT #1# (CDR #1#)
    |EUCDOM-;principalIdeal;LR;9|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #0#)))))
(QVELT |u| 2))))))))))
```

EUCDOM-;expressIdealMember;LSU;10

⟨EUCDOM-;expressIdealMember;LSU;10⟩≡

```
(DEFUN |EUCDOM-;expressIdealMember;LSU;10| (|l| |z| $)
 (PROG (#0=#:G1513 #1=#:G1514 |pid| |q| #2=#:G1515 |v| #3=#:G1516)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (|spadConstant| $ 16) (QREFELT $ 25))
      (CONS 0
       (PROGN
        (LETT #0# NIL |EUCDOM-;expressIdealMember;LSU;10|)
        (SEQ
         (LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
         (LETT #1# |l| |EUCDOM-;expressIdealMember;LSU;10|)
         G190
         (COND
          ((OR (ATOM #1#)
               (PROGN
                (LETT |v| (CAR #1#) |EUCDOM-;expressIdealMember;LSU;10|) NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT #0#
            (CONS (|spadConstant| $ 16) #0#)
            |EUCDOM-;expressIdealMember;LSU;10|)))
         (LETT #1# (CDR #1#) |EUCDOM-;expressIdealMember;LSU;10|)
         (GO G190)
         G191
        (EXIT (NREVERSE0 #0#))))))
     ((QUOTE T)
      (SEQ
       (LETT |pid|
        (SPADCALL |l| (QREFELT $ 42))
        |EUCDOM-;expressIdealMember;LSU;10|)
       (LETT |q|
        (SPADCALL |z| (QCDR |pid|) (QREFELT $ 33))
        |EUCDOM-;expressIdealMember;LSU;10|)
       (EXIT
        (COND
         ((QEQCAR |q| 1) (CONS 1 "failed"))
         ((QUOTE T)
          (CONS 0
           (PROGN
            (LETT #2# NIL |EUCDOM-;expressIdealMember;LSU;10|)
            (SEQ
(LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
(LETT #3# (QCAR |pid|) |EUCDOM-;expressIdealMember;LSU;10|)
G190
(COND
 ((OR (ATOM #3#)
      (PROGN
       (LETT |v| (CAR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
       NIL))
  (GO G191)))
(SEQ
 (EXIT
 (LETT #2#
  (CONS (SPADCALL (QCDR |q|) |v| (QREFELT $ 26))
     #2#)
  |EUCDOM-;expressIdealMember;LSU;10|)))
(LETT #3# (CDR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
(GO G190)
G191
(EXIT (NREVERSE0 #2#))))))))))))))
```

EUCDOM-;multiEuclidean;LSU;11

⟨EUCDOM-;multiEuclidean;LSU;11⟩≡

```
(DEFUN |EUCDOM-;multiEuclidean;LSU;11| (|l| |z| $)
 (PROG (|n| |l1| |l2| #0=#:G1405 #1=#:G1535 #2=#:G1522 #3=#:G1520
        #4=#:G1521 #5=#:G1406 #6=#:G1536 #7=#:G1525 #8=#:G1523 #9=#:G1524
        |u| |v1| |v2|)
  (RETURN
   (SEQ
    (LETT |n| (LENGTH |l|) |EUCDOM-;multiEuclidean;LSU;11|)
    (EXIT
     (COND
      ((ZEROP |n|) (|error| "empty list passed to multiEuclidean"))
      ((EQL |n| 1) (CONS 0 (LIST |z|)))
      ((QUOTE T)
       (SEQ
        (LETT |l1|
         (SPADCALL |l| (QREFELT $ 46)) |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |l2|
         (SPADCALL |l1| (QUOTIENT2 |n| 2) (QREFELT $ 48))
         |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |u|
         (SPADCALL
          (PROGN
           (LETT #4# NIL |EUCDOM-;multiEuclidean;LSU;11|)
           (SEQ
            (LETT #0# NIL |EUCDOM-;multiEuclidean;LSU;11|)
            (LETT #1# |l1| |EUCDOM-;multiEuclidean;LSU;11|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN
                   (LETT #0# (CAR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
                   NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (PROGN
               (LETT #2# #0# |EUCDOM-;multiEuclidean;LSU;11|)
               (COND
                (#4#
                 (LETT #3#
                  (SPADCALL #3# #2# (QREFELT $ 26))
                  |EUCDOM-;multiEuclidean;LSU;11|))
                ((QUOTE T)
                 (PROGN
          (LETT #3# #2# |EUCDOM-;multiEuclidean;LSU;11|)
          (LETT #4# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
    (LETT #1# (CDR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
    (GO G190)
    G191
    (EXIT NIL))
   (COND (#4# #3#) ((QUOTE T) (|spadConstant| $ 24))))
   (PROGN
   (LETT #9# NIL |EUCDOM-;multiEuclidean;LSU;11|)
   (SEQ
    (LETT #5# NIL |EUCDOM-;multiEuclidean;LSU;11|)
    (LETT #6# |l2| |EUCDOM-;multiEuclidean;LSU;11|)
    G190
    (COND
     ((OR (ATOM #6#)
          (PROGN
           (LETT #5# (CAR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
           NIL))
      (GO G191)))
    (SEQ
     (EXIT
      (PROGN
       (LETT #7# #5# |EUCDOM-;multiEuclidean;LSU;11|)
       (COND
        (#9#
         (LETT #8#
          (SPADCALL #8# #7# (QREFELT $ 26))
          |EUCDOM-;multiEuclidean;LSU;11|))
        ((QUOTE T)
         (PROGN
          (LETT #8# #7# |EUCDOM-;multiEuclidean;LSU;11|)
          (LETT #9# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
    (LETT #6# (CDR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
    (GO G190)
    G191
    (EXIT NIL))
   (COND (#9# #8#) ((QUOTE T) (|spadConstant| $ 24))))
  |z| (QREFELT $ 49))
 |EUCDOM-;multiEuclidean;LSU;11|)
(EXIT
 (COND
  ((QEQCAR |u| 1) (CONS 1 "failed"))
  ((QUOTE T)
   (SEQ
    (LETT |v1|
     (SPADCALL |l1| (QCDR (QCDR |u|)) (QREFELT $ 50))
```

```
             |EUCDOM-;multiEuclidean;LSU;11|)
            (EXIT
             (COND
              ((QEQCAR |v1| 1) (CONS 1 "failed"))
              ((QUOTE T)
               (SEQ
                (LETT |v2|
                 (SPADCALL |l2| (QCAR (QCDR |u|)) (QREFELT $ 50))
                 |EUCDOM-;multiEuclidean;LSU;11|)
                (EXIT
                 (COND
                  ((QEQCAR |v2| 1) (CONS 1 "failed"))
                  ((QUOTE T)
                   (CONS 0
                    (SPADCALL
                     (QCDR |v1|)
                     (QCDR |v2|)
                     (QREFELT $ 51))))))))))))))))))))))
```

EuclideanDomain&

⟨*EuclideanDomainAmp*⟩≡

```
  (DEFUN |EuclideanDomain&| (|#1|)
   (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
     (PROGN
      (LETT DV$1 (|devaluate| |#1|) . #0=(|EuclideanDomain&|))
      (LETT |dv$| (LIST (QUOTE |EuclideanDomain&|) DV$1) . #0#)
      (LETT $ (GETREFV 53) . #0#)
      (QSETREFV $ 0 |dv$|)
      (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
      (|stuffDomainSlots| $)
      (QSETREFV $ 6 |#1|)
      $))))
```

EUCDOM-;MAKEPROP

⟨*EUCDOM-;MAKEPROP*⟩≡

```
(MAKEPROP
 (QUOTE |EuclideanDomain&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|Boolean|) (0 . |zero?|)
  (|NonNegativeInteger|) (5 . |euclideanSize|) |EUCDOM-;sizeLess?;2SB;1|
  (|Record| (|:| |quotient| $) (|:| |remainder| $)) (10 . |divide|)
  |EUCDOM-;quo;3S;2| |EUCDOM-;rem;3S;3| (16 . |Zero|)
  (|Union| $ (QUOTE "failed")) |EUCDOM-;exquo;2SU;4| (20 . |unitCanonical|)
  (25 . |rem|) |EUCDOM-;gcd;3S;5|
  (|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
  (31 . |unitNormal|) (36 . |One|) (40 . =) (46 . *) (52 . -)
  (58 . |sizeLess?|) (64 . +)
  (|Record| (|:| |coef1| $) (|:| |coef2| $) (|:| |generator| $))
  |EUCDOM-;extendedEuclidean;2SR;7|
  (70 . |extendedEuclidean|) (76 . |exquo|)
  (|Record| (|:| |coef1| $) (|:| |coef2| $))
  (|Union| 34 (QUOTE "failed")) |EUCDOM-;extendedEuclidean;3SU;8|
  (|List| 6) (82 . =) (88 . |second|)
  (|Record| (|:| |coef| 41) (|:| |generator| $))
  (|List| $) (93 . |principalIdeal|) |EUCDOM-;principalIdeal;LR;9|
  (|Union| 41 (QUOTE "failed")) |EUCDOM-;expressIdealMember;LSU;10|
  (98 . |copy|) (|Integer|) (103 . |split!|) (109 . |extendedEuclidean|)
  (116 . |multiEuclidean|) (122 . |concat|) |EUCDOM-;multiEuclidean;LSU;11|))
  (QUOTE
   #(|sizeLess?| 128 |rem| 134 |quo| 140 |principalIdeal| 146
     |multiEuclidean| 151 |gcd| 157 |extendedEuclidean| 163
     |exquo| 176 |expressIdealMember| 182))
  (QUOTE NIL)
  (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS (QUOTE #())
    (CONS (QUOTE #())
     (|makeByteWordVec2| 52 (QUOTE (1 6 7 0 8 1 6 9 0 10 2 6 12 0 0 13 0
       6 0 16 1 6 0 0 19 2 6 0 0 0 20 1 6 22 0 23 0 6 0 24 2 6 7 0 0 25 2 6 0
       0 0 26 2 6 0 0 0 27 2 6 7 0 0 28 2 6 0 0 0 29 2 6 30 0 0 32 2 6 17 0 0
       33 2 37 7 0 0 38 1 37 6 0 39 1 6 40 41 42 1 37 0 0 46 2 37 0 0 47 48 3
       6 35 0 0 0 49 2 6 44 41 0 50 2 37 0 0 0 51 2 0 7 0 0 11 2 0 0 0 0 15 2
       0 0 0 0 14 1 0 40 41 43 2 0 44 41 0 52 2 0 0 0 0 21 3 0 35 0 0 0 36 2 0
       30 0 0 31 2 0 17 0 0 18 2 0 44 41 0 45))))))
  (QUOTE |lookupComplete|)))
```

```
⟨EUCDOM-.lsp BOOTSTRAP⟩≡

  ⟨EUCDOM-;VersionCheck⟩
  ⟨EUCDOM-;sizeLess?;2SB;1⟩
  ⟨EUCDOM-;quo;3S;2⟩
  ⟨EUCDOM-;rem;3S;3⟩
  ⟨EUCDOM-;exquo;2SU;4⟩
  ⟨EUCDOM-;gcd;3S;5⟩
  ⟨EUCDOM-;unitNormalizeIdealElt⟩
  ⟨EUCDOM-;extendedEuclidean;2SR;7⟩
  ⟨EUCDOM-;extendedEuclidean;3SU;8⟩
  ⟨EUCDOM-;principalIdeal;LR;9⟩
  ⟨EUCDOM-;expressIdealMember;LSU;10⟩
  ⟨EUCDOM-;multiEuclidean;LSU;11⟩
  ⟨EuclideanDomainAmp⟩
  ⟨EUCDOM-;MAKEPROP⟩
```

21.20 ENTIRER.lsp BOOTSTRAP

ENTIRER depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ENTIRER** category which we can write into the **MID** directory. We compile the lisp code and copy the **ENTIRER.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ENTIRER.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |EntireRing;AL| (QUOTE NIL))

(DEFUN |EntireRing| NIL
  (LET (#:G82841)
    (COND
      (|EntireRing;AL|)
      (T (SETQ |EntireRing;AL| (|EntireRing;|))))))

(DEFUN |EntireRing;| NIL
  (PROG (#1=#:G82839)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE ((|noZeroDivisors| T)))
              (QUOTE NIL)
              NIL))
          |EntireRing|)
        (SETELT #1# 0 (QUOTE (|EntireRing|)))))))

(MAKEPROP (QUOTE |EntireRing|) (QUOTE NILADIC) T)
```

21.21 FFIELDC.lsp BOOTSTRAP

FFIELDC depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FFIELDC.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |FiniteFieldCategory;AL| (QUOTE NIL))

(DEFUN |FiniteFieldCategory| NIL
 (LET (#:G83129)
  (COND
   (|FiniteFieldCategory;AL|)
   (T (SETQ |FiniteFieldCategory;AL| (|FiniteFieldCategory;|))))))

(DEFUN |FiniteFieldCategory;| NIL
 (PROG (#1=#:G83127)
  (RETURN
   (PROG1
    (LETT #1#
     (|Join|
      (|FieldOfPrimeCharacteristic|)
      (|Finite|)
      (|StepThrough|)
      (|DifferentialRing|)
      (|mkCategory|
       (QUOTE |domain|)
       (QUOTE (
        ((|charthRoot| (|$| |$|)) T)
        ((|conditionP| ((|Union| (|Vector| |$|) "failed") (|Matrix| |$|))) T)
        ((|factorsOfCyclicGroupSize|
         ((|List| (|Record|
                  (|:| |factor| (|Integer|))
                  (|:| |exponent| (|Integer|))))))
          T)
        ((|tableForDiscreteLogarithm|
          ((|Table| (|PositiveInteger|) (|NonNegativeInteger|))
           (|Integer|))) T)
        ((|createPrimitiveElement| (|$|)) T)
        ((|primitiveElement| (|$|)) T)
```

```
          ((|primitive?| ((|Boolean|) |$|)) T)
          ((|discreteLog| ((|NonNegativeInteger|) |$|)) T)
          ((|order| ((|PositiveInteger|) |$|)) T)
          ((|representationType|
            ((|Union| "prime" "polynomial" "normal" "cyclic"))) T)))
         NIL
         (QUOTE (
          (|PositiveInteger|)
          (|NonNegativeInteger|)
          (|Boolean|)
          (|Table| (|PositiveInteger|) (|NonNegativeInteger|))
          (|Integer|)
          (|List|
           (|Record| (|:| |factor| (|Integer|)) (|:| |exponent| (|Integer|))))
          (|Matrix| |$|)))
         NIL))
       |FiniteFieldCategory|)
      (SETELT #1# 0 (QUOTE (|FiniteFieldCategory|)))))))

(MAKEPROP (QUOTE |FiniteFieldCategory|) (QUOTE NILADIC) T)
```

21.22 FFIELDC-.lsp BOOTSTRAP

FFIELDC- depends on **FFIELDC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FFIELDC-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |FFIELDC-;differentiate;2S;1| (|x| |$|) (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;init;S;2| (|$|) (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;nextItem;SU;3| (|a| |$|)
 (COND
  ((SPADCALL
    (LETT |a|
     (SPADCALL (|+| (SPADCALL |a| (QREFELT |$| 11)) 1) (QREFELT |$| 12))
     |FFIELDC-;nextItem;SU;3|)
    (QREFELT |$| 14))
     (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 |a|))))

(DEFUN |FFIELDC-;order;SOpc;4| (|e| |$|)
 (SPADCALL (SPADCALL |e| (QREFELT |$| 17)) (QREFELT |$| 20)))

(DEFUN |FFIELDC-;conditionP;MU;5| (|mat| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |mat| (QREFELT |$| 24)) |FFIELDC-;conditionP;MU;5|)
    (COND
     ((OR
       (NULL |l|)
       (SPADCALL (ELT |$| 14) (|SPADfirst| |l|) (QREFELT |$| 27)))
         (EXIT (CONS 1 "failed"))))
    (EXIT
     (CONS 0
      (SPADCALL (ELT |$| 28) (|SPADfirst| |l|) (QREFELT |$| 30))))))))

(DEFUN |FFIELDC-;charthRoot;2S;6| (|x| |$|)
 (SPADCALL |x|
   (QUOTIENT2 (SPADCALL (QREFELT |$| 35)) (SPADCALL (QREFELT |$| 36)))
   (QREFELT |$| 37)))

 (DEFUN |FFIELDC-;charthRoot;SU;7| (|x| |$|)
  (CONS 0 (SPADCALL |x| (QREFELT |$| 28))))

 (DEFUN |FFIELDC-;createPrimitiveElement;S;8| (|$|)
  (PROG (|sm1| |start| |i| #1=#:G83175 |e| |found|)
   (RETURN
    (SEQ
     (LETT |sm1|
      (|-| (SPADCALL (QREFELT |$| 35)) 1)
      |FFIELDC-;createPrimitiveElement;S;8|)
     (LETT |start|
      (COND
       ((SPADCALL
         (SPADCALL (QREFELT |$| 42))
         (CONS 1 "polynomial")
         (QREFELT |$| 43))
        (SPADCALL (QREFELT |$| 36)))
       ((QUOTE T) 1))
      |FFIELDC-;createPrimitiveElement;S;8|)
     (LETT |found| (QUOTE NIL) |FFIELDC-;createPrimitiveElement;S;8|)
     (SEQ
      (LETT |i| |start| |FFIELDC-;createPrimitiveElement;S;8|)
      G190
      (COND
       ((NULL (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
         (GO G191)))
      (SEQ
       (LETT |e|
        (SPADCALL
         (PROG1
          (LETT #1# |i| |FFIELDC-;createPrimitiveElement;S;8|)
          (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#))
         (QREFELT |$| 12))
        |FFIELDC-;createPrimitiveElement;S;8|)
       (EXIT
        (LETT |found|
         (EQL (SPADCALL |e| (QREFELT |$| 17)) |sm1|)
         |FFIELDC-;createPrimitiveElement;S;8|)))
      (LETT |i| (|+| |i| 1) |FFIELDC-;createPrimitiveElement;S;8|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |e|)))))
```

```
(DEFUN |FFIELDC-;primitive?;SB;9| (|a| |$|)
 (PROG (|explist| |q| |exp| #1=#:G83187 |equalone|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |a| (QREFELT |$| 14)) (QUOTE NIL))
     ((QUOTE T)
      (SEQ
       (LETT |explist|
        (SPADCALL (QREFELT |$| 47)) |FFIELDC-;primitive?;SB;9|)
       (LETT |q|
        (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;primitive?;SB;9|)
       (LETT |equalone| (QUOTE NIL) |FFIELDC-;primitive?;SB;9|)
       (SEQ
        (LETT |exp| NIL |FFIELDC-;primitive?;SB;9|)
        (LETT #1# |explist| |FFIELDC-;primitive?;SB;9|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |exp| (CAR #1#) |FFIELDC-;primitive?;SB;9|) NIL)
           (NULL (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT |equalone|
           (SPADCALL
            (SPADCALL |a| (QUOTIENT2 |q| (QCAR |exp|)) (QREFELT |$| 48))
            (QREFELT |$| 49))
           |FFIELDC-;primitive?;SB;9|)))
        (LETT #1# (CDR #1#) |FFIELDC-;primitive?;SB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))))))))

(DEFUN |FFIELDC-;order;SPi;10| (|e| |$|)
 (PROG (|lof| |rec| #1=#:G83195 |primeDivisor|
        |j| #2=#:G83196 |a| |goon| |ord|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |e| (|spadConstant| |$| 7) (QREFELT |$| 51))
       (|error| "order(0) is not defined "))
     ((QUOTE T)
(SEQ
 (LETT |ord|
  (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;order;SPi;10|)
 (LETT |a| 0 |FFIELDC-;order;SPi;10|)
 (LETT |lof| (SPADCALL (QREFELT |$| 47)) |FFIELDC-;order;SPi;10|)
 (SEQ
  (LETT |rec| NIL |FFIELDC-;order;SPi;10|)
  (LETT #1# |lof| |FFIELDC-;order;SPi;10|)
  G190
  (COND
   ((OR
     (ATOM #1#)
     (PROGN (LETT |rec| (CAR #1#) |FFIELDC-;order;SPi;10|) NIL))
    (GO G191)))
  (SEQ
   (LETT |a|
    (QUOTIENT2 |ord|
     (LETT |primeDivisor| (QCAR |rec|) |FFIELDC-;order;SPi;10|))
    |FFIELDC-;order;SPi;10|)
   (LETT |goon|
    (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
    |FFIELDC-;order;SPi;10|)
   (SEQ
    (LETT |j| 0 |FFIELDC-;order;SPi;10|)
    (LETT #2# (|-| (QCDR |rec|) 2) |FFIELDC-;order;SPi;10|)
    G190
    (COND ((OR (QSGREATERP |j| #2#) (NULL |goon|)) (GO G191)))
    (SEQ
     (LETT |ord| |a| |FFIELDC-;order;SPi;10|)
     (LETT |a|
      (QUOTIENT2 |ord| |primeDivisor|)
      |FFIELDC-;order;SPi;10|)
     (EXIT
      (LETT |goon|
       (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
       |FFIELDC-;order;SPi;10|)))
    (LETT |j| (QSADD1 |j|) |FFIELDC-;order;SPi;10|)
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT (COND (|goon| (LETT |ord| |a| |FFIELDC-;order;SPi;10|)))))
  (LETT #1# (CDR #1#) |FFIELDC-;order;SPi;10|)
  (GO G190)
  G191
  (EXIT NIL))
 (EXIT |ord|))))))))
```

```
(DEFUN |FFIELDC-;discreteLog;SNni;11| (|b| |$|)
 (PROG (|faclist| |gen| |groupord| |f| #1=#:G83216 |fac| |t| #2=#:G83217
        |exp| |exptable| |n| |end| |i| |rho| |found| |disc1| |c| |mult|
        |disclog| |a|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |b| (QREFELT |$| 14))
       (|error| "discreteLog: logarithm of zero"))
     ((QUOTE T)
      (SEQ
       (LETT |faclist|
        (SPADCALL (QREFELT |$| 47))
        |FFIELDC-;discreteLog;SNni;11|)
       (LETT |a| |b| |FFIELDC-;discreteLog;SNni;11|)
       (LETT |gen|
        (SPADCALL (QREFELT |$| 53))
        |FFIELDC-;discreteLog;SNni;11|)
       (EXIT
        (COND
         ((SPADCALL |b| |gen| (QREFELT |$| 51)) 1)
         ((QUOTE T)
          (SEQ
           (LETT |disclog| 0 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |mult| 1 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |groupord|
            (|-| (SPADCALL (QREFELT |$| 35)) 1)
            |FFIELDC-;discreteLog;SNni;11|)
           (LETT |exp| |groupord| |FFIELDC-;discreteLog;SNni;11|)
           (SEQ
            (LETT |f| NIL |FFIELDC-;discreteLog;SNni;11|)
            (LETT #1# |faclist| |FFIELDC-;discreteLog;SNni;11|)
            G190
            (COND
             ((OR
               (ATOM #1#)
               (PROGN
                (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;SNni;11|)
                NIL))
             (GO G191)))
            (SEQ
             (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;SNni;11|)
             (EXIT
              (SEQ
               (LETT |t| 0 |FFIELDC-;discreteLog;SNni;11|)
```

```
(LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;SNni;11|)
G190
(COND ((QSGREATERP |t| #2#) (GO G191)))
(SEQ
 (LETT |exp|
  (QUOTIENT2 |exp| |fac|)
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |exptable|
  (SPADCALL |fac| (QREFELT |$| 55))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |n|
  (SPADCALL |exptable| (QREFELT |$| 56))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |c|
  (SPADCALL |a| |exp| (QREFELT |$| 48))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |end|
  (QUOTIENT2 (|-| |fac| 1) |n|)
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |found| (QUOTE NIL) |FFIELDC-;discreteLog;SNni;11|)
 (LETT |disc1| 0 |FFIELDC-;discreteLog;SNni;11|)
 (SEQ
  (LETT |i| 0 |FFIELDC-;discreteLog;SNni;11|)
  G190
  (COND
   ((OR
     (QSGREATERP |i| |end|)
     (NULL
      (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
    (GO G191)))
  (SEQ
   (LETT |rho|
    (SPADCALL
     (SPADCALL |c| (QREFELT |$| 11))
     |exptable|
     (QREFELT |$| 58))
    |FFIELDC-;discreteLog;SNni;11|)
   (EXIT
    (COND
     ((QEQCAR |rho| 0)
      (SEQ
       (LETT |found| (QUOTE T) |FFIELDC-;discreteLog;SNni;11|)
       (EXIT
        (LETT |disc1|
         (|*| (|+| (|*| |n| |i|) (QCDR |rho|)) |mult|)
         |FFIELDC-;discreteLog;SNni;11|))))
```

```
                    ((QUOTE T)
                     (LETT |c|
                      (SPADCALL |c|
                       (SPADCALL |gen|
                        (|*| (QUOTIENT2 |groupord| |fac|) (|-| |n|))
                        (QREFELT |$| 48))
                       (QREFELT |$| 59))
                      |FFIELDC-;discreteLog;SNni;11|)))))
                 (LETT |i| (QSADD1 |i|) |FFIELDC-;discreteLog;SNni;11|)
                 (GO G190)
                 G191
                 (EXIT NIL))
                (EXIT
                 (COND
                  (|found|
                   (SEQ
                    (LETT |mult|
                     (|*| |mult| |fac|)
                     |FFIELDC-;discreteLog;SNni;11|)
                    (LETT |disclog|
                     (|+| |disclog| |disc1|)
                     |FFIELDC-;discreteLog;SNni;11|)
                    (EXIT
                     (LETT |a|
                      (SPADCALL |a|
                       (SPADCALL |gen| (|-| |disc1|) (QREFELT |$| 48))
                       (QREFELT |$| 59))
                      |FFIELDC-;discreteLog;SNni;11|))))
                  ((QUOTE T)
                    (|error| "discreteLog: ?? discrete logarithm")))))
              (LETT |t|
               (QSADD1 |t|)
               |FFIELDC-;discreteLog;SNni;11|)
              (GO G190)
              G191
              (EXIT NIL))))
          (LETT #1#
           (CDR #1#)
           |FFIELDC-;discreteLog;SNni;11|)
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |disclog|))))))))))))

(DEFUN |FFIELDC-;discreteLog;2SU;12| (|logbase| |b| |$|)
 (PROG (|groupord| |faclist| |f| #1=#:G83235 |fac| |primroot|
        |t| #2=#:G83236 |exp| |rhoHelp| #3=#:G83234 |rho| |disclog|
        |mult| |a|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |b| (QREFELT |$| 14))
       (SEQ
        (SPADCALL "discreteLog: logarithm of zero" (QREFELT |$| 64))
        (EXIT (CONS 1 "failed"))))
      ((SPADCALL |logbase| (QREFELT |$| 14))
       (SEQ
        (SPADCALL "discreteLog: logarithm to base zero" (QREFELT |$| 64))
        (EXIT (CONS 1 "failed"))))
      ((SPADCALL |b| |logbase| (QREFELT |$| 51)) (CONS 0 1))
      ((QUOTE T)
       (COND
        ((NULL
          (ZEROP
           (REMAINDER2
            (LETT |groupord|
             (SPADCALL |logbase| (QREFELT |$| 17))
             |FFIELDC-;discreteLog;2SU;12|)
            (SPADCALL |b| (QREFELT |$| 17)))))
         (SEQ
          (SPADCALL
"discreteLog: second argument not in cyclic group generated by first argument"
           (QREFELT |$| 64))
          (EXIT (CONS 1 "failed"))))
        ((QUOTE T)
         (SEQ
          (LETT |faclist|
           (SPADCALL (SPADCALL |groupord| (QREFELT |$| 66)) (QREFELT |$| 68))
           |FFIELDC-;discreteLog;2SU;12|)
          (LETT |a| |b| |FFIELDC-;discreteLog;2SU;12|)
          (LETT |disclog| 0 |FFIELDC-;discreteLog;2SU;12|)
          (LETT |mult| 1 |FFIELDC-;discreteLog;2SU;12|)
          (LETT |exp| |groupord| |FFIELDC-;discreteLog;2SU;12|)
          (SEQ
           (LETT |f| NIL |FFIELDC-;discreteLog;2SU;12|)
           (LETT #1# |faclist| |FFIELDC-;discreteLog;2SU;12|)
           G190
           (COND
            ((OR
              (ATOM #1#)
              (PROGN (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;2SU;12|) NIL))
  (GO G191)))
(SEQ
 (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;2SU;12|)
 (LETT |primroot|
  (SPADCALL |logbase|
   (QUOTIENT2 |groupord| |fac|)
   (QREFELT |$| 48))
  |FFIELDC-;discreteLog;2SU;12|)
 (EXIT
  (SEQ
   (LETT |t| 0 |FFIELDC-;discreteLog;2SU;12|)
   (LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;2SU;12|)
   G190
   (COND ((QSGREATERP |t| #2#) (GO G191)))
   (SEQ
    (LETT |exp|
     (QUOTIENT2 |exp| |fac|)
     |FFIELDC-;discreteLog;2SU;12|)
    (LETT |rhoHelp|
     (SPADCALL |primroot|
      (SPADCALL |a| |exp| (QREFELT |$| 48))
      |fac|
      (QREFELT |$| 70))
      |FFIELDC-;discreteLog;2SU;12|)
    (EXIT
     (COND
      ((QEQCAR |rhoHelp| 1)
       (PROGN
        (LETT #3# (CONS 1 "failed") |FFIELDC-;discreteLog;2SU;12|)
        (GO #3#)))
      ((QUOTE T)
       (SEQ
        (LETT |rho|
         (|*| (QCDR |rhoHelp|) |mult|)
         |FFIELDC-;discreteLog;2SU;12|)
        (LETT |disclog|
         (|+| |disclog| |rho|)
         |FFIELDC-;discreteLog;2SU;12|)
        (LETT |mult|
         (|*| |mult| |fac|)
         |FFIELDC-;discreteLog;2SU;12|)
        (EXIT
         (LETT |a|
          (SPADCALL |a|
           (SPADCALL |logbase| (|-| |rho|) (QREFELT |$| 48))
           (QREFELT |$| 59))
```

```
                     |FFIELDC-;discreteLog;2SU;12|)))))))
              (LETT |t| (QSADD1 |t|) |FFIELDC-;discreteLog;2SU;12|)
              (GO G190)
              G191
              (EXIT NIL))))
           (LETT #1# (CDR #1#) |FFIELDC-;discreteLog;2SU;12|)
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT (CONS 0 |disclog|))))))))
    #3#
    (EXIT #3#)))))

(DEFUN |FFIELDC-;squareFreePolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 75)))

(DEFUN |FFIELDC-;factorPolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 77)))

(DEFUN |FFIELDC-;factorSquareFreePolynomial| (|f| |$|)
 (PROG (|flist| |u| #1=#:G83248 #2=#:G83245 #3=#:G83243 #4=#:G83244)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |f| (|spadConstant| |$| 78) (QREFELT |$| 79))
       (|spadConstant| |$| 80))
     ((QUOTE T)
      (SEQ
       (LETT |flist|
        (SPADCALL |f| (QUOTE T) (QREFELT |$| 83))
        |FFIELDC-;factorSquareFreePolynomial|)
       (EXIT
        (SPADCALL
         (SPADCALL (QCAR |flist|) (QREFELT |$| 84))
         (PROGN
          (LETT #4# NIL |FFIELDC-;factorSquareFreePolynomial|)
          (SEQ
           (LETT |u| NIL |FFIELDC-;factorSquareFreePolynomial|)
           (LETT #1# (QCDR |flist|) |FFIELDC-;factorSquareFreePolynomial|)
           G190
           (COND
            ((OR
              (ATOM #1#)
              (PROGN
               (LETT |u| (CAR #1#) |FFIELDC-;factorSquareFreePolynomial|)
               NIL))
```

```
                (GO G191)))
              (SEQ
               (EXIT
                (PROGN
                 (LETT #2#
                  (SPADCALL (QCAR |u|) (QCDR |u|) (QREFELT |$| 85))
                  |FFIELDC-;factorSquareFreePolynomial|)
                 (COND
                  (#4#
                   (LETT #3#
                    (SPADCALL #3# #2# (QREFELT |$| 86))
                    |FFIELDC-;factorSquareFreePolynomial|))
                  ((QUOTE T)
                   (PROGN
                    (LETT #3# #2# |FFIELDC-;factorSquareFreePolynomial|)
                    (LETT #4#
                     (QUOTE T)
                     |FFIELDC-;factorSquareFreePolynomial|)))))))
              (LETT #1# (CDR #1#) |FFIELDC-;factorSquareFreePolynomial|)
              (GO G190)
              G191
             (EXIT NIL))
             (COND (#4# #3#) ((QUOTE T) (|spadConstant| |$| 87))))
            (QREFELT |$| 88))))))))))

(DEFUN |FFIELDC-;gcdPolynomial;3Sup;16| (|f| |g| |$|)
 (SPADCALL |f| |g| (QREFELT |$| 90)))

(DEFUN |FiniteFieldCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|FiniteFieldCategory&|))
    (LETT |dv$| (LIST (QUOTE |FiniteFieldCategory&|) |DV$1|) . #1#)
    (LETT |$| (GETREFV 93) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|) |$|))))

(MAKEPROP
 (QUOTE |FiniteFieldCategory&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (0 . |Zero|)
  |FFIELDC-;differentiate;2S;1| |FFIELDC-;init;S;2| (|PositiveInteger|)
  (4 . |lookup|) (9 . |index|) (|Boolean|) (14 . |zero?|)
  (|Union| |$| (QUOTE "failed")) |FFIELDC-;nextItem;SU;3| (19 . |order|)
  (|Integer|) (|OnePointCompletion| 10) (24 . |coerce|)
  |FFIELDC-;order;SOpc;4| (|List| 26) (|Matrix| 6) (29 . |nullSpace|)
  (|Mapping| 13 6) (|Vector| 6) (34 . |every?|) (40 . |charthRoot|)
  (|Mapping| 6 6) (45 . |map|) (|Union| (|Vector| |$|) (QUOTE "failed"))
  (|Matrix| |$|) |FFIELDC-;conditionP;MU;5| (|NonNegativeInteger|)
  (51 . |size|) (55 . |characteristic|) (59 . |**|)
  |FFIELDC-;charthRoot;2S;6| |FFIELDC-;charthRoot;SU;7| (65 . |One|)
  (|Union| (QUOTE "prime") (QUOTE "polynomial") (QUOTE "normal")
  (QUOTE "cyclic")) (69 . |representationType|) (73 . |=|)
  |FFIELDC-;createPrimitiveElement;S;8| (|Record| (|:| |factor| 18)
  (|:| |exponent| 18)) (|List| 45) (79 . |factorsOfCyclicGroupSize|)
  (83 . |**|) (89 . |one?|) |FFIELDC-;primitive?;SB;9| (94 . |=|)
  |FFIELDC-;order;SPi;10| (100 . |primitiveElement|) (|Table| 10 34)
  (104 . |tableForDiscreteLogarithm|) (109 . |#|)
  (|Union| 34 (QUOTE "failed")) (114 . |search|) (120 . |*|)
  |FFIELDC-;discreteLog;SNni;11| (|Void|) (|String|) (|OutputForm|)
  (126 . |messagePrint|) (|Factored| |$|) (131 . |factor|)
  (|Factored| 18) (136 . |factors|) (|DiscreteLogarithmPackage| 6)
  (141 . |shanksDiscLogAlgorithm|) |FFIELDC-;discreteLog;2SU;12|
  (|Factored| 73) (|SparseUnivariatePolynomial| 6)
  (|UnivariatePolynomialSquareFree| 6 73) (148 . |squareFree|)
  (|DistinctDegreeFactorize| 6 73) (153 . |factor|) (158 . |Zero|)
  (162 . |=|) (168 . |Zero|) (|Record| (|:| |irr| 73) (|:| |pow| 18))
  (|Record| (|:| |cont| 6) (|:| |factors| (|List| 81)))
  (172 . |distdfact|) (178 . |coerce|) (183 . |primeFactor|)
  (189 . |*|) (195 . |One|) (199 . |*|) (|EuclideanDomain&| 73)
  (205 . |gcd|) (|SparseUnivariatePolynomial| |$|)
  |FFIELDC-;gcdPolynomial;3Sup;16|))
 (QUOTE
  #(|primitive?| 211 |order| 216 |nextItem| 226 |init| 231
    |gcdPolynomial| 235 |discreteLog| 241 |differentiate| 252
    |createPrimitiveElement| 257 |conditionP| 261 |charthRoot| 266))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 92
    (QUOTE
     (0 6 0 7 1 6 10 0 11 1 6 0 10 12 1 6 13 0 14 1 6 10 0 17 1 19
      0 18 20 1 23 22 0 24 2 26 13 25 0 27 1 6 0 0 28 2 26 0 29 0 30
```

```
            0 6 34 35 0 6 34 36 2 6 0 0 34 37 0 6 0 40 0 6 41 42 2 41 13
            0 0 43 0 6 46 47 2 6 0 0 18 48 1 6 13 0 49 2 6 13 0 0 51 0 6 0
            53 1 6 54 18 55 1 54 34 0 56 2 54 57 10 0 58 2 6 0 0 0 59 1
            63 61 62 64 1 18 65 0 66 1 67 46 0 68 3 69 57 6 6 34 70 1 74
            72 73 75 1 76 72 73 77 0 73 0 78 2 73 13 0 0 79 0 72 0 80 2
            76 82 73 13 83 1 73 0 6 84 2 72 0 73 18 85 2 72 0 0 0 86 0 72
            0 87 2 72 0 73 0 88 2 89 0 0 0 90 1 0 13 0 50 1 0 10 0 52 1 0
            19 0 21 1 0 15 0 16 0 0 0 9 2 0 91 91 91 92 1 0 34 0 60 2 0 57
            0 0 71 1 0 0 0 8 0 0 0 44 1 0 31 32 33 1 0 0 0 38 1 0 15 0 39))))))
      (QUOTE |lookupComplete|)))
```

21.23 FPS.lsp BOOTSTRAP

FPS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FPS.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |FloatingPointSystem;AL| (QUOTE NIL))

(DEFUN |FloatingPointSystem| NIL
  (LET (#:G105645)
    (COND
      (|FloatingPointSystem;AL|)
      (T (SETQ |FloatingPointSystem;AL| (|FloatingPointSystem;|))))))

(DEFUN |FloatingPointSystem;| NIL
  (PROG (#1=#:G105643)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|RealNumberSystem|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|float| (|$| (|Integer|) (|Integer|))) T)
                ((|float| (|$| (|Integer|) (|Integer|) (|PositiveInteger|))) T)
                ((|order| ((|Integer|) |$|)) T)
                ((|base| ((|PositiveInteger|))) T)
                ((|exponent| ((|Integer|) |$|)) T)
                ((|mantissa| ((|Integer|) |$|)) T)
                ((|bits| ((|PositiveInteger|))) T)
                ((|digits| ((|PositiveInteger|))) T)
                ((|precision| ((|PositiveInteger|))) T)
                ((|bits| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|digits| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|precision| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|increasePrecision| ((|PositiveInteger|) (|Integer|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|decreasePrecision| ((|PositiveInteger|) (|Integer|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|min| (|$|))
                  (AND
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|)))))
                ((|max| (|$|))
                  (AND
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|)))))))
              (QUOTE ((|approximate| T)))
              (QUOTE ((|PositiveInteger|) (|Integer|)))
              NIL))
          |FloatingPointSystem|)
        (SETELT #1# 0 (QUOTE (|FloatingPointSystem|)))))))

(MAKEPROP (QUOTE |FloatingPointSystem|) (QUOTE NILADIC) T)
```

21.24 FPS-.lsp BOOTSTRAP

FPS- depends **FPS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FPS-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |FPS-;float;2IS;1| (|ma| |ex| |$|)
  (SPADCALL |ma| |ex| (SPADCALL (QREFELT |$| 8)) (QREFELT |$| 10)))

(DEFUN |FPS-;digits;Pi;2| (|$|)
  (PROG (#1=#:G105654)
    (RETURN
      (PROG1
        (LETT #1#
          (MAX 1
            (QUOTIENT2
              (SPADCALL 4004
                (|-| (SPADCALL (QREFELT |$| 13)) 1)
                (QREFELT |$| 14))
              13301))
          |FPS-;digits;Pi;2|)
        (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#)))))

(DEFUN |FloatingPointSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|FloatingPointSystem&|))
        (LETT |dv$| (LIST (QUOTE |FloatingPointSystem&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 17) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST
                (|HasAttribute| |#1| (QUOTE |arbitraryExponent|))
                (|HasAttribute| |#1| (QUOTE |arbitraryPrecision|)))) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))
```

```
(MAKEPROP
  (QUOTE |FloatingPointSystem&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (0 . |base|)
        (|Integer|)
        (4 . |float|)
        |FPS-;float;2IS;1|
        (11 . |One|)
        (15 . |bits|)
        (19 . |*|)
        (25 . |max|)
        |FPS-;digits;Pi;2|))
    (QUOTE #(|float| 29 |digits| 35))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 16
            (QUOTE
              (0 6 7 8 3 6 0 9 9 7 10 0 6 0 12 0 6 7 13 2 9 0 7 0 14 0 6 0 15
               2 0 0 9 9 11 0 0 7 16))))))
     (QUOTE |lookupComplete|)))
```

21.25 GCDDOM.lsp BOOTSTRAP

GCDDOM needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM**. We break this chain with **GCDDOM.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*GCDDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |GcdDomain;AL| (QUOTE NIL))

(DEFUN |GcdDomain| NIL
  (LET (#:G83171)
    (COND
      (|GcdDomain;AL|)
      (T (SETQ |GcdDomain;AL| (|GcdDomain;|))))))

(DEFUN |GcdDomain;| NIL
  (PROG (#1=#:G83169)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|IntegralDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|gcd| (|$| |$| |$|)) T)
                ((|gcd| (|$| (|List| |$|))) T)
                ((|lcm| (|$| |$| |$|)) T)
                ((|lcm| (|$| (|List| |$|))) T)
                ((|gcdPolynomial|
                  ((|SparseUnivariatePolynomial| |$|)
                   (|SparseUnivariatePolynomial| |$|)
                   (|SparseUnivariatePolynomial| |$|)))
                  T)))
              NIL
              (QUOTE ((|SparseUnivariatePolynomial| |$|) (|List| |$|)))
              NIL))
              |GcdDomain|)
         (SETELT #1# 0 (QUOTE (|GcdDomain|)))))))

(MAKEPROP (QUOTE |GcdDomain|) (QUOTE NILADIC) T)
```

21.26 GCDDOM-.lsp BOOTSTRAP

GCDDOM- depends on **GCDDOM**. We break this chain with **GCDDOM-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*GCDDOM-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |GCDDOM-;lcm;3S;1| (|x| |y| |$|)
  (PROG (LCM)
    (RETURN
      (SEQ
        (COND
          ((OR
              (SPADCALL |y| (|spadConstant| |$| 7) (QREFELT |$| 9))
              (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))
            (|spadConstant| |$| 7))
          ((QUOTE T)
            (SEQ
              (LETT LCM
                (SPADCALL |y|
                  (SPADCALL |x| |y| (QREFELT |$| 10))
                  (QREFELT |$| 12))
                |GCDDOM-;lcm;3S;1|)
              (EXIT
                (COND
                  ((QEQCAR LCM 0) (SPADCALL |x| (QCDR LCM) (QREFELT |$| 13)))
                  ((QUOTE T) (|error| "bad gcd in lcm computation")))))))))))

(DEFUN |GCDDOM-;lcm;LS;2| (|l| |$|)
  (SPADCALL
    (ELT |$| 15)
    |l|
    (|spadConstant| |$| 16)
    (|spadConstant| |$| 7)
    (QREFELT |$| 19)))

(DEFUN |GCDDOM-;gcd;LS;3| (|l| |$|)
  (SPADCALL
    (ELT |$| 10)
    |1|
    (|spadConstant| |$| 7)
    (|spadConstant| |$| 16)
    (QREFELT |$| 19)))

(DEFUN |GCDDOM-;gcdPolynomial;3Sup;4| (|p1| |p2| |$|)
  (PROG (|e2| |e1| |c1| |p| |c2| #1=#:G83191)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p1| (QREFELT |$| 24)) (SPADCALL |p2| (QREFELT |$| 25)))
          ((SPADCALL |p2| (QREFELT |$| 24)) (SPADCALL |p1| (QREFELT |$| 25)))
          ((QUOTE T)
            (SEQ
              (LETT |c1|
                (SPADCALL |p1| (QREFELT |$| 26))
                |GCDDOM-;gcdPolynomial;3Sup;4|)
              (LETT |c2|
                (SPADCALL |p2| (QREFELT |$| 26))
                |GCDDOM-;gcdPolynomial;3Sup;4|)
              (LETT |p1|
                (PROG2
                  (LETT #1#
                    (SPADCALL |p1| |c1| (QREFELT |$| 27))
                    |GCDDOM-;gcdPolynomial;3Sup;4|)
                  (QCDR #1#)
                  (|check-union|
                    (QEQCAR #1# 0)
                    (|SparseUnivariatePolynomial| (QREFELT |$| 6))
                    #1#))
                |GCDDOM-;gcdPolynomial;3Sup;4|)
              (LETT |p2|
                (PROG2
                  (LETT #1#
                    (SPADCALL |p2| |c2| (QREFELT |$| 27))
                    |GCDDOM-;gcdPolynomial;3Sup;4|)
                  (QCDR #1#)
                  (|check-union|
                    (QEQCAR #1# 0)
                    (|SparseUnivariatePolynomial| (QREFELT |$| 6))
                    #1#))
                |GCDDOM-;gcdPolynomial;3Sup;4|)
              (SEQ
                (LETT |e1|
                  (SPADCALL |p1| (QREFELT |$| 29))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
  (EXIT
    (COND
      ((|<| 0 |e1|)
        (LETT |p1|
          (PROG2
            (LETT #1#
              (SPADCALL |p1|
                (SPADCALL
                 (|spadConstant| |$| 16) |e1| (QREFELT |$| 32))
                (QREFELT |$| 33))
              |GCDDOM-;gcdPolynomial;3Sup;4|)
            (QCDR #1#)
            (|check-union|
              (QEQCAR #1# 0)
              (|SparseUnivariatePolynomial| (QREFELT |$| 6))
              #1#))
          |GCDDOM-;gcdPolynomial;3Sup;4|)))))
(SEQ
  (LETT |e2|
    (SPADCALL |p2| (QREFELT |$| 29))
    |GCDDOM-;gcdPolynomial;3Sup;4|)
  (EXIT
    (COND
      ((|<| 0 |e2|)
        (LETT |p2|
          (PROG2
            (LETT #1#
              (SPADCALL |p2|
                (SPADCALL
                  (|spadConstant| |$| 16)
                  |e2|
                  (QREFELT |$| 32))
                (QREFELT |$| 33))
              |GCDDOM-;gcdPolynomial;3Sup;4|)
            (QCDR #1#)
            (|check-union|
              (QEQCAR #1# 0)
              (|SparseUnivariatePolynomial| (QREFELT |$| 6))
              #1#))
          |GCDDOM-;gcdPolynomial;3Sup;4|)))))
(LETT |e1|
  (MIN |e1| |e2|)
  |GCDDOM-;gcdPolynomial;3Sup;4|)
(LETT |c1|
  (SPADCALL |c1| |c2| (QREFELT |$| 10))
  |GCDDOM-;gcdPolynomial;3Sup;4|)
```

```
(LETT |p1|
  (COND
    ((OR
         (EQL (SPADCALL |p1| (QREFELT |$| 34)) 0)
         (EQL (SPADCALL |p2| (QREFELT |$| 34)) 0))
       (SPADCALL |c1| 0 (QREFELT |$| 32)))
    ((QUOTE T)
      (SEQ
        (LETT |p|
          (SPADCALL |p1| |p2| (QREFELT |$| 35))
          |GCDDOM-;gcdPolynomial;3Sup;4|)
        (EXIT
          (COND
            ((EQL (SPADCALL |p| (QREFELT |$| 34)) 0)
              (SPADCALL |c1| 0 (QREFELT |$| 32)))
            ((QUOTE T)
              (SEQ
                (LETT |c2|
                  (SPADCALL
                    (SPADCALL |p1| (QREFELT |$| 36))
                    (SPADCALL |p2| (QREFELT |$| 36))
                    (QREFELT |$| 10))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (EXIT
                  (SPADCALL
                    (SPADCALL |c1|
                      (SPADCALL
                        (PROG2
                          (LETT #1#
                            (SPADCALL
                              (SPADCALL
                                |c2|
                                |p|
                                (QREFELT |$| 37))
                              (SPADCALL |p| (QREFELT |$| 36))
                              (QREFELT |$| 27))
                            |GCDDOM-;gcdPolynomial;3Sup;4|)
                           (QCDR #1#)
                           (|check-union|
                             (QEQCAR #1# 0)
                             (|SparseUnivariatePolynomial|
                               (QREFELT |$| 6))
                             #1#))
                        (QREFELT |$| 38))
                      (QREFELT |$| 37))
                    (QREFELT |$| 25))))))))))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (EXIT
                 (COND
                  ((ZEROP |e1|) |p1|)
                  ((QUOTE T)
                    (SPADCALL
                     (SPADCALL (|spadConstant| |$| 16) |e1| (QREFELT |$| 32))
                     |p1| (QREFELT |$| 39)))))))))))

(DEFUN |GcdDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|GcdDomain&|))
        (LETT |dv$| (LIST (QUOTE |GcdDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 42) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |GcdDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |=|)
        (10 . |gcd|)
        (|Union| |$| (QUOTE "failed"))
        (16 . |exquo|)
        (22 . |*|)
        |GCDDOM-;lcm;3S;1|
        (28 . |lcm|)
        (34 . |One|)
        (|Mapping| 6 6 6)
        (|List| 6)
        (38 . |reduce|)
        (|List| |$|)
        |GCDDOM-;lcm;LS;2|
        |GCDDOM-;gcd;LS;3|
        (|SparseUnivariatePolynomial| 6)
```

```
          (46 . |zero?|)
          (51 . |unitCanonical|)
          (56 . |content|)
          (61 . |exquo|)
          (|NonNegativeInteger|)
          (67 . |minimumDegree|)
          (72 . |Zero|)
          (76 . |One|)
          (80 . |monomial|)
          (86 . |exquo|)
          (92 . |degree|)
          (97 . |subResultantGcd|)
          (103 . |leadingCoefficient|)
          (108 . |*|)
          (114 . |primitivePart|)
          (119 . |*|)
          (|SparseUnivariatePolynomial| |$|)
          |GCDDOM-;gcdPolynomial;3Sup;4|))
      (QUOTE #(|lcm| 125 |gcdPolynomial| 136 |gcd| 142))
      (QUOTE NIL)
      (CONS
        (|makeByteWordVec2| 1 (QUOTE NIL))
        (CONS
          (QUOTE #())
          (CONS
            (QUOTE #())
              (|makeByteWordVec2| 41
               (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 0 0 10 2 6 11 0 0 12 2 6 0 0 0
                       13 2 6 0 0 0 15 0 6 0 16 4 18 6 17 0 6 6 19 1 23 8 0 24
                       1 23 0 0 25 1 23 6 0 26 2 23 11 0 6 27 1 23 28 0 29 0 23
                       0 30 0 23 0 31 2 23 0 6 28 32 2 23 11 0 0 33 1 23 28 0
                       34 2 23 0 0 0 35 1 23 6 0 36 2 23 0 6 0 37 1 23 0 0 38 2
                       23 0 0 0 39 1 0 0 20 21 2 0 0 0 0 14 2 0 40 40 40 41 1 0
                       0 20 22))))))
     (QUOTE |lookupComplete|)))
```

21.27 HOAGG.lsp BOOTSTRAP

HOAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*HOAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |HomogeneousAggregate;CAT| (QUOTE NIL))

(SETQ |HomogeneousAggregate;AL| (QUOTE NIL))

(DEFUN |HomogeneousAggregate| (#1=#:G82375)
  (LET (#2=#:G82376)
    (COND
      ((SETQ #2# (|assoc| (|devaluate| #1#) |HomogeneousAggregate;AL|))
        (CDR #2#))
      (T
        (SETQ |HomogeneousAggregate;AL|
          (|cons5|
            (CONS (|devaluate| #1#) (SETQ #2# (|HomogeneousAggregate;| #1#)))
            |HomogeneousAggregate;AL|))
        #2#))))

(DEFUN |HomogeneousAggregate;| (|t#1|)
  (PROG (#1=#:G82374)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
            (COND
              (|HomogeneousAggregate;CAT|)
              ((QUOTE T)
                (LETT |HomogeneousAggregate;CAT|
                  (|Join|
                    (|Aggregate|)
                    (|mkCategory|
                      (QUOTE |domain|)
                      (QUOTE (
                        ((|map| (|$| (|Mapping| |t#1| |t#1|) |$|)) T)
                ((|map!| (|$| (|Mapping| |t#1| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
                ((|any?|
                   ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|every?|
                   ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|count|
                   ((|NonNegativeInteger|)
                    (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|parts| ((|List| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|members| ((|List| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|count| ((|NonNegativeInteger|) |t#1| |$|))
                  (AND
                    (|has| |t#1| (|SetCategory|))
                    (|has| |$| (ATTRIBUTE |finiteAggregate|))))
                ((|member?| ((|Boolean|) |t#1| |$|))
                  (AND
                    (|has| |t#1| (|SetCategory|))
                    (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
             (QUOTE (
              ((|SetCategory|) (|has| |t#1| (|SetCategory|)))
              ((|Evalable| |t#1|)
                (AND
                  (|has| |t#1| (|Evalable| |t#1|))
                  (|has| |t#1| (|SetCategory|))))))
            (QUOTE (
              (|Boolean|)
              (|NonNegativeInteger|)
              (|List| |t#1|)))
            NIL))
        . #2=(|HomogeneousAggregate|))))) . #2#)
(SETELT #1# 0
  (LIST (QUOTE |HomogeneousAggregate|) (|devaluate| |t#1|)))))))
```

21.28 HOAGG-.lsp BOOTSTRAP

HOAGG- depends on **HOAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*HOAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |HOAGG-;eval;ALA;1| (|u| |l| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;eval;ALA;1!0|) (VECTOR |$| |l|))
  |u|
  (QREFELT |$| 11)))

(DEFUN |HOAGG-;eval;ALA;1!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 9)))

(DEFUN |HOAGG-;#;ANni;2| (|c| |$|)
 (LENGTH (SPADCALL |c| (QREFELT |$| 14))))

(DEFUN |HOAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82396 #2=#:G82393 #3=#:G82391 #4=#:G82392)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |HOAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
             (COND
                (#3# (QUOTE T))
                ((QUOTE T) #2#))
               |HOAGG-;any?;MAB;3|))
            ((QUOTE T)
             (PROGN
              (LETT #3# #2# |HOAGG-;any?;MAB;3|)
              (LETT #4# (QUOTE T) |HOAGG-;any?;MAB;3|)))))))
       (LETT #1# (CDR #1#) |HOAGG-;any?;MAB;3|) (GO G190) G191 (EXIT NIL))
      (COND (#4# #3#) ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |HOAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82401 #2=#:G82399 #3=#:G82397 #4=#:G82398)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |HOAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;every?;MAB;4|)
      G190
      (COND
        ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;every?;MAB;4|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;every?;MAB;4|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
            |HOAGG-;every?;MAB;4|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |HOAGG-;every?;MAB;4|)
            (LETT #4# (QUOTE T) |HOAGG-;every?;MAB;4|)))))))
      (LETT #1# (CDR #1#) |HOAGG-;every?;MAB;4|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE T))))))))

(DEFUN |HOAGG-;count;MANni;5| (|f| |c| |$|)
 (PROG (|x| #1=#:G82406 #2=#:G82404 #3=#:G82402 #4=#:G82403)
  (RETURN
   (SEQ
     (PROGN
      (LETT #4# NIL |HOAGG-;count;MANni;5|)
      (SEQ
       (LETT |x| NIL |HOAGG-;count;MANni;5|)
       (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;count;MANni;5|)
       G190
       (COND
        ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;count;MANni;5|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (COND
          ((SPADCALL |x| |f|)
           (PROGN
            (LETT #2# 1 |HOAGG-;count;MANni;5|)
            (COND
             (#4# (LETT #3# (|+| #3# #2#) |HOAGG-;count;MANni;5|))
             ((QUOTE T)
              (PROGN
               (LETT #3# #2# |HOAGG-;count;MANni;5|)
               (LETT #4# (QUOTE T) |HOAGG-;count;MANni;5|)))))))))
       (LETT #1# (CDR #1#) |HOAGG-;count;MANni;5|)
       (GO G190)
       G191
       (EXIT NIL))
      (COND (#4# #3#) ((QUOTE T) 0)))))))

(DEFUN |HOAGG-;members;AL;6| (|x| |$|) (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |HOAGG-;count;SANni;7| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;count;SANni;7!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 24)))

(DEFUN |HOAGG-;count;SANni;7!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))

(DEFUN |HOAGG-;member?;SAB;8| (|e| |c| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;member?;SAB;8!0|) (VECTOR |$| |e|))
  |c|
  (QREFELT |$| 26)))

(DEFUN |HOAGG-;member?;SAB;8!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))
```

```
(DEFUN |HOAGG-;=;2AB;9| (|x| |y| |$|)
 (PROG (|b| #1=#:G82416 |a| #2=#:G82415 #3=#:G82412 #4=#:G82410 #5=#:G82411)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (SPADCALL |y| (QREFELT |$| 28)) (QREFELT |$| 29))
      (PROGN
       (LETT #5# NIL |HOAGG-;=;2AB;9|)
       (SEQ
        (LETT |b| NIL |HOAGG-;=;2AB;9|)
        (LETT #1# (SPADCALL |y| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        (LETT |a| NIL |HOAGG-;=;2AB;9|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;=;2AB;9|) NIL)
           (ATOM #1#)
           (PROGN (LETT |b| (CAR #1#) |HOAGG-;=;2AB;9|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (PROGN
           (LETT #3# (SPADCALL |a| |b| (QREFELT |$| 23)) |HOAGG-;=;2AB;9|)
           (COND
             (#5#
              (LETT #4#
               (COND (#4# #3#) ((QUOTE T) (QUOTE NIL)))
               |HOAGG-;=;2AB;9|))
             ((QUOTE T)
              (PROGN
               (LETT #4# #3# |HOAGG-;=;2AB;9|)
               (LETT #5# (QUOTE T) |HOAGG-;=;2AB;9|)))))))
        (LETT #2#
         (PROG1
          (CDR #2#)
          (LETT #1# (CDR #1#) |HOAGG-;=;2AB;9|))
         |HOAGG-;=;2AB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (COND (#5# #4#) ((QUOTE T) (QUOTE T)))))
     ((QUOTE T) (QUOTE NIL)))))))
```

```
(DEFUN |HOAGG-;coerce;AOf;10| (|x| |$|)
 (PROG (#1=#:G82420 |a| #2=#:G82421)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |HOAGG-;coerce;AOf;10|)
       (SEQ
        (LETT |a| NIL |HOAGG-;coerce;AOf;10|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;coerce;AOf;10|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;coerce;AOf;10|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |a| (QREFELT |$| 32)) #1#)
           |HOAGG-;coerce;AOf;10|)))
        (LETT #2# (CDR #2#) |HOAGG-;coerce;AOf;10|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 34))
     (QREFELT |$| 35))))))

(DEFUN |HomogeneousAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|HomogeneousAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |HomogeneousAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 38) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (LIST (QUOTE |Evalable|) (|devaluate| |#2|)))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))))
      . #1#))
     (|stuffDomainSlots| |$|)
     (QSETREFV |$| 6 |#1|)
     (QSETREFV |$| 7 |#2|)
     (COND
      ((|testBitVector| |pv$| 3)
        (QSETREFV |$| 12 (CONS (|dispatchFunction| |HOAGG-;eval;ALA;1|) |$|))))
     (COND
      ((|testBitVector| |pv$| 1)
       (PROGN
        (QSETREFV |$| 16 (CONS (|dispatchFunction| |HOAGG-;#;ANni;2|) |$|))
        (QSETREFV |$| 19 (CONS (|dispatchFunction| |HOAGG-;any?;MAB;3|) |$|))
        (QSETREFV |$| 20 (CONS (|dispatchFunction| |HOAGG-;every?;MAB;4|) |$|))
        (QSETREFV |$| 21 (CONS (|dispatchFunction| |HOAGG-;count;MANni;5|) |$|))
        (QSETREFV |$| 22 (CONS (|dispatchFunction| |HOAGG-;members;AL;6|) |$|))
        (COND
         ((|testBitVector| |pv$| 4)
          (PROGN
           (QSETREFV |$| 25
            (CONS (|dispatchFunction| |HOAGG-;count;SANni;7|) |$|))
           (QSETREFV |$| 27
            (CONS (|dispatchFunction| |HOAGG-;member?;SAB;8|) |$|))
           (QSETREFV |$| 30
            (CONS (|dispatchFunction| |HOAGG-;=;2AB;9|) |$|))
           (QSETREFV |$| 36
            (CONS (|dispatchFunction| |HOAGG-;coerce;AOf;10|) |$|))))))))
     |$|))))

(MAKEPROP
 (QUOTE |HomogeneousAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 37)
   (0 . |eval|) (|Mapping| 7 7) (6 . |map|) (12 . |eval|) (|List| 7)
   (18 . |parts|) (|NonNegativeInteger|) (23 . |#|) (|Boolean|)
   (|Mapping| 17 7) (28 . |any?|) (34 . |every?|) (40 . |count|)
   (46 . |members|) (51 . |=|) (57 . |count|) (63 . |count|) (69 . |any?|)
   (75 . |member?|) (81 . |#|) (86 . |size?|) (92 . |=|) (|OutputForm|)
   (98 . |coerce|) (|List| |$|) (103 . |commaSeparate|) (108 . |bracket|)
   (113 . |coerce|) (|Equation| 7)))
  (QUOTE #(|members| 118 |member?| 123 |every?| 129 |eval| 135 |count| 141
   |coerce| 153 |any?| 158 |=| 164 |#| 170))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 36
    (QUOTE (2 7 0 0 8 9 2 6 0 10 0 11 2 0 0 0 8 12 1 6 13 0 14 1 0 15 0
      16 2 0 17 18 0 19 2 0 17 18 0 20 2 0 15 18 0 21 1 0 13 0 22 2 7 17
      0 0 23 2 6 15 18 0 24 2 0 15 7 0 25 2 6 17 18 0 26 2 0 17 7 0 27 1
      6 15 0 28 2 6 17 0 15 29 2 0 17 0 0 30 1 7 31 0 32 1 31 0 33 34 1
      31 0 0 35 1 0 31 0 36 1 0 13 0 22 2 0 17 7 0 27 2 0 17 18 0 20 2 0
      0 0 8 12 2 0 15 7 0 25 2 0 15 18 0 21 1 0 31 0 36 2 0 17 18 0 19 2
      0 17 0 0 30 1 0 15 0 16))))))
(QUOTE |lookupComplete|)))
```

21.29 INS.lsp BOOTSTRAP

INS depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

⟨*INS.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(SETQ |IntegerNumberSystem;AL| (QUOTE NIL))

(DEFUN |IntegerNumberSystem| NIL
  (LET (#:G1068)
    (COND
      (|IntegerNumberSystem;AL|)
      (T (SETQ |IntegerNumberSystem;AL| (|IntegerNumberSystem;|))))))

(DEFUN |IntegerNumberSystem;| NIL (PROG (#0=#:G1066)
  (RETURN
    (PROG1
      (LETT #0#
        (|sublisV|
          (PAIR
            (QUOTE (#1=#:G1060 #2=#:G1061 #3=#:G1062
                    #4=#:G1063 #5=#:G1064 #6=#:G1065))
            (LIST
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|InputForm|))
              (QUOTE (|Pattern| (|Integer|)))
              (QUOTE (|Integer|))))
          (|Join|
            (|UniqueFactorizationDomain|)
            (|EuclideanDomain|)
            (|OrderedIntegralDomain|)
            (|DifferentialRing|)
            (|ConvertibleTo| (QUOTE #1#))
            (|RetractableTo| (QUOTE #2#))
            (|LinearlyExplicitRingOver| (QUOTE #3#))
            (|ConvertibleTo| (QUOTE #4#))
            (|ConvertibleTo| (QUOTE #5#))
            (|PatternMatchable| (QUOTE #6#))
            (|CombinatorialFunctionCategory|)
```

```
               (|RealConstant|)
               (|CharacteristicZero|)
               (|StepThrough|)
               (|mkCategory|
                 (QUOTE |domain|)
                 (QUOTE (
                   ((|odd?| ((|Boolean|) $)) T)
                   ((|even?| ((|Boolean|) $)) T)
                   ((|base| ($)) T)
                   ((|length| ($ $)) T)
                   ((|shift| ($ $ $)) T)
                   ((|bit?| ((|Boolean|) $ $)) T)
                   ((|positiveRemainder| ($ $ $)) T)
                   ((|symmetricRemainder| ($ $ $)) T)
                   ((|rational?| ((|Boolean|) $)) T)
                   ((|rational| ((|Fraction| (|Integer|)) $)) T)
                   ((|rationalIfCan|
                     ((|Union| (|Fraction| (|Integer|)) "failed") $)) T)
                   ((|random| ($)) T)
                   ((|random| ($ $)) T)
                   ((|hash| ($ $)) T)
                   ((|copy| ($ $)) T)
                   ((|inc| ($ $)) T)
                   ((|dec| ($ $)) T)
                   ((|mask| ($ $)) T)
                   ((|addmod| ($ $ $ $)) T)
                   ((|submod| ($ $ $ $)) T)
                   ((|mulmod| ($ $ $ $)) T)
                   ((|powmod| ($ $ $ $)) T)
                   ((|invmod| ($ $ $)) T)))
                 (QUOTE ((|multiplicativeValuation| T) (|canonicalUnitNormal| T)))
                 (QUOTE ((|Fraction| (|Integer|)) (|Boolean|))) NIL)))
             |IntegerNumberSystem|)
           (SETELT #0# 0 (QUOTE (|IntegerNumberSystem|)))))))

  (MAKEPROP (QUOTE |IntegerNumberSystem|) (QUOTE NILADIC) T)
```

21.30 INS-.lsp BOOTSTRAP

INS- depends on **INS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

⟨*INS-.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(PUT
  (QUOTE |INS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |INS-;characteristic;Nni;1| ($) 0)

(DEFUN |INS-;differentiate;2S;2| (|x| $)
  (|spadConstant| $ 9))

(DEFUN |INS-;even?;SB;3| (|x| $)
 (COND
   ((SPADCALL |x| (QREFELT $ 12)) (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |INS-;positive?;SB;4| (|x| $)
  (SPADCALL (|spadConstant| $ 9) |x| (QREFELT $ 14)))

(PUT
  (QUOTE |INS-;copy;2S;5|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INS-;copy;2S;5| (|x| $) |x|)

(DEFUN |INS-;bit?;2SB;6| (|x| |i| $)
  (SPADCALL
    (SPADCALL |x|
      (SPADCALL |i| (QREFELT $ 17))
      (QREFELT $ 18))
    (QREFELT $ 12)))

(DEFUN |INS-;mask;2S;7| (|n| $)
  (SPADCALL
    (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 18))
    (QREFELT $ 21)))

(PUT
  (QUOTE |INS-;rational?;SB;8|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (QUOTE T))))

(DEFUN |INS-;rational?;SB;8| (|x| $)
  (QUOTE T))

(DEFUN |INS-;euclideanSize;SNni;9| (|x| $)
  (PROG (#0=#:G1078 #1=#:G1079)
    (RETURN
      (COND
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 24))
          (|error| "euclideanSize called on zero"))
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 14))
          (PROG1
            (LETT #0#
              (- (SPADCALL |x| (QREFELT $ 26)))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #0# 0)
              (QUOTE (|NonNegativeInteger|))
              #0#)))
        ((QUOTE T)
          (PROG1
            (LETT #1#
              (SPADCALL |x| (QREFELT $ 26))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #1# 0)
              (QUOTE (|NonNegativeInteger|))
              #1#)))))))

(DEFUN |INS-;convert;SF;10| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 29)))

(DEFUN |INS-;convert;SDf;11| (|x| $)
  (FLOAT (SPADCALL |x| (QREFELT $ 26)) MOST-POSITIVE-LONG-FLOAT))

(DEFUN |INS-;convert;SIf;12| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 34)))

(DEFUN |INS-;retract;SI;13| (|x| $)
  (SPADCALL |x| (QREFELT $ 26)))
```

```
(DEFUN |INS-;convert;SP;14| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 38)))

(DEFUN |INS-;factor;SF;15| (|x| $)
  (SPADCALL |x| (QREFELT $ 42)))

(DEFUN |INS-;squareFree;SF;16| (|x| $)
  (SPADCALL |x| (QREFELT $ 45)))

(DEFUN |INS-;prime?;SB;17| (|x| $)
  (SPADCALL |x| (QREFELT $ 48)))

(DEFUN |INS-;factorial;2S;18| (|x| $)
  (SPADCALL |x| (QREFELT $ 51)))

(DEFUN |INS-;binomial;3S;19| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 53)))

(DEFUN |INS-;permutation;3S;20| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 55)))

(DEFUN |INS-;retractIfCan;SU;21| (|x| $)
  (CONS 0 (SPADCALL |x| (QREFELT $ 26))))

(DEFUN |INS-;init;S;22| ($)
  (|spadConstant| $ 9))

(DEFUN |INS-;nextItem;SU;23| (|n| $)
  (COND
    ((SPADCALL |n| (QREFELT $ 60))
       (CONS 0 (|spadConstant| $ 20)))
    ((SPADCALL (|spadConstant| $ 9) |n| (QREFELT $ 14))
       (CONS 0 (SPADCALL |n| (QREFELT $ 17))))
    ((QUOTE T)
       (CONS 0 (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 61))))))

(DEFUN |INS-;patternMatch;SP2Pmr;24| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 66)))

(DEFUN |INS-;rational;SF;25| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70)))

(DEFUN |INS-;rationalIfCan;SU;26| (|x| $)
  (CONS 0 (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70))))
```

```
(DEFUN |INS-;symmetricRemainder;3S;27| (|x| |n| $)
  (PROG (|r|)
    (RETURN
      (SEQ
        (LETT |r|
          (SPADCALL |x| |n| (QREFELT $ 74))
          |INS-;symmetricRemainder;3S;27|)
        (EXIT
          (COND
            ((SPADCALL |r| (|spadConstant| $ 9) (QREFELT $ 24)) |r|)
            ((QUOTE T)
              (SEQ
                (COND
                  ((SPADCALL |n| (|spadConstant| $ 9) (QREFELT $ 14))
                    (LETT |n|
                      (SPADCALL |n| (QREFELT $ 17))
                      |INS-;symmetricRemainder;3S;27|)))
                (EXIT
                  (COND
                    ((SPADCALL (|spadConstant| $ 9) |r| (QREFELT $ 14))
                      (COND
                        ((SPADCALL |n|
                            (SPADCALL 2 |r| (QREFELT $ 76))
                            (QREFELT $ 14))
                          (SPADCALL |r| |n| (QREFELT $ 61)))
                        ((QUOTE T) |r|)))
                    ((NULL
                      (SPADCALL
                         (|spadConstant| $ 9)
                         (SPADCALL
                           (SPADCALL 2 |r| (QREFELT $ 76))
                           |n|
                           (QREFELT $ 77))
                         (QREFELT $ 14)))
                       (SPADCALL |r| |n| (QREFELT $ 77)))
                    ((QUOTE T) |r|)))))))))))

(DEFUN |INS-;invmod;3S;28| (|a| |b| $)
  (PROG (|q| |r| |r1| |c| |c1| |d| |d1|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |a| (QREFELT $ 79))
            (LETT |a| (SPADCALL |a| |b| (QREFELT $ 80)) |INS-;invmod;3S;28|)))
        (LETT |c| |a| |INS-;invmod;3S;28|)
        (LETT |c1| (|spadConstant| $ 20) |INS-;invmod;3S;28|)
        (LETT |d| |b| |INS-;invmod;3S;28|)
        (LETT |d1| (|spadConstant| $ 9) |INS-;invmod;3S;28|)
        (SEQ G190
          (COND
            ((NULL
              (COND
                ((SPADCALL |d| (QREFELT $ 60)) (QUOTE NIL))
                ((QUOTE T) (QUOTE T))))
              (GO G191)))
          (SEQ
            (LETT |q| (SPADCALL |c| |d| (QREFELT $ 81)) |INS-;invmod;3S;28|)
            (LETT |r|
              (SPADCALL |c| (SPADCALL |q| |d| (QREFELT $ 82)) (QREFELT $ 61))
              |INS-;invmod;3S;28|)
            (LETT |r1|
              (SPADCALL |c1| (SPADCALL |q| |d1| (QREFELT $ 82)) (QREFELT $ 61))
              |INS-;invmod;3S;28|)
            (LETT |c| |d| |INS-;invmod;3S;28|)
            (LETT |c1| |d1| |INS-;invmod;3S;28|)
            (LETT |d| |r| |INS-;invmod;3S;28|)
            (EXIT (LETT |d1| |r1| |INS-;invmod;3S;28|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (COND
          ((NULL (SPADCALL |c| (QREFELT $ 83)))
            (EXIT (|error| "inverse does not exist"))))
        (EXIT
          (COND
            ((SPADCALL |c1| (QREFELT $ 79)) (SPADCALL |c1| |b| (QREFELT $ 77)))
            ((QUOTE T) |c1|)))))))

(DEFUN |INS-;powmod;4S;29| (|x| |n| |p| $)
  (PROG (|y| #0=#:G1137 |z|)
    (RETURN
      (SEQ
        (EXIT
          (SEQ
            (COND
              ((SPADCALL |x| (QREFELT $ 79))
                (LETT |x|
                  (SPADCALL |x| |p| (QREFELT $ 80))
                  |INS-;powmod;4S;29|)))
             (EXIT
              (COND
               ((SPADCALL |x| (QREFELT $ 60)) (|spadConstant| $ 9))
               ((SPADCALL |n| (QREFELT $ 60)) (|spadConstant| $ 20))
               ((QUOTE T)
                 (SEQ
                   (LETT |y| (|spadConstant| $ 20) |INS-;powmod;4S;29|)
                   (LETT |z| |x| |INS-;powmod;4S;29|)
                   (EXIT
                     (SEQ G190
                       NIL
                       (SEQ
                         (COND
                           ((SPADCALL |n| (QREFELT $ 12))
                             (LETT |y|
                               (SPADCALL |y| |z| |p| (QREFELT $ 85))
                               |INS-;powmod;4S;29|)))
                         (EXIT
                           (COND
                             ((SPADCALL
                               (LETT |n|
                                 (SPADCALL |n|
                                   (SPADCALL
                                     (|spadConstant| $ 20)
                                     (QREFELT $ 17))
                                   (QREFELT $ 18))
                                  |INS-;powmod;4S;29|)
                               (QREFELT $ 60))
                              (PROGN
                                (LETT #0# |y| |INS-;powmod;4S;29|)
                                (GO #0#)))
                             ((QUOTE T)
                               (LETT |z|
                                 (SPADCALL |z| |z| |p| (QREFELT $ 85))
                                 |INS-;powmod;4S;29|)))))
                       NIL
                       (GO G190)
                       G191
                       (EXIT NIL)))))))))
        #0#
        (EXIT #0#)))))

(DEFUN |IntegerNumberSystem&| (|#1|)
  (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) . #0=(|IntegerNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |IntegerNumberSystem&|) DV$1) . #0#)
```

```
          (LETT $ (GETREFV 87) . #0#)
          (QSETREFV $ 0 |dv$|)
          (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
          (|stuffDomainSlots| $)
          (QSETREFV $ 6 |#1|)
          $))))

  (MAKEPROP
    (QUOTE |IntegerNumberSystem&|)
    (QUOTE |infovec|)
    (LIST
      (QUOTE
        #(NIL NIL NIL NIL NIL NIL
          (|local| |#1|)
          (|NonNegativeInteger|)
          |INS-;characteristic;Nni;1|
          (0 . |Zero|)
          |INS-;differentiate;2S;2|
          (|Boolean|)
          (4 . |odd?|)
          |INS-;even?;SB;3|
          (9 . <)
          |INS-;positive?;SB;4|
          |INS-;copy;2S;5|
          (15 . -)
          (20 . |shift|)
          |INS-;bit?;2SB;6|
          (26 . |One|)
          (30 . |dec|)
          |INS-;mask;2S;7|
          |INS-;rational?;SB;8|
          (35 . =)
          (|Integer|)
          (41 . |convert|)
          |INS-;euclideanSize;SNni;9|
          (|Float|)
          (46 . |coerce|)
          |INS-;convert;SF;10|
          (|DoubleFloat|)
          |INS-;convert;SDf;11|
          (|InputForm|)
          (51 . |convert|)
          |INS-;convert;SIf;12|
          |INS-;retract;SI;13|
          (|Pattern| 25)
          (56 . |coerce|)
```

```
|INS-;convert;SP;14|
(|Factored| 6)
(|IntegerFactorizationPackage| 6)
(61 . |factor|)
(|Factored| $)
|INS-;factor;SF;15|
(66 . |squareFree|)
|INS-;squareFree;SF;16|
(|IntegerPrimesPackage| 6)
(71 . |prime?|)
|INS-;prime?;SB;17|
(|IntegerCombinatoricFunctions| 6)
(76 . |factorial|)
|INS-;factorial;2S;18|
(81 . |binomial|)
|INS-;binomial;3S;19|
(87 . |permutation|)
|INS-;permutation;3S;20|
(|Union| 25 (QUOTE "failed"))
|INS-;retractIfCan;SU;21|
|INS-;init;S;22|
(93 . |zero?|)
(98 . -)
(|Union| $ (QUOTE "failed"))
|INS-;nextItem;SU;23|
(|PatternMatchResult| 25 6)
(|PatternMatchIntegerNumberSystem| 6)
(104 . |patternMatch|)
(|PatternMatchResult| 25 $)
|INS-;patternMatch;SP2Pmr;24|
(|Fraction| 25)
(111 . |coerce|)
|INS-;rational;SF;25|
(|Union| 69 (QUOTE "failed"))
|INS-;rationalIfCan;SU;26|
(116 . |rem|)
(|PositiveInteger|)
(122 . *)
(128 . +)
|INS-;symmetricRemainder;3S;27|
(134 . |negative?|)
(139 . |positiveRemainder|)
(145 . |quo|)
(151 . *)
(157 . |one?|)
|INS-;invmod;3S;28|
    (162 . |mulmod|)
    |INS-;powmod;4S;29|))
 (QUOTE
   #(|symmetricRemainder| 169 |squareFree| 175 |retractIfCan| 180
     |retract| 185 |rationalIfCan| 190 |rational?| 195 |rational| 200
     |prime?| 205 |powmod| 210 |positive?| 217 |permutation| 222
     |patternMatch| 228 |nextItem| 235 |mask| 240 |invmod| 245 |init| 251
     |factorial| 255 |factor| 260 |even?| 265 |euclideanSize| 270
     |differentiate| 275 |copy| 280 |convert| 285 |characteristic| 305
     |bit?| 309 |binomial| 315))
 (QUOTE NIL)
 (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
     (QUOTE #())
     (CONS
       (QUOTE #())
       (|makeByteWordVec2| 86
         (QUOTE
           (0 6 0 9 1 6 11 0 12 2 6 11 0 0 14 1 6 0 0 17 2 6 0 0 0 18 0 6
            0 20 1 6 0 0 21 2 6 11 0 0 24 1 6 25 0 26 1 28 0 25 29 1 33 0
            25 34 1 37 0 25 38 1 41 40 6 42 1 41 40 6 45 1 47 11 6 48 1 50
            6 6 51 2 50 6 6 6 53 2 50 6 6 6 55 1 6 11 0 60 2 6 0 0 0 61 3
            65 64 6 37 64 66 1 69 0 25 70 2 6 0 0 0 74 2 6 0 75 0 76 2 6 0
            0 0 77 1 6 11 0 79 2 6 0 0 0 80 2 6 0 0 0 81 2 6 0 0 0 82 1 6
            11 0 83 3 6 0 0 0 0 85 2 0 0 0 0 78 1 0 43 0 46 1 0 57 0 58 1
            0 25 0 36 1 0 72 0 73 1 0 11 0 23 1 0 69 0 71 1 0 11 0 49 3 0
            0 0 0 0 86 1 0 11 0 15 2 0 0 0 0 56 3 0 67 0 37 67 68 1 0 62
            0 63 1 0 0 0 22 2 0 0 0 0 84 0 0 0 59 1 0 0 0 52 1 0 43 0 44
            1 0 11 0 13 1 0 7 0 27 1 0 0 0 10 1 0 0 0 16 1 0 31 0 32 1 0
            28 0 30 1 0 37 0 39 1 0 33 0 35 0 0 7 8 2 0 11 0 0 19 2 0 0
            0 0 54))))))
(QUOTE |lookupComplete|)))
```

21.31 INTDOM.lsp BOOTSTRAP

INTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*INTDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |IntegralDomain;AL| (QUOTE NIL))

(DEFUN |IntegralDomain| NIL
  (LET (#:G83060)
    (COND
      (|IntegralDomain;AL|)
      (T (SETQ |IntegralDomain;AL| (|IntegralDomain;|))))))

(DEFUN |IntegralDomain;| NIL
  (PROG (#1=#:G83058)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|CommutativeRing|)
            (|Algebra| (QUOTE |$|))
            (|EntireRing|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|exquo| ((|Union| |$| "failed") |$| |$|)) T)
                ((|unitNormal|
                  ((|Record|
                      (|:| |unit| |$|)
                      (|:| |canonical| |$|)
                      (|:| |associate| |$|)) |$|)) T)
                ((|unitCanonical| (|$| |$|)) T)
                ((|associates?| ((|Boolean|) |$| |$|)) T)
                ((|unit?| ((|Boolean|) |$|)) T)))
              NIL
              (QUOTE ((|Boolean|)))
              NIL))
           |IntegralDomain|)
          (SETELT #1# 0 (QUOTE (|IntegralDomain|)))))))

 (MAKEPROP (QUOTE |IntegralDomain|) (QUOTE NILADIC) T)
```

21.32 INTDOM-.lsp BOOTSTRAP

INTDOM- depends on **INTDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*INTDOM-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |INTDOM-;unitNormal;SR;1| (|x| |$|)
  (VECTOR (|spadConstant| |$| 7) |x| (|spadConstant| |$| 7)))

(DEFUN |INTDOM-;unitCanonical;2S;2| (|x| |$|)
  (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1))

(DEFUN |INTDOM-;recip;SU;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (CONS 1 "failed"))
    ((QUOTE T) (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 15)))))

(DEFUN |INTDOM-;unit?;SB;4| (|x| |$|)
  (COND
    ((QEQCAR (SPADCALL |x| (QREFELT |$| 17)) 1) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |INTDOM-;associates?;2SB;5| (|x| |y| |$|)
  (SPADCALL
    (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1)
    (QVELT (SPADCALL |y| (QREFELT |$| 10)) 1)
    (QREFELT |$| 19)))

(DEFUN |INTDOM-;associates?;2SB;6| (|x| |y| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |y| (QREFELT |$| 13)))
    ((OR
        (SPADCALL |y| (QREFELT |$| 13))
        (OR
          (QEQCAR (SPADCALL |x| |y| (QREFELT |$| 15)) 1)
          (QEQCAR (SPADCALL |y| |x| (QREFELT |$| 15)) 1)))
      (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))
```

```
(DEFUN |IntegralDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|IntegralDomain&|))
        (LETT |dv$| (LIST (QUOTE |IntegralDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 21) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Field|))))
          ((QUOTE T)
            (QSETREFV |$| 9
              (CONS (|dispatchFunction| |INTDOM-;unitNormal;SR;1|) |$|))))
        (COND
          ((|HasAttribute| |#1| (QUOTE |canonicalUnitNormal|))
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;5|) |$|)))
          ((QUOTE T)
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;6|) |$|))))
        |$|))))

(MAKEPROP
  (QUOTE |IntegralDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |One|)
        (|Record|
          (|:| |unit| |$|)
          (|:| |canonical| |$|)
          (|:| |associate| |$|))
        (4 . |unitNormal|)
        (9 . |unitNormal|)
        |INTDOM-;unitCanonical;2S;2|
        (|Boolean|)
        (14 . |zero?|)
        (|Union| |$| (QUOTE "failed"))
        (19 . |exquo|)
        |INTDOM-;recip;SU;3|
        (25 . |recip|)
      |INTDOM-;unit?;SB;4|
      (30 . |=|)
      (36 . |associates?|)))
  (QUOTE
    #(|unitNormal| 42 |unitCanonical| 47 |unit?| 52 |recip| 57
      |associates?| 62))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 20
          (QUOTE
            (0 6 0 7 1 0 8 0 9 1 6 8 0 10 1 6 12 0 13 2 6 14 0 0 15 1 6 14
             0 17 2 6 12 0 0 19 2 0 12 0 0 20 1 0 8 0 9 1 0 0 0 11 1 0 12 0
             18 1 0 14 0 16 2 0 12 0 0 20))))))
 (QUOTE |lookupComplete|)))
```

21.33 LNAGG.lsp BOOTSTRAP

LNAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LNAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |LinearAggregate;CAT| (QUOTE NIL))

(SETQ |LinearAggregate;AL| (QUOTE NIL))

(DEFUN |LinearAggregate| (#1=#:G85818)
 (LET (#2=#:G85819)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |LinearAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |LinearAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|LinearAggregate;| #1#)))
      |LinearAggregate;AL|))
    #2#))))

(DEFUN |LinearAggregate;| (|t#1|)
 (PROG (#1=#:G85817)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G85816)) (LIST (QUOTE (|Integer|))))
       (COND
        (|LinearAggregate;CAT|)
        ((QUOTE T)
         (LETT |LinearAggregate;CAT|
          (|Join|
           (|IndexedAggregate| (QUOTE #2#) (QUOTE |t#1|))
           (|Collection| (QUOTE |t#1|))
           (|mkCategory|
            (QUOTE |domain|)
          (QUOTE (
           ((|new| (|$| (|NonNegativeInteger|) |t#1|)) T)
           ((|concat| (|$| |$| |t#1|)) T)
           ((|concat| (|$| |t#1| |$|)) T)
           ((|concat| (|$| |$| |$|)) T)
           ((|concat| (|$| (|List| |$|))) T)
           ((|map| (|$| (|Mapping| |t#1| |t#1| |t#1|) |$| |$|)) T)
           ((|elt| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
           ((|delete| (|$| |$| (|Integer|))) T)
           ((|delete| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
           ((|insert| (|$| |t#1| |$| (|Integer|))) T)
           ((|insert| (|$| |$| |$| (|Integer|))) T)
           ((|setelt| (|t#1| |$| (|UniversalSegment| (|Integer|)) |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
          NIL
          (QUOTE (
            (|UniversalSegment| (|Integer|))
            (|Integer|)
            (|List| |$|)
            (|NonNegativeInteger|)))
           NIL))
        . #3=(|LinearAggregate|))))))
    . #3#)
   (SETELT #1# 0 (LIST (QUOTE |LinearAggregate|) (|devaluate| |t#1|)))))))
```

21.34 LNAGG-.lsp BOOTSTRAP

LNAGG- depends on **LNAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LNAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |LNAGG-;indices;AL;1| (|a| |$|)
 (PROG (#1=#:G85833 |i| #2=#:G85834)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |LNAGG-;indices;AL;1|)
     (SEQ
      (LETT |i| (SPADCALL |a| (QREFELT |$| 9)) |LNAGG-;indices;AL;1|)
      (LETT #2# (SPADCALL |a| (QREFELT |$| 10)) |LNAGG-;indices;AL;1|)
      G190
      (COND ((|>| |i| #2#) (GO G191)))
      (SEQ (EXIT (LETT #1# (CONS |i| #1#) |LNAGG-;indices;AL;1|)))
      (LETT |i| (|+| |i| 1) |LNAGG-;indices;AL;1|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#))))))))

(DEFUN |LNAGG-;index?;IAB;2| (|i| |a| |$|)
 (COND
  ((OR
     (|<| |i| (SPADCALL |a| (QREFELT |$| 9)))
     (|<| (SPADCALL |a| (QREFELT |$| 10)) |i|))
    (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |LNAGG-;concat;ASA;3| (|a| |x| |$|)
 (SPADCALL |a| (SPADCALL 1 |x| (QREFELT |$| 16)) (QREFELT |$| 17)))

(DEFUN |LNAGG-;concat;S2A;4| (|x| |y| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |y| (QREFELT |$| 17)))

(DEFUN |LNAGG-;insert;SAIA;5| (|x| |a| |i| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |a| |i| (QREFELT |$| 20)))
```

```
(DEFUN |LNAGG-;maxIndex;AI;6| (|l| |$|)
 (|+| (|-| (SPADCALL |l| (QREFELT |$| 22)) 1) (SPADCALL |l| (QREFELT |$| 9))))

(DEFUN |LinearAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|LinearAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |LinearAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 25) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
      (QSETREFV |$| 23
       (CONS (|dispatchFunction| |LNAGG-;maxIndex;AI;6|) |$|))))
    |$|))))

(MAKEPROP
 (QUOTE |LinearAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|Integer|)
    (0 . |minIndex|) (5 . |maxIndex|) (|List| 8) |LNAGG-;indices;AL;1|
    (|Boolean|) |LNAGG-;index?;IAB;2| (|NonNegativeInteger|) (10 . |new|)
    (16 . |concat|) |LNAGG-;concat;ASA;3| |LNAGG-;concat;S2A;4|
    (22 . |insert|) |LNAGG-;insert;SAIA;5| (29 . |#|) (34 . |maxIndex|)
    (|List| |$|)))
  (QUOTE #(|maxIndex| 39 |insert| 44 |indices| 51 |index?| 56 |concat| 62))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 23 (QUOTE (1 6 8 0 9 1 6 8 0 10 2 6 0 15 7
     16 2 6 0 0 0 17 3 6 0 0 0 8 20 1 6 15 0 22 1 0 8 0 23 1 0 8 0 23 3 0
     0 7 0 8 21 1 0 11 0 12 2 0 13 8 0 14 2 0 0 0 7 18 2 0 0 7 0 19))))))
  (QUOTE |lookupComplete|)))
```

21.35 LSAGG.lsp BOOTSTRAP

LSAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LSAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |ListAggregate;CAT| (QUOTE NIL))

(SETQ |ListAggregate;AL| (QUOTE NIL))

(DEFUN |ListAggregate| (#1=#:G87500)
 (LET (#2=#:G87501)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |ListAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |ListAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|ListAggregate;| #1#)))
      |ListAggregate;AL|))
    #2#))))

(DEFUN |ListAggregate;| (|t#1|)
 (PROG (#1=#:G87499)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|ListAggregate;CAT|)
       ((QUOTE T)
        (LETT |ListAggregate;CAT|
         (|Join|
          (|StreamAggregate| (QUOTE |t#1|))
          (|FiniteLinearAggregate| (QUOTE |t#1|))
          (|ExtensibleLinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (((|list| (|$| |t#1|)) T)))
```

```
             NIL
             (QUOTE NIL)
              NIL))
            . #2=(|ListAggregate|)))))
       . #2#)
      (SETELT #1# 0 (LIST (QUOTE |ListAggregate|) (|devaluate| |t#1|)))))))
```

21.36 LSAGG-.lsp BOOTSTRAP

LSAGG- depends on **LSAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LSAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |LSAGG-;sort!;M2A;1| (|f| |l| |$|)
 (|LSAGG-;mergeSort| |f| |l| (SPADCALL |l| (QREFELT |$| 9)) |$|))

(DEFUN |LSAGG-;list;SA;2| (|x| |$|)
 (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 13)))

(DEFUN |LSAGG-;reduce;MAS;3| (|f| |x| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 16))
    (|error| "reducing over an empty list needs the 3 argument form"))
  ((QUOTE T)
    (SPADCALL |f|
     (SPADCALL |x| (QREFELT |$| 17))
     (SPADCALL |x| (QREFELT |$| 18))
     (QREFELT |$| 20)))))

(DEFUN |LSAGG-;merge;M3A;4| (|f| |p| |q| |$|)
 (SPADCALL |f|
  (SPADCALL |p| (QREFELT |$| 22))
  (SPADCALL |q| (QREFELT |$| 22))
  (QREFELT |$| 23)))

(DEFUN |LSAGG-;select!;M2A;5| (|f| |x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (SEQ G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
```

```
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 16)) |x|)
      ((QUOTE T)
       (SEQ
        (LETT |y| |x| |LSAGG-;select!;M2A;5|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL |z| (QREFELT |$| 16)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((SPADCALL (SPADCALL |z| (QREFELT |$| 18)) |f|)
              (SEQ
               (LETT |y| |z| |LSAGG-;select!;M2A;5|)
               (EXIT
                (LETT |z|
                 (SPADCALL |z| (QREFELT |$| 17))
                 |LSAGG-;select!;M2A;5|))))
            ((QUOTE T)
             (SEQ
              (LETT |z| (SPADCALL |z| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
              (EXIT (SPADCALL |y| |z| (QREFELT |$| 25))))))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;merge!;M3A;6| (|f| |p| |q| |$|)
(PROG (|r| |t|)
 (RETURN
  (SEQ
   (COND
    ((SPADCALL |p| (QREFELT |$| 16)) |q|)
    ((SPADCALL |q| (QREFELT |$| 16)) |p|)
    ((SPADCALL |p| |q| (QREFELT |$| 28))
      (|error| "cannot merge a list into itself"))
    ((QUOTE T)
     (SEQ
      (COND
       ((SPADCALL
         (SPADCALL |p| (QREFELT |$| 18))
         (SPADCALL |q| (QREFELT |$| 18))
         |f|)
        (SEQ
         (LETT |r| (LETT |t| |p| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
         (EXIT
          (LETT |p| (SPADCALL |p| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|))))
       ((QUOTE T)
        (SEQ
         (LETT |r| (LETT |t| |q| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
         (EXIT
          (LETT |q| (SPADCALL |q| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|)))))
      (SEQ
       G190
       (COND
        ((NULL
           (COND
            ((OR
              (SPADCALL |p| (QREFELT |$| 16))
              (SPADCALL |q| (QREFELT |$| 16)))
             (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (EXIT
         (COND
          ((SPADCALL
             (SPADCALL |p| (QREFELT |$| 18))
             (SPADCALL |q| (QREFELT |$| 18))
             |f|)
           (SEQ
            (SPADCALL |t| |p| (QREFELT |$| 25))
            (LETT |t| |p| |LSAGG-;merge!;M3A;6|)
            (EXIT
              (LETT |p|
               (SPADCALL |p| (QREFELT |$| 17))
               |LSAGG-;merge!;M3A;6|))))
           ((QUOTE T)
            (SEQ
             (SPADCALL |t| |q| (QREFELT |$| 25))
             (LETT |t| |q| |LSAGG-;merge!;M3A;6|)
             (EXIT
              (LETT |q|
               (SPADCALL |q| (QREFELT |$| 17))
               |LSAGG-;merge!;M3A;6|)))))))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (SPADCALL |t|
        (COND
         ((SPADCALL |p| (QREFELT |$| 16)) |q|)
         ((QUOTE T) |p|))
        (QREFELT |$| 25))
       (EXIT |r|))))))))

(DEFUN |LSAGG-;insert!;SAIA;7| (|s| |x| |i| |$|)
 (PROG (|m| #1=#:G87547 |y| |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;SAIA;7|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |s| |x| (QREFELT |$| 13)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;SAIA;7|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;insert!;SAIA;7|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;SAIA;7|)
        (SPADCALL |y| (SPADCALL |s| |z| (QREFELT |$| 13)) (QREFELT |$| 25))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;insert!;2AIA;8| (|w| |x| |i| |$|)
 (PROG (|m| #1=#:G87551 |y| |z|)
    (RETURN
     (SEQ
      (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;2AIA;8|)
      (EXIT
       (COND
        ((|<| |i| |m|) (|error| "index out of range"))
        ((EQL |i| |m|) (SPADCALL |w| |x| (QREFELT |$| 34)))
        ((QUOTE T)
         (SEQ
          (LETT |y|
           (SPADCALL |x|
            (PROG1
             (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;2AIA;8|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 32))
           |LSAGG-;insert!;2AIA;8|)
          (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;2AIA;8|)
          (SPADCALL |y| |w| (QREFELT |$| 25))
          (SPADCALL |y| |z| (QREFELT |$| 34))
          (EXIT |x|)))))))))

  (DEFUN |LSAGG-;remove!;M2A;9| (|f| |x| |$|)
   (PROG (|p| |q|)
    (RETURN
     (SEQ
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
           ((QUOTE T) (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))))
         (GO G191)))
       (SEQ
        (EXIT
         (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 16)) |x|)
        ((QUOTE T)
         (SEQ
          (LETT |p| |x| |LSAGG-;remove!;M2A;9|)
          (LETT |q| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)
          (SEQ
           G190
           (COND
            ((NULL
               (COND
                ((SPADCALL |q| (QREFELT |$| 16)) (QUOTE NIL))
                ((QUOTE T) (QUOTE T))))
              (GO G191)))
           (SEQ
            (EXIT
             (COND
              ((SPADCALL (SPADCALL |q| (QREFELT |$| 18)) |f|)
                (LETT |q|
                 (SPADCALL |p| (SPADCALL |q| (QREFELT |$| 17)) (QREFELT |$| 25))
                 |LSAGG-;remove!;M2A;9|))
              ((QUOTE T)
                (SEQ
                 (LETT |p| |q| |LSAGG-;remove!;M2A;9|)
                 (EXIT
                  (LETT |q|
                    (SPADCALL |q| (QREFELT |$| 17))
                    |LSAGG-;remove!;M2A;9|)))))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |x|)))))))))

(DEFUN |LSAGG-;delete!;AIA;10| (|x| |i| |$|)
 (PROG (|m| #1=#:G87564 |y|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AIA;10|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |x| (QREFELT |$| 17)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;delete!;AIA;10|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
        |LSAGG-;delete!;AIA;10|)
       (SPADCALL |y| (SPADCALL |y| 2 (QREFELT |$| 32)) (QREFELT |$| 25))
       (EXIT |x|)))))))))

(DEFUN |LSAGG-;delete!;AUsA;11| (|x| |i| |$|)
 (PROG (|l| |m| |h| #1=#:G87569 #2=#:G87570 |t| #3=#:G87571)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |i| (QREFELT |$| 39)) |LSAGG-;delete!;AUsA;11|)
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AUsA;11|)
    (EXIT
     (COND
      ((|<| |l| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (COND
          ((SPADCALL |i| (QREFELT |$| 40)) (SPADCALL |i| (QREFELT |$| 41)))
          ((QUOTE T) (SPADCALL |x| (QREFELT |$| 42))))
         |LSAGG-;delete!;AUsA;11|)
        (EXIT
         (COND
          ((|<| |h| |l|) |x|)
          ((EQL |l| |m|)
           (SPADCALL |x|
            (PROG1
             (LETT #1# (|-| (|+| |h| 1) |m|) |LSAGG-;delete!;AUsA;11|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 32)))
          ((QUOTE T)
           (SEQ
            (LETT |t|
             (SPADCALL |x|
              (PROG1
               (LETT #2# (|-| (|-| |l| 1) |m|) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #2# 0)
                 (QUOTE (|NonNegativeInteger|)) #2#))
              (QREFELT |$| 32))
             |LSAGG-;delete!;AUsA;11|)
            (SPADCALL |t|
             (SPADCALL |t|
              (PROG1
               (LETT #3# (|+| (|-| |h| |l|) 2) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #3# 0)
                (QUOTE (|NonNegativeInteger|)) #3#))
              (QREFELT |$| 32))
              (QREFELT |$| 25))
             (EXIT |x|)))))))))))))

(DEFUN |LSAGG-;find;MAU;12| (|f| |x| |$|)
 (SEQ
  (SEQ
   G190
   (COND
    ((NULL
      (COND
       ((OR
          (SPADCALL |x| (QREFELT |$| 16))
          (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
         (QUOTE NIL))
       ((QUOTE T) (QUOTE T))))
      (GO G191)))
   (SEQ (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;find;MAU;12|)))
   NIL
   (GO G190)
   G191
   (EXIT NIL))
  (EXIT
   (COND
    ((SPADCALL |x| (QREFELT |$| 16)) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (SPADCALL |x| (QREFELT |$| 18))))))))

(DEFUN |LSAGG-;position;MAI;13| (|f| |x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;MAI;13|)
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;MAI;13|)))
     (LETT |k| (|+| |k| 1) |LSAGG-;position;MAI;13|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 16)) (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
      ((QUOTE T) |k|)))))))

(DEFUN |LSAGG-;mergeSort| (|f| |p| |n| |$|)
 (PROG (#1=#:G87593 |l| |q|)
  (RETURN
   (SEQ
    (COND
     ((EQL |n| 2)
      (COND
       ((SPADCALL
         (SPADCALL (SPADCALL |p| (QREFELT |$| 17)) (QREFELT |$| 18))
         (SPADCALL |p| (QREFELT |$| 18)) |f|)
        (LETT |p| (SPADCALL |p| (QREFELT |$| 47)) |LSAGG-;mergeSort|)))))
    (EXIT
     (COND
      ((|<| |n| 3) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |l|
         (PROG1
          (LETT #1# (QUOTIENT2 |n| 2) |LSAGG-;mergeSort|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         |LSAGG-;mergeSort|)
        (LETT |q| (SPADCALL |p| |l| (QREFELT |$| 48)) |LSAGG-;mergeSort|)
        (LETT |p| (|LSAGG-;mergeSort| |f| |p| |l| |$|) |LSAGG-;mergeSort|)
        (LETT |q|
          (|LSAGG-;mergeSort| |f| |q| (|-| |n| |l|) |$|)
          |LSAGG-;mergeSort|)
        (EXIT (SPADCALL |f| |p| |q| (QREFELT |$| 23))))))))))

(DEFUN |LSAGG-;sorted?;MAB;15| (|f| |l| |$|)
 (PROG (#1=#:G87603 |p|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |l| (QREFELT |$| 16)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (LETT |p| (SPADCALL |l| (QREFELT |$| 17)) |LSAGG-;sorted?;MAB;15|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((NULL
              (SPADCALL
               (SPADCALL |l| (QREFELT |$| 18))
               (SPADCALL |p| (QREFELT |$| 18))
               |f|))
              (PROGN (LETT #1# (QUOTE NIL) |LSAGG-;sorted?;MAB;15|) (GO #1#)))
            ((QUOTE T)
             (LETT |p|
              (SPADCALL
               (LETT |l| |p| |LSAGG-;sorted?;MAB;15|)
               (QREFELT |$| 17))
              |LSAGG-;sorted?;MAB;15|)))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT (QUOTE T))))))
    #1#
    (EXIT #1#)))))

(DEFUN |LSAGG-;reduce;MA2S;16| (|f| |x| |i| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA2S;16|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |r|
        (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
        |LSAGG-;reduce;MA2S;16|)
       (EXIT
        (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA2S;16|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |r|)))))

(DEFUN |LSAGG-;reduce;MA3S;17| (|f| |x| |i| |a| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA3S;17|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
            (SPADCALL |x| (QREFELT |$| 16))
            (SPADCALL |r| |a| (QREFELT |$| 51)))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T)))) (GO G191)))
     (SEQ
      (LETT |r|
       (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
       |LSAGG-;reduce;MA3S;17|)
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA3S;17|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |r|)))))

(DEFUN |LSAGG-;new;NniSA;18| (|n| |s| |$|)
 (PROG (|k| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL (QREFELT |$| 12)) |LSAGG-;new;NniSA;18|)
    (SEQ
     (LETT |k| 1 |LSAGG-;new;NniSA;18|)
     G190
     (COND ((QSGREATERP |k| |n|) (GO G191)))
```

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      (SEQ
       (EXIT
        (LETT |l| (SPADCALL |s| |l| (QREFELT |$| 13)) |LSAGG-;new;NniSA;18|)))
      (LETT |k| (QSADD1 |k|) |LSAGG-;new;NniSA;18|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |l|)))))

(DEFUN |LSAGG-;map;M3A;19| (|f| |x| |y| |$|)
 (PROG (|z|)
  (RETURN
   (SEQ
    (LETT |z| (SPADCALL (QREFELT |$| 12)) |LSAGG-;map;M3A;19|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR (SPADCALL |x| (QREFELT |$| 16)) (SPADCALL |y| (QREFELT |$| 16)))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |z|
       (SPADCALL
        (SPADCALL
          (SPADCALL |x| (QREFELT |$| 18))
          (SPADCALL |y| (QREFELT |$| 18))
          |f|)
         |z|
         (QREFELT |$| 13))
       |LSAGG-;map;M3A;19|)
      (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)
      (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |z| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;reverse!;2A;20| (|x| |$|)
 (PROG (|z| |y|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL |x| (QREFELT |$| 16))
       (SPADCALL
        (LETT |y| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
        (QREFELT |$| 16)))
      |x|)
     ((QUOTE T)
      (SEQ
       (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 25))
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
         (SPADCALL |y| |x| (QREFELT |$| 25))
         (LETT |x| |y| |LSAGG-;reverse!;2A;20|)
         (EXIT (LETT |y| |z| |LSAGG-;reverse!;2A;20|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |LSAGG-;copy;2A;21| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (LETT |y| (SPADCALL (QREFELT |$| 12)) |LSAGG-;copy;2A;21|)
    (SEQ
     (LETT |k| 0 |LSAGG-;copy;2A;21|)
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((EQL |k| 1000)
         (COND
           ((SPADCALL |x| (QREFELT |$| 56)) (EXIT (|error| "cyclic list"))))))
      (LETT |y|
       (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |y| (QREFELT |$| 13))
       |LSAGG-;copy;2A;21|)
      (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copy;2A;21|)))
     (LETT |k| (QSADD1 |k|) |LSAGG-;copy;2A;21|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |y| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;copyInto!;2AIA;22| (|y| |x| |s| |$|)
 (PROG (|m| #1=#:G87636 |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |y| (QREFELT |$| 31)) |LSAGG-;copyInto!;2AIA;22|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |z|
         (SPADCALL |y|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;copyInto!;2AIA;22|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;copyInto!;2AIA;22|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |z| (QREFELT |$| 16))
               (SPADCALL |x| (QREFELT |$| 16)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (SPADCALL |z| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 58))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copyInto!;2AIA;22|)
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 17))
            |LSAGG-;copyInto!;2AIA;22|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |y|)))))))))

(DEFUN |LSAGG-;position;SA2I;23| (|w| |x| |s| |$|)
 (PROG (|m| #1=#:G87644 |k|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;SA2I;23|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;position;SA2I;23|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;position;SA2I;23|)
        (SEQ
         (LETT |k| |s| |LSAGG-;position;SA2I;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 16))
               (SPADCALL |w| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 51)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |x|
            (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;SA2I;23|)))
         (LETT |k| (|+| |k| 1) |LSAGG-;position;SA2I;23|)
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT
         (COND
          ((SPADCALL |x| (QREFELT |$| 16))
             (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
           ((QUOTE T) |k|)))))))))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24| (|l| |$|)
 (PROG (|p|)
  (RETURN
   (SEQ
    (LETT |p| |l| |LSAGG-;removeDuplicates!;2A;24|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |p|
        (SPADCALL |p|
         (SPADCALL
          (CONS
           (FUNCTION |LSAGG-;removeDuplicates!;2A;24!0|)
           (VECTOR |$| |p|))
          (SPADCALL |p| (QREFELT |$| 17))
          (QREFELT |$| 61))
         (QREFELT |$| 25))
        |LSAGG-;removeDuplicates!;2A;24|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |l|)))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24!0| (|#1| |$$|)
 (PROG (|$|)
  (LETT |$| (QREFELT |$$| 0) |LSAGG-;removeDuplicates!;2A;24|)
  (RETURN
   (PROGN
    (SPADCALL |#1|
     (SPADCALL (QREFELT |$$| 1) (QREFELT |$| 18))
     (QREFELT |$| 51))))))

(DEFUN |LSAGG-;<;2AB;25| (|x| |y| |$|)
 (PROG (#1=#:G87662)
  (RETURN
    (SEQ
     (EXIT
      (SEQ
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((OR
               (SPADCALL |x| (QREFELT |$| 16))
               (SPADCALL |y| (QREFELT |$| 16)))
              (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((NULL
             (SPADCALL
              (SPADCALL |x| (QREFELT |$| 18))
              (SPADCALL |y| (QREFELT |$| 18))
              (QREFELT |$| 51)))
             (PROGN
              (LETT #1#
                (SPADCALL
                 (SPADCALL |x| (QREFELT |$| 18))
                 (SPADCALL |y| (QREFELT |$| 18))
                (QREFELT |$| 63))
               |LSAGG-;<;2AB;25|)
              (GO #1#)))
           ((QUOTE T)
            (SEQ
             (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)
             (EXIT
              (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)))))))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT
        (COND
         ((SPADCALL |x| (QREFELT |$| 16))
           (COND
            ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))))
     #1#
     (EXIT #1#)))))

(DEFUN |ListAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ListAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |ListAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (QSETREFV |$| 52
         (CONS (|dispatchFunction| |LSAGG-;reduce;MA3S;17|) |$|))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (PROGN
        (QSETREFV |$| 60
         (CONS (|dispatchFunction| |LSAGG-;position;SA2I;23|) |$|))
        (QSETREFV |$| 62
         (CONS (|dispatchFunction| |LSAGG-;removeDuplicates!;2A;24|) |$|)))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|OrderedSet|)))
       (QSETREFV |$| 64 (CONS (|dispatchFunction| |LSAGG-;<;2AB;25|) |$|))))
    |$|))))

(MAKEPROP
 (QUOTE |ListAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|NonNegativeInteger|) (0 . |#|) (|Mapping| 15 7 7) |LSAGG-;sort!;M2A;1|
   (5 . |empty|) (9 . |concat|) |LSAGG-;list;SA;2| (|Boolean|)
   (15 . |empty?|) (20 . |rest|) (25 . |first|) (|Mapping| 7 7 7)
   (30 . |reduce|) |LSAGG-;reduce;MAS;3| (37 . |copy|) (42 . |merge!|)
   |LSAGG-;merge;M3A;4| (49 . |setrest!|) (|Mapping| 15 7)
   |LSAGG-;select!;M2A;5| (55 . |eq?|) |LSAGG-;merge!;M3A;6|
   (|Integer|) (61 . |minIndex|) (66 . |rest|) |LSAGG-;insert!;SAIA;7|
   (72 . |concat!|) |LSAGG-;insert!;2AIA;8| |LSAGG-;remove!;M2A;9|
   |LSAGG-;delete!;AIA;10| (|UniversalSegment| 30) (78 . |lo|)
 (83 . |hasHi|) (88 . |hi|) (93 . |maxIndex|) |LSAGG-;delete!;AUsA;11|
 (|Union| 7 (QUOTE "failed")) |LSAGG-;find;MAU;12|
 |LSAGG-;position;MAI;13| (98 . |reverse!|) (103 . |split!|)
 |LSAGG-;sorted?;MAB;15| |LSAGG-;reduce;MA2S;16| (109 . |=|)
 (115 . |reduce|) |LSAGG-;new;NniSA;18| |LSAGG-;map;M3A;19|
 |LSAGG-;reverse!;2A;20| (123 . |cyclic?|) |LSAGG-;copy;2A;21|
 (128 . |setfirst!|) |LSAGG-;copyInto!;2AIA;22| (134 . |position|)
 (141 . |remove!|) (147 . |removeDuplicates!|) (152 . |<|) (158 . |<|)
 (|Mapping| 7 7)))
(QUOTE #(|sorted?| 164 |sort!| 170 |select!| 176 |reverse!| 182
  |removeDuplicates!| 187 |remove!| 192 |reduce| 198 |position| 219
  |new| 232 |merge!| 238 |merge| 245 |map| 252 |list| 259 |insert!|
  264 |find| 278 |delete!| 284 |copyInto!| 296 |copy| 303 |<| 308))
 (QUOTE NIL)
 (CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 64 (QUOTE (1 6 8 0 9 0 6 0 12 2 6 0 7 0 13 1 6
     15 0 16 1 6 0 0 17 1 6 7 0 18 3 6 7 19 0 7 20 1 6 0 0 22 3 6 0 10
     0 0 23 2 6 0 0 0 25 2 6 15 0 0 28 1 6 30 0 31 2 6 0 0 8 32 2 6 0 0
     0 34 1 38 30 0 39 1 38 15 0 40 1 38 30 0 41 1 6 30 0 42 1 6 0 0 47
     2 6 0 0 30 48 2 7 15 0 0 51 4 0 7 19 0 7 7 52 1 6 15 0 56 2 6 7 0
     7 58 3 0 30 7 0 30 60 2 6 0 26 0 61 1 0 0 0 62 2 7 15 0 0 63 2 0 15
     0 0 64 2 0 15 10 0 49 2 0 0 10 0 11 2 0 0 26 0 27 1 0 0 0 55 1 0 0
     0 62 2 0 0 26 0 36 3 0 7 19 0 7 50 4 0 7 19 0 7 7 52 2 0 7 19 0 21
     2 0 30 26 0 46 3 0 30 7 0 30 60 2 0 0 8 7 53 3 0 0 10 0 0 29 3 0 0
     10 0 0 24 3 0 0 19 0 0 54 1 0 0 7 14 3 0 0 7 0 30 33 3 0 0 0 0 30
     35 2 0 44 26 0 45 2 0 0 0 38 43 2 0 0 0 30 37 3 0 0 0 0 30 59 1 0
     0 0 57 2 0 15 0 0 64))))))
 (QUOTE |lookupComplete|)))
```

21.37 MONOID.lsp BOOTSTRAP

MONOID depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MONOID.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Monoid;AL| (QUOTE NIL))

(DEFUN |Monoid| NIL
  (LET (#:G82434)
    (COND
      (|Monoid;AL|)
      (T (SETQ |Monoid;AL| (|Monoid;|))))))

(DEFUN |Monoid;| NIL
  (PROG (#1=#:G82432)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|One| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|one?| ((|Boolean|) |$|)) T)
                ((|**| (|$| |$| (|NonNegativeInteger|))) T)
                ((|^| (|$| |$| (|NonNegativeInteger|))) T)
                ((|recip| ((|Union| |$| "failed") |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |Monoid|)
          (SETELT #1# 0 (QUOTE (|Monoid|)))))))

(MAKEPROP (QUOTE |Monoid|) (QUOTE NILADIC) T)
```

21.38 MONOID-.lsp BOOTSTRAP

MONOID- depends on **MONOID**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID-** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MONOID-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |MONOID-;^;SNniS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |MONOID-;one?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 10) (QREFELT |$| 12)))

(DEFUN |MONOID-;sample;S;3| (|$|)
  (|spadConstant| |$| 10))

(DEFUN |MONOID-;recip;SU;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 15)) (CONS 0 |x|))
    ((QUOTE T) (CONS 1 "failed"))))

(DEFUN |MONOID-;**;SNniS;5| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 20)))))

(DEFUN |Monoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Monoid&|))
        (LETT |dv$| (LIST (QUOTE |Monoid&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |Monoid&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|NonNegativeInteger|)
        (0 . |**|)
        |MONOID-;^;SNniS;1|
        (6 . |One|)
        (|Boolean|)
        (10 . |=|)
        |MONOID-;one?;SB;2|
        |MONOID-;sample;S;3|
        (16 . |one?|)
        (|Union| |$| (QUOTE "failed"))
        |MONOID-;recip;SU;4|
        (|PositiveInteger|)
        (|RepeatedSquaring| 6)
        (21 . |expt|)
        |MONOID-;**;SNniS;5|))
    (QUOTE #(|sample| 27 |recip| 31 |one?| 36 |^| 41 |**| 47))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 21
            (QUOTE
              (2 6 0 0 7 8 0 6 0 10 2 6 11 0 0 12 1 6 11 0 15 2 19 6 6 18 20
              0 0 0 14 1 0 16 0 17 1 0 11 0 13 2 0 0 0 7 9 2 0 0 0 7 21))))))
  (QUOTE |lookupComplete|)))
```

21.39 MTSCAT.lsp BOOTSTRAP

MTSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MTSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **MTSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MTSCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |MultivariateTaylorSeriesCategory;CAT| (QUOTE NIL))

(SETQ |MultivariateTaylorSeriesCategory;AL| (QUOTE NIL))

(DEFUN |MultivariateTaylorSeriesCategory|
 (|&REST| #1=#:G83334 |&AUX| #2=#:G83332)
   (DSETQ #2# #1#)
   (LET (#3=#:G83333)
    (COND
     ((SETQ #3#
       (|assoc| (|devaluateList| #2#) |MultivariateTaylorSeriesCategory;AL|))
        (CDR #3#))
     (T
      (SETQ |MultivariateTaylorSeriesCategory;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |MultivariateTaylorSeriesCategory;|) #2#)))
        |MultivariateTaylorSeriesCategory;AL|))
       #3#))))

(DEFUN |MultivariateTaylorSeriesCategory;| (|t#1| |t#2|)
 (PROG (#1=#:G83331)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83330)) (LIST (QUOTE (|IndexedExponents| |t#2|))))
       (COND
        (|MultivariateTaylorSeriesCategory;CAT|)
        ((QUOTE T)
(LETT |MultivariateTaylorSeriesCategory;CAT|
 (|Join|
  (|PartialDifferentialRing| (QUOTE |t#2|))
  (|PowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#) (QUOTE |t#2|))
  (|InnerEvalable| (QUOTE |t#2|) (QUOTE |$|))
  (|Evalable| (QUOTE |$|))
  (|mkCategory|
   (QUOTE |domain|)
   (QUOTE (
    ((|coefficient| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
    ((|coefficient|
        (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
        T)
    ((|extend| (|$| |$| (|NonNegativeInteger|))) T)
    ((|monomial| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
    ((|monomial|
       (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
        T)
    ((|order| ((|NonNegativeInteger|) |$| |t#2|)) T)
    ((|order|
       ((|NonNegativeInteger|) |$| |t#2| (|NonNegativeInteger|)))
        T)
    ((|polynomial|
      ((|Polynomial| |t#1|) |$| (|NonNegativeInteger|)))
       T)
    ((|polynomial|
      ((|Polynomial| |t#1|)
       |$|
       (|NonNegativeInteger|)
       (|NonNegativeInteger|)))
        T)
    ((|integrate| (|$| |$| |t#2|))
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
   (QUOTE (
    ((|RadicalCategory|)
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
    ((|TranscendentalFunctionCategory|)
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
   (QUOTE
    ((|Polynomial| |t#1|)
     (|NonNegativeInteger|)
     (|List| |t#2|)
     (|List| (|NonNegativeInteger|))))
     NIL))
  . #3=(|MultivariateTaylorSeriesCategory|))))))
. #3#)
```

```
    (SETELT #1# 0
     (LIST
      (QUOTE |MultivariateTaylorSeriesCategory|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)))))))
```

21.40 OINTDOM.lsp BOOTSTRAP

OINTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **OINTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **OINTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*OINTDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedIntegralDomain;AL| (QUOTE NIL))

(DEFUN |OrderedIntegralDomain| NIL
  (LET (#:G84531)
    (COND
      (|OrderedIntegralDomain;AL|)
      (T (SETQ |OrderedIntegralDomain;AL| (|OrderedIntegralDomain;|))))))

(DEFUN |OrderedIntegralDomain;| NIL
  (PROG (#1=#:G84529)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join| (|IntegralDomain|) (|OrderedRing|)) |OrderedIntegralDomain|)
        (SETELT #1# 0 (QUOTE (|OrderedIntegralDomain|)))))))

(MAKEPROP (QUOTE |OrderedIntegralDomain|) (QUOTE NILADIC) T)
```

21.41 ORDRING.lsp BOOTSTRAP

ORDRING depends on **INT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Technically I can't justify this bootstrap stanza based on the lattice since **INT** is already bootstrapped. However using **INT** naked generates a "value stack overflow" error suggesting an infinite recursive loop. This code is here to experiment with breaking that loop.

Note that this code is not included in the generated catdef.spad file.

⟨*ORDRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedRing;AL| (QUOTE NIL))

(DEFUN |OrderedRing| NIL
  (LET (#:G84457)
    (COND
      (|OrderedRing;AL|)
      (T (SETQ |OrderedRing;AL| (|OrderedRing;|))))))

(DEFUN |OrderedRing;| NIL
  (PROG (#1=#:G84455)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|OrderedAbelianGroup|)
            (|Ring|)
            (|Monoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|positive?| ((|Boolean|) |$|)) T)
                ((|negative?| ((|Boolean|) |$|)) T)
                ((|sign| ((|Integer|) |$|)) T)
                ((|abs| (|$| |$|)) T)))
              NIL
              (QUOTE ((|Integer|) (|Boolean|)))
              NIL))
           |OrderedRing|)
          (SETELT #1# 0 (QUOTE (|OrderedRing|)))))))

(MAKEPROP (QUOTE |OrderedRing|) (QUOTE NILADIC) T)
```

21.42 ORDRING-.lsp BOOTSTRAP

ORDRING- depends on **ORDRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ORDRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ORDRING-;positive?;SB;1| (|x| |$|)
  (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 9)))

(DEFUN |ORDRING-;negative?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ORDRING-;sign;SI;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) 1)
    ((SPADCALL |x| (QREFELT |$| 13)) -1)
    ((SPADCALL |x| (QREFELT |$| 15)) 0)
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |ORDRING-;abs;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) |x|)
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |x| (QREFELT |$| 18)))
    ((SPADCALL |x| (QREFELT |$| 15)) (|spadConstant| |$| 7))
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |OrderedRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|OrderedRing&|))
        (LETT |dv$| (LIST (QUOTE |OrderedRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 20) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |OrderedRing&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |<|)
        |ORDRING-;positive?;SB;1|
        |ORDRING-;negative?;SB;2|
        (10 . |positive?|)
        (15 . |negative?|)
        (20 . |One|)
        (24 . |zero?|)
        (|Integer|)
        |ORDRING-;sign;SI;3|
        (29 . |-|)
        |ORDRING-;abs;2S;4|))
    (QUOTE #(|sign| 34 |positive?| 39 |negative?| 44 |abs| 49))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 19
            (QUOTE
              (0 6 0 7 2 6 8 0 0 9 1 6 8 0 12 1 6 8 0 13 0 6 0 14 1 6 8 0 15
               1 6 0 0 18 1 0 16 0 17 1 0 8 0 10 1 0 8 0 11 1 0 0 0 19))))))
   (QUOTE |lookupComplete|)))
```

21.43 POLYCAT.lsp BOOTSTRAP

POLYCAT depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*POLYCAT.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(SETQ |PolynomialCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialCategory;AL| (QUOTE NIL))

(DEFUN |PolynomialCategory| (&REST #0=#:G1430 &AUX #1=#:G1428)
 (DSETQ #1# #0#)
 (LET (#2=#:G1429)
  (COND
   ((SETQ #2# (|assoc| (|devaluateList| #1#) |PolynomialCategory;AL|))
     (CDR #2#))
   (T
     (SETQ |PolynomialCategory;AL|
      (|cons5|
       (CONS (|devaluateList| #1#)
        (SETQ #2# (APPLY (FUNCTION |PolynomialCategory;|) #1#)))
       |PolynomialCategory;AL|))
     #2#))))

(DEFUN |PolynomialCategory;| (|t#1| |t#2| |t#3|)
 (PROG (#0=#:G1427)
  (RETURN
   (PROG1
    (LETT #0#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2| |t#3|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|) (|deva
      (COND
       (|PolynomialCategory;CAT|)
       ((QUOTE T)
        (LETT |PolynomialCategory;CAT|
         (|Join|
          (|PartialDifferentialRing| (QUOTE |t#3|))
          (|FiniteAbelianMonoidRing| (QUOTE |t#1|) (QUOTE |t#2|))
          (|Evalable| (QUOTE $))
```

```
(|InnerEvalable| (QUOTE |t#3|) (QUOTE |t#1|))
(|InnerEvalable| (QUOTE |t#3|) (QUOTE $))
(|RetractableTo| (QUOTE |t#3|))
(|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
(|mkCategory| (QUOTE |domain|)
 (QUOTE
  (((|degree| ((|NonNegativeInteger|) $ |t#3|)) T)
   ((|degree| ((|List| (|NonNegativeInteger|)) $ (|List| |t#3|))) T)
   ((|coefficient| ($ $ |t#3| (|NonNegativeInteger|))) T)
   ((|coefficient| ($ $ (|List| |t#3|)
     (|List| (|NonNegativeInteger|)))) T)
   ((|monomials| ((|List| $) $)) T)
   ((|univariate| ((|SparseUnivariatePolynomial| $) $ |t#3|)) T)
   ((|univariate| ((|SparseUnivariatePolynomial| |t#1|) $)) T)
   ((|mainVariable| ((|Union| |t#3| "failed") $)) T)
   ((|minimumDegree| ((|NonNegativeInteger|) $ |t#3|)) T)
   ((|minimumDegree| ((|List| (|NonNegativeInteger|)) $
     (|List| |t#3|))) T)
   ((|monicDivide|
     ((|Record| (|:| |quotient| $) (|:| |remainder| $)) $ $ |t#3|))
     T)
   ((|monomial| ($ $ |t#3| (|NonNegativeInteger|))) T)
   ((|monomial| ($ $ (|List| |t#3|) (|List| (|NonNegativeInteger|))))
     T)
   ((|multivariate| ($ (|SparseUnivariatePolynomial| |t#1|) |t#3|))
     T)
   ((|multivariate| ($ (|SparseUnivariatePolynomial| $) |t#3|)) T)
   ((|isPlus| ((|Union| (|List| $) "failed") $)) T)
   ((|isTimes| ((|Union| (|List| $) "failed") $)) T)
   ((|isExpt|
     ((|Union|
      (|Record| (|:| |var| |t#3|)
                (|:| |exponent| (|NonNegativeInteger|)))
      "failed") $))
     T)
   ((|totalDegree| ((|NonNegativeInteger|) $)) T)
   ((|totalDegree| ((|NonNegativeInteger|) $ (|List| |t#3|))) T)
   ((|variables| ((|List| |t#3|) $)) T)
   ((|primitiveMonomials| ((|List| $) $)) T)
   ((|resultant| ($ $ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
   ((|discriminant| ($ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
   ((|content| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
   ((|primitivePart| ($ $)) (|has| |t#1| (|GcdDomain|)))
   ((|primitivePart| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
   ((|squareFree| ((|Factored| $) $)) (|has| |t#1| (|GcdDomain|)))
   ((|squareFreePart| ($ $)) (|has| |t#1| (|GcdDomain|)))))
```

```
           (QUOTE
            (((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
             ((|ConvertibleTo| (|InputForm|))
              (AND (|has| |t#3| (|ConvertibleTo| (|InputForm|)))
                   (|has| |t#1| (|ConvertibleTo| (|InputForm|)))))
             ((|ConvertibleTo| (|Pattern| (|Integer|)))
              (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Integer|))))
                   (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Integer|))))))
             ((|ConvertibleTo| (|Pattern| (|Float|)))
              (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Float|))))
                   (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Float|))))))
             ((|PatternMatchable| (|Integer|))
              (AND
               (|has| |t#3| (|PatternMatchable| (|Integer|)))
               (|has| |t#1| (|PatternMatchable| (|Integer|)))))
             ((|PatternMatchable| (|Float|))
              (AND
               (|has| |t#3| (|PatternMatchable| (|Float|)))
               (|has| |t#1| (|PatternMatchable| (|Float|)))))
             ((|GcdDomain|) (|has| |t#1| (|GcdDomain|)))
             (|canonicalUnitNormal|
               (|has| |t#1| (ATTRIBUTE |canonicalUnitNormal|)))
             ((|PolynomialFactorizationExplicit|)
               (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
           (QUOTE
            ((|Factored| $)
             (|List| $)
             (|List| |t#3|)
             (|NonNegativeInteger|)
             (|SparseUnivariatePolynomial| $)
             (|SparseUnivariatePolynomial| |t#1|)
             (|List| (|NonNegativeInteger|))))
            NIL))
          . #1=(|PolynomialCategory|)))))
      . #1#)
    (SETELT #0# 0
     (LIST (QUOTE |PolynomialCategory|)
      (|devaluate| |t#1|) (|devaluate| |t#2|) (|devaluate| |t#3|)))))))
```

21.44 POLYCAT-.lsp BOOTSTRAP

POLYCAT- depends on **POLYCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*POLYCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)


(/VERSIONCHECK 2)

(DEFUN |POLYCAT-;eval;SLS;1| (|p| |l| $)
 (PROG (#0=#:G1444 #1=#:G1438 #2=#:G1445 #3=#:G1446 |lvar| #4=#:G1447
        |e| #5=#:G1448)
  (RETURN
   (SEQ
    (COND
     ((NULL |l|) |p|)
     ((QUOTE T)
      (SEQ
       (SEQ
        (EXIT
         (SEQ
          (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
          (LETT #0# |l| |POLYCAT-;eval;SLS;1|)
          G190
          (COND
           ((OR (ATOM #0#)
                (PROGN (LETT |e| (CAR #0#) |POLYCAT-;eval;SLS;1|) NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (COND
             ((QEQCAR
               (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 13)) 1)
              (PROGN
               (LETT #1#
                (|error| "cannot find a variable to evaluate")
                |POLYCAT-;eval;SLS;1|)
               (GO #1#))))))
          (LETT #0# (CDR #0#) |POLYCAT-;eval;SLS;1|)
      (GO G190)
      G191
      (EXIT NIL)))
    #1# (EXIT #1#))
   (LETT |lvar|
    (PROGN
     (LETT #2# NIL |POLYCAT-;eval;SLS;1|)
     (SEQ
      (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
      (LETT #3# |l| |POLYCAT-;eval;SLS;1|)
      G190
      (COND
       ((OR (ATOM #3#)
            (PROGN (LETT |e| (CAR #3#) |POLYCAT-;eval;SLS;1|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (LETT #2#
         (CONS (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 14))
               #2#)
         |POLYCAT-;eval;SLS;1|)))
      (LETT #3# (CDR #3#) |POLYCAT-;eval;SLS;1|)
      (GO G190)
      G191
      (EXIT (NREVERSEO #2#))))
     |POLYCAT-;eval;SLS;1|)
   (EXIT
    (SPADCALL |p| |lvar|
     (PROGN
      (LETT #4# NIL |POLYCAT-;eval;SLS;1|)
      (SEQ
       (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
       (LETT #5# |l| |POLYCAT-;eval;SLS;1|)
       G190
       (COND
        ((OR (ATOM #5#)
             (PROGN (LETT |e| (CAR #5#) |POLYCAT-;eval;SLS;1|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #4# (CONS (SPADCALL |e| (QREFELT $ 15)) #4#)
          |POLYCAT-;eval;SLS;1|)))
       (LETT #5# (CDR #5#) |POLYCAT-;eval;SLS;1|)
       (GO G190)
       G191
       (EXIT (NREVERSEO #4#))))
```

```
          (QREFELT $ 18))))))))))

  (DEFUN |POLYCAT-;monomials;SL;2| (|p| $)
   (PROG (|ml|)
    (RETURN
     (SEQ
      (LETT |ml| NIL |POLYCAT-;monomials;SL;2|)
      (SEQ G190
       (COND
        ((NULL
          (COND
           ((SPADCALL |p| (|spadConstant| $ 21) (QREFELT $ 24)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (LETT |ml|
         (CONS (SPADCALL |p| (QREFELT $ 25)) |ml|)
         |POLYCAT-;monomials;SL;2|)
        (EXIT
         (LETT |p| (SPADCALL |p| (QREFELT $ 26)) |POLYCAT-;monomials;SL;2|)))
        NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (REVERSE |ml|))))))

  (DEFUN |POLYCAT-;isPlus;SU;3| (|p| $)
   (PROG (|l|)
    (RETURN
     (COND
      ((NULL
         (CDR (LETT |l| (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;isPlus;SU;3|)))
        (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 |l|))))))

  (DEFUN |POLYCAT-;isTimes;SU;4| (|p| $)
   (PROG (|lv| #0=#:G1470 |v| #1=#:G1471 |l| |r|)
    (RETURN
     (SEQ
      (COND
       ((OR (NULL
             (LETT |lv| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;isTimes;SU;4|))
            (NULL (SPADCALL |p| (QREFELT $ 32))))
         (CONS 1 "failed"))
       ((QUOTE T)
        (SEQ
       (LETT |l|
        (PROGN
         (LETT #0# NIL |POLYCAT-;isTimes;SU;4|)
         (SEQ
          (LETT |v| NIL |POLYCAT-;isTimes;SU;4|)
          (LETT #1# |lv| |POLYCAT-;isTimes;SU;4|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN (LETT |v| (CAR #1#) |POLYCAT-;isTimes;SU;4|) NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (LETT #0#
             (CONS
              (SPADCALL (|spadConstant| $ 33) |v|
               (SPADCALL |p| |v| (QREFELT $ 36)) (QREFELT $ 37))
              #0#)
              |POLYCAT-;isTimes;SU;4|)))
          (LETT #1# (CDR #1#) |POLYCAT-;isTimes;SU;4|)
          (GO G190)
          G191
         (EXIT (NREVERSE0 #0#))))
        |POLYCAT-;isTimes;SU;4|)
       (LETT |r| (SPADCALL |p| (QREFELT $ 38)) |POLYCAT-;isTimes;SU;4|)
       (EXIT
        (COND
         ((SPADCALL |r| (|spadConstant| $ 34) (QREFELT $ 39))
           (COND
            ((NULL (CDR |lv|)) (CONS 1 "failed"))
            ((QUOTE T) (CONS 0 |l|))))
         ((QUOTE T)
           (CONS 0 (CONS (SPADCALL |r| (QREFELT $ 40)) |l|))))))))))))

(DEFUN |POLYCAT-;isExpt;SU;5| (|p| $)
 (PROG (|u| |d|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;isExpt;SU;5|)
    (EXIT
     (COND
      ((OR (QEQCAR |u| 1)
           (NULL
            (SPADCALL |p|
             (SPADCALL (|spadConstant| $ 33)
              (QCDR |u|)
              (LETT |d| (SPADCALL |p| (QCDR |u|) (QREFELT $ 36))
               |POLYCAT-;isExpt;SU;5|)
              (QREFELT $ 37))
             (QREFELT $ 24))))
       (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (CONS (QCDR |u|) |d|)))))))))

(DEFUN |POLYCAT-;coefficient;SVarSetNniS;6| (|p| |v| |n| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) |n| (QREFELT $ 49)))

(DEFUN |POLYCAT-;coefficient;SLLS;7| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in coefficient"))))
  ((NULL |ln|) (|error| "mismatched lists in coefficient"))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL
     (SPADCALL |p| (|SPADfirst| |lv|) (QREFELT $ 47))
     (|SPADfirst| |ln|)
     (QREFELT $ 49))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 52)))))

(DEFUN |POLYCAT-;monomial;SLLS;8| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in monomial"))))
  ((NULL |ln|) (|error| "mismatched lists in monomial"))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |ln|) (QREFELT $ 37))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 54)))))

(DEFUN |POLYCAT-;retract;SVarSet;9| (|p| $)
 (PROG (#0=#:G1496 |q|)
  (RETURN
   (SEQ
    (LETT |q|
     (PROG2
      (LETT #0# (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retract;SVarSet;9|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
     |POLYCAT-;retract;SVarSet;9|)
    (EXIT
     (COND
      ((SPADCALL (SPADCALL |q| (QREFELT $ 56)) |p| (QREFELT $ 24)) |q|)
      ((QUOTE T) (|error| "Polynomial is not a single variable"))))))))

(DEFUN |POLYCAT-;retractIfCan;SU;10| (|p| $)
 (PROG (|q| #0=#:G1504)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       (LETT |q| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retractIfCan;SU;10|)
       (EXIT
        (COND
         ((QEQCAR |q| 0)
          (COND
           ((SPADCALL (SPADCALL (QCDR |q|) (QREFELT $ 56)) |p| (QREFELT $ 24))
             (PROGN
              (LETT #0# |q| |POLYCAT-;retractIfCan;SU;10|)
              (GO #0#))))))))
      (EXIT (CONS 1 "failed"))))
     #0#
     (EXIT #0#)))))

(DEFUN |POLYCAT-;mkPrim| (|p| $)
 (SPADCALL
  (|spadConstant| $ 34)
  (SPADCALL |p| (QREFELT $ 59))
  (QREFELT $ 60)))

(DEFUN |POLYCAT-;primitiveMonomials;SL;12| (|p| $)
 (PROG (#0=#:G1509 |q| #1=#:G1510)
  (RETURN
   (SEQ
    (PROGN
     (LETT #0# NIL |POLYCAT-;primitiveMonomials;SL;12|)
     (SEQ
      (LETT |q| NIL |POLYCAT-;primitiveMonomials;SL;12|)
      (LETT #1# (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;primitiveMonomials;SL;12|)
      G190
      (COND
       ((OR (ATOM #1#)
            (PROGN
             (LETT |q| (CAR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
             NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (LETT #0# (CONS (|POLYCAT-;mkPrim| |q| $) #0#)
         |POLYCAT-;primitiveMonomials;SL;12|)))
      (LETT #1# (CDR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #0#))))))))

(DEFUN |POLYCAT-;totalDegree;SNni;13| (|p| $)
 (PROG (#0=#:G1512 |d| |u|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 62)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |u|
        (SPADCALL |p|
         (PROG2
          (LETT #0#
           (SPADCALL |p| (QREFELT $ 42))
           |POLYCAT-;totalDegree;SNni;13|)
          (QCDR #0#)
          (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
         (QREFELT $ 47))
        |POLYCAT-;totalDegree;SNni;13|)
       (LETT |d| 0 |POLYCAT-;totalDegree;SNni;13|)
       (SEQ G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T)))) (GO G191)))
        (SEQ
         (LETT |d|
          (MAX |d|
           (+
            (SPADCALL |u| (QREFELT $ 65))
            (SPADCALL (SPADCALL |u| (QREFELT $ 66)) (QREFELT $ 67))))
           |POLYCAT-;totalDegree;SNni;13|)
          (EXIT
           (LETT |u|
            (SPADCALL |u| (QREFELT $ 68)) |POLYCAT-;totalDegree;SNni;13|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |d|))))))))

(DEFUN |POLYCAT-;totalDegree;SLNni;14| (|p| |lv| $)
 (PROG (#0=#:G1520 |v| |w| |d| |u|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 62)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |u|
        (SPADCALL |p|
         (LETT |v|
          (PROG2
           (LETT #0#
            (SPADCALL |p| (QREFELT $ 42))
            |POLYCAT-;totalDegree;SLNni;14|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
          |POLYCAT-;totalDegree;SLNni;14|)
         (QREFELT $ 47))
        |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |d| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |w| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (COND
        ((SPADCALL |v| |lv| (QREFELT $ 70))
          (LETT |w| 1 |POLYCAT-;totalDegree;SLNni;14|)))
       (SEQ G190
        (COND
         ((NULL
          (COND
           ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T)))) (GO G191)))
        (SEQ
         (LETT |d|
          (MAX |d|
           (+
            (* |w| (SPADCALL |u| (QREFELT $ 65)))
            (SPADCALL (SPADCALL |u| (QREFELT $ 66)) |lv| (QREFELT $ 71))))
          |POLYCAT-;totalDegree;SLNni;14|)
         (EXIT
          (LETT |u|
           (SPADCALL |u| (QREFELT $ 68))
           |POLYCAT-;totalDegree;SLNni;14|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
       (EXIT |d|))))))))

(DEFUN |POLYCAT-;resultant;2SVarSetS;15| (|p1| |p2| |mvar| $)
 (SPADCALL
  (SPADCALL |p1| |mvar| (QREFELT $ 47))
  (SPADCALL |p2| |mvar| (QREFELT $ 47))
  (QREFELT $ 73)))

(DEFUN |POLYCAT-;discriminant;SVarSetS;16| (|p| |var| $)
 (SPADCALL (SPADCALL |p| |var| (QREFELT $ 47)) (QREFELT $ 75)))

(DEFUN |POLYCAT-;allMonoms| (|l| $)
 (PROG (#0=#:G1532 |p| #1=#:G1533)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #0# NIL |POLYCAT-;allMonoms|)
       (SEQ
        (LETT |p| NIL |POLYCAT-;allMonoms|)
        (LETT #1# |l| |POLYCAT-;allMonoms|)
        G190
        (COND
         ((OR (ATOM #1#) (PROGN (LETT |p| (CAR #1#) |POLYCAT-;allMonoms|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0#
           (CONS (SPADCALL |p| (QREFELT $ 77)) #0#)
           |POLYCAT-;allMonoms|)))
        (LETT #1# (CDR #1#) |POLYCAT-;allMonoms|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))
      (QREFELT $ 79))
      (QREFELT $ 80))))))

  (DEFUN |POLYCAT-;P2R| (|p| |b| |n| $)
   (PROG (|w| |bj| #0=#:G1538 |i| #1=#:G1537)
    (RETURN
     (SEQ
      (LETT |w|
       (SPADCALL |n| (|spadConstant| $ 22) (QREFELT $ 82))
       |POLYCAT-;P2R|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;P2R|)
       (LETT #0# |b| |POLYCAT-;P2R|)
       (LETT |i| (SPADCALL |w| (QREFELT $ 84)) |POLYCAT-;P2R|)
       (LETT #1# (QVSIZE |w|) |POLYCAT-;P2R|)
       G190
       (COND
        ((OR (> |i| #1#)
             (ATOM #0#)
             (PROGN (LETT |bj| (CAR #0#) |POLYCAT-;P2R|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (SPADCALL |w| |i| (SPADCALL |p| |bj| (QREFELT $ 85)) (QREFELT $ 86))))
       (LETT |i|
        (PROG1 (+ |i| 1) (LETT #0# (CDR #0#) |POLYCAT-;P2R|)) |POLYCAT-;P2R|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |w|)))))

  (DEFUN |POLYCAT-;eq2R| (|l| |b| $)
   (PROG (#0=#:G1542 |bj| #1=#:G1543 #2=#:G1544 |p| #3=#:G1545)
    (RETURN
     (SEQ
      (SPADCALL
       (PROGN
        (LETT #0# NIL |POLYCAT-;eq2R|)
        (SEQ
         (LETT |bj| NIL |POLYCAT-;eq2R|)
         (LETT #1# |b| |POLYCAT-;eq2R|)
         G190
         (COND
          ((OR (ATOM #1#)
               (PROGN (LETT |bj| (CAR #1#) |POLYCAT-;eq2R|) NIL)) (GO G191)))
         (SEQ
          (EXIT
         (LETT #0#
          (CONS
           (PROGN
            (LETT #2# NIL |POLYCAT-;eq2R|)
            (SEQ
             (LETT |p| NIL |POLYCAT-;eq2R|)
             (LETT #3# |l| |POLYCAT-;eq2R|)
             G190
             (COND
              ((OR (ATOM #3#) (PROGN (LETT |p| (CAR #3#) |POLYCAT-;eq2R|) NIL))
               (GO G191)))
             (SEQ
              (EXIT
               (LETT #2#
                (CONS (SPADCALL |p| |bj| (QREFELT $ 85)) #2#)
                |POLYCAT-;eq2R|)))
             (LETT #3# (CDR #3#) |POLYCAT-;eq2R|)
             (GO G190)
             G191
             (EXIT (NREVERSE0 #2#))))
           #0#)
          |POLYCAT-;eq2R|)))
       (LETT #1# (CDR #1#) |POLYCAT-;eq2R|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #0#))))
     (QREFELT $ 89))))))

(DEFUN |POLYCAT-;reducedSystem;MM;20| (|m| $)
 (PROG (#0=#:G1555 |r| #1=#:G1556 |b| #2=#:G1557 |bj| #3=#:G1558 |d| |mm| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MM;20|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (PROGN
        (LETT #0# NIL |POLYCAT-;reducedSystem;MM;20|)
        (SEQ
         (LETT |r| NIL |POLYCAT-;reducedSystem;MM;20|)
         (LETT #1# |l| |POLYCAT-;reducedSystem;MM;20|)
         G190
        (COND
         ((OR (ATOM #1#)
              (PROGN (LETT |r| (CAR #1#) |POLYCAT-;reducedSystem;MM;20|) NIL))
          (GO G191)))
      (SEQ
       (EXIT
        (LETT #0#
         (CONS (|POLYCAT-;allMonoms| |r| $) #0#)
         |POLYCAT-;reducedSystem;MM;20|)))
      (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MM;20|)
      (GO G190)
      G191
      (EXIT (NREVERSEO #0#))))
     (QREFELT $ 79))
    (QREFELT $ 80))
   |POLYCAT-;reducedSystem;MM;20|)
  (LETT |d|
   (PROGN
    (LETT #2# NIL |POLYCAT-;reducedSystem;MM;20|)
    (SEQ
     (LETT |bj| NIL |POLYCAT-;reducedSystem;MM;20|)
     (LETT #3# |b| |POLYCAT-;reducedSystem;MM;20|)
     G190
     (COND
      ((OR (ATOM #3#)
           (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MM;20|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT #2#
        (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
        |POLYCAT-;reducedSystem;MM;20|)))
     (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MM;20|)
     (GO G190)
     G191
     (EXIT (NREVERSEO #2#))))
   |POLYCAT-;reducedSystem;MM;20|)
  (LETT |mm|
   (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) |POLYCAT-;reducedSystem;MM;20|)
  (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)
  (SEQ G190
   (COND
    ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
   (SEQ
    (LETT |mm|
     (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
     |POLYCAT-;reducedSystem;MM;20|)
    (EXIT (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)))
   NIL
   (GO G190)
```

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     G191
     (EXIT NIL))
    (EXIT |mm|)))))

(DEFUN |POLYCAT-;reducedSystem;MVR;21| (|m| |v| $)
 (PROG (#0=#:G1570 |s| #1=#:G1571 |b| #2=#:G1572 |bj| #3=#:G1573 |d| |n|
        |mm| |w| |l| |r|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |r| (SPADCALL |v| (QREFELT $ 97)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (|POLYCAT-;allMonoms| |r| $)
       (SPADCALL
        (PROGN
         (LETT #0# NIL |POLYCAT-;reducedSystem;MVR;21|)
         (SEQ
          (LETT |s| NIL |POLYCAT-;reducedSystem;MVR;21|)
          (LETT #1# |l| |POLYCAT-;reducedSystem;MVR;21|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN
                 (LETT |s| (CAR #1#) |POLYCAT-;reducedSystem;MVR;21|)
                  NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (LETT #0#
             (CONS (|POLYCAT-;allMonoms| |s| $) #0#)
             |POLYCAT-;reducedSystem;MVR;21|)))
          (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MVR;21|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #0#))))
        (QREFELT $ 79))
       (QREFELT $ 98))
      (QREFELT $ 80))
     |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |d|
     (PROGN
      (LETT #2# NIL |POLYCAT-;reducedSystem;MVR;21|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;reducedSystem;MVR;21|)
   (LETT #3# |b| |POLYCAT-;reducedSystem;MVR;21|)
   G190
   (COND
    ((OR (ATOM #3#)
         (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MVR;21|) NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (LETT #2#
      (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
      |POLYCAT-;reducedSystem;MVR;21|)))
   (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MVR;21|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #2#))))
  |POLYCAT-;reducedSystem;MVR;21|)
(LETT |n| (LENGTH |d|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |mm|
 (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |w|
 (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)
(SEQ G190
 (COND
  ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |mm|
   (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
   |POLYCAT-;reducedSystem;MVR;21|)
  (LETT |w|
   (SPADCALL |w|
    (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
    (QREFELT $ 99))
   |POLYCAT-;reducedSystem;MVR;21|)
  (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
  (EXIT (LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
(EXIT (CONS |mm| |w|))))))
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(DEFUN |POLYCAT-;gcdPolynomial;3Sup;22| (|pp| |qq| $)
 (SPADCALL |pp| |qq| (QREFELT $ 104)))

(DEFUN |POLYCAT-;solveLinearPolynomialEquation;LSupU;23| (|lpp| |pp| $)
 (SPADCALL |lpp| |pp| (QREFELT $ 109)))

(DEFUN |POLYCAT-;factorPolynomial;SupF;24| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 114)))

(DEFUN |POLYCAT-;factorSquareFreePolynomial;SupF;25| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 117)))

(DEFUN |POLYCAT-;factor;SF;26| (|p| $)
 (PROG (|v| |ansR| #0=#:G1615 |w| #1=#:G1616 |up| |ansSUP| #2=#:G1617
        |ww| #3=#:G1618)
  (RETURN
   (SEQ
    (LETT |v| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;factor;SF;26|)
    (EXIT
     (COND
      ((QEQCAR |v| 1)
       (SEQ
        (LETT |ansR|
         (SPADCALL (SPADCALL |p| (QREFELT $ 38)) (QREFELT $ 120))
         |POLYCAT-;factor;SF;26|)
        (EXIT
         (SPADCALL
          (SPADCALL (SPADCALL |ansR| (QREFELT $ 122)) (QREFELT $ 40))
          (PROGN
           (LETT #0# NIL |POLYCAT-;factor;SF;26|)
           (SEQ
            (LETT |w| NIL |POLYCAT-;factor;SF;26|)
            (LETT #1#
             (SPADCALL |ansR| (QREFELT $ 126))
             |POLYCAT-;factor;SF;26|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN (LETT |w| (CAR #1#) |POLYCAT-;factor;SF;26|) NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (LETT #0#
               (CONS
                (VECTOR (QVELT |w| 0)
                  (SPADCALL (QVELT |w| 1) (QREFELT $ 40)) (QVELT |w| 2))
                  #0#)
                 |POLYCAT-;factor;SF;26|)))
               (LETT #1# (CDR #1#) |POLYCAT-;factor;SF;26|)
               (GO G190)
               G191
               (EXIT (NREVERSE0 #0#))))
             (QREFELT $ 130)))))
         ((QUOTE T)
          (SEQ
           (LETT |up|
            (SPADCALL |p| (QCDR |v|) (QREFELT $ 47)) |POLYCAT-;factor;SF;26|)
           (LETT |ansSUP| (SPADCALL |up| (QREFELT $ 114)) |POLYCAT-;factor;SF;26|)
           (EXIT
            (SPADCALL
             (SPADCALL
              (SPADCALL |ansSUP| (QREFELT $ 131)) (QCDR |v|) (QREFELT $ 132))
             (PROGN
              (LETT #2# NIL |POLYCAT-;factor;SF;26|)
              (SEQ
               (LETT |ww| NIL |POLYCAT-;factor;SF;26|)
               (LETT #3#
                (SPADCALL |ansSUP| (QREFELT $ 135))
                |POLYCAT-;factor;SF;26|)
               G190
               (COND
                ((OR (ATOM #3#)
                     (PROGN (LETT |ww| (CAR #3#) |POLYCAT-;factor;SF;26|) NIL))
                 (GO G191)))
               (SEQ
                (EXIT
                 (LETT #2#
                  (CONS
                   (VECTOR (QVELT |ww| 0) (SPADCALL (QVELT |ww| 1) (QCDR |v|)
                           (QREFELT $ 132)) (QVELT |ww| 2))
                   #2#)
                  |POLYCAT-;factor;SF;26|)))
               (LETT #3# (CDR #3#) |POLYCAT-;factor;SF;26|)
               (GO G190)
               G191
               (EXIT (NREVERSE0 #2#))))
             (QREFELT $ 130)))))))))))

(DEFUN |POLYCAT-;conditionP;MU;27| (|mat| $)
 (PROG (|ll| #0=#:G1653 |z| #1=#:G1654 |ch| |l| #2=#:G1655 #3=#:G1656
        #4=#:G1625 #5=#:G1623 #6=#:G1624 #7=#:G1657 |vars| |degs|
        #8=#:G1658 |d| #9=#:G1659 |nd| #10=#:G1652 #11=#:G1632 |deg1|
      |redmons| #12=#:G1660 |v| #13=#:G1662 |u| #14=#:G1661 |llR|
      |monslist| |ans| #15=#:G1663 #16=#:G1664 |mons| #17=#:G1665 |m|
      #18=#:G1666 |i| #19=#:G1648 #20=#:G1646 #21=#:G1647)
(RETURN
 (SEQ
  (EXIT
   (SEQ
    (LETT |ll|
     (SPADCALL (SPADCALL |mat| (QREFELT $ 137)) (QREFELT $ 92))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |llR|
     (PROGN
      (LETT #0# NIL |POLYCAT-;conditionP;MU;27|)
      (SEQ
       (LETT |z| NIL |POLYCAT-;conditionP;MU;27|)
       (LETT #1# (|SPADfirst| |ll|) |POLYCAT-;conditionP;MU;27|)
       G190
       (COND
        ((OR (ATOM #1#)
             (PROGN (LETT |z| (CAR #1#) |POLYCAT-;conditionP;MU;27|) NIL))
         (GO G191)))
       (SEQ (EXIT (LETT #0# (CONS NIL #0#) |POLYCAT-;conditionP;MU;27|)))
       (LETT #1# (CDR #1#) |POLYCAT-;conditionP;MU;27|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #0#))))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |monslist| NIL |POLYCAT-;conditionP;MU;27|)
    (LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;conditionP;MU;27|)
    (SEQ
     (LETT |l| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #2# |ll| |POLYCAT-;conditionP;MU;27|)
     G190
     (COND
      ((OR (ATOM #2#)
           (PROGN (LETT |l| (CAR #2#) |POLYCAT-;conditionP;MU;27|) NIL))
       (GO G191)))
     (SEQ
      (LETT |mons|
       (PROGN
        (LETT #6# NIL |POLYCAT-;conditionP;MU;27|)
        (SEQ
         (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
         (LETT #3# |l| |POLYCAT-;conditionP;MU;27|)
         G190
         (COND
    ((OR (ATOM #3#)
         (PROGN (LETT |u| (CAR #3#) |POLYCAT-;conditionP;MU;27|) NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (PROGN
      (LETT #4#
       (SPADCALL |u| (QREFELT $ 77))
       |POLYCAT-;conditionP;MU;27|)
      (COND
       (#6#
        (LETT #5#
         (SPADCALL #5# #4# (QREFELT $ 139))
         |POLYCAT-;conditionP;MU;27|))
       ((QUOTE T)
        (PROGN
         (LETT #5# #4# |POLYCAT-;conditionP;MU;27|)
         (LETT #6# (QUOTE T) |POLYCAT-;conditionP;MU;27|)))))))
   (LETT #3# (CDR #3#) |POLYCAT-;conditionP;MU;27|)
   (GO G190)
   G191
   (EXIT NIL))
  (COND (#6# #5#) ((QUOTE T) (|IdentityError| (QUOTE |setUnion|)))))
 |POLYCAT-;conditionP;MU;27|)
(LETT |redmons| NIL |POLYCAT-;conditionP;MU;27|)
(SEQ
 (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
 (LETT #7# |mons| |POLYCAT-;conditionP;MU;27|)
 G190
 (COND
  ((OR (ATOM #7#)
       (PROGN (LETT |m| (CAR #7#) |POLYCAT-;conditionP;MU;27|) NIL))
    (GO G191)))
 (SEQ
  (LETT |vars|
   (SPADCALL |m| (QREFELT $ 31))
   |POLYCAT-;conditionP;MU;27|)
  (LETT |degs|
   (SPADCALL |m| |vars| (QREFELT $ 140))
   |POLYCAT-;conditionP;MU;27|)
  (LETT |deg1|
   (PROGN
    (LETT #8# NIL |POLYCAT-;conditionP;MU;27|)
    (SEQ
     (LETT |d| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #9# |degs| |POLYCAT-;conditionP;MU;27|)
   G190
   (COND
    ((OR (ATOM #9#)
         (PROGN
          (LETT |d| (CAR #9#) |POLYCAT-;conditionP;MU;27|)
          NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (LETT #8#
      (CONS
       (SEQ
        (LETT |nd|
         (SPADCALL |d| |ch| (QREFELT $ 142))
         |POLYCAT-;conditionP;MU;27|)
        (EXIT
         (COND
          ((QEQCAR |nd| 1)
            (PROGN
              (LETT #10#
               (CONS 1 "failed") |POLYCAT-;conditionP;MU;27|)
              (GO #10#)))
          ((QUOTE T)
            (PROG1
             (LETT #11# (QCDR |nd|) |POLYCAT-;conditionP;MU;27|)
             (|check-subtype|
              (>= #11# 0) (QUOTE (|NonNegativeInteger|)) #11#))))))
       #8#)
      |POLYCAT-;conditionP;MU;27|)))
  (LETT #9# (CDR #9#) |POLYCAT-;conditionP;MU;27|)
  (GO G190)
  G191
  (EXIT (NREVERSE0 #8#))))
 |POLYCAT-;conditionP;MU;27|)
(LETT |redmons|
 (CONS
  (SPADCALL (|spadConstant| $ 33) |vars| |deg1| (QREFELT $ 54))
  |redmons|)
 |POLYCAT-;conditionP;MU;27|)
(EXIT
 (LETT |llR|
  (PROGN
   (LETT #12# NIL |POLYCAT-;conditionP;MU;27|)
   (SEQ
    (LETT |v| NIL |POLYCAT-;conditionP;MU;27|)
    (LETT #13# |llR| |POLYCAT-;conditionP;MU;27|)
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        (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
        (LETT #14# |l| |POLYCAT-;conditionP;MU;27|)
        G190
        (COND
         ((OR (ATOM #14#)
           (PROGN
            (LETT |u| (CAR #14#) |POLYCAT-;conditionP;MU;27|)
             NIL)
           (ATOM #13#)
           (PROGN (LETT |v| (CAR #13#) |POLYCAT-;conditionP;MU;27|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #12#
           (CONS
            (CONS
             (SPADCALL
              (SPADCALL |u| |vars| |degs| (QREFELT $ 52))
              (QREFELT $ 143))
             |v|)
            #12#)
           |POLYCAT-;conditionP;MU;27|)))
        (LETT #14#
         (PROG1
          (CDR #14#)
          (LETT #13# (CDR #13#) |POLYCAT-;conditionP;MU;27|))
         |POLYCAT-;conditionP;MU;27|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #12#))))
      |POLYCAT-;conditionP;MU;27|)))
   (LETT #7# (CDR #7#) |POLYCAT-;conditionP;MU;27|)
   (GO G190)
   G191
   (EXIT NIL))
  (EXIT
   (LETT |monslist|
    (CONS |redmons| |monslist|)
    |POLYCAT-;conditionP;MU;27|)))
 (LETT #2# (CDR #2#) |POLYCAT-;conditionP;MU;27|)
 (GO G190)
 G191
 (EXIT NIL))
(LETT |ans|
 (SPADCALL (SPADCALL (SPADCALL |llR| (QREFELT $ 89)) (QREFELT $ 144))
           (QREFELT $ 146))
 |POLYCAT-;conditionP;MU;27|)
(EXIT
 (COND
  ((QEQCAR |ans| 1) (CONS 1 "failed"))
  ((QUOTE T)
   (SEQ
    (LETT |i| 0 |POLYCAT-;conditionP;MU;27|)
    (EXIT
     (CONS 0
      (PRIMVEC2ARR
       (PROGN
        (LETT #15# (GETREFV (SIZE |monslist|))
         |POLYCAT-;conditionP;MU;27|)
        (SEQ
         (LETT #16# 0 |POLYCAT-;conditionP;MU;27|)
         (LETT |mons| NIL |POLYCAT-;conditionP;MU;27|)
         (LETT #17# |monslist| |POLYCAT-;conditionP;MU;27|)
         G190
         (COND
          ((OR (ATOM #17#)
               (PROGN
                (LETT |mons| (CAR #17#) |POLYCAT-;conditionP;MU;27|)
                NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (SETELT #15# #16#
            (PROGN
             (LETT #21# NIL |POLYCAT-;conditionP;MU;27|)
             (SEQ
              (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
              (LETT #18# |mons| |POLYCAT-;conditionP;MU;27|)
              G190
              (COND
               ((OR (ATOM #18#)
                  (PROGN
                   (LETT |m| (CAR #18#) |POLYCAT-;conditionP;MU;27|)
                    NIL))
                (GO G191)))
              (SEQ
               (EXIT
                (PROGN
                 (LETT #19#
                  (SPADCALL |m|
                   (SPADCALL
                    (SPADCALL
                            (QCDR |ans|)
                            (LETT |i| (+ |i| 1) |POLYCAT-;conditionP;MU;27|)
                            (QREFELT $ 147))
                           (QREFELT $ 40))
                          (QREFELT $ 148))
                         |POLYCAT-;conditionP;MU;27|)
                        (COND
                         (#21#
                          (LETT #20#
                           (SPADCALL #20# #19# (QREFELT $ 149))
                           |POLYCAT-;conditionP;MU;27|))
                         ((QUOTE T)
                           (PROGN
                            (LETT #20# #19# |POLYCAT-;conditionP;MU;27|)
                            (LETT #21#
                             (QUOTE T)
                             |POLYCAT-;conditionP;MU;27|)))))))
                     (LETT #18# (CDR #18#) |POLYCAT-;conditionP;MU;27|)
                     (GO G190)
                     G191
                     (EXIT NIL))
                    (COND (#21# #20#) ((QUOTE T) (|spadConstant| $ 21)))))))
                (LETT #17#
                 (PROG1
                  (CDR #17#)
                  (LETT #16# (QSADD1 #16#) |POLYCAT-;conditionP;MU;27|))
                 |POLYCAT-;conditionP;MU;27|)
                (GO G190)
                G191
                (EXIT NIL))
               #15#))))))))))
   #10#
   (EXIT #10#)))))

(DEFUN |POLYCAT-;charthRoot;SU;28| (|p| $)
 (PROG (|vars| |ans| |ch|)
  (RETURN
   (SEQ
    (LETT |vars| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;charthRoot;SU;28|)
    (EXIT
     (COND
      ((NULL |vars|)
        (SEQ
         (LETT |ans|
          (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
          |POLYCAT-;charthRoot;SU;28|)
         (EXIT
          (COND
           ((QEQCAR |ans| 1) (CONS 1 "failed"))
           ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
      ((QUOTE T)
       (SEQ
        (LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;charthRoot;SU;28|)
        (EXIT (|POLYCAT-;charthRootlv| |p| |vars| |ch| $))))))))))

(DEFUN |POLYCAT-;charthRootlv| (|p| |vars| |ch| $)
 (PROG (|v| |dd| |cp| |d| #0=#:G1687 |ans| |ansx| #1=#:G1694)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((NULL |vars|)
       (SEQ
        (LETT |ans|
         (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
         |POLYCAT-;charthRootlv|)
        (EXIT
         (COND
          ((QEQCAR |ans| 1) (CONS 1 "failed"))
          ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
      ((QUOTE T)
       (SEQ
        (LETT |v| (|SPADfirst| |vars|) |POLYCAT-;charthRootlv|)
        (LETT |vars| (CDR |vars|) |POLYCAT-;charthRootlv|)
        (LETT |d| (SPADCALL |p| |v| (QREFELT $ 36)) |POLYCAT-;charthRootlv|)
        (LETT |ans| (|spadConstant| $ 21) |POLYCAT-;charthRootlv|)
        (SEQ G190
         (COND ((NULL (< 0 |d|)) (GO G191)))
         (SEQ
          (LETT |dd|
           (SPADCALL |d| |ch| (QREFELT $ 142))
           |POLYCAT-;charthRootlv|)
          (EXIT
           (COND
            ((QEQCAR |dd| 1)
              (PROGN
               (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
               (GO #1#)))
            ((QUOTE T)
             (SEQ
              (LETT |cp|
               (SPADCALL |p| |v| |d| (QREFELT $ 154))
       |POLYCAT-;charthRootlv|)
      (LETT |p|
       (SPADCALL |p|
        (SPADCALL |cp| |v| |d| (QREFELT $ 37))
        (QREFELT $ 155))
       |POLYCAT-;charthRootlv|)
      (LETT |ansx|
       (|POLYCAT-;charthRootlv| |cp| |vars| |ch| $)
       |POLYCAT-;charthRootlv|)
      (EXIT
       (COND
        ((QEQCAR |ansx| 1)
          (PROGN
           (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
           (GO #1#)))
        ((QUOTE T)
         (SEQ
          (LETT |d|
           (SPADCALL |p| |v| (QREFELT $ 36))
           |POLYCAT-;charthRootlv|)
          (EXIT
           (LETT |ans|
            (SPADCALL |ans|
             (SPADCALL (QCDR |ansx|) |v|
              (PROG1
               (LETT #0# (QCDR |dd|) |POLYCAT-;charthRootlv|)
               (|check-subtype| (>= #0# 0)
                (QUOTE (|NonNegativeInteger|)) #0#))
              (QREFELT $ 37))
             (QREFELT $ 149))
            |POLYCAT-;charthRootlv|)))))))))))
 NIL
 (GO G190)
 G191
 (EXIT NIL))
(LETT |ansx|
 (|POLYCAT-;charthRootlv| |p| |vars| |ch| $)
 |POLYCAT-;charthRootlv|)
(EXIT
 (COND
  ((QEQCAR |ansx| 1)
   (PROGN
    (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
    (GO #1#)))
  ((QUOTE T)
   (PROGN
            (LETT #1#
             (CONS 0 (SPADCALL |ans| (QCDR |ansx|) (QREFELT $ 149)))
             |POLYCAT-;charthRootlv|)
            (GO #1#))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |POLYCAT-;monicDivide;2SVarSetR;30| (|p1| |p2| |mvar| $)
 (PROG (|result|)
  (RETURN
   (SEQ
    (LETT |result|
     (SPADCALL
      (SPADCALL |p1| |mvar| (QREFELT $ 47))
      (SPADCALL |p2| |mvar| (QREFELT $ 47))
      (QREFELT $ 157))
     |POLYCAT-;monicDivide;2SVarSetR;30|)
    (EXIT
     (CONS
      (SPADCALL (QCAR |result|) |mvar| (QREFELT $ 132))
      (SPADCALL (QCDR |result|) |mvar| (QREFELT $ 132)))))))))

(DEFUN |POLYCAT-;squareFree;SF;31| (|p| $)
 (SPADCALL |p| (QREFELT $ 160)))

(DEFUN |POLYCAT-;squareFree;SF;32| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFree;SF;33| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFreePart;2S;34| (|p| $)
 (PROG (|s| |f| #0=#:G1710 #1=#:G1708 #2=#:G1706 #3=#:G1707)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (LETT |s| (SPADCALL |p| (QREFELT $ 164)) |POLYCAT-;squareFreePart;2S;34|)
      (QREFELT $ 165))
     (PROGN
      (LETT #3# NIL |POLYCAT-;squareFreePart;2S;34|)
      (SEQ
       (LETT |f| NIL |POLYCAT-;squareFreePart;2S;34|)
       (LETT #0# (SPADCALL |s| (QREFELT $ 168)) |POLYCAT-;squareFreePart;2S;34|)
       G190
       (COND
         ((OR (ATOM #0#)
              (PROGN (LETT |f| (CAR #0#) |POLYCAT-;squareFreePart;2S;34|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (PROGN
           (LETT #1# (QCAR |f|) |POLYCAT-;squareFreePart;2S;34|)
           (COND
            (#3#
             (LETT #2#
              (SPADCALL #2# #1# (QREFELT $ 148))
              |POLYCAT-;squareFreePart;2S;34|))
            ((QUOTE T)
             (PROGN
              (LETT #2# #1# |POLYCAT-;squareFreePart;2S;34|)
              (LETT #3# (QUOTE T) |POLYCAT-;squareFreePart;2S;34|)))))))
        (LETT #0# (CDR #0#) |POLYCAT-;squareFreePart;2S;34|)
        (GO G190)
        G191
        (EXIT NIL))
       (COND (#3# #2#) ((QUOTE T) (|spadConstant| $ 33))))
      (QREFELT $ 148))))))

(DEFUN |POLYCAT-;content;SVarSetS;35| (|p| |v| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) (QREFELT $ 170)))

(DEFUN |POLYCAT-;primitivePart;2S;36| (|p| $)
 (PROG (#0=#:G1713)
  (RETURN
   (COND
    ((SPADCALL |p| (QREFELT $ 172)) |p|)
    ((QUOTE T)
     (QVELT
      (SPADCALL
       (PROG2
        (LETT #0#
         (SPADCALL |p| (SPADCALL |p| (QREFELT $ 173)) (QREFELT $ 174))
         |POLYCAT-;primitivePart;2S;36|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
       (QREFELT $ 176))
      1))))))

(DEFUN |POLYCAT-;primitivePart;SVarSetS;37| (|p| |v| $)
 (PROG (#0=#:G1720)
  (RETURN
   (COND
    ((SPADCALL |p| (QREFELT $ 172)) |p|)
    ((QUOTE T)
     (QVELT
      (SPADCALL
       (PROG2
        (LETT #0#
         (SPADCALL |p| (SPADCALL |p| |v| (QREFELT $ 178)) (QREFELT $ 179))
         |POLYCAT-;primitivePart;SVarSetS;37|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
       (QREFELT $ 176))
      1))))))

(DEFUN |POLYCAT-;<;2SB;38| (|p| |q| $)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp| (SPADCALL |p| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (LETT |dq| (SPADCALL |q| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (EXIT
     (COND
      ((SPADCALL |dp| |dq| (QREFELT $ 181))
        (SPADCALL
         (|spadConstant| $ 22)
         (SPADCALL |q| (QREFELT $ 38))
         (QREFELT $ 182)))
      ((SPADCALL |dq| |dp| (QREFELT $ 181))
        (SPADCALL
         (SPADCALL |p| (QREFELT $ 38))
         (|spadConstant| $ 22)
         (QREFELT $ 182)))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL (SPADCALL |p| |q| (QREFELT $ 155)) (QREFELT $ 38))
        (|spadConstant| $ 22)
        (QREFELT $ 182)))))))))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;39| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 187)))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;40| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 193)))

(DEFUN |POLYCAT-;convert;SP;41| (|x| $)
 (SPADCALL (ELT $ 196) (ELT $ 197) |x| (QREFELT $ 201)))
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(DEFUN |POLYCAT-;convert;SP;42| (|x| $)
 (SPADCALL (ELT $ 203) (ELT $ 204) |x| (QREFELT $ 208)))

(DEFUN |POLYCAT-;convert;SIf;43| (|p| $)
 (SPADCALL (ELT $ 211) (ELT $ 212) |p| (QREFELT $ 216)))

(DEFUN |PolynomialCategory&| (|#1| |#2| |#3| |#4|)
 (PROG (DV$1 DV$2 DV$3 DV$4 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|PolynomialCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT |dv$| (LIST (QUOTE |PolynomialCategory&|) DV$1 DV$2 DV$3 DV$4) . #0#)
    (LETT $ (GETREFV 226) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
        (|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
        (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
        (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|OrderedSet|)))))
     . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    (QSETREFV $ 7 |#2|)
    (QSETREFV $ 8 |#3|)
    (QSETREFV $ 9 |#4|)
    (COND
     ((|testBitVector| |pv$| 4)
      (PROGN
   (QSETREFV $ 74
    (CONS (|dispatchFunction| |POLYCAT-;resultant;2SVarSetS;15|) $))
   (QSETREFV $ 76
    (CONS (|dispatchFunction| |POLYCAT-;discriminant;SVarSetS;16|) $)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
  (PROGN
   (QSETREFV $ 95
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MM;20|) $))
   (QSETREFV $ 102
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MVR;21|) $)))))
(COND
 ((|testBitVector| |pv$| 1)
  (PROGN
   (QSETREFV $ 105
    (CONS (|dispatchFunction| |POLYCAT-;gcdPolynomial;3Sup;22|) $))
   (QSETREFV $ 112
    (CONS
     (|dispatchFunction|
       |POLYCAT-;solveLinearPolynomialEquation;LSupU;23|)
     $))
   (QSETREFV $ 116
    (CONS (|dispatchFunction| |POLYCAT-;factorPolynomial;SupF;24|) $))
   (QSETREFV $ 118
    (CONS
     (|dispatchFunction| |POLYCAT-;factorSquareFreePolynomial;SupF;25|)
     $))
   (QSETREFV $ 136 (CONS (|dispatchFunction| |POLYCAT-;factor;SF;26|) $))
   (COND
    ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
     (PROGN
      (QSETREFV $ 150
       (CONS (|dispatchFunction| |POLYCAT-;conditionP;MU;27|) $))))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
  (PROGN
   (QSETREFV $ 152
    (CONS (|dispatchFunction| |POLYCAT-;charthRoot;SU;28|) $)))))
(COND
 ((|testBitVector| |pv$| 3)
  (PROGN
   (COND
    ((|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
     (COND
      ((|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (QSETREFV $ 161
        (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;31|) $)))
      ((QUOTE T)
       (QSETREFV $ 161
        (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;32|) $)))))
    ((QUOTE T)
     (QSETREFV $ 161
      (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;33|) $))))
  (QSETREFV $ 169
   (CONS (|dispatchFunction| |POLYCAT-;squareFreePart;2S;34|) $))
  (QSETREFV $ 171
   (CONS (|dispatchFunction| |POLYCAT-;content;SVarSetS;35|) $))
  (QSETREFV $ 177
   (CONS (|dispatchFunction| |POLYCAT-;primitivePart;2S;36|) $))
  (QSETREFV $ 180
   (CONS (|dispatchFunction| |POLYCAT-;primitivePart;SVarSetS;37|) $)))))
(COND
 ((|testBitVector| |pv$| 15)
  (PROGN
   (QSETREFV $ 183 (CONS (|dispatchFunction| |POLYCAT-;<;2SB;38|) $))
   (COND
    ((|testBitVector| |pv$| 8)
     (COND
      ((|testBitVector| |pv$| 7)
       (QSETREFV $ 189
        (CONS
         (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;39|)
         $))))))
   (COND
    ((|testBitVector| |pv$| 6)
     (COND
     ((|testBitVector| |pv$| 5)
      (QSETREFV $ 195
       (CONS
        (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;40|)
        $)))))))))
(COND
 ((|testBitVector| |pv$| 12)
  (COND
   ((|testBitVector| |pv$| 11)
    (QSETREFV $ 202
     (CONS (|dispatchFunction| |POLYCAT-;convert;SP;41|) $))))))
(COND
 ((|testBitVector| |pv$| 10)
  (COND
   ((|testBitVector| |pv$| 9)
    (QSETREFV $ 209
         (CONS (|dispatchFunction| |POLYCAT-;convert;SP;42|) $))))))
    (COND
     ((|testBitVector| |pv$| 14)
      (COND
       ((|testBitVector| |pv$| 13)
         (QSETREFV $ 217
          (CONS (|dispatchFunction| |POLYCAT-;convert;SIf;43|) $))))))
    $))))

(MAKEPROP
 (QUOTE |PolynomialCategory&|)
 (QUOTE |infovec|)
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
    (|local| |#4|) (|Equation| 6) (0 . |lhs|) (|Union| 9 (QUOTE "failed"))
    (5 . |retractIfCan|) (10 . |retract|) (15 . |rhs|) (|List| 9) (|List| $)
    (20 . |eval|) (|List| 221) |POLYCAT-;eval;SLS;1| (27 . |Zero|)
    (31 . |Zero|) (|Boolean|) (35 . =) (41 . |leadingMonomial|)
    (46 . |reductum|) |POLYCAT-;monomials;SL;2| (51 . |monomials|)
    (|Union| 17 (QUOTE "failed")) |POLYCAT-;isPlus;SU;3| (56 . |variables|)
    (61 . |monomial?|) (66 . |One|) (70 . |One|) (|NonNegativeInteger|)
    (74 . |degree|) (80 . |monomial|) (87 . |leadingCoefficient|) (92 . =)
    (98 . |coerce|) |POLYCAT-;isTimes;SU;4| (103 . |mainVariable|)
    (|Record| (|:| |var| 9) (|:| |exponent| 35))
    (|Union| 43 (QUOTE "failed")) |POLYCAT-;isExpt;SU;5|
    (|SparseUnivariatePolynomial| $) (108 . |univariate|)
    (|SparseUnivariatePolynomial| 6) (114 . |coefficient|)
    |POLYCAT-;coefficient;SVarSetNniS;6| (|List| 35) (120 . |coefficient|)
    |POLYCAT-;coefficient;SLLS;7| (127 . |monomial|)
    |POLYCAT-;monomial;SLLS;8| (134 . |coerce|)
    |POLYCAT-;retract;SVarSet;9| |POLYCAT-;retractIfCan;SU;10|
    (139 . |degree|) (144 . |monomial|) |POLYCAT-;primitiveMonomials;SL;12|
    (150 . |ground?|) (155 . |Zero|) (159 . =) (165 . |degree|)
    (170 . |leadingCoefficient|) (175 . |totalDegree|) (180 . |reductum|)
    |POLYCAT-;totalDegree;SNni;13| (185 . |member?|) (191 . |totalDegree|)
    |POLYCAT-;totalDegree;SLNni;14| (197 . |resultant|) (203 . |resultant|)
    (210 . |discriminant|) (215 . |discriminant|) (221 . |primitiveMonomials|)
    (|List| 6) (226 . |concat|) (231 . |removeDuplicates!|) (|Vector| 7)
    (236 . |new|) (|Integer|) (242 . |minIndex|) (247 . |coefficient|)
    (253 . |qsetelt!|) (|List| 220) (|Matrix| 7) (260 . |matrix|)
    (|List| 78) (|Matrix| 6) (265 . |listOfLists|) (270 . |vertConcat|)
    (|Matrix| $) (276 . |reducedSystem|) (|Vector| 6) (281 . |entries|)
    (286 . |concat|) (292 . |concat|)
    (|Record| (|:| |mat| 88) (|:| |vec| 81)) (|Vector| $)
    (298 . |reducedSystem|) (|GeneralPolynomialGcdPackage| 8 9 7 6)
    (304 . |gcdPolynomial|) (310 . |gcdPolynomial|)
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(|Union| 107 (QUOTE "failed")) (|List| 48)
(|PolynomialFactorizationByRecursion| 7 8 9 6)
(316 . |solveLinearPolynomialEquationByRecursion|)
(|Union| 111 (QUOTE "failed")) (|List| 46)
(322 . |solveLinearPolynomialEquation|) (|Factored| 48)
(328 . |factorByRecursion|) (|Factored| 46) (333 . |factorPolynomial|)
(338 . |factorSquareFreeByRecursion|)
(343 . |factorSquareFreePolynomial|) (|Factored| $) (348 . |factor|)
(|Factored| 7) (353 . |unit|)
(|Union| (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
(|Record| (|:| |flg| 123) (|:| |fctr| 7) (|:| |xpnt| 83))
(|List| 124) (358 . |factorList|)
(|Record| (|:| |flg| 123) (|:| |fctr| 6) (|:| |xpnt| 83))
(|List| 127) (|Factored| 6) (363 . |makeFR|) (369 . |unit|)
(374 . |multivariate|)
(|Record| (|:| |flg| 123) (|:| |fctr| 48) (|:| |xpnt| 83))
(|List| 133) (380 . |factorList|) (385 . |factor|) (390 . |transpose|)
(395 . |characteristic|) (399 . |setUnion|) (405 . |degree|)
(|Union| $ (QUOTE "failed")) (411 . |exquo|) (417 . |ground|)
(422 . |transpose|) (|Union| 101 (QUOTE "failed")) (427 . |conditionP|)
(432 . |elt|) (438 . *) (444 . +) (450 . |conditionP|)
(455 . |charthRoot|) (460 . |charthRoot|) (465 . |Zero|)
(469 . |coefficient|) (476 . -)
(|Record| (|:| |quotient| $) (|:| |remainder| $))
(482 . |monicDivide|) |POLYCAT-;monicDivide;2SVarSetR;30|
(|MultivariateSquareFree| 8 9 7 6) (488 . |squareFree|)
(493 . |squareFree|) (|PolynomialSquareFree| 9 8 7 6)
(498 . |squareFree|) (503 . |squareFree|) (508 . |unit|)
(|Record| (|:| |factor| 6) (|:| |exponent| 83)) (|List| 166)
(513 . |factors|) (518 . |squareFreePart|) (523 . |content|)
(528 . |content|) (534 . |zero?|) (539 . |content|) (544 . |exquo|)
(|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
(550 . |unitNormal|) (555 . |primitivePart|) (560 . |content|)
(566 . |exquo|) (572 . |primitivePart|) (578 . <) (584 . <) (590 . <)
(|PatternMatchResult| 83 6) (|Pattern| 83)
(|PatternMatchPolynomialCategory| 83 8 9 7 6) (596 . |patternMatch|)
(|PatternMatchResult| 83 $) (603 . |patternMatch|)
(|PatternMatchResult| (|Float|) 6) (|Pattern| (|Float|))
(|PatternMatchPolynomialCategory| (|Float|) 8 9 7 6)
(610 . |patternMatch|) (|PatternMatchResult| (|Float|) $)
(617 . |patternMatch|) (624 . |convert|) (629 . |convert|)
(|Mapping| 185 9) (|Mapping| 185 7)
(|PolynomialCategoryLifting| 8 9 7 6 185) (634 . |map|)
(641 . |convert|) (646 . |convert|) (651 . |convert|) (|Mapping| 191 9)
(|Mapping| 191 7) (|PolynomialCategoryLifting| 8 9 7 6 191)
(656 . |map|) (663 . |convert|) (|InputForm|) (668 . |convert|)
```

```
  (673 . |convert|) (|Mapping| 210 9) (|Mapping| 210 7)
  (|PolynomialCategoryLifting| 8 9 7 6 210) (678 . |map|)
  (685 . |convert|) (|Record| (|:| |mat| 219) (|:| |vec| (|Vector| 83)))
  (|Matrix| 83) (|List| 7) (|Equation| $) (|Union| 83 (QUOTE "failed"))
  (|Union| 224 (QUOTE "failed")) (|Fraction| 83)
  (|Union| 7 (QUOTE "failed"))))
(QUOTE #(|totalDegree| 690 |squareFreePart| 701 |squareFree| 706
 |solveLinearPolynomialEquation| 711 |retractIfCan| 717 |retract| 722
 |resultant| 727 |reducedSystem| 734 |primitivePart| 745
 |primitiveMonomials| 756 |patternMatch| 761 |monomials| 775
 |monomial| 780 |monicDivide| 787 |isTimes| 794 |isPlus| 799
 |isExpt| 804 |gcdPolynomial| 809 |factorSquareFreePolynomial| 815
 |factorPolynomial| 820 |factor| 825 |eval| 830 |discriminant| 836
 |convert| 842 |content| 857 |conditionP| 863 |coefficient| 868
 |charthRoot| 882 < 887))
(QUOTE NIL)
(CONS (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 217 (QUOTE
    (1 10 6 0 11 1 6 12 0 13 1 6 9 0 14 1 10 6 0 15 3 6 0 0 16 17 18 0 6 0
     21 0 7 0 22 2 6 23 0 0 24 1 6 0 0 25 1 6 0 0 26 1 6 17 0 28 1 6 16 0
     31 1 6 23 0 32 0 6 0 33 0 7 0 34 2 6 35 0 9 36 3 6 0 0 9 35 37 1 6 7
     0 38 2 7 23 0 0 39 1 6 0 7 40 1 6 12 0 42 2 6 46 0 9 47 2 48 6 0 35
     49 3 6 0 0 16 51 52 3 6 0 0 16 51 54 1 6 0 9 56 1 6 8 0 59 2 6 0 7 8
     60 1 6 23 0 62 0 48 0 63 2 48 23 0 0 64 1 48 35 0 65 1 48 6 0 66 1 6
     35 0 67 1 48 0 0 68 2 16 23 9 0 70 2 6 35 0 16 71 2 48 6 0 0 73 3 0
     0 0 0 9 74 1 48 6 0 75 2 0 0 0 9 76 1 6 17 0 77 1 78 0 17 79 1 78 0
     0 80 2 81 0 35 7 82 1 81 83 0 84 2 6 7 0 8 85 3 81 7 0 83 7 86 1 88
     0 87 89 1 91 90 0 92 2 88 0 0 0 93 1 0 88 94 95 1 96 78 0 97 2 78 0
     0 0 98 2 81 0 0 0 99 2 0 100 94 101 102 2 103 48 48 48 104 2 0 46 46
     46 105 2 108 106 107 48 109 2 0 110 111 46 112 1 108 113 48 114 1 0
     115 46 116 1 108 113 48 117 1 0 115 46 118 1 7 119 0 120 1 121 7 0
     122 1 121 125 0 126 2 129 0 6 128 130 1 113 48 0 131 2 6 0 46 9 132
     1 113 134 0 135 1 0 119 0 136 1 91 0 0 137 0 6 35 138 2 78 0 0 0 139
     2 6 51 0 16 140 2 83 141 0 0 142 1 6 7 0 143 1 88 0 0 144 1 7 145 94
     146 2 81 7 0 83 147 2 6 0 0 0 148 2 6 0 0 0 149 1 0 145 94 150 1 7
     141 0 151 1 0 141 0 152 0 8 0 153 3 6 0 0 9 35 154 2 6 0 0 0 155 2
     48 156 0 0 157 1 159 129 6 160 1 0 119 0 161 1 162 129 6 163 1 6 119
     0 164 1 129 6 0 165 1 129 167 0 168 1 0 0 0 169 1 48 6 0 170 2 0 0 0
     9 171 1 6 23 0 172 1 6 7 0 173 2 6 141 0 7 174 1 6 175 0 176 1 0 0 0
     177 2 6 0 0 9 178 2 6 141 0 0 179 2 0 0 0 9 180 2 8 23 0 0 181 2 7 23
     0 0 182 2 0 23 0 0 183 3 186 184 6 185 184 187 3 0 188 0 185 188 189
     3 192 190 6 191 190 193 3 0 194 0 191 194 195 1 9 185 0 196 1 7 185
     0 197 3 200 185 198 199 6 201 1 0 185 0 202 1 9 191 0 203 1 7 191 0
     204 3 207 191 205 206 6 208 1 0 191 0 209 1 9 210 0 211 1 7 210 0
```

```
          212 3 215 210 213 214 6 216 1 0 210 0 217 2 0 35 0 16 72 1 0 35 0 69
          1 0 0 0 169 1 0 119 0 161 2 0 110 111 46 112 1 0 12 0 58 1 0 9 0 57
          3 0 0 0 0 9 74 1 0 88 94 95 2 0 100 94 101 102 2 0 0 0 9 180 1 0 0 0
          177 1 0 17 0 61 3 0 188 0 185 188 189 3 0 194 0 191 194 195 1 0 17 0
          27 3 0 0 0 16 51 55 3 0 156 0 0 9 158 1 0 29 0 41 1 0 29 0 30 1 0 44
          0 45 2 0 46 46 46 105 1 0 115 46 118 1 0 115 46 116 1 0 119 0 136 2
          0 0 0 19 20 2 0 0 0 9 76 1 0 210 0 217 1 0 185 0 202 1 0 191 0 209 2
          0 0 0 9 171 1 0 145 94 150 3 0 0 0 16 51 53 3 0 0 0 9 35 50 1 0 141
          0 152 2 0 23 0 0 183))))))
     (QUOTE |lookupComplete|)))
```

21.45 PSETCAT.lsp BOOTSTRAP

PSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*PSETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |PolynomialSetCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialSetCategory;AL| (QUOTE NIL))

(DEFUN |PolynomialSetCategory| (|&REST| #1=#:G82375 |&AUX| #2=#:G82373)
 (DSETQ #2# #1#)
 (LET (#3=#:G82374)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |PolynomialSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |PolynomialSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |PolynomialSetCategory;|) #2#)))
      |PolynomialSetCategory;AL|))
    #3#))))

(DEFUN |PolynomialSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (PROG (#1=#:G82372)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
       (LIST
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)
        (|devaluate| |t#3|)
        (|devaluate| |t#4|)))
      (|sublisV|
(PAIR (QUOTE (#2=#:G82371)) (LIST (QUOTE (|List| |t#4|))))
(COND
 (|PolynomialSetCategory;CAT|)
 ((QUOTE T)
  (LETT |PolynomialSetCategory;CAT|
   (|Join|
    (|SetCategory|)
    (|Collection| (QUOTE |t#4|))
    (|CoercibleTo| (QUOTE #2#))
    (|mkCategory|
     (QUOTE |domain|)
     (QUOTE (
      ((|retractIfCan| ((|Union| |$| "failed") (|List| |t#4|))) T)
      ((|retract| (|$| (|List| |t#4|))) T)
      ((|mvar| (|t#3| |$|)) T)
      ((|variables| ((|List| |t#3|) |$|)) T)
      ((|mainVariables| ((|List| |t#3|) |$|)) T)
      ((|mainVariable?| ((|Boolean|) |t#3| |$|)) T)
      ((|collectUnder| (|$| |$| |t#3|)) T)
      ((|collect| (|$| |$| |t#3|)) T)
      ((|collectUpper| (|$| |$| |t#3|)) T)
      ((|sort|
       ((|Record|
         (|:| |under| |$|)
         (|:| |floor| |$|)
         (|:| |upper| |$|)) |$| |t#3|))
        T)
      ((|trivialIdeal?| ((|Boolean|) |$|)) T)
      ((|roughBase?| ((|Boolean|) |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughSubIdeal?| ((|Boolean|) |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughEqualIdeals?| ((|Boolean|) |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughUnitIdeal?| ((|Boolean|) |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|headRemainder|
        ((|Record|
          (|:| |num| |t#4|)
          (|:| |den| |t#1|)) |t#4| |$|))
         (|has| |t#1| (|IntegralDomain|)))
      ((|remainder|
        ((|Record|
          (|:| |rnum| |t#1|)
          (|:| |polnum| |t#4|)
          (|:| |den| |t#1|)) |t#4| |$|))
```

```
             (|has| |t#1| (|IntegralDomain|)))
          ((|rewriteIdealWithHeadRemainder|
           ((|List| |t#4|) (|List| |t#4|) |$|))
             (|has| |t#1| (|IntegralDomain|)))
          ((|rewriteIdealWithRemainder| ((|List| |t#4|) (|List| |t#4|) |$|))
            (|has| |t#1| (|IntegralDomain|)))
          ((|triangular?| ((|Boolean|) |$|))
            (|has| |t#1| (|IntegralDomain|)))))
         (QUOTE ((|finiteAggregate| T)))
         (QUOTE ((|Boolean|) (|List| |t#4|) (|List| |t#3|)))
         NIL))
    . #3=(|PolynomialSetCategory|))))))
  . #3#)
(SETELT #1# 0
  (LIST
   (QUOTE |PolynomialSetCategory|)
   (|devaluate| |t#1|)
   (|devaluate| |t#2|)
   (|devaluate| |t#3|)
   (|devaluate| |t#4|)))))))
```

21.46 PSETCAT-.lsp BOOTSTRAP

PSETCAT- depends on **PSETCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*PSETCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |PSETCAT-;elements| (|ps| |$|)
 (PROG (|lp|)
  (RETURN
   (LETT |lp| (SPADCALL |ps| (QREFELT |$| 12)) |PSETCAT-;elements|))))

(DEFUN |PSETCAT-;variables1| (|lp| |$|)
 (PROG (#1=#:G82392 |p| #2=#:G82393 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables1|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables1|)
       (LETT #2# |lp| |PSETCAT-;variables1|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables1|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 14)) #1#) |PSETCAT-;variables1|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables1|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables1|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables1!0|) |$|)
      (SPADCALL (SPADCALL |lvars| (QREFELT |$| 18)) (QREFELT |$| 19))
      (QREFELT |$| 21)))))))
```

```
(DEFUN |PSETCAT-;variables1!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables2| (|lp| |$|)
 (PROG (#1=#:G82397 |p| #2=#:G82398 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables2|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables2|)
       (LETT #2# |lp| |PSETCAT-;variables2|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables2|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 22)) #1#) |PSETCAT-;variables2|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables2|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables2|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables2!0|) |$|)
      (SPADCALL |lvars| (QREFELT |$| 19))
      (QREFELT |$| 21)))))))

(DEFUN |PSETCAT-;variables2!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables;SL;4| (|ps| |$|)
 (|PSETCAT-;variables1| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;mainVariables;SL;5| (|ps| |$|)
 (|PSETCAT-;variables2|
  (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  |$|))

(DEFUN |PSETCAT-;mainVariable?;VarSetSB;6| (|v| |ps| |$|)
 (PROG (|lp|)
    (RETURN
     (SEQ
      (LETT |lp|
       (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
       |PSETCAT-;mainVariable?;VarSetSB;6|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((OR
             (NULL |lp|)
             (SPADCALL
              (SPADCALL (|SPADfirst| |lp|) (QREFELT |$| 22))
              |v|
              (QREFELT |$| 28)))
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;mainVariable?;VarSetSB;6|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))))))

  (DEFUN |PSETCAT-;collectUnder;SVarSetS;7| (|ps| |v| |$|)
   (PROG (|p| |lp| |lq|)
    (RETURN
     (SEQ
      (LETT |lp|
       (|PSETCAT-;elements| |ps| |$|)
       |PSETCAT-;collectUnder;SVarSetS;7|)
      (LETT |lq| NIL |PSETCAT-;collectUnder;SVarSetS;7|)
      (SEQ
       G190
       (COND
        ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
          (GO G191)))
       (SEQ
        (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
        (LETT |lp| (CDR |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
        (EXIT
         (COND
          ((OR
            (SPADCALL |p| (QREFELT |$| 24))
          (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
         (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUnder;SVarSetsS;7|)))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collectUpper;SVarSetsS;8| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp|
     (|PSETCAT-;elements| |ps| |$|)
     |PSETCAT-;collectUpper;SVarSetsS;8|)
    (LETT |lq| NIL |PSETCAT-;collectUpper;SVarSetsS;8|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUpper;SVarSetsS;8|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;collectUpper;SVarSetsS;8|)
      (EXIT
       (COND
        ((NULL (SPADCALL |p| (QREFELT |$| 24)))
         (COND
          ((SPADCALL |v| (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16))
           (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUpper;SVarSetsS;8|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collect;SVarSetsS;9| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;collect;SVarSetsS;9|)
    (LETT |lq| NIL |PSETCAT-;collect;SVarSetsS;9|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
          (GO G191)))
       (SEQ
        (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collect;SVarSetS;9|)
        (LETT |lp| (CDR |lp|) |PSETCAT-;collect;SVarSetS;9|)
        (EXIT
         (COND
          ((NULL (SPADCALL |p| (QREFELT |$| 24)))
           (COND
            ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
              (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collect;SVarSetS;9|)))))))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;sort;SVarSetR;10| (|ps| |v| |$|)
 (PROG (|p| |lp| |us| |vs| |ws|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;sort;SVarSetR;10|)
    (LETT |us| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |vs| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |ws| NIL |PSETCAT-;sort;SVarSetR;10|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (EXIT
       (COND
        ((OR
           (SPADCALL |p| (QREFELT |$| 24))
           (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
          (LETT |us| (CONS |p| |us|) |PSETCAT-;sort;SVarSetR;10|))
        ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
          (LETT |vs| (CONS |p| |vs|) |PSETCAT-;sort;SVarSetR;10|))
        ((QUOTE T) (LETT |ws| (CONS |p| |ws|) |PSETCAT-;sort;SVarSetR;10|)))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
      (VECTOR
       (SPADCALL |us| (QREFELT |$| 30))
       (SPADCALL |vs| (QREFELT |$| 30))
       (SPADCALL |ws| (QREFELT |$| 30))))))))

(DEFUN |PSETCAT-;=;2SB;11| (|ps1| |ps2| |$|)
 (PROG (#1=#:G82439 #2=#:G82440 #3=#:G82437 |p| #4=#:G82438)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #2# (|PSETCAT-;elements| |ps1| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #1# (CONS |p| #1#) |PSETCAT-;=;2SB;11|)))
        (LETT #2# (CDR #2#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 37))
     (SPADCALL
      (PROGN
       (LETT #3# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #4# (|PSETCAT-;elements| |ps2| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #4#) (PROGN (LETT |p| (CAR #4#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #3# (CONS |p| #3#) |PSETCAT-;=;2SB;11|)))
        (LETT #4# (CDR #4#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #3#))))
      (QREFELT |$| 37))
     (QREFELT |$| 38))))))

(DEFUN |PSETCAT-;localInf?| (|p| |q| |$|)
 (SPADCALL
    (SPADCALL |p| (QREFELT |$| 40))
    (SPADCALL |q| (QREFELT |$| 40))
    (QREFELT |$| 41)))

  (DEFUN |PSETCAT-;localTriangular?| (|lp| |$|)
   (PROG (|q| |p|)
    (RETURN
     (SEQ
      (LETT |lp|
       (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26)) |PSETCAT-;localTriangular?|)
      (EXIT
       (COND
        ((NULL |lp|) (QUOTE T))
        ((SPADCALL (ELT |$| 24) |lp| (QREFELT |$| 43)) (QUOTE NIL))
        ((QUOTE T)
         (SEQ
          (LETT |lp|
           (SPADCALL
            (CONS (FUNCTION |PSETCAT-;localTriangular?!0|) |$|)
            |lp|
            (QREFELT |$| 45))
           |PSETCAT-;localTriangular?|)
          (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;localTriangular?|)
          (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)
          (SEQ
           G190
           (COND
            ((NULL
              (COND
               ((NULL |lp|) (QUOTE NIL))
               ((QUOTE T)
                 (SPADCALL
                  (SPADCALL
                   (LETT |q|
                    (|SPADfirst| |lp|)
                    |PSETCAT-;localTriangular?|)
                   (QREFELT |$| 22))
                  (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16)))))
             (GO G191)))
           (SEQ
            (LETT |p| |q| |PSETCAT-;localTriangular?|)
            (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
         (EXIT (NULL |lp|))))))))))

(DEFUN |PSETCAT-;localTriangular?!0| (|#1| |#2| |$|)
 (SPADCALL
  (SPADCALL |#2| (QREFELT |$| 22))
  (SPADCALL |#1| (QREFELT |$| 22))
  (QREFELT |$| 16)))

(DEFUN |PSETCAT-;triangular?;SB;14| (|ps| |$|)
 (|PSETCAT-;localTriangular?| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;trivialIdeal?;SB;15| (|ps| |$|)
 (NULL
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))))

(DEFUN |PSETCAT-;roughUnitIdeal?;SB;16| (|ps| |$|)
 (SPADCALL
  (ELT |$| 24)
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  (QREFELT |$| 43)))

(DEFUN |PSETCAT-;relativelyPrimeLeadingMonomials?| (|p| |q| |$|)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp|
     (SPADCALL |p| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (LETT |dq|
     (SPADCALL |q| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (EXIT
     (SPADCALL
      (SPADCALL |dp| |dq| (QREFELT |$| 49))
      (SPADCALL |dp| |dq| (QREFELT |$| 50))
      (QREFELT |$| 51)))))))

(DEFUN |PSETCAT-;roughBase?;SB;18| (|ps| |$|)
 (PROG (|p| |lp| |rB?| |copylp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;roughBase?;SB;18|)
    (EXIT
     (COND
      ((NULL |lp|) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (LETT |rB?| (QUOTE T) |PSETCAT-;roughBase?;SB;18|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
           (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;roughBase?;SB;18|)
          (LETT |lp| (CDR |lp|) |PSETCAT-;roughBase?;SB;18|)
          (LETT |copylp| |lp| |PSETCAT-;roughBase?;SB;18|)
          (EXIT
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |copylp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
              (GO G191)))
            (SEQ
             (LETT |rB?|
              (|PSETCAT-;relativelyPrimeLeadingMonomials?| |p|
               (|SPADfirst| |copylp|) |$|)
              |PSETCAT-;roughBase?;SB;18|)
             (EXIT (LETT |copylp| (CDR |copylp|) |PSETCAT-;roughBase?;SB;18|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |rB?|)))))))))

(DEFUN |PSETCAT-;roughSubIdeal?;2SB;19| (|ps1| |ps2| |$|)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (|PSETCAT-;elements| |ps1| |$|) |ps2| (QREFELT |$| 53))
     |PSETCAT-;roughSubIdeal?;2SB;19|)
    (EXIT (NULL (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26))))))))

(DEFUN |PSETCAT-;roughEqualIdeals?;2SB;20| (|ps1| |ps2| |$|)
 (COND
  ((SPADCALL |ps1| |ps2| (QREFELT |$| 55)) (QUOTE T))
  ((SPADCALL |ps1| |ps2| (QREFELT |$| 56))
    (SPADCALL |ps2| |ps1| (QREFELT |$| 56)))
  ((QUOTE T) (QUOTE NIL))))

(DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
 (SPADCALL |r| |s| (QREFELT |$| 58)))

(DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
 (PROG (#1=#:G82473)
  (RETURN
   (PROG2
    (LETT #1# (SPADCALL |r| |s| (QREFELT |$| 60)) |PSETCAT-;exactQuo|)
    (QCDR #1#)
    (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#)))))

(DEFUN |PSETCAT-;headRemainder;PSR;23| (|a| |ps| |$|)
 (PROG (|lp1| |p| |e| |g| |#G47| |#G48| |lca| |lcp| |r| |lp2|)
  (RETURN
   (SEQ
    (LETT |lp1|
     (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;headRemainder;PSR;23|)
    (EXIT
     (COND
      ((NULL |lp1|) (CONS |a| (|spadConstant| |$| 61)))
      ((SPADCALL (ELT |$| 24) |lp1| (QREFELT |$| 43))
        (CONS (SPADCALL |a| (QREFELT |$| 62)) (|spadConstant| |$| 61)))
      ((QUOTE T)
       (SEQ
        (LETT |r| (|spadConstant| |$| 61) |PSETCAT-;headRemainder;PSR;23|)
        (LETT |lp1|
         (SPADCALL
          (CONS (|function| |PSETCAT-;localInf?|) |$|)
          (REVERSE (|PSETCAT-;elements| |ps| |$|))
          (QREFELT |$| 45))
         |PSETCAT-;headRemainder;PSR;23|)
        (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((OR (SPADCALL |a| (QREFELT |$| 42)) (NULL |lp2|)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
```

```
(SEQ
 (LETT |p| (|SPADfirst| |lp2|) |PSETCAT-;headRemainder;PSR;23|)
 (LETT |e|
  (SPADCALL
   (SPADCALL |a| (QREFELT |$| 40))
   (SPADCALL |p| (QREFELT |$| 40))
   (QREFELT |$| 63))
  |PSETCAT-;headRemainder;PSR;23|)
 (EXIT
  (COND
   ((QEQCAR |e| 0)
    (SEQ
     (LETT |g|
      (SPADCALL
       (LETT |lca|
        (SPADCALL |a| (QREFELT |$| 64))
        |PSETCAT-;headRemainder;PSR;23|)
       (LETT |lcp|
        (SPADCALL |p| (QREFELT |$| 64))
        |PSETCAT-;headRemainder;PSR;23|)
       (QREFELT |$| 65))
      |PSETCAT-;headRemainder;PSR;23|)
     (PROGN
      (LETT |#G47|
       (|PSETCAT-;exactQuo| |lca| |g| |$|)
       |PSETCAT-;headRemainder;PSR;23|)
      (LETT |#G48|
       (|PSETCAT-;exactQuo| |lcp| |g| |$|)
       |PSETCAT-;headRemainder;PSR;23|)
      (LETT |lca| |#G47| |PSETCAT-;headRemainder;PSR;23|)
      (LETT |lcp| |#G48| |PSETCAT-;headRemainder;PSR;23|))
     (LETT |a|
      (SPADCALL
       (SPADCALL |lcp|
        (SPADCALL |a| (QREFELT |$| 62))
        (QREFELT |$| 66))
       (SPADCALL
        (SPADCALL |lca| (QCDR |e|) (QREFELT |$| 67))
        (SPADCALL |p| (QREFELT |$| 62)) (QREFELT |$| 68))
       (QREFELT |$| 69))
      |PSETCAT-;headRemainder;PSR;23|)
     (LETT |r|
      (SPADCALL |r| |lcp| (QREFELT |$| 70))
      |PSETCAT-;headRemainder;PSR;23|)
     (EXIT (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|))))
   ((QUOTE T)
             (LETT |lp2| (CDR |lp2|) |PSETCAT-;headRemainder;PSR;23|)))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (CONS |a| |r|))))))))))

(DEFUN |PSETCAT-;makeIrreducible!| (|frac| |$|)
 (PROG (|g|)
  (RETURN
   (SEQ
    (LETT |g|
     (SPADCALL (QCDR |frac|) (QCAR |frac|) (QREFELT |$| 73))
     |PSETCAT-;makeIrreducible!|)
    (EXIT
     (COND
      ((SPADCALL |g| (QREFELT |$| 74)) |frac|)
      ((QUOTE T)
       (SEQ
        (PROGN
         (RPLACA |frac| (SPADCALL (QCAR |frac|) |g| (QREFELT |$| 75)))
         (QCAR |frac|))
        (PROGN
         (RPLACD |frac| (|PSETCAT-;exactQuo| (QCDR |frac|) |g| |$|))
         (QCDR |frac|))
        (EXIT |frac|)))))))))

(DEFUN |PSETCAT-;remainder;PSR;25| (|a| |ps| |$|)
 (PROG (|hRa| |r| |lca| |g| |b| |c|)
  (RETURN
   (SEQ
    (LETT |hRa|
     (|PSETCAT-;makeIrreducible!| (SPADCALL |a| |ps| (QREFELT |$| 76)) |$|)
     |PSETCAT-;remainder;PSR;25|)
    (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (LETT |r| (QCDR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (EXIT
     (COND
      ((SPADCALL |a| (QREFELT |$| 42))
        (VECTOR (|spadConstant| |$| 61) |a| |r|))
      ((QUOTE T)
       (SEQ
        (LETT |b|
         (SPADCALL
          (|spadConstant| |$| 61)
          (SPADCALL |a| (QREFELT |$| 40))
      (QREFELT |$| 67))
     |PSETCAT-;remainder;PSR;25|)
    (LETT |c| (SPADCALL |a| (QREFELT |$| 64)) |PSETCAT-;remainder;PSR;25|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL
            (LETT |a|
             (SPADCALL |a| (QREFELT |$| 62))
             |PSETCAT-;remainder;PSR;25|)
            (QREFELT |$| 42))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |hRa|
       (|PSETCAT-;makeIrreducible!|
        (SPADCALL |a| |ps| (QREFELT |$| 76))
        |$|)
       |PSETCAT-;remainder;PSR;25|)
      (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
      (LETT |r|
       (SPADCALL |r| (QCDR |hRa|) (QREFELT |$| 70))
       |PSETCAT-;remainder;PSR;25|)
      (LETT |g|
       (SPADCALL |c|
        (LETT |lca|
         (SPADCALL |a| (QREFELT |$| 64))
         |PSETCAT-;remainder;PSR;25|)
        (QREFELT |$| 65))
       |PSETCAT-;remainder;PSR;25|)
      (LETT |b|
       (SPADCALL
        (SPADCALL
         (SPADCALL
          (QCDR |hRa|)
          (|PSETCAT-;exactQuo| |c| |g| |$|)
          (QREFELT |$| 70))
         |b|
         (QREFELT |$| 66))
        (SPADCALL
         (|PSETCAT-;exactQuo| |lca| |g| |$|)
         (SPADCALL |a| (QREFELT |$| 40))
         (QREFELT |$| 67))
```

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              (QREFELT |$| 77))
             |PSETCAT-;remainder;PSR;25|)
            (EXIT (LETT |c| |g| |PSETCAT-;remainder;PSR;25|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT (VECTOR |c| |b| |r|))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
           (SEQ
            (LETT |rs| NIL |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
            (SEQ
             G190
             (COND
              ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                (GO G191)))
             (SEQ
              (LETT |p|
               (|SPADfirst| |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |ps|
               (CDR |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |p|
               (QCAR (SPADCALL |p| |cs| (QREFELT |$| 76)))
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (EXIT
               (COND
                ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                 (COND
                  ((SPADCALL |p| (QREFELT |$| 24))
                   (SEQ
                    (LETT |ps| NIL
                     |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
                    (EXIT
                     (LETT |rs|
                      (LIST (|spadConstant| |$| 83))
                      |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|))))
                  ((QUOTE T)
                   (SEQ
                    (SPADCALL |p| (QREFELT |$| 84))
                    (EXIT
                     (LETT |rs|
                      (CONS |p| |rs|)
                      |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)))))))))
             NIL
             (GO G190)
             G191
             (EXIT NIL))
            (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithRemainder;LSL;27| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
           (SEQ
            G190
            (COND
               ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                 (GO G191)))
              (SEQ
               (LETT |p|
                (|SPADfirst| |ps|)
                |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
               (LETT |ps| (CDR |ps|) |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
               (LETT |p|
                (QVELT (SPADCALL |p| |cs| (QREFELT |$| 87)) 1)
                |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
               (EXIT
                (COND
                 ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                  (COND
                   ((SPADCALL |p| (QREFELT |$| 24))
                    (SEQ
                     (LETT |ps| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                     (EXIT
                      (LETT |rs|
                       (LIST (|spadConstant| |$| 83))
                       |PSETCAT-;rewriteIdealWithRemainder;LSL;27|))))
                   ((QUOTE T)
                    (LETT |rs|
                     (CONS (SPADCALL |p| (QREFELT |$| 88)) |rs|)
                     |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)))))))
              NIL
              (GO G190)
              G191
              (EXIT NIL))
             (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PolynomialSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (|DV$1| |DV$2| |DV$3| |DV$4| |DV$5| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|PolynomialSetCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |DV$3| (|devaluate| |#3|) . #1#)
    (LETT |DV$4| (|devaluate| |#4|) . #1#)
    (LETT |DV$5| (|devaluate| |#5|) . #1#)
    (LETT |dv$|
     (LIST
      (QUOTE |PolynomialSetCategory&|)
      |DV$1| |DV$2| |DV$3| |DV$4| |DV$5|) . #1#)
    (LETT |$| (GETREFV 90) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasCategory| |#2| (QUOTE (|IntegralDomain|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (QSETREFV |$| 8 |#3|)
    (QSETREFV |$| 9 |#4|)
    (QSETREFV |$| 10 |#5|)
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 48
        (CONS (|dispatchFunction| |PSETCAT-;roughUnitIdeal?;SB;16|) |$|))
       (QSETREFV |$| 52
        (CONS (|dispatchFunction| |PSETCAT-;roughBase?;SB;18|) |$|))
       (QSETREFV |$| 54
        (CONS (|dispatchFunction| |PSETCAT-;roughSubIdeal?;2SB;19|) |$|))
       (QSETREFV |$| 57
        (CONS (|dispatchFunction| |PSETCAT-;roughEqualIdeals?;2SB;20|) |$|)))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|GcdDomain|)))
     (COND
      ((|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Symbol|))))
       (PROGN
        (QSETREFV |$| 72
         (CONS (|dispatchFunction| |PSETCAT-;headRemainder;PSR;23|) |$|))
        (QSETREFV |$| 79
         (CONS (|dispatchFunction| |PSETCAT-;remainder;PSR;25|) |$|))
        (QSETREFV |$| 86
         (CONS
          (|dispatchFunction| |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
          |$|))
        (QSETREFV |$| 89
         (CONS
          (|dispatchFunction| |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
          |$|)))))))
    |$|))))

(MAKEPROP
 (QUOTE |PolynomialSetCategory&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE
   #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
   (|local| |#4|) (|local| |#5|) (|List| 10) (0 . |members|) (|List| 9)
   (5 . |variables|) (|Boolean|) (10 . |<|) (|List| |$|) (16 . |concat|)
   (21 . |removeDuplicates|) (|Mapping| 15 9 9) (26 . |sort|)
   (32 . |mvar|) |PSETCAT-;variables;SL;4| (37 . |ground?|)
   (|Mapping| 15 10) (42 . |remove|) |PSETCAT-;mainVariables;SL;5|
   (48 . |=|) |PSETCAT-;mainVariable?;VarSetSB;6| (54 . |construct|)
   |PSETCAT-;collectUnder;SVarSetS;7| |PSETCAT-;collectUpper;SVarSetS;8|
   |PSETCAT-;collect;SVarSetS;9| (|Record| (|:| |under| |$|)
   (|:| |floor| |$|) (|:| |upper| |$|)) |PSETCAT-;sort;SVarSetR;10|
   (|Set| 10) (59 . |brace|) (64 . |=|) |PSETCAT-;=;2SB;11|
   (70 . |degree|) (75 . |<|) (81 . |zero?|) (86 . |any?|)
   (|Mapping| 15 10 10) (92 . |sort|) |PSETCAT-;triangular?;SB;14|
   |PSETCAT-;trivialIdeal?;SB;15| (98 . |roughUnitIdeal?|)
   (103 . |sup|) (109 . |+|) (115 . |=|) (121 . |roughBase?|)
   (126 . |rewriteIdealWithRemainder|) (132 . |roughSubIdeal?|)
   (138 . |=|) (144 . |roughSubIdeal?|) (150 . |roughEqualIdeals?|)
   (156 . |quo|) (|Union| |$| (QUOTE "failed")) (162 . |exquo|)
   (168 . |One|) (172 . |reductum|) (177 . |subtractIfCan|)
   (183 . |leadingCoefficient|) (188 . |gcd|) (194 . |*|)
   (200 . |monomial|) (206 . |*|) (212 . |-|) (218 . |*|)
   (|Record| (|:| |num| 10) (|:| |den| 7)) (224 . |headRemainder|)
   (230 . |gcd|) (236 . |one?|) (241 . |exactQuotient!|)
   (247 . |headRemainder|) (253 . |+|) (|Record| (|:| |rnum| 7)
   (|:| |polnum| 10) (|:| |den| 7)) (259 . |remainder|)
   (265 . |trivialIdeal?|) (270 . |roughUnitIdeal?|)
   (275 . |Zero|) (279 . |One|) (283 . |primitivePart!|)
   (288 . |removeDuplicates|) (293 . |rewriteIdealWithHeadRemainder|)
   (299 . |remainder|) (305 . |unitCanonical|)
   (310 . |rewriteIdealWithRemainder|)))
(QUOTE #(|variables| 316 |trivialIdeal?| 321 |triangular?| 326
   |sort| 331 |roughUnitIdeal?| 337 |roughSubIdeal?| 342
   |roughEqualIdeals?| 348 |roughBase?| 354 |rewriteIdealWithRemainder|
   359 |rewriteIdealWithHeadRemainder| 365 |remainder| 371 |mainVariables|
   377 |mainVariable?| 382 |headRemainder| 388 |collectUpper| 394
   |collectUnder| 400 |collect| 406 |=| 412))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 89 (QUOTE (1 6 11 0 12 1 10 13 0 14 2 9 15 0
    0 16 1 13 0 17 18 1 13 0 0 19 2 13 0 20 0 21 1 10 9 0 22 1 10 15 0 24
    2 11 0 25 0 26 2 9 15 0 0 28 1 6 0 11 30 1 36 0 11 37 2 36 15 0 0 38 1
    10 8 0 40 2 8 15 0 0 41 1 10 15 0 42 2 11 15 25 0 43 2 11 0 44 0 45 1
```

```
        0 15 0 48 2 8 0 0 0 49 2 8 0 0 0 50 2 8 15 0 0 51 1 0 15 0 52 2 6 11 11
        0 53 2 0 15 0 0 54 2 6 15 0 0 55 2 6 15 0 0 56 2 0 15 0 0 57 2 7 0 0 0
        58 2 7 59 0 0 60 0 7 0 61 1 10 0 0 62 2 8 59 0 0 63 1 10 7 0 64 2 7 0
        0 0 65 2 10 0 7 0 66 2 10 0 7 8 67 2 10 0 0 0 68 2 10 0 0 0 69 2 7 0
        0 0 70 2 0 71 10 0 72 2 10 7 7 0 73 1 7 15 0 74 2 10 0 0 7 75 2 6 71
        10 0 76 2 10 0 0 0 77 2 0 78 10 0 79 1 6 15 0 80 1 6 15 0 81 0 10 0
        82 0 10 0 83 1 10 0 0 84 1 11 0 0 85 2 0 11 11 0 86 2 6 78 10 0 87 1
        10 0 0 88 2 0 11 11 0 89 1 0 13 0 23 1 0 15 0 47 1 0 15 0 46 2 0 34
        0 9 35 1 0 15 0 48 2 0 15 0 0 54 2 0 15 0 0 57 1 0 15 0 52 2 0 11 11
        0 89 2 0 11 11 0 86 2 0 78 10 0 79 1 0 13 0 27 2 0 15 9 0 29 2 0 71
        10 0 72 2 0 0 0 9 32 2 0 0 0 9 31 2 0 0 0 9 33 2 0 15 0 0 39))))))
    (QUOTE |lookupComplete|)))
```

21.47 QFCAT.lsp BOOTSTRAP

QFCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*QFCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |QuotientFieldCategory;CAT| (QUOTE NIL))

(SETQ |QuotientFieldCategory;AL| (QUOTE NIL))

(DEFUN |QuotientFieldCategory| (#1=#:G103631)
 (LET (#2=#:G103632)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |QuotientFieldCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |QuotientFieldCategory;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|QuotientFieldCategory;| #1#)))
      |QuotientFieldCategory;AL|))
    #2#))))

(DEFUN |QuotientFieldCategory;| (|t#1|)
 (PROG (#1=#:G103630)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|QuotientFieldCategory;CAT|)
       ((QUOTE T)
        (LETT |QuotientFieldCategory;CAT|
         (|Join|
          (|Field|)
          (|Algebra| (QUOTE |t#1|))
          (|RetractableTo| (QUOTE |t#1|))
          (|FullyEvalableOver| (QUOTE |t#1|))
          (|DifferentialExtension| (QUOTE |t#1|))
          (|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
          (|Patternable| (QUOTE |t#1|))
          (|FullyPatternMatchable| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|/| (|$| |t#1| |t#1|)) T)
            ((|numer| (|t#1| |$|)) T)
            ((|denom| (|t#1| |$|)) T)
            ((|numerator| (|$| |$|)) T)
            ((|denominator| (|$| |$|)) T)
            ((|wholePart| (|t#1| |$|)) (|has| |t#1| (|EuclideanDomain|)))
            ((|fractionPart| (|$| |$|)) (|has| |t#1| (|EuclideanDomain|)))
            ((|random| (|$|)) (|has| |t#1| (|IntegerNumberSystem|)))
            ((|ceiling| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))
            ((|floor| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))))
          (QUOTE (
            ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
            ((|RetractableTo| (|Integer|))
               (|has| |t#1| (|RetractableTo| (|Integer|))))
            ((|RetractableTo| (|Fraction| (|Integer|)))
               (|has| |t#1| (|RetractableTo| (|Integer|))))
            ((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
            ((|OrderedIntegralDomain|) (|has| |t#1| (|OrderedIntegralDomain|)))
            ((|RealConstant|) (|has| |t#1| (|RealConstant|)))
            ((|ConvertibleTo| (|InputForm|))
               (|has| |t#1| (|ConvertibleTo| (|InputForm|))))
            ((|CharacteristicZero|) (|has| |t#1| (|CharacteristicZero|)))
            ((|CharacteristicNonZero|) (|has| |t#1| (|CharacteristicNonZero|)))
            ((|RetractableTo| (|Symbol|))
               (|has| |t#1| (|RetractableTo| (|Symbol|))))
            ((|PolynomialFactorizationExplicit|)
               (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
          (QUOTE NIL) NIL)) . #2=(|QuotientFieldCategory|))))) . #2#)
    (SETELT #1# 0
      (LIST (QUOTE |QuotientFieldCategory|) (|devaluate| |t#1|)))))))
```

21.48 QFCAT-.lsp BOOTSTRAP

QFCAT- depends on **QFCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*QFCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |QFCAT-;numerator;2A;1| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;denominator;2A;2| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;init;A;3| (|$|)
 (SPADCALL (|spadConstant| |$| 13) (|spadConstant| |$| 14) (QREFELT |$| 15)))

(DEFUN |QFCAT-;nextItem;AU;4| (|n| |$|)
 (PROG (|m|)
  (RETURN
   (SEQ
    (LETT |m|
     (SPADCALL
      (SPADCALL |n| (QREFELT |$| 8))
      (QREFELT |$| 18))
     |QFCAT-;nextItem;AU;4|)
    (EXIT
     (COND
      ((QEQCAR |m| 1)
       (|error| "We seem to have a Fraction of a finite object"))
      ((QUOTE T)
       (CONS 0
        (SPADCALL (QCDR |m|) (|spadConstant| |$| 14) (QREFELT |$| 15))))))))))

(DEFUN |QFCAT-;map;M2A;5| (|fn| |x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) |fn|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) |fn|)
  (QREFELT |$| 15)))

(DEFUN |QFCAT-;reducedSystem;MM;6| (|m| |$|)
 (SPADCALL |m| (QREFELT |$| 26)))

(DEFUN |QFCAT-;characteristic;Nni;7| (|$|)
 (SPADCALL (QREFELT |$| 30)))

(DEFUN |QFCAT-;differentiate;AMA;8| (|x| |deriv| |$|)
 (PROG (|n| |d|)
  (RETURN
   (SEQ
    (LETT |n| (SPADCALL |x| (QREFELT |$| 8)) |QFCAT-;differentiate;AMA;8|)
    (LETT |d| (SPADCALL |x| (QREFELT |$| 11)) |QFCAT-;differentiate;AMA;8|)
    (EXIT
     (SPADCALL
      (SPADCALL
       (SPADCALL (SPADCALL |n| |deriv|) |d| (QREFELT |$| 32))
       (SPADCALL |n| (SPADCALL |d| |deriv|) (QREFELT |$| 32))
       (QREFELT |$| 33))
      (SPADCALL |d| 2 (QREFELT |$| 35)) (QREFELT |$| 15)))))))

(DEFUN |QFCAT-;convert;AIf;9| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 38))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 38))
  (QREFELT |$| 39)))

(DEFUN |QFCAT-;convert;AF;10| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 42))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 42))
  (QREFELT |$| 43)))

(DEFUN |QFCAT-;convert;ADf;11| (|x| |$|)
 (|/|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 46))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 46))))

(DEFUN |QFCAT-;<;2AB;12| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL
   (SPADCALL |x| (QREFELT |$| 8))
   (SPADCALL |y| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (SPADCALL
   (SPADCALL |y| (QREFELT |$| 8))
   (SPADCALL |x| (QREFELT |$| 11))
   (QREFELT |$| 32))
```

```
    (QREFELT |$| 49)))

  (DEFUN |QFCAT-;<;2AB;13| (|x| |y| |$|)
   (PROG (|#G19| |#G20| |#G21| |#G22|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL
         (SPADCALL |x| (QREFELT |$| 11))
         (|spadConstant| |$| 51)
         (QREFELT |$| 49))
        (PROGN
         (LETT |#G19| |y| |QFCAT-;<;2AB;13|)
         (LETT |#G20| |x| |QFCAT-;<;2AB;13|)
         (LETT |x| |#G19| |QFCAT-;<;2AB;13|)
         (LETT |y| |#G20| |QFCAT-;<;2AB;13|))))
      (COND
       ((SPADCALL
          (SPADCALL |y| (QREFELT |$| 11))
          (|spadConstant| |$| 51)
          (QREFELT |$| 49))
         (PROGN
          (LETT |#G21| |y| |QFCAT-;<;2AB;13|)
          (LETT |#G22| |x| |QFCAT-;<;2AB;13|)
          (LETT |x| |#G21| |QFCAT-;<;2AB;13|)
          (LETT |y| |#G22| |QFCAT-;<;2AB;13|))))
      (EXIT
       (SPADCALL
        (SPADCALL
         (SPADCALL |x| (QREFELT |$| 8))
         (SPADCALL |y| (QREFELT |$| 11))
         (QREFELT |$| 32))
        (SPADCALL
         (SPADCALL |y| (QREFELT |$| 8))
         (SPADCALL |x| (QREFELT |$| 11))
         (QREFELT |$| 32))
        (QREFELT |$| 49)))))))

  (DEFUN |QFCAT-;<;2AB;14| (|x| |y| |$|)
   (SPADCALL
    (SPADCALL
     (SPADCALL |x| (QREFELT |$| 8))
     (SPADCALL |y| (QREFELT |$| 11))
     (QREFELT |$| 32))
    (SPADCALL
     (SPADCALL |y| (QREFELT |$| 8))
    (SPADCALL |x| (QREFELT |$| 11))
    (QREFELT |$| 32))
   (QREFELT |$| 49)))

(DEFUN |QFCAT-;fractionPart;2A;15| (|x| |$|)
 (SPADCALL |x|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 52)) (QREFELT |$| 9))
  (QREFELT |$| 53)))

(DEFUN |QFCAT-;coerce;SA;16| (|s| |$|)
 (SPADCALL (SPADCALL |s| (QREFELT |$| 56)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;retract;AS;17| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 59)))

(DEFUN |QFCAT-;retractIfCan;AU;18| (|x| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;18|)
    (EXIT
     (COND
      ((QEQCAR |r| 1) (CONS 1 "failed"))
      ((QUOTE T) (SPADCALL (QCDR |r|) (QREFELT |$| 64)))))))))

(DEFUN |QFCAT-;convert;AP;19| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 67))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 67))
  (QREFELT |$| 68)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;20| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 72)))

(DEFUN |QFCAT-;convert;AP;21| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 76))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 76))
  (QREFELT |$| 77)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;22| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 81)))

(DEFUN |QFCAT-;coerce;FA;23| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 86)) (QREFELT |$| 87))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 88)) (QREFELT |$| 87))
  (QREFELT |$| 89)))

(DEFUN |QFCAT-;retract;AI;24| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 91)))

(DEFUN |QFCAT-;retractIfCan;AU;25| (|x| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;25|)
    (EXIT
     (COND
      ((QEQCAR |u| 1) (CONS 1 "failed"))
      ((QUOTE T) (SPADCALL (QCDR |u|) (QREFELT |$| 94)))))))))

(DEFUN |QFCAT-;random;A;26| (|$|)
 (PROG (|d|)
  (RETURN
   (SEQ
    (SEQ
      G190
      (COND
        ((NULL
          (SPADCALL
           (LETT |d| (SPADCALL (QREFELT |$| 96)) |QFCAT-;random;A;26|)
           (QREFELT |$| 97)))
         (GO G191)))
      (SEQ (EXIT |d|))
      NIL
      (GO G190)
      G191
     (EXIT NIL))
    (EXIT (SPADCALL (SPADCALL (QREFELT |$| 96)) |d| (QREFELT |$| 15)))))))

(DEFUN |QFCAT-;reducedSystem;MVR;27| (|m| |v| |$|)
 (PROG (|n|)
  (RETURN
   (SEQ
    (LETT |n|
     (SPADCALL
      (SPADCALL (SPADCALL |v| (QREFELT |$| 100)) |m| (QREFELT |$| 101))
      (QREFELT |$| 102))
     |QFCAT-;reducedSystem;MVR;27|)
    (EXIT
     (CONS
      (SPADCALL |n|
       (SPADCALL |n| (QREFELT |$| 103))
       (SPADCALL |n| (QREFELT |$| 104))
       (|+| 1 (SPADCALL |n| (QREFELT |$| 105)))
       (SPADCALL |n| (QREFELT |$| 106))
       (QREFELT |$| 107))
      (SPADCALL |n| (SPADCALL |n| (QREFELT |$| 105)) (QREFELT |$| 109))))))))

(DEFUN |QuotientFieldCategory&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|QuotientFieldCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |QuotientFieldCategory&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 119) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0 (LIST
       (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
       (|HasCategory| |#2| (QUOTE (|IntegerNumberSystem|)))
       (|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Symbol|))))
       (|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
       (|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
       (|HasCategory| |#2| (QUOTE (|RealConstant|)))
       (|HasCategory| |#2| (QUOTE (|OrderedIntegralDomain|)))
       (|HasCategory| |#2| (QUOTE (|OrderedSet|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Integer|))))
       (|HasCategory| |#2| (QUOTE (|StepThrough|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 12)
       (PROGN
        (QSETREFV |$| 16 (CONS (|dispatchFunction| |QFCAT-;init;A;3|) |$|))
        (QSETREFV |$| 20
         (CONS (|dispatchFunction| |QFCAT-;nextItem;AU;4|) |$|)))))
    (COND
     ((|testBitVector| |pv$| 7)
       (QSETREFV |$| 40
        (CONS (|dispatchFunction| |QFCAT-;convert;AIf;9|) |$|))))
    (COND
 ((|testBitVector| |pv$| 8)
   (PROGN
    (QSETREFV |$| 44
     (CONS (|dispatchFunction| |QFCAT-;convert;AF;10|) |$|))
    (QSETREFV |$| 47
     (CONS (|dispatchFunction| |QFCAT-;convert;ADf;11|) |$|)))))
(COND
 ((|testBitVector| |pv$| 9)
  (COND
   ((|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;12|) |$|)))
   ((QUOTE T)
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;13|) |$|)))))
 ((|testBitVector| |pv$| 10)
  (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;14|) |$|))))
(COND
 ((|testBitVector| |pv$| 3)
   (QSETREFV |$| 54
    (CONS (|dispatchFunction| |QFCAT-;fractionPart;2A;15|) |$|))))
(COND
 ((|testBitVector| |pv$| 4)
  (PROGN
   (QSETREFV |$| 57 (CONS (|dispatchFunction| |QFCAT-;coerce;SA;16|) |$|))
   (QSETREFV |$| 60 (CONS (|dispatchFunction| |QFCAT-;retract;AS;17|) |$|))
   (QSETREFV |$| 65
    (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;18|) |$|)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
  (PROGN
   (QSETREFV |$| 69 (CONS (|dispatchFunction| |QFCAT-;convert;AP;19|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
      (QSETREFV |$| 74
       (CONS
        (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;20|) |$|)))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
  (PROGN
   (QSETREFV |$| 78 (CONS (|dispatchFunction| |QFCAT-;convert;AP;21|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
     (QSETREFV |$| 83
      (CONS (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;22|) |$|)))))))
(COND
 ((|testBitVector| |pv$| 11)
  (PROGN
        (QSETREFV |$| 90 (CONS (|dispatchFunction| |QFCAT-;coerce;FA;23|) |$|))
        (COND
         ((|domainEqual| |#2| (|Integer|)))
         ((QUOTE T)
          (PROGN
           (QSETREFV |$| 92
            (CONS (|dispatchFunction| |QFCAT-;retract;AI;24|) |$|))
           (QSETREFV |$| 95
            (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;25|) |$|))))))))
     (COND
      ((|testBitVector| |pv$| 2)
        (QSETREFV |$| 98 (CONS (|dispatchFunction| |QFCAT-;random;A;26|) |$|))))
     |$|))))

(MAKEPROP
 (QUOTE |QuotientFieldCategory&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |numer|) (5 . |coerce|) |QFCAT-;numerator;2A;1| (10 . |denom|)
   |QFCAT-;denominator;2A;2| (15 . |init|) (19 . |One|) (23 . |/|)
   (29 . |init|) (|Union| |$| (QUOTE "failed")) (33 . |nextItem|)
   (38 . |One|) (42 . |nextItem|) (|Mapping| 7 7) |QFCAT-;map;M2A;5|
   (|Matrix| 7) (|Matrix| 6) (|MatrixCommonDenominator| 7 6)
   (47 . |clearDenominator|) (|Matrix| |$|) |QFCAT-;reducedSystem;MM;6|
   (|NonNegativeInteger|) (52 . |characteristic|)
   |QFCAT-;characteristic;Nni;7| (56 . |*|) (62 . |-|)
   (|PositiveInteger|) (68 . |**|) |QFCAT-;differentiate;AMA;8|
   (|InputForm|) (74 . |convert|) (79 . |/|) (85 . |convert|)
   (|Float|) (90 . |convert|) (95 . |/|) (101 . |convert|)
   (|DoubleFloat|) (106 . |convert|) (111 . |convert|) (|Boolean|)
   (116 . |<|) (122 . |<|) (128 . |Zero|) (132 . |wholePart|)
   (137 . |-|) (143 . |fractionPart|) (|Symbol|) (148 . |coerce|)
   (153 . |coerce|) (158 . |retract|) (163 . |retract|)
   (168 . |retract|) (|Union| 7 (QUOTE "failed")) (173 . |retractIfCan|)
   (|Union| 55 (QUOTE "failed")) (178 . |retractIfCan|)
   (183 . |retractIfCan|) (|Pattern| 84) (188 . |convert|)
   (193 . |/|) (199 . |convert|) (|PatternMatchResult| 84 6)
   (|PatternMatchQuotientFieldCategory| 84 7 6) (204 . |patternMatch|)
   (|PatternMatchResult| 84 |$|) (211 . |patternMatch|) (|Pattern| 41)
   (218 . |convert|) (223 . |/|) (229 . |convert|)
   (|PatternMatchResult| 41 6)
   (|PatternMatchQuotientFieldCategory| 41 7 6) (234 . |patternMatch|)
   (|PatternMatchResult| 41 |$|) (241 . |patternMatch|) (|Integer|)
   (|Fraction| 84) (248 . |numer|) (253 . |coerce|) (258 . |denom|)
   (263 . |/|) (269 . |coerce|) (274 . |retract|) (279 . |retract|)
 (|Union| 84 (QUOTE "failed")) (284 . |retractIfCan|)
 (289 . |retractIfCan|) (294 . |random|) (298 . |zero?|)
 (303 . |random|) (|Vector| 6) (307 . |coerce|) (312 . |horizConcat|)
 (318 . |reducedSystem|) (323 . |minRowIndex|) (328 . |maxRowIndex|)
 (333 . |minColIndex|) (338 . |maxColIndex|) (343 . |subMatrix|)
 (|Vector| 7) (352 . |column|) (|Record| (|:| |mat| 23)
 (|:| |vec| 108)) (|Vector| |$|) |QFCAT-;reducedSystem;MVR;27|
 (|Union| 85 (QUOTE "failed")) (|Record| (|:| |mat| 115)
 (|:| |vec| (|Vector| 84))) (|Matrix| 84) (|List| 55) (|List| 29)
 (|OutputForm|)))
(QUOTE #(|retractIfCan| 358 |retract| 368 |reducedSystem| 378
 |random| 389 |patternMatch| 393 |numerator| 407 |nextItem| 412
 |map| 417 |init| 423 |fractionPart| 427 |differentiate| 432
 |denominator| 438 |convert| 443 |coerce| 468 |characteristic| 478
 |<| 482))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 112 (QUOTE
    (1 6 7 0 8 1 6 0 7 9 1 6 7 0 11 0 7 0 13 0 7 0 14 2 6 0 7 7 15 0
     0 0 16 1 7 17 0 18 0 6 0 19 1 0 17 0 20 1 25 23 24 26 0 7 29 30 2
     7 0 0 0 32 2 7 0 0 0 33 2 7 0 0 34 35 1 7 37 0 38 2 37 0 0 0 39 1
     0 37 0 40 1 7 41 0 42 2 41 0 0 0 43 1 0 41 0 44 1 7 45 0 46 1 0 45
     0 47 2 7 48 0 0 49 2 0 48 0 0 50 0 7 0 51 1 6 7 0 52 2 6 0 0 0 53 1
     0 0 0 54 1 7 0 55 56 1 0 0 55 57 1 6 7 0 58 1 7 55 0 59 1 0 55 0 60
     1 6 61 0 62 1 7 63 0 64 1 0 63 0 65 1 7 66 0 67 2 66 0 0 0 68 1 0
     66 0 69 3 71 70 6 66 70 72 3 0 73 0 66 73 74 1 7 75 0 76 2 75 0 0
     0 77 1 0 75 0 78 3 80 79 6 75 79 81 3 0 82 0 75 82 83 1 85 84 0 86
     1 6 0 84 87 1 85 84 0 88 2 6 0 0 0 89 1 0 0 85 90 1 7 84 0 91 1 0
     84 0 92 1 7 93 0 94 1 0 93 0 95 0 7 0 96 1 7 48 0 97 0 0 0 98 1 24
     0 99 100 2 24 0 0 0 101 1 6 23 27 102 1 23 84 0 103 1 23 84 0 104
     1 23 84 0 105 1 23 84 0 106 5 23 0 0 84 84 84 84 107 2 23 108 0 84
     109 1 0 93 0 95 1 0 63 0 65 1 0 84 0 92 1 0 55 0 60 2 0 110 27 111
     112 1 0 23 27 28 0 0 0 98 3 0 82 0 75 82 83 3 0 73 0 66 73 74 1 0
     0 0 10 1 0 17 0 20 2 0 0 21 0 22 0 0 0 16 1 0 0 0 54 2 0 0 0 21 36
     1 0 0 0 12 1 0 45 0 47 1 0 37 0 40 1 0 41 0 44 1 0 66 0 69 1 0 75 0
     78 1 0 0 55 57 1 0 0 85 90 0 0 29 31 2 0 48 0 0 50))))))
(QUOTE |lookupComplete|)))
```

21.49 RCAGG.lsp BOOTSTRAP

RCAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RCAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |RecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |RecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |RecursiveAggregate| (#1=#:G84501)
 (LET (#2=#:G84502)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |RecursiveAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |RecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|RecursiveAggregate;| #1#)))
      |RecursiveAggregate;AL|))
    #2#))))

(DEFUN |RecursiveAggregate;| (|t#1|)
 (PROG (#1=#:G84500)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|RecursiveAggregate;CAT|)
       ((QUOTE T)
        (LETT |RecursiveAggregate;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|children| ((|List| |$|) |$|)) T)
            ((|nodes| ((|List| |$|) |$|)) T)
```

```
             ((|leaf?| ((|Boolean|) |$|)) T)
             ((|value| (|t#1| |$|)) T)
             ((|elt| (|t#1| |$| "value")) T)
             ((|cyclic?| ((|Boolean|) |$|)) T)
             ((|leaves| ((|List| |t#1|) |$|)) T)
             ((|distance| ((|Integer|) |$| |$|)) T)
             ((|child?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
             ((|node?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
             ((|setchildren!| (|$| |$| (|List| |$|)))
               (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
             ((|setelt| (|t#1| |$| "value" |t#1|))
               (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
             ((|setvalue!| (|t#1| |$| |t#1|))
               (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
            NIL
            (QUOTE ((|List| |$|) (|Boolean|) (|Integer|) (|List| |t#1|)))
            NIL))
          . #2=(|RecursiveAggregate|)))))
       . #2#)
      (SETELT #1# 0 (LIST (QUOTE |RecursiveAggregate|) (|devaluate| |t#1|)))))))
```

21.50 RCAGG-.lsp BOOTSTRAP

RCAGG- depends on **RCAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RCAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |RCAGG-;elt;AvalueS;1| (|x| G84515 |$|)
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |RCAGG-;setelt;Avalue2S;2| (|x| G84517 |y| |$|)
 (SPADCALL |x| |y| (QREFELT |$| 11)))

(DEFUN |RCAGG-;child?;2AB;3| (|x| |l| |$|)
 (SPADCALL |x| (SPADCALL |l| (QREFELT |$| 14)) (QREFELT |$| 17)))

(DEFUN |RecursiveAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|RecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |RecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 19) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 1)
      (QSETREFV |$| 12
       (CONS (|dispatchFunction| |RCAGG-;setelt;Avalue2S;2|) |$|))))
    (COND
     ((|testBitVector| |pv$| 2)
      (QSETREFV |$| 18 (CONS (|dispatchFunction| |RCAGG-;child?;2AB;3|) |$|))))
    |$|))))

 (MAKEPROP
  (QUOTE |RecursiveAggregate&|)
  (QUOTE |infovec|)
  (LIST
   (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
    (0 . |value|) (QUOTE "value") |RCAGG-;elt;AvalueS;1| (5 . |setvalue!|)
    (11 . |setelt|) (|List| |$|) (18 . |children|) (|Boolean|) (|List| 6)
    (23 . |member?|) (29 . |child?|)))
   (QUOTE #(|setelt| 35 |elt| 42 |child?| 48))
   (QUOTE NIL)
   (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 18 (QUOTE (1 6 7 0 8 2 6 7 0 7 11 3 0 7 0 9 7 12
      1 6 13 0 14 2 16 15 6 0 17 2 0 15 0 0 18 3 0 7 0 9 7 12 2 0 7 0 9
      10 2 0 15 0 0 18))))))
   (QUOTE |lookupComplete|)))
```

21.51 RING.lsp BOOTSTRAP

RING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Ring;AL| (QUOTE NIL))

(DEFUN |Ring| NIL
  (LET (#:G82789)
    (COND
      (|Ring;AL|)
      (T (SETQ |Ring;AL| (|Ring;|))))))

(DEFUN |Ring;| NIL
  (PROG (#1=#:G82787)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Rng|)
            (|Monoid|)
            (|LeftModule| (QUOTE |$|))
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|characteristic| ((|NonNegativeInteger|))) T)
                ((|coerce| (|$| (|Integer|))) T)))
              (QUOTE ((|unitsKnown| T)))
              (QUOTE ((|Integer|) (|NonNegativeInteger|)))
              NIL))
           |Ring|)
        (SETELT #1# 0 (QUOTE (|Ring|)))))))

(MAKEPROP (QUOTE |Ring|) (QUOTE NILADIC) T)
```

21.52 RING-.lsp BOOTSTRAP

RING- depends on **RING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |RING-;coerce;IS;1| (|n| |$|)
  (SPADCALL |n| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |Ring&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Ring&|))
        (LETT |dv$| (LIST (QUOTE |Ring&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 12) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |Ring&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |One|)
        (|Integer|)
        (4 . |*|)
        |RING-;coerce;IS;1|
        (|OutputForm|)))
  (QUOTE #(|coerce| 10))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 10 (QUOTE (0 6 0 7 2 6 0 8 0 9 1 0 0 8 10))))))
   (QUOTE |lookupComplete|)))
```

21.53 RNG.lsp BOOTSTRAP

RNG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNG.lsp BOOTSTRAP*⟩≡

```

(|/VERSIONCHECK| 2)

(SETQ |Rng;AL| (QUOTE NIL))

(DEFUN |Rng| NIL
  (LET (#:G82722)
    (COND
      (|Rng;AL|)
      (T (SETQ |Rng;AL| (|Rng;|))))))

(DEFUN |Rng;| NIL
  (PROG (#1=#:G82720)
    (RETURN
      (PROG1
        (LETT #1# (|Join| (|AbelianGroup|) (|SemiGroup|)) |Rng|)
        (SETELT #1# 0 (QUOTE (|Rng|)))))))

(MAKEPROP (QUOTE |Rng|) (QUOTE NILADIC) T)
```

21.54 RNS.lsp BOOTSTRAP

RNS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNS.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |RealNumberSystem;AL| (QUOTE NIL))

(DEFUN |RealNumberSystem| NIL
  (LET (#:G105478)
    (COND
      (|RealNumberSystem;AL|)
      (T (SETQ |RealNumberSystem;AL| (|RealNumberSystem;|))))))

(DEFUN |RealNumberSystem;| NIL
  (PROG (#1=#:G105476)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G105472 #3=#:G105473 #4=#:G105474 #5=#:G105475))
              (LIST
                (QUOTE (|Integer|))
                (QUOTE (|Fraction| (|Integer|)))
                (QUOTE (|Pattern| (|Float|)))
                (QUOTE (|Float|))))
            (|Join|
              (|Field|)
              (|OrderedRing|)
              (|RealConstant|)
              (|RetractableTo| (QUOTE #2#))
              (|RetractableTo| (QUOTE #3#))
              (|RadicalCategory|)
              (|ConvertibleTo| (QUOTE #4#))
              (|PatternMatchable| (QUOTE #5#))
              (|CharacteristicZero|)
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|norm| (|$| |$|)) T)
                  ((|ceiling| (|$| |$|)) T)
                  ((|floor| (|$| |$|)) T)
                  ((|wholePart| ((|Integer|) |$|)) T)
                  ((|fractionPart| (|$| |$|)) T)
                  ((|truncate| (|$| |$|)) T)
                  ((|round| (|$| |$|)) T)
                  ((|abs| (|$| |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL)))
          |RealNumberSystem|)
        (SETELT #1# 0 (QUOTE (|RealNumberSystem|)))))))

(MAKEPROP (QUOTE |RealNumberSystem|) (QUOTE NILADIC) T)
```

21.55 RNS-.lsp BOOTSTRAP

RNS- depends **RNS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNS-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |RNS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |RNS-;characteristic;Nni;1| (|$|) 0)

(DEFUN |RNS-;fractionPart;2S;2| (|x| |$|)
  (SPADCALL |x| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 10)))

(DEFUN |RNS-;truncate;2S;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL
          (SPADCALL |x| (QREFELT |$| 14))
          (QREFELT |$| 15))
        (QREFELT |$| 14)))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 15)))))

(DEFUN |RNS-;round;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL |x|
          (SPADCALL
            (|spadConstant| |$| 17)
            (SPADCALL 2 (QREFELT |$| 19))
            (QREFELT |$| 20))
          (QREFELT |$| 10))
        (QREFELT |$| 9)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL |x|
          (SPADCALL
            (|spadConstant| |$| 17)
            (SPADCALL 2 (QREFELT |$| 19))
            (QREFELT |$| 20))
          (QREFELT |$| 21))
        (QREFELT |$| 9)))))

(DEFUN |RNS-;norm;2S;5| (|x| |$|)
  (SPADCALL |x| (QREFELT |$| 23)))

(DEFUN |RNS-;coerce;FS;6| (|x| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 26))
      (QREFELT |$| 19))
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 27))
      (QREFELT |$| 19))
    (QREFELT |$| 20)))

(DEFUN |RNS-;convert;SP;7| (|x| |$|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 30)) (QREFELT |$| 32)))

(DEFUN |RNS-;floor;2S;8| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;floor;2S;8|)
        (EXIT
          (COND
            ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
            ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37))
              (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 10)))
            ((QUOTE T) |x1|)))))))

(DEFUN |RNS-;ceiling;2S;9| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;ceiling;2S;9|)
        (EXIT
          (COND
          ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
          ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37)) |x1|)
          ((QUOTE T)
            (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 21)))))))))

(DEFUN |RNS-;patternMatch;SP2Pmr;10| (|x| |p| |l| |$|)
  (PROG (|r|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 40))
            (SPADCALL |p| |x| |l| (QREFELT |$| 42)))
          ((SPADCALL |p| (QREFELT |$| 43))
            (SEQ
              (LETT |r|
                (SPADCALL |p| (QREFELT |$| 45))
                |RNS-;patternMatch;SP2Pmr;10|)
              (EXIT
                (COND
                  ((QEQCAR |r| 0)
                    (COND
                      ((SPADCALL
                         (SPADCALL |x| (QREFELT |$| 30))
                         (QCDR |r|)
                         (QREFELT |$| 46))
                        |l|)
                      ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))
                  ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))))
          ((QUOTE T) (SPADCALL (QREFELT |$| 47))))))))

(DEFUN |RealNumberSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|RealNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |RealNumberSystem&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 52) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |RealNumberSystem&|)
  (QUOTE |infovec|)
```

```
(LIST
  (QUOTE
    #(NIL NIL NIL NIL NIL NIL
      (|local| |#1|)
      (|NonNegativeInteger|)
      |RNS-;characteristic;Nni;1|
      (0 . |truncate|)
      (5 . |-|)
      |RNS-;fractionPart;2S;2|
      (|Boolean|)
      (11 . |negative?|)
      (16 . |-|)
      (21 . |floor|)
      |RNS-;truncate;2S;3|
      (26 . |One|)
      (|Integer|)
      (30 . |coerce|)
      (35 . |/|)
      (41 . |+|)
      |RNS-;round;2S;4|
      (47 . |abs|)
      |RNS-;norm;2S;5|
      (|Fraction| 18)
      (52 . |numer|)
      (57 . |denom|)
      |RNS-;coerce;FS;6|
      (|Float|)
      (62 . |convert|)
      (|Pattern| 29)
      (67 . |coerce|)
      |RNS-;convert;SP;7|
      (72 . |wholePart|)
      (77 . |=|)
      (83 . |Zero|)
      (87 . |<|)
      |RNS-;floor;2S;8|
      |RNS-;ceiling;2S;9|
      (93 . |generic?|)
      (|PatternMatchResult| 29 6)
      (98 . |addMatch|)
      (105 . |constant?|)
      (|Union| 29 (QUOTE "failed"))
      (110 . |retractIfCan|)
      (115 . |=|)
      (121 . |failed|)
      (|PatternMatchResult| 29 |$|)
    |RNS-;patternMatch;SP2Pmr;10|
    (|DoubleFloat|)
    (|OutputForm|)))
(QUOTE
  #(|truncate| 125 |round| 130 |patternMatch| 135 |norm| 142
    |fractionPart| 147 |floor| 152 |convert| 157 |coerce| 162
    |characteristic| 172 |ceiling| 176))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 49
        (QUOTE
          (1 6 0 0 9 2 6 0 0 0 10 1 6 12 0 13 1 6 0 0 14 1 6 0 0 15 0 6 0
           17 1 6 0 18 19 2 6 0 0 0 20 2 6 0 0 0 21 1 6 0 0 23 1 25 18 0
           26 1 25 18 0 27 1 6 29 0 30 1 31 0 29 32 1 6 18 0 34 2 6 12 0
           0 35 0 6 0 36 2 6 12 0 0 37 1 31 12 0 40 3 41 0 31 6 0 42 1 31
           12 0 43 1 31 44 0 45 2 29 12 0 0 46 0 41 0 47 1 0 0 0 16 1 0 0
           0 22 3 0 48 0 31 48 49 1 0 0 0 24 1 0 0 0 11 1 0 0 0 38 1 0 31
           0 33 1 0 0 25 28 1 0 0 25 28 0 0 7 8 1 0 0 0 39))))))
(QUOTE |lookupComplete|)))
```

21.56 SETAGG.lsp BOOTSTRAP

SETAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |SetAggregate;CAT| (QUOTE NIL))

(SETQ |SetAggregate;AL| (QUOTE NIL))

(DEFUN |SetAggregate| (#1=#:G83200)
 (LET (#2=#:G83201)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |SetAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |SetAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|SetAggregate;| #1#)))
      |SetAggregate;AL|))
    #2#))))

(DEFUN |SetAggregate;| (|t#1|)
 (PROG (#1=#:G83199)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|SetAggregate;CAT|)
       ((QUOTE T)
        (LETT |SetAggregate;CAT|
         (|Join|
          (|SetCategory|)
          (|Collection| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|<| ((|Boolean|) |$| |$|)) T)
        ((|brace| (|$|)) T)
        ((|brace| (|$| (|List| |t#1|))) T)
        ((|set| (|$|)) T)
        ((|set| (|$| (|List| |t#1|))) T)
        ((|intersect| (|$| |$| |$|)) T)
        ((|difference| (|$| |$| |$|)) T)
        ((|difference| (|$| |$| |t#1|)) T)
        ((|symmetricDifference| (|$| |$| |$|)) T)
        ((|subset?| ((|Boolean|) |$| |$|)) T)
        ((|union| (|$| |$| |$|)) T)
        ((|union| (|$| |$| |t#1|)) T)
        ((|union| (|$| |t#1| |$|)) T)))
       (QUOTE ((|partiallyOrderedSet| T)))
       (QUOTE ((|Boolean|) (|List| |t#1|)))
       NIL))
     . #2=(|SetAggregate|)))))
  . #2#)
(SETELT #1# 0 (LIST (QUOTE |SetAggregate|) (|devaluate| |t#1|)))))))
```

21.57 SETAGG-.lsp BOOTSTRAP

SETAGG- depends on **SETAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |SETAGG-;symmetricDifference;3A;1| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL |x| |y| (QREFELT |$| 8))
  (SPADCALL |y| |x| (QREFELT |$| 8))
  (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;ASA;2| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;S2A;3| (|x| |s| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))

(DEFUN |SETAGG-;difference;ASA;4| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 8)))

(DEFUN |SetAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |SetAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 16) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    |$|))))

(MAKEPROP
 (QUOTE |SetAggregate&|)
 (QUOTE |infovec|)
(LIST
 (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |difference|) (6 . |union|) |SETAGG-;symmetricDifference;3A;1|
   (|List| 7) (12 . |brace|) |SETAGG-;union;ASA;2| |SETAGG-;union;S2A;3|
   |SETAGG-;difference;ASA;4|))
 (QUOTE #(|union| 17 |symmetricDifference| 29 |difference| 35))
 (QUOTE NIL)
 (CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 15 (QUOTE (2 6 0 0 0 8 2 6 0 0 0 9 1 6 0 11 12 2
      0 0 7 0 14 2 0 0 0 7 13 2 0 0 0 0 10 2 0 0 0 7 15))))))
 (QUOTE |lookupComplete|)))
```

21.58 SETCAT.lsp BOOTSTRAP

SETCAT needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT**. We break this chain with **SETCAT.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |SetCategory;AL| (QUOTE NIL))

(DEFUN |SetCategory| NIL
  (LET (#:G82359)
    (COND
      (|SetCategory;AL|)
      (T (SETQ |SetCategory;AL| (|SetCategory;|))))))

(DEFUN |SetCategory;| NIL
  (PROG (#1=#:G82357)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G82356))
              (LIST (QUOTE (|OutputForm|))))
            (|Join|
              (|BasicType|)
              (|CoercibleTo| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|hash| ((|SingleInteger|) |$|)) T)
                  ((|latex| ((|String|) |$|)) T)))
                NIL
                (QUOTE ((|String|) (|SingleInteger|)))
                NIL)))
          |SetCategory|)
        (SETELT #1# 0 (QUOTE (|SetCategory|)))))))
```

```
(MAKEPROP (QUOTE |SetCategory|) (QUOTE NILADIC) T)
```

21.59 SETCAT-.lsp BOOTSTRAP

SETCAT- is the implementation of the operations exported by **SETCAT**. It comes into existance whenever **SETCAT** gets compiled by Axiom. However this will not happen at the lisp level so we also cache this information here. See the explanation under the **SETCAT.lsp** section for more details.

Note that this code is not included in the generated catdef.spad file.

⟨*SETCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |SETCAT-;hash;SSi;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) 0)))

(DEFUN |SETCAT-;hash;SSi;1| (|s| |$|) 0)

(PUT
  (QUOTE |SETCAT-;latex;SS;2|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) "\\mbox{\\bf Unimplemented}")))

(DEFUN |SETCAT-;latex;SS;2| (|s| |$|)
  "\\mbox{\\bf Unimplemented}")

(DEFUN |SetCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
   (RETURN
     (PROGN
       (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetCategory&|))
       (LETT |dv$| (LIST (QUOTE |SetCategory&|) |DV$1|) . #1#)
       (LETT |$| (GETREFV 11) . #1#)
       (QSETREFV |$| 0 |dv$|)
       (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
       (|stuffDomainSlots| |$|)
       (QSETREFV |$| 6 |#1|)
       |$|))))

(MAKEPROP
  (QUOTE |SetCategory&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
    (|local| |#1|)
    (|SingleInteger|)
    |SETCAT-;hash;SSi;1|
    (|String|)
    |SETCAT-;latex;SS;2|))
(QUOTE
  #(|latex| 0 |hash| 5))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2|
        10
        (QUOTE (1 0 9 0 10 1 0 7 0 8))))))
(QUOTE |lookupComplete|)))
```

21.60 STAGG.lsp BOOTSTRAP

STAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*STAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |StreamAggregate;CAT| (QUOTE NIL))

(SETQ |StreamAggregate;AL| (QUOTE NIL))

(DEFUN |StreamAggregate| (#1=#:G87035)
 (LET (#2=#:G87036)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |StreamAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |StreamAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|StreamAggregate;| #1#)))
      |StreamAggregate;AL|))
    #2#))))
v
(DEFUN |StreamAggregate;| (|t#1|)
 (PROG (#1=#:G87034)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|StreamAggregate;CAT|)
       ((QUOTE T)
        (LETT |StreamAggregate;CAT|
         (|Join|
          (|UnaryRecursiveAggregate| (QUOTE |t#1|))
          (|LinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|explicitlyFinite?| ((|Boolean|) |$|)) T)
          ((|possiblyInfinite?| ((|Boolean|) |$|)) T)))
         NIL
         (QUOTE ((|Boolean|)))
         NIL))
       . #2=(|StreamAggregate|)))))
     . #2#)
   (SETELT #1# 0 (LIST (QUOTE |StreamAggregate|) (|devaluate| |t#1|)))))))
```

21.61 STAGG-.lsp BOOTSTRAP

STAGG- depends on **STAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*STAGG-.lsp BOOTSTRAP*⟩≡

```

(|/VERSIONCHECK| 2)

(DEFUN |STAGG-;explicitlyFinite?;AB;1| (|x| |$|)
 (COND ((SPADCALL |x| (QREFELT |$| 9)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))

(DEFUN |STAGG-;possiblyInfinite?;AB;2| (|x| |$|)
 (SPADCALL |x| (QREFELT |$| 9)))

(DEFUN |STAGG-;first;ANniA;3| (|x| |n| |$|)
 (PROG (#1=#:G87053 |i|)
  (RETURN
   (SEQ
    (SPADCALL
     (PROGN
      (LETT #1# NIL |STAGG-;first;ANniA;3|)
      (SEQ
       (LETT |i| 1 |STAGG-;first;ANniA;3|)
       G190
       (COND ((QSGREATERP |i| |n|) (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS
           (|STAGG-;c2| |x|
            (LETT |x| (SPADCALL |x| (QREFELT |$| 12)) |STAGG-;first;ANniA;3|)
            |$|)
           #1#)
          |STAGG-;first;ANniA;3|)))
       (LETT |i| (QSADD1 |i|) |STAGG-;first;ANniA;3|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 14)))))

(DEFUN |STAGG-;c2| (|x| |r| |$|)
 (COND
   ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Index out of range"))
   ((QUOTE T) (SPADCALL |x| (QREFELT |$| 18)))))

(DEFUN |STAGG-;elt;AIS;5| (|x| |i| |$|)
 (PROG (#1=#:G87056)
  (RETURN
   (SEQ
    (LETT |i| (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;elt;AIS;5|)
    (COND
     ((OR
       (|<| |i| 0)
       (SPADCALL
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# |i| |STAGG-;elt;AIS;5|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         |STAGG-;elt;AIS;5|)
        (QREFELT |$| 17)))
      (EXIT (|error| "index out of range"))))
    (EXIT (SPADCALL |x| (QREFELT |$| 18)))))))

(DEFUN |STAGG-;elt;AUsA;6| (|x| |i| |$|)
 (PROG (|l| #1=#:G87060 |h| #2=#:G87062 #3=#:G87063)
  (RETURN
   (SEQ
    (LETT |l|
     (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
     |STAGG-;elt;AUsA;6|)
    (EXIT
     (COND
      ((|<| |l| 0) (|error| "index out of range"))
      ((NULL (SPADCALL |i| (QREFELT |$| 25)))
        (SPADCALL
         (SPADCALL |x|
          (PROG1
           (LETT #1# |l| |STAGG-;elt;AUsA;6|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         (QREFELT |$| 26)))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (|-| (SPADCALL |i| (QREFELT |$| 27)) (SPADCALL |x| (QREFELT |$| 20)))
```

```
          |STAGG-;elt;AUsA;6|)
         (EXIT
          (COND
           ((|<| |h| |l|) (SPADCALL (QREFELT |$| 28)))
           ((QUOTE T)
            (SPADCALL
             (SPADCALL |x|
              (PROG1
               (LETT #2# |l| |STAGG-;elt;AUsA;6|)
               (|check-subtype|
                 (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
              (QREFELT |$| 21))
             (PROG1
              (LETT #3# (|+| (|-| |h| |l|) 1) |STAGG-;elt;AUsA;6|)
              (|check-subtype| (|>=| #3# 0) (QUOTE (|NonNegativeInteger|)) #3#))
             (QREFELT |$| 29)))))))))))))

(DEFUN |STAGG-;concat;3A;7| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 26)) |y| (QREFELT |$| 31)))

(DEFUN |STAGG-;concat;LA;8| (|l| |$|)
 (COND
  ((NULL |l|) (SPADCALL (QREFELT |$| 28)))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL (|SPADfirst| |l|) (QREFELT |$| 26))
    (SPADCALL (CDR |l|) (QREFELT |$| 34))
    (QREFELT |$| 31)))))

(DEFUN |STAGG-;map!;M2A;9| (|f| |l| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |l| |STAGG-;map!;M2A;9|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |l| (QREFELT |$| 17)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |l|
       (SPADCALL (SPADCALL |l| (QREFELT |$| 18)) |f|) (QREFELT |$| 36))
      (EXIT (LETT |l| (SPADCALL |l| (QREFELT |$| 12)) |STAGG-;map!;M2A;9|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |y|)))))

(DEFUN |STAGG-;fill!;ASA;10| (|x| |s| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |x| |STAGG-;fill!;ASA;10|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |y| (QREFELT |$| 17)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |y| |s| (QREFELT |$| 36))
      (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 12)) |STAGG-;fill!;ASA;10|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |STAGG-;setelt;AI2S;11| (|x| |i| |s| |$|)
 (PROG (#1=#:G87081)
  (RETURN
   (SEQ
    (LETT |i|
     (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;setelt;AI2S;11|)
    (COND
     ((OR
       (|<| |i| 0)
       (SPADCALL
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# |i| |STAGG-;setelt;AI2S;11|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         |STAGG-;setelt;AI2S;11|)
        (QREFELT |$| 17)))
      (EXIT (|error| "index out of range"))))
      (EXIT (SPADCALL |x| |s| (QREFELT |$| 36)))))))

  (DEFUN |STAGG-;setelt;AUs2S;12| (|x| |i| |s| |$|)
   (PROG (|l| |h| #1=#:G87086 #2=#:G87087 |z| |y|)
    (RETURN
     (SEQ
      (LETT |l|
       (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
        |STAGG-;setelt;AUs2S;12|)
      (EXIT
       (COND
        ((|<| |l| 0) (|error| "index out of range"))
        ((QUOTE T)
         (SEQ
          (LETT |h|
           (COND
            ((SPADCALL |i| (QREFELT |$| 25))
              (|-|
               (SPADCALL |i| (QREFELT |$| 27))
               (SPADCALL |x| (QREFELT |$| 20))))
            ((QUOTE T) (SPADCALL |x| (QREFELT |$| 41))))
           |STAGG-;setelt;AUs2S;12|)
          (EXIT
           (COND
            ((|<| |h| |l|) |s|)
            ((QUOTE T)
              (SEQ
               (LETT |y|
                (SPADCALL |x|
                 (PROG1
                  (LETT #1# |l| |STAGG-;setelt;AUs2S;12|)
                  (|check-subtype|
                   (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
                 (QREFELT |$| 21))
                |STAGG-;setelt;AUs2S;12|)
               (LETT |z|
                (SPADCALL |y|
                 (PROG1
                  (LETT #2# (|+| (|-| |h| |l|) 1) |STAGG-;setelt;AUs2S;12|)
                  (|check-subtype|
                    (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
                 (QREFELT |$| 21))
                |STAGG-;setelt;AUs2S;12|)
               (SEQ
                 G190
                 (COND
                  ((NULL
                    (COND
                     ((SPADCALL |y| |z| (QREFELT |$| 42)) (QUOTE NIL))
                     ((QUOTE T) (QUOTE T))))
                   (GO G191)))
                 (SEQ
                  (SPADCALL |y| |s| (QREFELT |$| 36))
                  (EXIT
                   (LETT |y|
                    (SPADCALL |y| (QREFELT |$| 12))
                    |STAGG-;setelt;AUs2S;12|)))
                 NIL
                 (GO G190)
                 G191
                 (EXIT NIL))
               (EXIT |s|)))))))))))))

(DEFUN |STAGG-;concat!;3A;13| (|x| |y| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 17)) |y|)
   ((QUOTE T)
     (SEQ
      (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 45))
      (EXIT |x|))))))

(DEFUN |StreamAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|StreamAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |StreamAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 51) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
      (PROGN
       (QSETREFV |$| 32 (CONS (|dispatchFunction| |STAGG-;concat;3A;7|) |$|))
       (QSETREFV |$| 35 (CONS (|dispatchFunction| |STAGG-;concat;LA;8|) |$|))
       (QSETREFV |$| 38 (CONS (|dispatchFunction| |STAGG-;map!;M2A;9|) |$|))
       (QSETREFV |$| 39 (CONS (|dispatchFunction| |STAGG-;fill!;ASA;10|) |$|))
       (QSETREFV |$| 40
         (CONS (|dispatchFunction| |STAGG-;setelt;AI2S;11|) |$|))
       (QSETREFV |$| 43
         (CONS (|dispatchFunction| |STAGG-;setelt;AUs2S;12|) |$|))
       (QSETREFV |$| 46
         (CONS (|dispatchFunction| |STAGG-;concat!;3A;13|) |$|)))))
    |$|))))

(MAKEPROP
 (QUOTE |StreamAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|Boolean|) (0 . |cyclic?|) |STAGG-;explicitlyFinite?;AB;1|
   |STAGG-;possiblyInfinite?;AB;2| (5 . |rest|) (|List| 7)
   (10 . |construct|) (|NonNegativeInteger|) |STAGG-;first;ANniA;3|
   (15 . |empty?|) (20 . |first|) (|Integer|) (25 . |minIndex|)
   (30 . |rest|) |STAGG-;elt;AIS;5| (|UniversalSegment| 19) (36 . |lo|)
   (41 . |hasHi|) (46 . |copy|) (51 . |hi|) (56 . |empty|) (60 . |first|)
   |STAGG-;elt;AUsA;6| (66 . |concat!|) (72 . |concat|) (|List| |$|)
   (78 . |concat|) (83 . |concat|) (88 . |setfirst!|) (|Mapping| 7 7)
   (94 . |map!|) (100 . |fill!|) (106 . |setelt|) (113 . |maxIndex|)
   (118 . |eq?|) (124 . |setelt|) (131 . |tail|) (136 . |setrest!|)
   (142 . |concat!|) (QUOTE "rest") (QUOTE "last") (QUOTE "first")
   (QUOTE "value")))
  (QUOTE #(|setelt| 148 |possiblyInfinite?| 162 |map!| 167 |first| 173
   |fill!| 179 |explicitlyFinite?| 185 |elt| 190 |concat!| 202 |concat| 208))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 46 (QUOTE (1 6 8 0 9 1 6 0 0 12 1 6 0 13 14 1 6
       8 0 17 1 6 7 0 18 1 6 19 0 20 2 6 0 0 15 21 1 23 19 0 24 1 23 8
       0 25 1 6 0 0 26 1 23 19 0 27 0 6 0 28 2 6 0 0 15 29 2 6 0 0 0 31
       2 0 0 0 0 32 1 6 0 33 34 1 0 0 33 35 2 6 7 0 7 36 2 0 0 37 0 38 2
       0 0 0 7 39 3 0 7 0 19 7 40 1 6 19 0 41 2 6 8 0 0 42 3 0 7 0 23 7 43
       1 6 0 0 44 2 6 0 0 0 45 2 0 0 0 0 46 3 0 7 0 19 7 40 3 0 7 0 23 7 43
       1 0 8 0 11 2 0 0 37 0 38 2 0 0 0 15 16 2 0 0 0 7 39 1 0 8 0 10 2 0 7
       0 19 22 2 0 0 0 23 30 2 0 0 0 0 46 1 0 0 33 35 2 0 0 0 0 32))))))
  (QUOTE |lookupComplete|)))
```

21.62 TSETCAT.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*TSETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |TriangularSetCategory;CAT| (QUOTE NIL))

(SETQ |TriangularSetCategory;AL| (QUOTE NIL))

(DEFUN |TriangularSetCategory| (|&REST| #1=#:G82394 |&AUX| #2=#:G82392)
 (DSETQ #2# #1#)
 (LET (#3=#:G82393)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |TriangularSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |TriangularSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |TriangularSetCategory;|) #2#)))
      |TriangularSetCategory;AL|))
     #3#))))

(DEFUN |TriangularSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (PROG (#1=#:G82391)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
       (LIST
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)
        (|devaluate| |t#3|)
        (|devaluate| |t#4|)))
      (COND
(|TriangularSetCategory;CAT|)
((QUOTE T)
 (LETT |TriangularSetCategory;CAT|
  (|Join|
   (|PolynomialSetCategory|
    (QUOTE |t#1|)
    (QUOTE |t#2|)
    (QUOTE |t#3|)
    (QUOTE |t#4|))
   (|mkCategory|
    (QUOTE |domain|)
    (QUOTE (
     ((|infRittWu?| ((|Boolean|) |$| |$|)) T)
     ((|basicSet| (
        (|Union|
         (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
        (|List| |t#4|)
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|basicSet| (
        (|Union|
         (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
        (|List| |t#4|)
        (|Mapping| (|Boolean|) |t#4|)
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|initials| ((|List| |t#4|) |$|)) T)
     ((|degree| ((|NonNegativeInteger|) |$|)) T)
     ((|quasiComponent| (
        (|Record|
         (|:| |close| (|List| |t#4|))
         (|:| |open| (|List| |t#4|)))
        |$|))
       T)
     ((|normalized?| ((|Boolean|) |t#4| |$|)) T)
     ((|normalized?| ((|Boolean|) |$|)) T)
     ((|reduced?| (
        (|Boolean|)
        |t#4|
        |$|
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|stronglyReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|headReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|initiallyReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|autoReduced?| (
      (|Boolean|)
      |$|
      (|Mapping| (|Boolean|) |t#4| (|List| |t#4|))))
      T)
    ((|stronglyReduced?| ((|Boolean|) |$|)) T)
    ((|headReduced?| ((|Boolean|) |$|)) T)
    ((|initiallyReduced?| ((|Boolean|) |$|)) T)
    ((|reduce| (
       |t#4|
       |t#4|
       |$|
       (|Mapping| |t#4| |t#4| |t#4|)
       (|Mapping| (|Boolean|) |t#4| |t#4|)))
      T)
    ((|rewriteSetWithReduction| (
      (|List| |t#4|)
      (|List| |t#4|)
      |$|
      (|Mapping| |t#4| |t#4| |t#4|)
      (|Mapping| (|Boolean|) |t#4| |t#4|)))
      T)
    ((|stronglyReduce| (|t#4| |t#4| |$|)) T)
    ((|headReduce| (|t#4| |t#4| |$|)) T)
    ((|initiallyReduce| (|t#4| |t#4| |$|)) T)
    ((|removeZero| (|t#4| |t#4| |$|)) T)
    ((|collectQuasiMonic| (|$| |$|)) T)
    ((|reduceByQuasiMonic| (|t#4| |t#4| |$|)) T)
    ((|zeroSetSplit| ((|List| |$|) (|List| |t#4|))) T)
    ((|zeroSetSplitIntoTriangularSystems|
      ((|List|
        (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
       (|List| |t#4|)))
      T)
    ((|first| ((|Union| |t#4| "failed") |$|)) T)
    ((|last| ((|Union| |t#4| "failed") |$|)) T)
    ((|rest| ((|Union| |$| "failed") |$|)) T)
    ((|algebraicVariables| ((|List| |t#3|) |$|)) T)
    ((|algebraic?| ((|Boolean|) |t#3| |$|)) T)
    ((|select| ((|Union| |t#4| "failed") |$| |t#3|)) T)
    ((|extendIfCan| ((|Union| |$| "failed") |$| |t#4|)) T)
    ((|extend| (|$| |$| |t#4|)) T)
    ((|coHeight| ((|NonNegativeInteger|) |$|))
      (|has| |t#3| (|Finite|)))))
   (QUOTE (
    (|finiteAggregate| T)
    (|shallowlyMutable| T)))
```

```
            (QUOTE ((|NonNegativeInteger|)
             (|Boolean|)
             (|List| |t#3|)
             (|List| (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
             (|List| |t#4|)
             (|List| |$|)))
            NIL))
      . #2=(|TriangularSetCategory|)))))
     . #2#)
    (SETELT #1# 0
     (LIST
      (QUOTE |TriangularSetCategory|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)
      (|devaluate| |t#3|)
      (|devaluate| |t#4|)))))))
```

21.63 TSETCAT-.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*TSETCAT-.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(DEFUN |TSETCAT-;=;2SB;1| (|ts| |us| $)
 (PROG (#0=#:G1475 #1=#:G1481)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |us| (QREFELT $ 12)))
    ((OR
      (SPADCALL |us| (QREFELT $ 12))
      (NULL
       (SPADCALL
        (PROG2
         (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (PROG2
         (LETT #0# (SPADCALL |us| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (QREFELT $ 15))))
      (QUOTE NIL))
    ((QUOTE T)
     (SPADCALL
      (PROG2
       (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
       (QCDR #1#)
       (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
      (PROG2
       (LETT #1# (SPADCALL |us| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
       (QCDR #1#)
       (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
      (QREFELT $ 18)))))))

(DEFUN |TSETCAT-;infRittWu?;2SB;2| (|ts| |us| $)
 (PROG (|p| #0=#:G1489 |q| |v|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL |us| (QREFELT $ 12))
         (COND
          ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
          ((QUOTE T) (QUOTE T))))
       ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
       ((QUOTE T)
        (SEQ
         (LETT |p|
          (PROG2
           (LETT #0# (SPADCALL |ts| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;infRittWu?;2SB;2|)
         (LETT |q|
          (PROG2
           (LETT #0# (SPADCALL |us| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;infRittWu?;2SB;2|)
         (EXIT
          (COND
           ((SPADCALL |p| |q| (QREFELT $ 21)) (QUOTE T))
           ((SPADCALL |p| |q| (QREFELT $ 22)) (QUOTE NIL))
           ((QUOTE T)
            (SEQ
             (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;infRittWu?;2SB;2|)
             (EXIT
              (SPADCALL
               (SPADCALL |ts| |v| (QREFELT $ 24))
               (SPADCALL |us| |v| (QREFELT $ 24)) (QREFELT $ 25)))))))))))))

  (DEFUN |TSETCAT-;reduced?;PSMB;3| (|p| |ts| |redOp?| $)
   (PROG (|lp|)
    (RETURN
     (SEQ
      (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;reduced?;PSMB;3|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((NULL |lp|) (QUOTE NIL))
           ((QUOTE T) (SPADCALL |p| (|SPADfirst| |lp|) |redOp?|))))
          (GO G191)))
        (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;reduced?;PSMB;3|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (NULL |lp|))))))

(DEFUN |TSETCAT-;basicSet;LMU;4| (|ps| |redOp?| $)
 (PROG (|b| |bs| |p| |ts|)
  (RETURN
   (SEQ
    (LETT |ps|
     (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
     |TSETCAT-;basicSet;LMU;4|)
    (EXIT
     (COND
      ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |ps|
         (SPADCALL (ELT $ 21) |ps| (QREFELT $ 36))
         |TSETCAT-;basicSet;LMU;4|)
        (LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMU;4|)
        (LETT |ts| NIL |TSETCAT-;basicSet;LMU;4|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |b| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
          (LETT |bs|
           (SPADCALL |bs| |b| (QREFELT $ 38))
           |TSETCAT-;basicSet;LMU;4|)
          (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)
          (EXIT
           (SEQ
            G190
            (COND
             ((NULL
               (COND
                ((OR
                  (NULL |ps|)
                  (SPADCALL
                   (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
                  |bs| |redOp?| (QREFELT $ 39)))
                 (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMU;4|)
             (EXIT (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT (CONS 0 (CONS |bs| |ts|)))))))))))

(DEFUN |TSETCAT-;basicSet;LMMU;5| (|ps| |pred?| |redOp?| $)
 (PROG (|bps| |b| |bs| |p| |gps| |ts|)
  (RETURN
   (SEQ
    (LETT |ps|
     (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
     |TSETCAT-;basicSet;LMMU;5|)
    (EXIT
     (COND
      ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |gps| NIL |TSETCAT-;basicSet;LMMU;5|)
        (LETT |bps| NIL |TSETCAT-;basicSet;LMMU;5|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMMU;5|)
          (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMMU;5|)
          (EXIT
           (COND
            ((SPADCALL |p| |pred?|)
             (LETT |gps| (CONS |p| |gps|) |TSETCAT-;basicSet;LMMU;5|))
            ((QUOTE T)
             (LETT |bps| (CONS |p| |bps|) |TSETCAT-;basicSet;LMMU;5|)))))
          NIL
  (GO G190)
  G191
  (EXIT NIL))
(LETT |gps|
 (SPADCALL (ELT $ 21) |gps| (QREFELT $ 36)) |TSETCAT-;basicSet;LMMU;5|)
(LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMMU;5|)
(LETT |ts| NIL |TSETCAT-;basicSet;LMMU;5|)
(SEQ
 G190
 (COND
  ((NULL (COND ((NULL |gps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |b| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
  (LETT |bs|
   (SPADCALL |bs| |b| (QREFELT $ 38)) |TSETCAT-;basicSet;LMMU;5|)
  (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)
  (EXIT
   (SEQ
    G190
    (COND
     ((NULL
      (COND
       ((OR
         (NULL |gps|)
         (SPADCALL
          (LETT |p| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
          |bs| |redOp?| (QREFELT $ 39)))
        (QUOTE NIL))
       ((QUOTE T) (QUOTE T))))
      (GO G191)))
    (SEQ
     (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMMU;5|)
     (EXIT (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)))
    NIL
    (GO G190)
    G191
    (EXIT NIL))))
 NIL
 (GO G190)
 G191
 (EXIT NIL))
(LETT |ts|
 (SPADCALL
  (ELT $ 21)
  (SPADCALL |ts| |bps| (QREFELT $ 43))
```

```
          (QREFELT $ 36))
         |TSETCAT-;basicSet;LMMU;5|)
        (EXIT (CONS 0 (CONS |bs| |ts|)))))))))))

(DEFUN |TSETCAT-;initials;SL;6| (|ts| $)
 (PROG (|p| |ip| |lip| |lp|)
  (RETURN
   (SEQ
    (LETT |lip| NIL |TSETCAT-;initials;SL;6|)
    (EXIT
     (COND
      ((SPADCALL |ts| (QREFELT $ 12)) |lip|)
      ((QUOTE T)
       (SEQ
        (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initials;SL;6|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;initials;SL;6|)
          (COND
           ((NULL
             (SPADCALL
              (LETT |ip|
               (SPADCALL |p| (QREFELT $ 45))
               |TSETCAT-;initials;SL;6|)
              (QREFELT $ 34)))
            (LETT |lip|
             (CONS (SPADCALL |ip| (QREFELT $ 46)) |lip|)
             |TSETCAT-;initials;SL;6|)))
          (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initials;SL;6|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (SPADCALL |lip| (QREFELT $ 47)))))))))))

(DEFUN |TSETCAT-;degree;SNni;7| (|ts| $)
 (PROG (|lp| |d|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;degree;SNni;7|)
       (LETT |d|
        (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50))
        |TSETCAT-;degree;SNni;7|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((NULL
              (LETT |lp| (CDR |lp|) |TSETCAT-;degree;SNni;7|)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT |d|
           (* |d| (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50)))
           |TSETCAT-;degree;SNni;7|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |d|))))))))

(DEFUN |TSETCAT-;quasiComponent;SR;8| (|ts| $)
 (CONS (SPADCALL |ts| (QREFELT $ 28)) (SPADCALL |ts| (QREFELT $ 52))))

(DEFUN |TSETCAT-;normalized?;PSB;9| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 56)))

(DEFUN |TSETCAT-;stronglyReduced?;PSB;10| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 58)))

(DEFUN |TSETCAT-;headReduced?;PSB;11| (|p| |ts| $)
 (SPADCALL (SPADCALL |p| (QREFELT $ 60)) |ts| (QREFELT $ 61)))

(DEFUN |TSETCAT-;initiallyReduced?;PSB;12| (|p| |ts| $)
 (PROG (|lp| |red|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initiallyReduced?;PSB;12|)
    (LETT |red| (QUOTE T) |TSETCAT-;initiallyReduced?;PSB;12|)
    (SEQ
     G190
```

```
(COND
 ((NULL
   (COND
    ((OR (NULL |lp|) (SPADCALL |p| (QREFELT $ 34))) (QUOTE NIL))
    ((QUOTE T) |red|)))
  (GO G191)))
(SEQ
 (SEQ
  G190
  (COND
   ((NULL
     (COND
      ((NULL |lp|) (QUOTE NIL))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL |p| (QREFELT $ 23))
        (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
        (QREFELT $ 63)))))
    (GO G191)))
  (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
 (EXIT
  (COND
   ((NULL (NULL |lp|))
    (COND
     ((SPADCALL
       (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
       (SPADCALL |p| (QREFELT $ 23))
       (QREFELT $ 64))
      (COND
       ((SPADCALL |p| (|SPADfirst| |lp|) (QREFELT $ 65))
        (SEQ
         (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)
         (EXIT
          (LETT |p|
           (SPADCALL |p| (QREFELT $ 45))
           |TSETCAT-;initiallyReduced?;PSB;12|))))
       ((QUOTE T)
        (LETT |red| (QUOTE NIL) |TSETCAT-;initiallyReduced?;PSB;12|))))
     ((QUOTE T)
      (LETT |p|
       (SPADCALL |p| (QREFELT $ 45))
       |TSETCAT-;initiallyReduced?;PSB;12|)))))))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |red|)))))

(DEFUN |TSETCAT-;reduce;PSMMP;13| (|p| |ts| |redOp| |redOp?| $)
 (PROG (|ts0| #0=#:G1572 |reductor| #1=#:G1575)
  (RETURN
   (SEQ
    (COND
     ((OR (SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |p| (QREFELT $ 34))) |p|)
     ((QUOTE T)
      (SEQ
       (LETT |ts0| |ts| |TSETCAT-;reduce;PSMMP;13|)
       (SEQ
        G190
        (COND
         ((NULL
          (COND
           ((OR
             (SPADCALL |ts| (QREFELT $ 12))
             (SPADCALL |p| (QREFELT $ 34)))
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |reductor|
          (PROG2
           (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;reduce;PSMMP;13|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;reduce;PSMMP;13|)
         (LETT |ts|
          (PROG2
           (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;reduce;PSMMP;13|)
           (QCDR #1#)
           (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
          |TSETCAT-;reduce;PSMMP;13|)
         (EXIT
          (COND
           ((NULL (SPADCALL |p| |reductor| |redOp?|))
            (SEQ
             (LETT |p|
              (SPADCALL |p| |reductor| |redOp|) |TSETCAT-;reduce;PSMMP;13|)
             (EXIT (LETT |ts| |ts0| |TSETCAT-;reduce;PSMMP;13|)))))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |p|))))))))

(DEFUN |TSETCAT-;rewriteSetWithReduction;LSMML;14|
    (|lp| |ts| |redOp| |redOp?| $)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 69)) |lp|)
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 31) |lp| (QREFELT $ 33))
        |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
       (EXIT
        (COND
         ((NULL |lp|) |lp|)
         ((SPADCALL (ELT $ 34) |lp| (QREFELT $ 35))
           (LIST (|spadConstant| $ 70)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |p|
              (|SPADfirst| |lp|)
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |lp| (CDR |lp|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |p|
              (SPADCALL
               (SPADCALL |p| |ts| |redOp| |redOp?| (QREFELT $ 71))
               (QREFELT $ 46))
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (EXIT
              (COND
               ((NULL (SPADCALL |p| (QREFELT $ 31)))
                 (COND
                  ((SPADCALL |p| (QREFELT $ 34))
                   (SEQ
                    (LETT |lp| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
                    (EXIT
                     (LETT |rs|
                      (LIST (|spadConstant| $ 70))
                      |TSETCAT-;rewriteSetWithReduction;LSMML;14|))))
                  ((QUOTE T)
                   (LETT |rs|
                    (CONS |p| |rs|)
                    |TSETCAT-;rewriteSetWithReduction;LSMML;14|)))))))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
           (EXIT (SPADCALL |rs| (QREFELT $ 47)))))))))))))

(DEFUN |TSETCAT-;stronglyReduce;PSP;15| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 73) (ELT $ 65) (QREFELT $ 71)))

(DEFUN |TSETCAT-;headReduce;PSP;16| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 75) (ELT $ 76) (QREFELT $ 71)))

(DEFUN |TSETCAT-;initiallyReduce;PSP;17| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 78) (ELT $ 79) (QREFELT $ 71)))

(DEFUN |TSETCAT-;removeZero;PSP;18| (|p| |ts| $)
 (PROG (|v| |tsv-| #0=#:G1599 #1=#:G1608 |q|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;removeZero;PSP;18|)
        (LETT |tsv-|
         (SPADCALL |ts| |v| (QREFELT $ 81))
         |TSETCAT-;removeZero;PSP;18|)
        (COND
         ((SPADCALL |v| |ts| (QREFELT $ 82))
          (SEQ
           (LETT |q|
            (SPADCALL |p|
             (PROG2
              (LETT #0#
               (SPADCALL |ts| |v| (QREFELT $ 83))
       |TSETCAT-;removeZero;PSP;18|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
     (QREFELT $ 73))
    |TSETCAT-;removeZero;PSP;18|)
   (EXIT
    (COND
     ((SPADCALL |q| (QREFELT $ 31))
       (PROGN (LETT #1# |q| |TSETCAT-;removeZero;PSP;18|) (GO #1#)))
      ((SPADCALL (SPADCALL |q| |tsv-| (QREFELT $ 84)) (QREFELT $ 31))
        (PROGN
         (LETT #1#
          (|spadConstant| $ 85)
          |TSETCAT-;removeZero;PSP;18|) (GO #1#))))))))
(EXIT
 (COND
  ((SPADCALL |tsv-| (QREFELT $ 12)) |p|)
  ((QUOTE T)
   (SEQ
    (LETT |q| (|spadConstant| $ 85) |TSETCAT-;removeZero;PSP;18|)
    (SEQ
     G190
     (COND
      ((NULL
        (SPADCALL (SPADCALL |p| |v| (QREFELT $ 86)) (QREFELT $ 88)))
       (GO G191)))
     (SEQ
      (LETT |q|
       (SPADCALL
        (SPADCALL
         (SPADCALL
          (SPADCALL |p| (QREFELT $ 45))
           |tsv-| (QREFELT $ 84))
          (SPADCALL |p| (QREFELT $ 89))
         (QREFELT $ 90))
        |q|
        (QREFELT $ 91))
       |TSETCAT-;removeZero;PSP;18|)
      (EXIT
       (LETT |p|
        (SPADCALL |p| (QREFELT $ 92))
        |TSETCAT-;removeZero;PSP;18|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
```

```
              (EXIT
               (SPADCALL |q|
                (SPADCALL |p| |tsv-| (QREFELT $ 84))
                (QREFELT $ 91)))))))))))
      #1#
      (EXIT #1#)))))

(DEFUN |TSETCAT-;reduceByQuasiMonic;PSP;19| (|p| |ts| $)
 (COND
  ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
  ((QUOTE T)
    (QVELT (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 94)) (QREFELT $ 96)) 1))))

(DEFUN |TSETCAT-;autoReduced?;SMB;20| (|ts| |redOp?| $)
 (PROG (|p| |lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE T))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((NULL |lp|) (QUOTE NIL))
            ((QUOTE T) (SPADCALL |p| |lp| |redOp?|))))
           (GO G191)))
        (SEQ
         (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
         (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (NULL |lp|)))))))))

(DEFUN |TSETCAT-;stronglyReduced?;SB;21| (|ts| $)
 (SPADCALL |ts| (ELT $ 58) (QREFELT $ 100)))

(DEFUN |TSETCAT-;normalized?;SB;22| (|ts| $)
 (SPADCALL |ts| (ELT $ 56) (QREFELT $ 100)))

(DEFUN |TSETCAT-;headReduced?;SB;23| (|ts| $)
 (SPADCALL |ts| (ELT $ 103) (QREFELT $ 100)))

(DEFUN |TSETCAT-;initiallyReduced?;SB;24| (|ts| $)
 (SPADCALL |ts| (ELT $ 105) (QREFELT $ 100)))

(DEFUN |TSETCAT-;mvar;SV;25| (|ts| $)
 (PROG (#0=#:G1627)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12))
      (|error| "Error from TSETCAT in mvar : #1 is empty"))
    ((QUOTE T)
      (SPADCALL
       (PROG2
        (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;mvar;SV;25|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
       (QREFELT $ 23))))))

(DEFUN |TSETCAT-;first;SU;26| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;first;SU;26|)
       (EXIT (CONS 0 (|SPADfirst| |lp|))))))))))

(DEFUN |TSETCAT-;last;SU;27| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 21) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;last;SU;27|)
        (EXIT (CONS 0 (|SPADfirst| |lp|))))))))))

  (DEFUN |TSETCAT-;rest;SU;28| (|ts| $)
   (PROG (|lp|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
       ((QUOTE T)
        (SEQ
         (LETT |lp|
          (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
          |TSETCAT-;rest;SU;28|)
         (EXIT (CONS 0 (SPADCALL (CDR |lp|) (QREFELT $ 110))))))))))

  (DEFUN |TSETCAT-;coerce;SL;29| (|ts| $)
   (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36)))

  (DEFUN |TSETCAT-;algebraicVariables;SL;30| (|ts| $)
   (PROG (#0=#:G1652 |p| #1=#:G1653)
    (RETURN
     (SEQ
      (PROGN
       (LETT #0# NIL |TSETCAT-;algebraicVariables;SL;30|)
       (SEQ
        (LETT |p| NIL |TSETCAT-;algebraicVariables;SL;30|)
        (LETT #1#
         (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;algebraicVariables;SL;30|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |p| (CAR #1#) |TSETCAT-;algebraicVariables;SL;30|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0#
           (CONS (SPADCALL |p| (QREFELT $ 23)) #0#)
           |TSETCAT-;algebraicVariables;SL;30|)))
        (LETT #1# (CDR #1#) |TSETCAT-;algebraicVariables;SL;30|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))))))

  (DEFUN |TSETCAT-;algebraic?;VSB;31| (|v| |ts| $)
   (SPADCALL |v| (SPADCALL |ts| (QREFELT $ 115)) (QREFELT $ 116)))
```

```
(DEFUN |TSETCAT-;select;SVU;32| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;select;SVU;32|)
    (SEQ
     G190
     (COND
      ((NULL
       (COND
        ((OR
          (NULL |lp|)
          (SPADCALL |v|
           (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
           (QREFELT $ 64)))
          (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;select;SVU;32|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((NULL |lp|) (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (|SPADfirst| |lp|)))))))))

(DEFUN |TSETCAT-;collectQuasiMonic;2S;33| (|ts| $)
 (PROG (|newlp| |lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28))
     |TSETCAT-;collectQuasiMonic;2S;33|)
    (LETT |newlp| NIL |TSETCAT-;collectQuasiMonic;2S;33|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (COND
       ((SPADCALL (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 45)) (QREFELT $ 34))
        (LETT |newlp|
         (CONS (|SPADfirst| |lp|) |newlp|)
         |TSETCAT-;collectQuasiMonic;2S;33|)))
      (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectQuasiMonic;2S;33|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |newlp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUnder;SVS;34| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUnder;SVS;34|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (NULL |lp|)
           (SPADCALL
            (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
            |v|
            (QREFELT $ 63)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectUnder;SVS;34|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUpper;SVS;35| (|ts| |v| $)
 (PROG (|lp2| |lp1|)
  (RETURN
   (SEQ
    (LETT |lp1|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUpper;SVS;35|)
```

```
      (LETT |lp2| NIL |TSETCAT-;collectUpper;SVS;35|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((NULL |lp1|) (QUOTE NIL))
           ((QUOTE T)
            (SPADCALL |v|
             (SPADCALL (|SPADfirst| |lp1|) (QREFELT $ 23))
             (QREFELT $ 63)))))
         (GO G191)))
       (SEQ
        (LETT |lp2|
         (CONS (|SPADfirst| |lp1|) |lp2|)
         |TSETCAT-;collectUpper;SVS;35|)
        (EXIT (LETT |lp1| (CDR |lp1|) |TSETCAT-;collectUpper;SVS;35|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (SPADCALL (REVERSE |lp2|) (QREFELT $ 110)))))))

  (DEFUN |TSETCAT-;construct;LS;36| (|lp| $)
   (PROG (|rif|)
    (RETURN
     (SEQ
      (LETT |rif| (SPADCALL |lp| (QREFELT $ 122)) |TSETCAT-;construct;LS;36|)
      (EXIT
       (COND
        ((QEQCAR |rif| 0) (QCDR |rif|))
        ((QUOTE T)
         (|error| "in construct : LP -> $ from TSETCAT : bad arg"))))))))

  (DEFUN |TSETCAT-;retractIfCan;LU;37| (|lp| $)
   (PROG (|rif|)
    (RETURN
     (SEQ
      (COND
       ((NULL |lp|) (CONS 0 (SPADCALL (QREFELT $ 37))))
       ((QUOTE T)
        (SEQ
         (LETT |lp|
          (SPADCALL (ELT $ 22) |lp| (QREFELT $ 36))
          |TSETCAT-;retractIfCan;LU;37|)
         (LETT |rif|
        (SPADCALL (CDR |lp|) (QREFELT $ 122))
        |TSETCAT-;retractIfCan;LU;37|)
       (EXIT
        (COND
         ((QEQCAR |rif| 0)
           (SPADCALL (QCDR |rif|) (|SPADfirst| |lp|) (QREFELT $ 124)))
         ((QUOTE T)
           (|error| "in retractIfCan : LP -> ... from TSETCAT : bad arg")
 ))))))))))

(DEFUN |TSETCAT-;extend;SPS;38| (|ts| |p| $)
 (PROG (|eif|)
  (RETURN
   (SEQ
    (LETT |eif| (SPADCALL |ts| |p| (QREFELT $ 124)) |TSETCAT-;extend;SPS;38|)
    (EXIT
     (COND
      ((QEQCAR |eif| 0) (QCDR |eif|))
      ((QUOTE T)
       (|error| "in extend : ($,P) -> $ from TSETCAT : bad ars"))))))))

(DEFUN |TSETCAT-;coHeight;SNni;39| (|ts| $)
 (PROG (|n| |m| #0=#:G1696)
  (RETURN
   (SEQ
    (LETT |n| (SPADCALL (QREFELT $ 127)) |TSETCAT-;coHeight;SNni;39|)
    (LETT |m|
     (LENGTH (SPADCALL |ts| (QREFELT $ 28)))
     |TSETCAT-;coHeight;SNni;39|)
    (EXIT
     (PROG2
      (LETT #0# (SPADCALL |n| |m| (QREFELT $ 128)) |TSETCAT-;coHeight;SNni;39|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (|NonNegativeInteger|) #0#)))))))

(DEFUN |TriangularSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (DV$1 DV$2 DV$3 DV$4 DV$5 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|TriangularSetCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT DV$5 (|devaluate| |#5|) . #0#)
    (LETT |dv$|
     (LIST (QUOTE |TriangularSetCategory&|) DV$1 DV$2 DV$3 DV$4 DV$5) . #0#)
```

```
      (LETT $ (GETREFV 131) . #0#)
      (QSETREFV $ 0 |dv$|)
      (QSETREFV $ 3
       (LETT |pv$|
        (|buildPredVector| 0 0 (LIST (|HasCategory| |#4| (QUOTE (|Finite|)))))
        . #0#))
      (|stuffDomainSlots| $)
      (QSETREFV $ 6 |#1|)
      (QSETREFV $ 7 |#2|)
      (QSETREFV $ 8 |#3|)
      (QSETREFV $ 9 |#4|)
      (QSETREFV $ 10 |#5|)
      (COND
       ((|testBitVector| |pv$| 1)
         (QSETREFV $ 129
          (CONS (|dispatchFunction| |TSETCAT-;coHeight;SNni;39|) $))))
       $))))

(MAKEPROP
 (QUOTE |TriangularSetCategory&|)
 (QUOTE |infovec|)
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
    (|local| |#4|) (|local| |#5|) (|Boolean|) (0 . |empty?|)
    (|Union| 10 (QUOTE "failed")) (5 . |first|) (10 . =)
    (|Union| $ (QUOTE "failed")) (16 . |rest|) (21 . =) |TSETCAT-;=;2SB;1|
    (27 . |last|) (32 . |infRittWu?|) (38 . |supRittWu?|) (44 . |mvar|)
    (49 . |collectUpper|) (55 . |infRittWu?|) |TSETCAT-;infRittWu?;2SB;2|
    (|List| 10) (61 . |members|) (|Mapping| 11 10 10)
    |TSETCAT-;reduced?;PSMB;3| (66 . |zero?|) (|Mapping| 11 10)
    (71 . |remove|) (77 . |ground?|) (82 . |any?|) (88 . |sort|)
    (94 . |empty|) (98 . |extend|) (104 . |reduced?|)
    (|Record| (|:| |bas| $) (|:| |top| 27))
    (|Union| 40 (QUOTE "failed")) |TSETCAT-;basicSet;LMU;4|
    (111 . |concat|) |TSETCAT-;basicSet;LMMU;5| (117 . |init|)
    (122 . |primPartElseUnitCanonical|) (127 . |removeDuplicates|)
    |TSETCAT-;initials;SL;6| (|NonNegativeInteger|) (132 . |mdeg|)
    |TSETCAT-;degree;SNni;7| (137 . |initials|)
    (|Record| (|:| |close| 27) (|:| |open| 27))
    |TSETCAT-;quasiComponent;SR;8| (|List| $) (142 . |normalized?|)
    |TSETCAT-;normalized?;PSB;9| (148 . |reduced?|)
    |TSETCAT-;stronglyReduced?;PSB;10| (154 . |head|)
    (159 . |stronglyReduced?|) |TSETCAT-;headReduced?;PSB;11|
    (165 . <) (171 . =) (177 . |reduced?|)
    |TSETCAT-;initiallyReduced?;PSB;12| (|Mapping| 10 10 10)
    |TSETCAT-;reduce;PSMMP;13| (183 . |trivialIdeal?|) (188 . |One|)
```

```
   (192 . |reduce|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|
   (200 . |lazyPrem|) |TSETCAT-;stronglyReduce;PSP;15|
   (206 . |headReduce|) (212 . |headReduced?|)
   |TSETCAT-;headReduce;PSP;16| (218 . |initiallyReduce|)
   (224 . |initiallyReduced?|) |TSETCAT-;initiallyReduce;PSP;17|
   (230 . |collectUnder|) (236 . |algebraic?|) (242 . |select|)
   (248 . |removeZero|) (254 . |Zero|) (258 . |degree|) (|Integer|)
   (264 . |positive?|) (269 . |mainMonomial|) (274 . *) (280 . +)
   (286 . |tail|) |TSETCAT-;removeZero;PSP;18| (291 . |collectQuasiMonic|)
   (|Record| (|:| |rnum| 7) (|:| |polnum| 10) (|:| |den| 7))
   (296 . |remainder|) |TSETCAT-;reduceByQuasiMonic;PSP;19|
   (|Mapping| 11 10 27) |TSETCAT-;autoReduced?;SMB;20|
   (302 . |autoReduced?|) |TSETCAT-;stronglyReduced?;SB;21|
   |TSETCAT-;normalized?;SB;22| (308 . |headReduced?|)
   |TSETCAT-;headReduced?;SB;23| (314 . |initiallyReduced?|)
   |TSETCAT-;initiallyReduced?;SB;24| |TSETCAT-;mvar;SV;25|
   |TSETCAT-;first;SU;26| |TSETCAT-;last;SU;27| (320 . |construct|)
   |TSETCAT-;rest;SU;28| |TSETCAT-;coerce;SL;29| (|List| 9)
   |TSETCAT-;algebraicVariables;SL;30| (325 . |algebraicVariables|)
   (330 . |member?|) |TSETCAT-;algebraic?;VSB;31|
   |TSETCAT-;select;SVU;32| |TSETCAT-;collectQuasiMonic;2S;33|
   |TSETCAT-;collectUnder;SVS;34| |TSETCAT-;collectUpper;SVS;35|
   (336 . |retractIfCan|) |TSETCAT-;construct;LS;36|
   (341 . |extendIfCan|) |TSETCAT-;retractIfCan;LU;37|
   |TSETCAT-;extend;SPS;38| (347 . |size|) (351 . |subtractIfCan|)
   (357 . |coHeight|) (|OutputForm|)))
(QUOTE #(|stronglyReduced?| 362 |stronglyReduce| 373 |select| 379
   |rewriteSetWithReduction| 385 |retractIfCan| 393 |rest| 398 |removeZero|
   403 |reduced?| 409 |reduceByQuasiMonic| 416 |reduce| 422
   |quasiComponent| 430 |normalized?| 435 |mvar| 446 |last| 451
   |initials| 456 |initiallyReduced?| 461 |initiallyReduce| 472
   |infRittWu?| 478 |headReduced?| 484 |headReduce| 495 |first| 501
   |extend| 506 |degree| 512 |construct| 517 |collectUpper| 522
   |collectUnder| 528 |collectQuasiMonic| 534 |coerce| 539 |coHeight|
   544 |basicSet| 549 |autoReduced?| 562 |algebraicVariables| 568
   |algebraic?| 573 = 579))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #()) (CONS (QUOTE #())
  (|makeByteWordVec2| 129 (QUOTE (1 6 11 0 12 1 6 13 0 14 2 10 11 0 0 15
    1 6 16 0 17 2 6 11 0 0 18 1 6 13 0 20 2 10 11 0 0 21 2 10 11 0 0 22
    1 10 9 0 23 2 6 0 0 9 24 2 6 11 0 0 25 1 6 27 0 28 1 10 11 0 31 2 27
    0 32 0 33 1 10 11 0 34 2 27 11 32 0 35 2 27 0 29 0 36 0 6 0 37 2 6 0
    0 10 38 3 6 11 10 0 29 39 2 27 0 0 0 43 1 10 0 0 45 1 10 0 0 46 1 27
    0 0 47 1 10 49 0 50 1 6 27 0 52 2 10 11 0 55 56 2 10 11 0 55 58 1 10
   0 0 60 2 6 11 10 0 61 2 9 11 0 0 63 2 9 11 0 0 64 2 10 11 0 0 65 1 6
   11 0 69 0 10 0 70 4 6 10 10 0 67 29 71 2 10 0 0 0 73 2 10 0 0 0 75 2
   10 11 0 0 76 2 10 0 0 0 78 2 10 11 0 0 79 2 6 0 0 9 81 2 6 11 9 0 82
   2 6 13 0 9 83 2 6 10 10 0 84 0 10 0 85 2 10 49 0 9 86 1 87 11 0 88 1
   10 0 0 89 2 10 0 0 0 90 2 10 0 0 0 91 1 10 0 0 92 1 6 0 0 94 2 6 95
   10 0 96 2 6 11 0 98 100 2 10 11 0 55 103 2 10 11 0 55 105 1 6 0 27
   110 1 6 113 0 115 2 113 11 9 0 116 1 6 16 27 122 2 6 16 0 10 124 0
   9 49 127 2 49 16 0 0 128 1 0 49 0 129 1 0 11 0 101 2 0 11 10 0 59 2
   0 10 10 0 74 2 0 13 0 9 118 4 0 27 27 0 67 29 72 1 0 16 27 125 1 0
   16 0 111 2 0 10 10 0 93 3 0 11 10 0 29 30 2 0 10 10 0 97 4 0 10 10 0
   67 29 68 1 0 53 0 54 1 0 11 0 102 2 0 11 10 0 57 1 0 9 0 107 1 0 13
   0 109 1 0 27 0 48 1 0 11 0 106 2 0 11 10 0 66 2 0 10 10 0 80 2 0 11
   0 0 26 1 0 11 0 104 2 0 11 10 0 62 2 0 10 10 0 77 1 0 13 0 108 2 0 0
   0 10 126 1 0 49 0 51 1 0 0 27 123 2 0 0 0 9 121 2 0 0 0 9 120 1 0 0
   0 119 1 0 27 0 112 1 0 49 0 129 3 0 41 27 32 29 44 2 0 41 27 29 42 2
   0 11 0 98 99 1 0 113 0 114 2 0 11 9 0 117 2 0 11 0 0 19))))))
(QUOTE |lookupComplete|)))
```

21.64 UFD.lsp BOOTSTRAP

UFD needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UFD.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UniqueFactorizationDomain;AL| (QUOTE NIL))

(DEFUN |UniqueFactorizationDomain| NIL
  (LET (#:G83334)
    (COND
      (|UniqueFactorizationDomain;AL|)
      (T
        (SETQ
          |UniqueFactorizationDomain;AL|
          (|UniqueFactorizationDomain;|))))))

(DEFUN |UniqueFactorizationDomain;| NIL
  (PROG (#1=#:G83332)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|GcdDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|prime?| ((|Boolean|) |$|)) T)
                ((|squareFree| ((|Factored| |$|) |$|)) T)
                ((|squareFreePart| (|$| |$|)) T)
                ((|factor| ((|Factored| |$|) |$|)) T)))
              NIL
              (QUOTE ((|Factored| |$|) (|Boolean|)))
              NIL))
          |UniqueFactorizationDomain|)
        (SETELT #1# 0 (QUOTE (|UniqueFactorizationDomain|)))))))
  (MAKEPROP (QUOTE |UniqueFactorizationDomain|) (QUOTE NILADIC) T)
```

21.65 UFD-.lsp BOOTSTRAP

UFD- needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UFD-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |UFD-;squareFreePart;2S;1| (|x| |$|)
  (PROG (|s| |f| #1=#:G83349 #2=#:G83347 #3=#:G83345 #4=#:G83346)
    (RETURN
      (SEQ
        (SPADCALL
          (SPADCALL
            (LETT |s|
              (SPADCALL |x| (QREFELT |$| 8))
              |UFD-;squareFreePart;2S;1|)
            (QREFELT |$| 10))
          (PROGN
            (LETT #4# NIL |UFD-;squareFreePart;2S;1|)
            (SEQ
              (LETT |f| NIL |UFD-;squareFreePart;2S;1|)
              (LETT #1#
                (SPADCALL |s| (QREFELT |$| 13))
                |UFD-;squareFreePart;2S;1|)
              G190
              (COND
                ((OR
                   (ATOM #1#)
                   (PROGN
                     (LETT |f| (CAR #1#) |UFD-;squareFreePart;2S;1|)
                     NIL))
                  (GO G191)))
              (SEQ
                (EXIT
                  (PROGN
                    (LETT #2# (QCAR |f|) |UFD-;squareFreePart;2S;1|)
                    (COND
                      (#4#
                        (LETT #3#
                          (SPADCALL #3# #2# (QREFELT |$| 14))
                          |UFD-;squareFreePart;2S;1|))
                    ((QUOTE T)
                      (PROGN
                        (LETT #3# #2# |UFD-;squareFreePart;2S;1|)
                        (LETT #4# (QUOTE T)
                          |UFD-;squareFreePart;2S;1|)))))))
              (LETT #1# (CDR #1#) |UFD-;squareFreePart;2S;1|)
              (GO G190)
              G191
              (EXIT NIL))
            (COND
              (#4# #3#)
              ((QUOTE T) (|spadConstant| |$| 15))))
         (QREFELT |$| 14))))))

(DEFUN |UFD-;prime?;SB;2| (|x| |$|)
  (EQL
    (LENGTH (SPADCALL (SPADCALL |x| (QREFELT |$| 17)) (QREFELT |$| 21))) 1))

(DEFUN |UniqueFactorizationDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|UniqueFactorizationDomain&|))
        (LETT |dv$| (LIST (QUOTE |UniqueFactorizationDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 24) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |UniqueFactorizationDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|Factored| |$|)
        (0 . |squareFree|)
        (|Factored| 6)
        (5 . |unit|)
        (|Record| (|:| |factor| 6) (|:| |exponent| (|Integer|)))
        (|List| 11)
        (10 . |factors|)
        (15 . |*|)
```

```
   (21 . |One|)
   |UFD-;squareFreePart;2S;1|
   (25 . |factor|)
   (|Union|
     (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
   (|Record| (|:| |flg| 18) (|:| |fctr| 6) (|:| |xpnt| (|Integer|)))
   (|List| 19)
   (30 . |factorList|)
   (|Boolean|)
   |UFD-;prime?;SB;2|))
(QUOTE #(|squareFreePart| 35 |prime?| 40))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 23
        (QUOTE
          (1 6 7 0 8 1 9 6 0 10 1 9 12 0 13 2 6 0 0 0 14 0 6 0 15 1 6 7
           0 17 1 9 20 0 21 1 0 0 0 16 1 0 22 0 23))))))
 (QUOTE |lookupComplete|)))
```

21.66 ULSCAT.lsp BOOTSTRAP

ULSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ULSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **ULSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ULSCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnivariateLaurentSeriesCategory;CAT| (QUOTE NIL))

(SETQ |UnivariateLaurentSeriesCategory;AL| (QUOTE NIL))

(DEFUN |UnivariateLaurentSeriesCategory| (#1=#:G83278)
 (LET (#2=#:G83279)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnivariateLaurentSeriesCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |UnivariateLaurentSeriesCategory;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|UnivariateLaurentSeriesCategory;| #1#)))
      |UnivariateLaurentSeriesCategory;AL|))
     #2#))))

(DEFUN |UnivariateLaurentSeriesCategory;| (|t#1|)
 (PROG (#1=#:G83277)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83276)) (LIST (QUOTE (|Integer|))))
       (COND
        (|UnivariateLaurentSeriesCategory;CAT|)
        ((QUOTE T)
         (LETT |UnivariateLaurentSeriesCategory;CAT|
          (|Join|
           (|UnivariatePowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#))
       (|mkCategory| (QUOTE |domain|)
        (QUOTE (
         ((|series|
           (|$|
            (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|)))))
            T)
         ((|multiplyCoefficients| (|$| (|Mapping| |t#1| (|Integer|)) |$|))
            T)
         ((|rationalFunction|
           ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|)))
            (|has| |t#1| (|IntegralDomain|)))
         ((|rationalFunction|
           ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|) (|Integer|)))
            (|has| |t#1| (|IntegralDomain|)))
         ((|integrate| (|$| |$|))
           (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
         ((|integrate| (|$| |$| (|Symbol|)))
           (AND
            (|has| |t#1|
              (SIGNATURE |variables| ((|List| (|Symbol|)) |t#1|)))
            (|has| |t#1| (SIGNATURE |integrate| (|t#1| |t#1| (|Symbol|))))
            (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))
         ((|integrate| (|$| |$| (|Symbol|)))
           (AND
            (|has| |t#1| (|AlgebraicallyClosedFunctionSpace| (|Integer|)))
            (|has| |t#1| (|PrimitiveFunctionCategory|))
            (|has| |t#1| (|TranscendentalFunctionCategory|))
            (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))))
       (QUOTE (
        ((|RadicalCategory|)
          (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
        ((|TranscendentalFunctionCategory|)
          (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
        ((|Field|) (|has| |t#1| (|Field|)))))
       (QUOTE (
        (|Symbol|)
        (|Fraction| (|Polynomial| |t#1|))
        (|Integer|)
        (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|)))))
        NIL))
       . #3=(|UnivariateLaurentSeriesCategory|))))))
      . #3#)
(SETELT #1# 0
 (LIST (QUOTE |UnivariateLaurentSeriesCategory|) (|devaluate| |t#1|)))))))
```

21.67 UPOLYC.lsp BOOTSTRAP

UPOLYC depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UPOLYC.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnivariatePolynomialCategory;CAT| (QUOTE NIL))

(SETQ |UnivariatePolynomialCategory;AL| (QUOTE NIL))

(DEFUN |UnivariatePolynomialCategory| (#1=#:G103214)
  (LET (#2=#:G103215)
    (COND
      ((SETQ #2# (|assoc| (|devaluate| #1#) |UnivariatePolynomialCategory;AL|))
         (CDR #2#))
      (T
        (SETQ |UnivariatePolynomialCategory;AL|
          (|cons5|
            (CONS
              (|devaluate| #1#)
              (SETQ #2# (|UnivariatePolynomialCategory;| #1#)))
            |UnivariatePolynomialCategory;AL|))
         #2#))))

(DEFUN |UnivariatePolynomialCategory;| (|t#1|)
  (PROG (#1=#:G103213)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
            (|sublisV|
              (PAIR
                (QUOTE (#2=#:G103211 #3=#:G103212))
                (LIST
                  (QUOTE (|NonNegativeInteger|))
                  (QUOTE (|SingletonAsOrderedSet|))))
              (COND
                (|UnivariatePolynomialCategory;CAT|)
```

```
((QUOTE T)
  (LETT |UnivariatePolynomialCategory;CAT|
    (|Join|
      (|PolynomialCategory|
        (QUOTE |t#1|) (QUOTE #2#) (QUOTE #3#))
      (|Eltable| (QUOTE |t#1|) (QUOTE |t#1|))
      (|Eltable| (QUOTE |$|) (QUOTE |$|))
      (|DifferentialRing|)
      (|DifferentialExtension| (QUOTE |t#1|))
      (|mkCategory|
        (QUOTE |domain|)
        (QUOTE (
          ((|vectorise|
            ((|Vector| |t#1|) |$| (|NonNegativeInteger|))) T)
          ((|makeSUP|
            ((|SparseUnivariatePolynomial| |t#1|) |$|)) T)
          ((|unmakeSUP|
            (|$| (|SparseUnivariatePolynomial| |t#1|))) T)
          ((|multiplyExponents|
            (|$| |$| (|NonNegativeInteger|))) T)
          ((|divideExponents|
            ((|Union| |$| "failed")
             |$|
             (|NonNegativeInteger|))) T)
          ((|monicDivide|
            ((|Record|
               (|:| |quotient| |$|)
               (|:| |remainder| |$|))
             |$|
             |$|)) T)
          ((|karatsubaDivide|
            ((|Record|
               (|:| |quotient| |$|)
               (|:| |remainder| |$|))
             |$|
             (|NonNegativeInteger|))) T)
          ((|shiftRight| (|$| |$| (|NonNegativeInteger|))) T)
          ((|shiftLeft| (|$| |$| (|NonNegativeInteger|))) T)
          ((|pseudoRemainder| (|$| |$| |$|)) T)
          ((|differentiate|
            (|$| |$| (|Mapping| |t#1| |t#1|) |$|)) T)
          ((|discriminant| (|t#1| |$|))
            (|has| |t#1| (|CommutativeRing|)))
          ((|resultant| (|t#1| |$| |$|))
            (|has| |t#1| (|CommutativeRing|)))
          ((|elt|
         ((|Fraction| |$|)
          (|Fraction| |$|)
          (|Fraction| |$|)))
        (|has| |t#1| (|IntegralDomain|)))
      ((|order| ((|NonNegativeInteger|) |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|subResultantGcd| (|$| |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|composite| ((|Union| |$| "failed") |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|composite|
         ((|Union| (|Fraction| |$|) "failed")
          (|Fraction| |$|)
          |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|pseudoQuotient| (|$| |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|pseudoDivide|
         ((|Record|
            (|:| |coef| |t#1|)
            (|:| |quotient| |$|)
            (|:| |remainder| |$|))
         |$|
         |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|separate|
         ((|Record|
            (|:| |primePart| |$|)
            (|:| |commonPart| |$|))
         |$|
         |$|))
        (|has| |t#1| (|GcdDomain|)))
      ((|elt| (|t#1| (|Fraction| |$|) |t#1|))
        (|has| |t#1| (|Field|)))
      ((|integrate| (|$| |$|))
        (|has| |t#1|
          (|Algebra| (|Fraction| (|Integer|)))))))
     (QUOTE (
       ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
       ((|Eltable|
          (|Fraction| |$|)
          (|Fraction| |$|))
         (|has| |t#1| (|IntegralDomain|)))
       ((|EuclideanDomain|) (|has| |t#1| (|Field|)))
       (|additiveValuation| (|has| |t#1| (|Field|)))))
     (QUOTE (
                        (|Fraction| |$|)
                        (|NonNegativeInteger|)
                        (|SparseUnivariatePolynomial| |t#1|)
                        (|Vector| |t#1|)))
                      NIL))
                 . #4=(|UnivariatePolynomialCategory|))))))
        . #4#)
     (SETELT #1# 0
       (LIST
         (QUOTE |UnivariatePolynomialCategory|)
         (|devaluate| |t#1|)))))))
```

21.68 UPOLYC-.lsp BOOTSTRAP

UPOLYC- depends on **UPOLYC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UPOLYC-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |UPOLYC-;variables;SL;1| (|p| |$|)
  (COND
    ((OR
      (SPADCALL |p| (QREFELT |$| 9))
      (ZEROP (SPADCALL |p| (QREFELT |$| 11))))
     NIL)
    ((QUOTE T) (LIST (SPADCALL (QREFELT |$| 13))))))

(DEFUN |UPOLYC-;degree;SSaosNni;2| (|p| |v| |$|)
  (SPADCALL |p| (QREFELT |$| 11)))

(DEFUN |UPOLYC-;totalDegree;SLNni;3| (|p| |lv| |$|)
  (COND ((NULL |lv|) 0) ((QUOTE T) (SPADCALL |p| (QREFELT |$| 17)))))

(DEFUN |UPOLYC-;degree;SLL;4| (|p| |lv| |$|)
  (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT |$| 11))))))

(DEFUN |UPOLYC-;eval;SLLS;5| (|p| |lv| |lq| |$|)
  (COND
    ((NULL |lv|) |p|)
    ((NULL (NULL (CDR |lv|)))
      (|error| "can only eval a univariate polynomial once"))
    ((QUOTE T)
      (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lq|) (QREFELT |$| 21)))))

(DEFUN |UPOLYC-;eval;SSaos2S;6| (|p| |v| |q| |$|)
  (SPADCALL |p| |q| (QREFELT |$| 24)))

(DEFUN |UPOLYC-;eval;SLLS;7| (|p| |lv| |lr| |$|)
  (COND
    ((NULL |lv|) |p|)
    ((NULL (NULL (CDR |lv|)))
      (|error| "can only eval a univariate polynomial once"))
    ((QUOTE T)
      (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lr|) (QREFELT |$| 26)))))

(DEFUN |UPOLYC-;eval;SSaosRS;8| (|p| |v| |r| |$|)
  (SPADCALL (SPADCALL |p| |r| (QREFELT |$| 29)) (QREFELT |$| 30)))

(DEFUN |UPOLYC-;eval;SLS;9| (|p| |le| |$|)
  (COND
    ((NULL |le|) |p|)
    ((NULL (NULL (CDR |le|)))
      (|error| "can only eval a univariate polynomial once"))
    ((QUOTE T)
      (COND
        ((QEQCAR
           (SPADCALL
             (SPADCALL (|SPADfirst| |le|) (QREFELT |$| 33))
             (QREFELT |$| 35))
            1)
          |p|)
        ((QUOTE T)
          (SPADCALL |p|
            (SPADCALL (|SPADfirst| |le|) (QREFELT |$| 36))
            (QREFELT |$| 24)))))))

(DEFUN |UPOLYC-;mainVariable;SU;10| (|p| |$|)
  (COND
    ((ZEROP (SPADCALL |p| (QREFELT |$| 11))) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (SPADCALL (QREFELT |$| 13))))))

(DEFUN |UPOLYC-;minimumDegree;SSaosNni;11| (|p| |v| |$|)
  (SPADCALL |p| (QREFELT |$| 40)))

(DEFUN |UPOLYC-;minimumDegree;SLL;12| (|p| |lv| |$|)
  (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT |$| 40))))))

(DEFUN |UPOLYC-;monomial;SSaosNniS;13| (|p| |v| |n| |$|)
  (SPADCALL
    (CONS (FUNCTION |UPOLYC-;monomial;SSaosNniS;13!0|) (VECTOR |$| |n|))
    |p|
    (QREFELT |$| 45)))

(DEFUN |UPOLYC-;monomial;SSaosNniS;13!0| (|#1| |$$|)
  (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 43)))

(DEFUN |UPOLYC-;coerce;SaosS;14| (|v| |$|)
  (SPADCALL (|spadConstant| |$| 48) 1 (QREFELT |$| 49)))
```

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(DEFUN |UPOLYC-;makeSUP;SSup;15| (|p| |$|)
  (COND
    ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 52))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL |p| (QREFELT |$| 53))
          (SPADCALL |p| (QREFELT |$| 11))
          (QREFELT |$| 54))
        (SPADCALL
          (SPADCALL |p| (QREFELT |$| 55))
          (QREFELT |$| 56))
        (QREFELT |$| 57)))))

(DEFUN |UPOLYC-;unmakeSUP;SupS;16| (|sp| |$|)
  (COND
    ((SPADCALL |sp| (QREFELT |$| 59)) (|spadConstant| |$| 60))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL |sp| (QREFELT |$| 61))
          (SPADCALL |sp| (QREFELT |$| 62))
          (QREFELT |$| 49))
        (SPADCALL (SPADCALL |sp| (QREFELT |$| 63)) (QREFELT |$| 64))
      (QREFELT |$| 65)))))

(DEFUN |UPOLYC-;karatsubaDivide;SNniR;17| (|p| |n| |$|)
  (SPADCALL |p|
    (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49))
    (QREFELT |$| 68)))

(DEFUN |UPOLYC-;shiftRight;SNniS;18| (|p| |n| |$|)
  (QCAR
    (SPADCALL |p|
      (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49))
      (QREFELT |$| 68))))

(DEFUN |UPOLYC-;shiftLeft;SNniS;19| (|p| |n| |$|)
  (SPADCALL |p|
    (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49)) (QREFELT |$| 71)))

(DEFUN |UPOLYC-;solveLinearPolynomialEquation;LSupU;20| (|lpp| |pp| |$|)
  (SPADCALL |lpp| |pp| (QREFELT |$| 77)))

(DEFUN |UPOLYC-;factorPolynomial;SupF;21| (|pp| |$|)
  (SPADCALL |pp| (QREFELT |$| 83)))

(DEFUN |UPOLYC-;factorSquareFreePolynomial;SupF;22| (|pp| |$|)
  (SPADCALL |pp| (QREFELT |$| 86)))

(DEFUN |UPOLYC-;factor;SF;23| (|p| |$|)
  (PROG (|ansR| #1=#:G103310 |w| #2=#:G103311)
    (RETURN
      (SEQ
        (COND
          ((ZEROP (SPADCALL |p| (QREFELT |$| 11)))
            (SEQ
              (LETT |ansR|
                (SPADCALL
                  (SPADCALL |p| (QREFELT |$| 53))
                  (QREFELT |$| 89))
                |UPOLYC-;factor;SF;23|)
              (EXIT
                (SPADCALL
                  (SPADCALL
                    (SPADCALL |ansR| (QREFELT |$| 91))
                    (QREFELT |$| 30))
                  (PROGN
                    (LETT #1# NIL |UPOLYC-;factor;SF;23|)
                    (SEQ
                      (LETT |w| NIL |UPOLYC-;factor;SF;23|)
                      (LETT #2#
                        (SPADCALL |ansR| (QREFELT |$| 95))
                        |UPOLYC-;factor;SF;23|)
                      G190
                      (COND
                        ((OR
                          (ATOM #2#)
                          (PROGN
                            (LETT |w| (CAR #2#) |UPOLYC-;factor;SF;23|)
                            NIL))
                         (GO G191)))
                      (SEQ
                        (EXIT
                          (LETT #1#
                            (CONS
                              (VECTOR
                                (QVELT |w| 0)
                                (SPADCALL (QVELT |w| 1) (QREFELT |$| 30))
                                (QVELT |w| 2))
                             #1#)
```

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                           |UPOLYC-;factor;SF;23|)))
                    (LETT #2# (CDR #2#) |UPOLYC-;factor;SF;23|)
                    (GO G190)
                    G191
                    (EXIT (NREVERSE0 #1#))))
                (QREFELT |$| 99)))))
        ((QUOTE T)
          (SPADCALL
            (ELT |$| 64)
            (SPADCALL (SPADCALL |p| (QREFELT |$| 56)) (QREFELT |$| 100))
            (QREFELT |$| 104))))))))

(DEFUN |UPOLYC-;vectorise;SNniV;24| (|p| |n| |$|)
  (PROG (|v| |m| |i| #1=#:G103316 #2=#:G103312)
    (RETURN
      (SEQ
        (LETT |m|
          (SPADCALL
            (LETT |v|
              (SPADCALL |n| (|spadConstant| |$| 106) (QREFELT |$| 108))
              |UPOLYC-;vectorise;SNniV;24|)
            (QREFELT |$| 110))
          |UPOLYC-;vectorise;SNniV;24|)
        (SEQ
          (LETT |i|
            (SPADCALL |v| (QREFELT |$| 110))
            |UPOLYC-;vectorise;SNniV;24|)
          (LETT #1# (QVSIZE |v|) |UPOLYC-;vectorise;SNniV;24|)
          G190
          (COND ((|>| |i| #1#) (GO G191)))
          (SEQ
            (EXIT
              (SPADCALL |v| |i|
                (SPADCALL |p|
                  (PROG1
                    (LETT #2# (|-| |i| |m|) |UPOLYC-;vectorise;SNniV;24|)
                    (|check-subtype|
                      (|>=| #2# 0)
                      (QUOTE (|NonNegativeInteger|))
                      #2#))
                  (QREFELT |$| 111))
                (QREFELT |$| 112))))
          (LETT |i| (|+| |i| 1) |UPOLYC-;vectorise;SNniV;24|)
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT |v|)))))

(DEFUN |UPOLYC-;retract;SR;25| (|p| |$|)
  (COND
    ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 106))
    ((ZEROP (SPADCALL |p| (QREFELT |$| 11))) (SPADCALL |p| (QREFELT |$| 53)))
    ((QUOTE T) (|error| "Polynomial is not of degree 0"))))

(DEFUN |UPOLYC-;retractIfCan;SU;26| (|p| |$|)
  (COND
    ((SPADCALL |p| (QREFELT |$| 9)) (CONS 0 (|spadConstant| |$| 106)))
    ((ZEROP (SPADCALL |p| (QREFELT |$| 11)))
       (CONS 0 (SPADCALL |p| (QREFELT |$| 53))))
    ((QUOTE T) (CONS 1 "failed"))))

(DEFUN |UPOLYC-;init;S;27| (|$|)
  (SPADCALL (|spadConstant| |$| 117) (QREFELT |$| 30)))

(DEFUN |UPOLYC-;nextItemInner| (|n| |$|)
  (PROG (|nn| |n1| |n2| #1=#:G103337 |n3|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |n| (QREFELT |$| 9))
            (CONS
              0
              (SPADCALL
                (PROG2
                  (LETT #1#
                    (SPADCALL (|spadConstant| |$| 106) (QREFELT |$| 120))
                    |UPOLYC-;nextItemInner|)
                  (QCDR #1#)
                  (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
                (QREFELT |$| 30))))
          ((ZEROP (SPADCALL |n| (QREFELT |$| 11)))
            (SEQ
              (LETT |nn|
                (SPADCALL (SPADCALL |n| (QREFELT |$| 53)) (QREFELT |$| 120))
                |UPOLYC-;nextItemInner|)
              (EXIT
                (COND
                  ((QEQCAR |nn| 1) (CONS 1 "failed"))
                  ((QUOTE T)
                    (CONS 0 (SPADCALL (QCDR |nn|) (QREFELT |$| 30))))))))
          ((QUOTE T)
            (SEQ
              (LETT |n1|
                (SPADCALL |n| (QREFELT |$| 55))
                |UPOLYC-;nextItemInner|)
              (LETT |n2|
                (|UPOLYC-;nextItemInner| |n1| |$|)
                |UPOLYC-;nextItemInner|)
              (EXIT
                (COND
                  ((QEQCAR |n2| 0)
                    (CONS
                      0
                      (SPADCALL
                        (SPADCALL
                          (SPADCALL |n| (QREFELT |$| 53))
                          (SPADCALL |n| (QREFELT |$| 11))
                          (QREFELT |$| 49))
                        (QCDR |n2|)
                        (QREFELT |$| 65))))
                  ((|<|
                     (|+| 1 (SPADCALL |n1| (QREFELT |$| 11)))
                     (SPADCALL |n| (QREFELT |$| 11)))
                    (CONS
                      0
                      (SPADCALL
                        (SPADCALL
                          (SPADCALL |n| (QREFELT |$| 53))
                          (SPADCALL |n| (QREFELT |$| 11))
                          (QREFELT |$| 49))
                        (SPADCALL
                          (PROG2
                            (LETT #1#
                              (SPADCALL
                                (|spadConstant| |$| 117)
                                (QREFELT |$| 120))
                              |UPOLYC-;nextItemInner|)
                            (QCDR #1#)
                            (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
                          (|+| 1 (SPADCALL |n1| (QREFELT |$| 11)))
                          (QREFELT |$| 49))
                        (QREFELT |$| 65))))
                  ((QUOTE T)
                    (SEQ
                      (LETT |n3|
                        (SPADCALL
                          (SPADCALL |n| (QREFELT |$| 53))
                          (QREFELT |$| 120))
                        |UPOLYC-;nextItemInner|)
                      (EXIT
                        (COND
                          ((QEQCAR |n3| 1) (CONS 1 "failed"))
                          ((QUOTE T)
                            (CONS
                              0
                              (SPADCALL
                                (QCDR |n3|)
                                (SPADCALL |n| (QREFELT |$| 11))
                                (QREFELT |$| 49)))))))))))))))))

(DEFUN |UPOLYC-;nextItem;SU;29| (|n| |$|)
  (PROG (|n1| #1=#:G103350)
    (RETURN
      (SEQ
        (LETT |n1| (|UPOLYC-;nextItemInner| |n| |$|) |UPOLYC-;nextItem;SU;29|)
        (EXIT
          (COND
            ((QEQCAR |n1| 1)
              (CONS
                0
                (SPADCALL
                  (PROG2
                    (LETT #1#
                      (SPADCALL (|spadConstant| |$| 117) (QREFELT |$| 120))
                      |UPOLYC-;nextItem;SU;29|)
                    (QCDR #1#)
                    (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
                 (|+| 1 (SPADCALL |n| (QREFELT |$| 11)))
               (QREFELT |$| 49))))
            ((QUOTE T) |n1|)))))))

(DEFUN |UPOLYC-;content;SSaosS;30| (|p| |v| |$|)
  (SPADCALL (SPADCALL |p| (QREFELT |$| 123)) (QREFELT |$| 30)))

(DEFUN |UPOLYC-;primeFactor| (|p| |q| |$|)
  (PROG (#1=#:G103356 |p1|)
    (RETURN
      (SEQ
        (LETT |p1|
           (PROG2
             (LETT #1#
               (SPADCALL |p|
                 (SPADCALL |p| |q| (QREFELT |$| 125))
                 (QREFELT |$| 126))
                |UPOLYC-;primeFactor|)
              (QCDR #1#)
              (|check-union| (QEQCAR #1# 0) (QREFELT |$| 6) #1#))
            |UPOLYC-;primeFactor|)
         (EXIT
           (COND
             ((SPADCALL |p1| |p| (QREFELT |$| 127)) |p|)
             ((QUOTE T) (|UPOLYC-;primeFactor| |p1| |q| |$|))))))))

(DEFUN |UPOLYC-;separate;2SR;32| (|p| |q| |$|)
  (PROG (|a| #1=#:G103362)
    (RETURN
      (SEQ
        (LETT |a|
          (|UPOLYC-;primeFactor| |p| |q| |$|)
          |UPOLYC-;separate;2SR;32|)
        (EXIT
          (CONS
            |a|
            (PROG2
              (LETT #1#
                (SPADCALL |p| |a| (QREFELT |$| 126))
                |UPOLYC-;separate;2SR;32|)
              (QCDR #1#)
              (|check-union| (QEQCAR #1# 0) (QREFELT |$| 6) #1#)))))))

(DEFUN |UPOLYC-;differentiate;SM2S;33| (|x| |deriv| |x'| |$|)
  (PROG (|dg| |lc| #1=#:G103367 |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;SM2S;33|)
        (SEQ G190
          (COND
            ((NULL
               (|<| 0
                 (LETT |dg|
                   (SPADCALL |x| (QREFELT |$| 11))
                   |UPOLYC-;differentiate;SM2S;33|)))
              (GO G191)))
          (SEQ
            (LETT |lc|
              (SPADCALL |x| (QREFELT |$| 53))
              |UPOLYC-;differentiate;SM2S;33|)
            (LETT |d|
              (SPADCALL
                (SPADCALL |d|
                    (SPADCALL |x'|
                      (SPADCALL
                        (SPADCALL |dg| |lc| (QREFELT |$| 131))
                        (PROG1
                          (LETT #1# (|-| |dg| 1) |UPOLYC-;differentiate;SM2S;33|)
                          (|check-subtype|
                            (|>=| #1# 0)
                            (QUOTE (|NonNegativeInteger|))
                            #1#))
                        (QREFELT |$| 49))
                      (QREFELT |$| 71))
                    (QREFELT |$| 65))
                    (SPADCALL
                      (SPADCALL |lc| |deriv|)
                      |dg|
                      (QREFELT |$| 49))
                  (QREFELT |$| 65))
                |UPOLYC-;differentiate;SM2S;33|)
              (EXIT
                (LETT |x|
                  (SPADCALL |x| (QREFELT |$| 55))
                  |UPOLYC-;differentiate;SM2S;33|)))
             NIL
             (GO G190)
             G191
             (EXIT NIL))
          (EXIT
            (SPADCALL |d|
              (SPADCALL
                (SPADCALL
                  (SPADCALL |x| (QREFELT |$| 53))
                  |deriv|)
                (QREFELT |$| 30))
              (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;ncdiff| (|n| |x'| |$|)
  (PROG (#1=#:G103385 |n1|)
    (RETURN
      (COND
        ((ZEROP |n|) (|spadConstant| |$| 60))
        ((ZEROP
          (LETT |n1|
            (PROG1
              (LETT #1# (|-| |n| 1) |UPOLYC-;ncdiff|)
              (|check-subtype|
                (|>=| #1# 0)
                (QUOTE (|NonNegativeInteger|))
                #1#))
              |UPOLYC-;ncdiff|))
            |x'|)
          ((QUOTE T)
            (SPADCALL
              (SPADCALL |x'|
                (SPADCALL (|spadConstant| |$| 48) |n1| (QREFELT |$| 49))
                (QREFELT |$| 71))
              (SPADCALL
                (SPADCALL (|spadConstant| |$| 48) 1 (QREFELT |$| 49))
                (|UPOLYC-;ncdiff| |n1| |x'| |$|)
                (QREFELT |$| 71))
              (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;differentiate;SM2S;35| (|x| |deriv| |x'| |$|)
  (PROG (|dg| |lc| |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;SM2S;35|)
        (SEQ G190
          (COND
            ((NULL
               (|<| 0
                 (LETT |dg|
                   (SPADCALL |x| (QREFELT |$| 11))
                   |UPOLYC-;differentiate;SM2S;35|)))
              (GO G191)))
          (SEQ
            (LETT |lc|
              (SPADCALL |x| (QREFELT |$| 53))
              |UPOLYC-;differentiate;SM2S;35|)
            (LETT |d|
              (SPADCALL
                (SPADCALL |d|
                  (SPADCALL
                    (SPADCALL |lc| |deriv|)
                    |dg|
                    (QREFELT |$| 49))
                  (QREFELT |$| 65))
                (SPADCALL |lc|
                  (|UPOLYC-;ncdiff| |dg| |x'| |$|)
                  (QREFELT |$| 134))
              (QREFELT |$| 65))
              |UPOLYC-;differentiate;SM2S;35|)
            (EXIT
              (LETT |x|
                (SPADCALL |x| (QREFELT |$| 55))
                |UPOLYC-;differentiate;SM2S;35|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
         (EXIT
           (SPADCALL |d|
             (SPADCALL
               (SPADCALL (SPADCALL |x| (QREFELT |$| 53)) |deriv|)
               (QREFELT |$| 30))
             (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;differentiate;SMS;36| (|x| |deriv| |$|)
  (SPADCALL |x| |deriv| (|spadConstant| |$| 47) (QREFELT |$| 135)))

(DEFUN |UPOLYC-;differentiate;2S;37| (|x| |$|)
  (PROG (|dg| #1=#:G103394 |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;2S;37|)
        (SEQ G190
          (COND
            ((NULL
              (|<| 0
                (LETT |dg|
                  (SPADCALL |x| (QREFELT |$| 11))
                  |UPOLYC-;differentiate;2S;37|)))
            (GO G191)))
          (SEQ
            (LETT |d|
              (SPADCALL |d|
                (SPADCALL
                  (SPADCALL |dg|
                    (SPADCALL |x| (QREFELT |$| 53)) (QREFELT |$| 131))
                  (PROG1
                    (LETT #1# (|-| |dg| 1) |UPOLYC-;differentiate;2S;37|)
                    (|check-subtype|
                      (|>=| #1# 0)
                      (QUOTE (|NonNegativeInteger|))
                      #1#))
                  (QREFELT |$| 49))
                (QREFELT |$| 65))
              |UPOLYC-;differentiate;2S;37|)
            (EXIT
              (LETT |x|
                (SPADCALL |x| (QREFELT |$| 55))
                |UPOLYC-;differentiate;2S;37|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
        (EXIT |d|)))))

(DEFUN |UPOLYC-;differentiate;SSaosS;38| (|x| |v| |$|)
  (SPADCALL |x| (QREFELT |$| 138)))

(DEFUN |UPOLYC-;elt;3F;39| (|g| |f| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |g| (QREFELT |$| 141))
      |f|
      (QREFELT |$| 143))
    (SPADCALL (SPADCALL |g| (QREFELT |$| 144)) |f| (QREFELT |$| 143))
    (QREFELT |$| 145)))

(DEFUN |UPOLYC-;pseudoQuotient;3S;40| (|p| |q| |$|)
  (PROG (|n| #1=#:G103440 #2=#:G103442)
    (RETURN
      (SEQ
        (LETT |n|
          (|+|
            (|-|
              (SPADCALL |p| (QREFELT |$| 11))
              (SPADCALL |q| (QREFELT |$| 11))) 1)
           |UPOLYC-;pseudoQuotient;3S;40|)
        (EXIT
          (COND
            ((|<| |n| 1) (|spadConstant| |$| 60))
            ((QUOTE T)
              (PROG2
                (LETT #2#
                  (SPADCALL
                    (SPADCALL
                      (SPADCALL
                        (SPADCALL
                          (SPADCALL |q| (QREFELT |$| 53))
                          (PROG1
                            (LETT #1# |n| |UPOLYC-;pseudoQuotient;3S;40|)
                            (|check-subtype|
                              (|>=| #1# 0)
```

```
                                  (QUOTE (|NonNegativeInteger|))
                                  #1#))
                              (QREFELT |$| 147))
                            |p|
                            (QREFELT |$| 134))
                          (SPADCALL |p| |q| (QREFELT |$| 148))
                          (QREFELT |$| 149))
                        |q|
                        (QREFELT |$| 126))
                      |UPOLYC-;pseudoQuotient;3S;40|)
                    (QCDR #2#)
                    (|check-union| (QEQCAR #2# 0) (QREFELT |$| 6) #2#)))))))))

(DEFUN |UPOLYC-;pseudoDivide;2SR;41| (|p| |q| |$|)
  (PROG (|n| |prem| #1=#:G103448 |lc| #2=#:G103450)
    (RETURN
      (SEQ
        (LETT |n|
          (|+|
            (|-|
              (SPADCALL |p| (QREFELT |$| 11))
              (SPADCALL |q| (QREFELT |$| 11))) 1)
          |UPOLYC-;pseudoDivide;2SR;41|)
        (EXIT
          (COND
            ((|<| |n| 1)
              (VECTOR (|spadConstant| |$| 48) (|spadConstant| |$| 60) |p|))
            ((QUOTE T)
              (SEQ
                (LETT |prem|
                  (SPADCALL |p| |q| (QREFELT |$| 148))
                  |UPOLYC-;pseudoDivide;2SR;41|)
                (LETT |lc|
                  (SPADCALL
                    (SPADCALL |q| (QREFELT |$| 53))
                    (PROG1
                      (LETT #1# |n| |UPOLYC-;pseudoDivide;2SR;41|)
                      (|check-subtype|
                        (|>=| #1# 0)
                        (QUOTE (|NonNegativeInteger|)) #1#))
                    (QREFELT |$| 147))
                  |UPOLYC-;pseudoDivide;2SR;41|)
                (EXIT
                  (VECTOR |lc|
                    (PROG2
                      (LETT #2#
                            (SPADCALL
                              (SPADCALL
                                (SPADCALL |lc| |p| (QREFELT |$| 134))
                                |prem|
                                (QREFELT |$| 149))
                              |q|
                              (QREFELT |$| 126))
                            |UPOLYC-;pseudoDivide;2SR;41|)
                          (QCDR #2#)
                          (|check-union| (QEQCAR #2# 0) (QREFELT |$| 6) #2#))
                        |prem|))))))))))

(DEFUN |UPOLYC-;composite;FSU;42| (|f| |q| |$|)
  (PROG (|n| |d|)
    (RETURN
      (SEQ
        (LETT |n|
          (SPADCALL (SPADCALL |f| (QREFELT |$| 141)) |q| (QREFELT |$| 153))
          |UPOLYC-;composite;FSU;42|)
        (EXIT
          (COND
            ((QEQCAR |n| 1) (CONS 1 "failed"))
            ((QUOTE T)
              (SEQ
                (LETT |d|
                  (SPADCALL
                    (SPADCALL |f| (QREFELT |$| 144)) |q| (QREFELT |$| 153))
                  |UPOLYC-;composite;FSU;42|)
                (EXIT
                  (COND
                    ((QEQCAR |d| 1) (CONS 1 "failed"))
                    ((QUOTE T)
                      (CONS
                        0
                        (SPADCALL
                          (QCDR |n|)
                          (QCDR |d|)
                          (QREFELT |$| 154)))))))))))))

(DEFUN |UPOLYC-;composite;2SU;43| (|p| |q| |$|)
  (PROG (|cqr| |v| |u| |w| #1=#:G103476)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 157)) (CONS 0 |p|))
          ((QUOTE T)
(SEQ
  (EXIT
    (SEQ
      (LETT |cqr|
        (SPADCALL |p| |q| (QREFELT |$| 158))
        |UPOLYC-;composite;2SU;43|)
      (COND
        ((SPADCALL (QVELT |cqr| 2) (QREFELT |$| 157))
          (SEQ
            (LETT |v|
              (SPADCALL
                (QVELT |cqr| 2)
                (QVELT |cqr| 0)
                (QREFELT |$| 159))
              |UPOLYC-;composite;2SU;43|)
            (EXIT
              (COND
                ((QEQCAR |v| 0)
                  (SEQ
                    (LETT |u|
                      (SPADCALL
                        (QVELT |cqr| 1)
                        |q|
                        (QREFELT |$| 153))
                      |UPOLYC-;composite;2SU;43|)
                    (EXIT
                      (COND
                        ((QEQCAR |u| 0)
                          (SEQ
                            (LETT |w|
                              (SPADCALL
                                (QCDR |u|)
                                (QVELT |cqr| 0)
                                (QREFELT |$| 159))
                              |UPOLYC-;composite;2SU;43|)
                            (EXIT
                              (COND
                                ((QEQCAR |w| 0)
                                  (PROGN
                                    (LETT #1#
                                      (CONS
                                        0
                                        (SPADCALL
                                          (QCDR |v|)
                                          (SPADCALL
                                            (SPADCALL
                                                       (|spadConstant| |$| 48)
                                                       1
                                                       (QREFELT |$| 49))
                                                     (QCDR |w|)
                                                     (QREFELT |$| 71))
                                                    (QREFELT |$| 65)))
                                                  |UPOLYC-;composite;2SU;43|)
                                                (GO #1#))))))))))))))))
                  (EXIT (CONS 1 "failed"))))
              #1#
              (EXIT #1#))))))))

(DEFUN |UPOLYC-;elt;S2F;44| (|p| |f| |$|)
  (PROG (|n| #1=#:G103483 |ans|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 161))
          ((QUOTE T)
            (SEQ
              (LETT |ans|
                (SPADCALL
                  (SPADCALL (SPADCALL |p| (QREFELT |$| 53)) (QREFELT |$| 30))
                  (QREFELT |$| 162))
                |UPOLYC-;elt;S2F;44|)
              (LETT |n| (SPADCALL |p| (QREFELT |$| 11)) |UPOLYC-;elt;S2F;44|)
              (SEQ G190
                (COND
                  ((NULL
                    (COND
                      ((SPADCALL
                        (LETT |p|
                          (SPADCALL |p| (QREFELT |$| 55))
                          |UPOLYC-;elt;S2F;44|)
                        (QREFELT |$| 9))
                       (QUOTE NIL))
                      ((QUOTE T) (QUOTE T))))
                   (GO G191)))
                (SEQ
                  (EXIT
                    (LETT |ans|
                      (SPADCALL
                        (SPADCALL |ans|
                          (SPADCALL |f|
                            (PROG1
                              (LETT #1#
                                (|-| |n|
                                  (LETT |n|
                                    (SPADCALL |p| (QREFELT |$| 11))
                                    |UPOLYC-;elt;S2F;44|))
                                |UPOLYC-;elt;S2F;44|)
                              (|check-subtype|
                                (|>=| #1# 0)
                                (QUOTE (|NonNegativeInteger|))
                                #1#))
                            (QREFELT |$| 163))
                          (QREFELT |$| 164))
                        (SPADCALL
                          (SPADCALL
                            (SPADCALL |p| (QREFELT |$| 53))
                            (QREFELT |$| 30))
                          (QREFELT |$| 162))
                        (QREFELT |$| 165))
                      |UPOLYC-;elt;S2F;44|)))
                NIL
                (GO G190)
                G191
                (EXIT NIL))
              (EXIT
                (COND
                  ((ZEROP |n|) |ans|)
                  ((QUOTE T)
                    (SPADCALL |ans|
                      (SPADCALL |f| |n| (QREFELT |$| 166))
                      (QREFELT |$| 164)))))))))))

(DEFUN |UPOLYC-;order;2SNni;45| (|p| |q| |$|)
  (PROG (|u| #1=#:G103497 |ans|)
    (RETURN
      (SEQ
        (EXIT
          (COND
            ((SPADCALL |p| (QREFELT |$| 9))
              (|error| "order: arguments must be nonzero"))
            ((|<| (SPADCALL |q| (QREFELT |$| 11)) 1)
              (|error| "order: place must be non-trivial"))
            ((QUOTE T)
              (SEQ
                (LETT |ans| 0 |UPOLYC-;order;2SNni;45|)
                (EXIT
                  (SEQ G190
                    NIL
                    (SEQ
                      (LETT |u|
                        (SPADCALL |p| |q| (QREFELT |$| 126))
                        |UPOLYC-;order;2SNni;45|)
                      (EXIT
                        (COND
                          ((QEQCAR |u| 1)
                            (PROGN
                              (LETT #1# |ans| |UPOLYC-;order;2SNni;45|)
                              (GO #1#)))
                          ((QUOTE T)
                            (SEQ
                              (LETT |p| (QCDR |u|) |UPOLYC-;order;2SNni;45|)
                              (EXIT
                                (LETT
                                  |ans|
                                  (|+| |ans| 1)
                                  |UPOLYC-;order;2SNni;45|)))))))
                    NIL
                    (GO G190)
                    G191
                    (EXIT NIL)))))))
          #1#
          (EXIT #1#)))))

(DEFUN |UPOLYC-;squareFree;SF;46| (|p| |$|)
  (SPADCALL |p| (QREFELT |$| 170)))

(DEFUN |UPOLYC-;squareFreePart;2S;47| (|p| |$|)
  (SPADCALL |p| (QREFELT |$| 172)))

(DEFUN |UPOLYC-;gcdPolynomial;3Sup;48| (|pp| |qq| |$|)
  (COND
    ((SPADCALL |pp| (QREFELT |$| 174)) (SPADCALL |qq| (QREFELT |$| 175)))
    ((SPADCALL |qq| (QREFELT |$| 174)) (SPADCALL |pp| (QREFELT |$| 175)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL
            (SPADCALL |pp| (QREFELT |$| 176))
            (SPADCALL |qq| (QREFELT |$| 176)) (QREFELT |$| 125))
          (SPADCALL
            (SPADCALL
              (SPADCALL |pp| (QREFELT |$| 177))
              (SPADCALL |qq| (QREFELT |$| 177)) (QREFELT |$| 178))
            (QREFELT |$| 177))
          (QREFELT |$| 179))
        (QREFELT |$| 175)))))

(DEFUN |UPOLYC-;squareFreePolynomial;SupF;49| (|pp| |$|)
  (SPADCALL |pp| (QREFELT |$| 182)))

(DEFUN |UPOLYC-;elt;F2R;50| (|f| |r| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |f| (QREFELT |$| 141))
      |r|
      (QREFELT |$| 29))
    (SPADCALL (SPADCALL |f| (QREFELT |$| 144)) |r| (QREFELT |$| 29))
    (QREFELT |$| 184)))

(DEFUN |UPOLYC-;euclideanSize;SNni;51| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 9))
      (|error| "euclideanSize called on 0 in Univariate Polynomial"))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 11)))))

(DEFUN |UPOLYC-;divide;2SR;52| (|x| |y| |$|)
  (PROG (|lc| |f| #1=#:G103510 |n| |quot|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |y| (QREFELT |$| 9))
            (|error| "division by 0 in Univariate Polynomials"))
          ((QUOTE T)
            (SEQ
              (LETT |quot| (|spadConstant| |$| 60) |UPOLYC-;divide;2SR;52|)
              (LETT |lc|
                (SPADCALL (SPADCALL |y| (QREFELT |$| 53)) (QREFELT |$| 187))
                |UPOLYC-;divide;2SR;52|)
              (SEQ G190
                (COND
                  ((NULL
                     (COND
                       ((OR
                          (SPADCALL |x| (QREFELT |$| 9))
                          (|<|
                            (SPADCALL |x| (QREFELT |$| 11))
                            (SPADCALL |y| (QREFELT |$| 11))))
                         (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                    (GO G191)))
                (SEQ
                  (LETT |f|
                    (SPADCALL |lc|
                      (SPADCALL |x| (QREFELT |$| 53))
                      (QREFELT |$| 188))
                    |UPOLYC-;divide;2SR;52|)
                  (LETT |n|
                    (PROG1
                      (LETT #1#
                        (|-|
                          (SPADCALL |x| (QREFELT |$| 11))
                          (SPADCALL |y| (QREFELT |$| 11)))
                         |UPOLYC-;divide;2SR;52|)
                      (|check-subtype|
                        (|>=| #1# 0)
                        (QUOTE (|NonNegativeInteger|))
                        #1#))
                    |UPOLYC-;divide;2SR;52|)
                  (LETT |quot|
                    (SPADCALL |quot|
                      (SPADCALL |f| |n| (QREFELT |$| 49))
                      (QREFELT |$| 65))
                    |UPOLYC-;divide;2SR;52|)
                  (EXIT
                    (LETT |x|
                      (SPADCALL |x|
                        (SPADCALL
                          (SPADCALL |f| |n| (QREFELT |$| 49))
                          |y|
                          (QREFELT |$| 71))
                        (QREFELT |$| 149))
                      |UPOLYC-;divide;2SR;52|)))
                 NIL
                 (GO G190)
                 G191
                 (EXIT NIL))
              (EXIT (CONS |quot| |x|)))))))))

(DEFUN |UPOLYC-;integrate;2S;53| (|p| |$|)
  (PROG (|l| |d| |ans|)
    (RETURN
      (SEQ
        (LETT |ans| (|spadConstant| |$| 60) |UPOLYC-;integrate;2S;53|)
        (SEQ G190
          (COND
            ((NULL
              (COND
                  ((SPADCALL |p| (|spadConstant| |$| 60) (QREFELT |$| 127))
                    (QUOTE NIL))
                  ((QUOTE T) (QUOTE T)))) (GO G191)))
           (SEQ
             (LETT |l|
               (SPADCALL |p| (QREFELT |$| 53))
               |UPOLYC-;integrate;2S;53|)
             (LETT |d|
               (|+| 1 (SPADCALL |p| (QREFELT |$| 11)))
               |UPOLYC-;integrate;2S;53|)
             (LETT |ans|
               (SPADCALL |ans|
                 (SPADCALL
                   (SPADCALL (SPADCALL |d| (QREFELT |$| 191)) (QREFELT |$| 192))
                   (SPADCALL |l| |d| (QREFELT |$| 49)) (QREFELT |$| 193))
                 (QREFELT |$| 65))
               |UPOLYC-;integrate;2S;53|)
             (EXIT
               (LETT |p|
                 (SPADCALL |p| (QREFELT |$| 55))
                 |UPOLYC-;integrate;2S;53|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
         (EXIT |ans|)))))

(DEFUN |UnivariatePolynomialCategory&| (|#1| |#2|)
  (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|UnivariatePolynomialCategory&|))
        (LETT |DV$2| (|devaluate| |#2|) . #1#)
        (LETT |dv$|
          (LIST (QUOTE |UnivariatePolynomialCategory&|) |DV$1| |DV$2|) . #1#)
        (LETT |$| (GETREFV 201) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST
                (|HasCategory| |#2|
                  (QUOTE (|Algebra| (|Fraction| (|Integer|)))))
                (|HasCategory| |#2| (QUOTE (|Field|)))
                (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
                (|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
        (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
        (|HasCategory| |#2| (QUOTE (|StepThrough|))))) . #1#))
(|stuffDomainSlots| |$|)
(QSETREFV |$| 6 |#1|)
(QSETREFV |$| 7 |#2|)
(COND
  ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
    (PROGN
      (QSETREFV |$| 81
        (CONS
          (|dispatchFunction|
             |UPOLYC-;solveLinearPolynomialEquation;LSupU;20|)
          |$|))
      (QSETREFV |$| 85
        (CONS
          (|dispatchFunction| |UPOLYC-;factorPolynomial;SupF;21|)
          |$|))
      (QSETREFV |$| 87
         (CONS
            (|dispatchFunction|
               |UPOLYC-;factorSquareFreePolynomial;SupF;22|)
            |$|))
      (QSETREFV |$| 105
        (CONS (|dispatchFunction| |UPOLYC-;factor;SF;23|) |$|)))))
(COND
  ((|testBitVector| |pv$| 6)
    (PROGN
      (QSETREFV |$| 118
        (CONS (|dispatchFunction| |UPOLYC-;init;S;27|) |$|))
      NIL
      (QSETREFV |$| 122
        (CONS (|dispatchFunction| |UPOLYC-;nextItem;SU;29|) |$|)))))
(COND
  ((|testBitVector| |pv$| 3)
    (PROGN
      (QSETREFV |$| 124
        (CONS (|dispatchFunction| |UPOLYC-;content;SSaosS;30|) |$|))
      NIL
      (QSETREFV |$| 129
        (CONS (|dispatchFunction| |UPOLYC-;separate;2SR;32|) |$|)))))
(COND
  ((|testBitVector| |pv$| 5)
    (QSETREFV |$| 133
      (CONS
        (|dispatchFunction| |UPOLYC-;differentiate;SM2S;33|)
        |$|)))
  ((QUOTE T)
    (PROGN
      (QSETREFV |$| 133
        (CONS
          (|dispatchFunction| |UPOLYC-;differentiate;SM2S;35|)
          |$|)))))
(COND
  ((|testBitVector| |pv$| 4)
    (PROGN
      (QSETREFV |$| 146
        (CONS (|dispatchFunction| |UPOLYC-;elt;3F;39|) |$|))
      (QSETREFV |$| 150
        (CONS (|dispatchFunction| |UPOLYC-;pseudoQuotient;3S;40|) |$|))
      (QSETREFV |$| 152
        (CONS (|dispatchFunction| |UPOLYC-;pseudoDivide;2SR;41|) |$|))
      (QSETREFV |$| 156
        (CONS (|dispatchFunction| |UPOLYC-;composite;FSU;42|) |$|))
      (QSETREFV |$| 160
        (CONS (|dispatchFunction| |UPOLYC-;composite;2SU;43|) |$|))
      (QSETREFV |$| 167
        (CONS (|dispatchFunction| |UPOLYC-;elt;S2F;44|) |$|))
      (QSETREFV |$| 168
        (CONS (|dispatchFunction| |UPOLYC-;order;2SNni;45|) |$|)))))
(COND
  ((|testBitVector| |pv$| 3)
    (PROGN
      (QSETREFV |$| 171
        (CONS (|dispatchFunction| |UPOLYC-;squareFree;SF;46|) |$|))
      (QSETREFV |$| 173
        (CONS
          (|dispatchFunction| |UPOLYC-;squareFreePart;2S;47|)
          |$|)))))
(COND
  ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
    (PROGN
      (QSETREFV |$| 180
        (CONS
          (|dispatchFunction| |UPOLYC-;gcdPolynomial;3Sup;48|)
          |$|))
      (QSETREFV |$| 183
        (CONS
          (|dispatchFunction| |UPOLYC-;squareFreePolynomial;SupF;49|)
          |$|)))))
(COND
  ((|testBitVector| |pv$| 2)
    (PROGN
              (QSETREFV |$| 185
                (CONS (|dispatchFunction| |UPOLYC-;elt;F2R;50|) |$|))
              (QSETREFV |$| 186
                (CONS
                  (|dispatchFunction| |UPOLYC-;euclideanSize;SNni;51|)
                  |$|))
              (QSETREFV |$| 189
                (CONS (|dispatchFunction| |UPOLYC-;divide;2SR;52|) |$|)))))
        (COND
          ((|testBitVector| |pv$| 1)
             (QSETREFV |$| 194
               (CONS
                 (|dispatchFunction| |UPOLYC-;integrate;2S;53|)
                 |$|)))) |$|))))

(MAKEPROP
  (QUOTE |UnivariatePolynomialCategory&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|local| |#2|)
        (|Boolean|)
        (0 . |zero?|)
        (|NonNegativeInteger|)
        (5 . |degree|)
        (|SingletonAsOrderedSet|)
        (10 . |create|)
        (|List| 12)
        |UPOLYC-;variables;SL;1|
        |UPOLYC-;degree;SSaosNni;2|
        (14 . |totalDegree|)
        |UPOLYC-;totalDegree;SLNni;3|
        (|List| 10)
        |UPOLYC-;degree;SLL;4|
        (19 . |eval|)
        (|List| |$|)
        |UPOLYC-;eval;SLLS;5|
        (26 . |elt|)
        |UPOLYC-;eval;SSaos2S;6|
        (32 . |eval|)
        (|List| 7)
        |UPOLYC-;eval;SLLS;7|
        (39 . |elt|)
        (45 . |coerce|)
```

```
|UPOLYC-;eval;SSaosRS;8|
(|Equation| 6)
(50 . |lhs|)
(|Union| 12 (QUOTE "failed"))
(55 . |mainVariable|)
(60 . |rhs|)
(|List| 197)
|UPOLYC-;eval;SLS;9|
|UPOLYC-;mainVariable;SU;10|
(65 . |minimumDegree|)
|UPOLYC-;minimumDegree;SSaosNni;11|
|UPOLYC-;minimumDegree;SLL;12|
(70 . |+|)
(|Mapping| 10 10)
(76 . |mapExponents|)
|UPOLYC-;monomial;SSaosNniS;13|
(82 . |One|)
(86 . |One|)
(90 . |monomial|)
|UPOLYC-;coerce;SaosS;14|
(|SparseUnivariatePolynomial| 7)
(96 . |Zero|)
(100 . |leadingCoefficient|)
(105 . |monomial|)
(111 . |reductum|)
(116 . |makeSUP|)
(121 . |+|)
|UPOLYC-;makeSUP;SSup;15|
(127 . |zero?|)
(132 . |Zero|)
(136 . |leadingCoefficient|)
(141 . |degree|)
(146 . |reductum|)
(151 . |unmakeSUP|)
(156 . |+|)
|UPOLYC-;unmakeSUP;SupS;16|
(|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
(162 . |monicDivide|)
|UPOLYC-;karatsubaDivide;SNniR;17|
|UPOLYC-;shiftRight;SNniS;18|
(168 . |*|)
|UPOLYC-;shiftLeft;SNniS;19|
(|Union| 74 (QUOTE "failed"))
(|List| 75)
(|SparseUnivariatePolynomial| 6)
(|PolynomialFactorizationByRecursionUnivariate| 7 6)
```

```
(174 . |solveLinearPolynomialEquationByRecursion|)
(|Union| 79 (QUOTE "failed"))
(|List| 80)
(|SparseUnivariatePolynomial| |$|)
(180 . |solveLinearPolynomialEquation|)
(|Factored| 75)
(186 . |factorByRecursion|)
(|Factored| 80)
(191 . |factorPolynomial|)
(196 . |factorSquareFreeByRecursion|)
(201 . |factorSquareFreePolynomial|)
(|Factored| |$|)
(206 . |factor|)
(|Factored| 7)
(211 . |unit|)
(|Union| (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
(|Record| (|:| |flg| 92) (|:| |fctr| 7) (|:| |xpnt| 109))
(|List| 93)
(216 . |factorList|)
(|Record| (|:| |flg| 92) (|:| |fctr| 6) (|:| |xpnt| 109))
(|List| 96)
(|Factored| 6)
(221 . |makeFR|)
(227 . |factorPolynomial|)
(|Mapping| 6 51)
(|Factored| 51)
(|FactoredFunctions2| 51 6)
(232 . |map|)
(238 . |factor|)
(243 . |Zero|)
(|Vector| 7)
(247 . |new|)
(|Integer|)
(253 . |minIndex|)
(258 . |coefficient|)
(264 . |qsetelt!|)
|UPOLYC-;vectorise;SNniV;24|
|UPOLYC-;retract;SR;25|
(|Union| 7 (QUOTE "failed"))
|UPOLYC-;retractIfCan;SU;26|
(271 . |init|)
(275 . |init|)
(|Union| |$| (QUOTE "failed"))
(279 . |nextItem|)
(284 . |One|)
(288 . |nextItem|)
```

```
(293 . |content|)
(298 . |content|)
(304 . |gcd|)
(310 . |exquo|)
(316 . |=|)
(|Record| (|:| |primePart| |$|) (|:| |commonPart| |$|))
(322 . |separate|)
(328 . |Zero|)
(332 . |*|)
(|Mapping| 7 7)
(338 . |differentiate|)
(345 . |*|)
(351 . |differentiate|)
|UPOLYC-;differentiate;SMS;36|
|UPOLYC-;differentiate;2S;37|
(358 . |differentiate|)
|UPOLYC-;differentiate;SSaosS;38|
(|Fraction| 6)
(363 . |numer|)
(|Fraction| |$|)
(368 . |elt|)
(374 . |denom|)
(379 . |/|)
(385 . |elt|)
(391 . |**|)
(397 . |pseudoRemainder|)
(403 . |-|)
(409 . |pseudoQuotient|)
(|Record| (|:| |coef| 7) (|:| |quotient| |$|) (|:| |remainder| |$|))
(415 . |pseudoDivide|)
(421 . |composite|)
(427 . |/|)
(|Union| 142 (QUOTE "failed"))
(433 . |composite|)
(439 . |ground?|)
(444 . |pseudoDivide|)
(450 . |exquo|)
(456 . |composite|)
(462 . |Zero|)
(466 . |coerce|)
(471 . |**|)
(477 . |*|)
(483 . |+|)
(489 . |**|)
(495 . |elt|)
(501 . |order|)
```

```
      (|UnivariatePolynomialSquareFree| 7 6)
      (507 . |squareFree|)
      (512 . |squareFree|)
      (517 . |squareFreePart|)
      (522 . |squareFreePart|)
      (527 . |zero?|)
      (532 . |unitCanonical|)
      (537 . |content|)
      (542 . |primitivePart|)
      (547 . |subResultantGcd|)
      (553 . |*|)
      (559 . |gcdPolynomial|)
      (|UnivariatePolynomialSquareFree| 6 75)
      (565 . |squareFree|)
      (570 . |squareFreePolynomial|)
      (575 . |/|)
      (581 . |elt|)
      (587 . |euclideanSize|)
      (592 . |inv|)
      (597 . |*|)
      (603 . |divide|)
      (|Fraction| 109)
      (609 . |coerce|)
      (614 . |inv|)
      (619 . |*|)
      (625 . |integrate|)
      (|Symbol|)
      (|List| 195)
      (|Equation| |$|)
      (|Union| 109 (QUOTE "failed"))
      (|Union| 190 (QUOTE "failed"))
      (|OutputForm|)))
  (QUOTE
    #(|vectorise| 630 |variables| 636 |unmakeSUP| 641 |totalDegree| 646
      |squareFreePolynomial| 652 |squareFreePart| 657 |squareFree| 662
      |solveLinearPolynomialEquation| 667 |shiftRight| 673 |shiftLeft| 679
      |separate| 685 |retractIfCan| 691 |retract| 696 |pseudoQuotient| 701
      |pseudoDivide| 707 |order| 713 |nextItem| 719 |monomial| 724
      |minimumDegree| 731 |makeSUP| 743 |mainVariable| 748
      |karatsubaDivide| 753 |integrate| 759 |init| 764 |gcdPolynomial| 768
      |factorSquareFreePolynomial| 774 |factorPolynomial| 779 |factor| 784
      |eval| 789 |euclideanSize| 823 |elt| 828 |divide| 846
      |differentiate| 852 |degree| 876 |content| 888 |composite| 894
      |coerce| 906))
  (QUOTE NIL)
  (CONS
(|makeByteWordVec2| 1 (QUOTE NIL))
(CONS
  (QUOTE #())
  (CONS
    (QUOTE #())
    (|makeByteWordVec2| 194
      (QUOTE
        (1 6 8 0 9 1 6 10 0 11 0 12 0 13 1 6 10 0 17 3 6 0 0 12 0 21 2
         6 0 0 0 24 3 6 0 0 12 7 26 2 6 7 0 7 29 1 6 0 7 30 1 32 6 0 33
         1 6 34 0 35 1 32 6 0 36 1 6 10 0 40 2 10 0 0 0 43 2 6 0 44 0
         45 0 6 0 47 0 7 0 48 2 6 0 7 10 49 0 51 0 52 1 6 7 0 53 2 51
         0 7 10 54 1 6 0 0 55 1 6 51 0 56 2 51 0 0 0 57 1 51 8 0 59 0
         6 0 60 1 51 7 0 61 1 51 10 0 62 1 51 0 0 63 1 6 0 51 64 2 6 0
         0 0 65 2 6 67 0 0 68 2 6 0 0 0 71 2 76 73 74 75 77 2 0 78 79
         80 81 1 76 82 75 83 1 0 84 80 85 1 76 82 75 86 1 0 84 80 87 1
         7 88 0 89 1 90 7 0 91 1 90 94 0 95 2 98 0 6 97 99 1 7 84 80
         100 2 103 98 101 102 104 1 0 88 0 105 0 7 0 106 2 107 0 10 7
         108 1 107 109 0 110 2 6 7 0 10 111 3 107 7 0 109 7 112 0 7 0
         117 0 0 0 118 1 7 119 0 120 0 75 0 121 1 0 119 0 122 1 6 7 0
         123 2 0 0 0 12 124 2 6 0 0 0 125 2 6 119 0 0 126 2 6 8 0 0 127
         2 0 128 0 0 129 0 75 0 130 2 7 0 10 0 131 3 0 0 0 132 0 133 2
         6 0 7 0 134 3 6 0 0 132 0 135 1 6 0 0 138 1 140 6 0 141 2 6
         142 0 142 143 1 140 6 0 144 2 140 0 0 0 145 2 0 142 142 142
         146 2 7 0 0 10 147 2 6 0 0 0 148 2 6 0 0 0 149 2 0 0 0 0 150
         2 0 151 0 0 152 2 6 119 0 0 153 2 140 0 6 6 154 2 0 155 142 0
         156 1 6 8 0 157 2 6 151 0 0 158 2 6 119 0 7 159 2 0 119 0 0
         160 0 140 0 161 1 140 0 6 162 2 140 0 0 109 163 2 140 0 0 0
         164 2 140 0 0 0 165 2 140 0 0 10 166 2 0 142 0 142 167 2 0 10
         0 0 168 1 169 98 6 170 1 0 88 0 171 1 169 6 6 172 1 0 0 0 173
         1 75 8 0 174 1 75 0 0 175 1 75 6 0 176 1 75 0 0 177 2 75 0 0
         0 178 2 75 0 6 0 179 2 0 80 80 80 180 1 181 82 75 182 1 0 84
         80 183 2 7 0 0 0 184 2 0 7 142 7 185 1 0 10 0 186 1 7 0 0 187
         2 7 0 0 0 188 2 0 67 0 0 189 1 190 0 109 191 1 190 0 0 192 2
         6 0 190 0 193 1 0 0 0 194 2 0 107 0 10 113 1 0 14 0 15 1 0 0
         51 66 2 0 10 0 14 18 1 0 84 80 183 1 0 0 0 173 1 0 88 0 171 2
         0 78 79 80 81 2 0 0 0 10 70 2 0 0 0 10 72 2 0 128 0 0 129 1 0
         115 0 116 1 0 7 0 114 2 0 0 0 0 150 2 0 151 0 0 152 2 0 10 0
         0 168 1 0 119 0 122 3 0 0 0 12 10 46 2 0 19 0 14 42 2 0 10 0
         12 41 1 0 51 0 58 1 0 34 0 39 2 0 67 0 10 69 1 0 0 0 194 0 0
         0 118 2 0 80 80 80 180 1 0 84 80 87 1 0 84 80 85 1 0 88 0 105
         3 0 0 0 12 0 25 3 0 0 0 14 22 23 3 0 0 0 14 27 28 3 0 0 0 12
         7 31 2 0 0 0 37 38 1 0 10 0 186 2 0 142 0 142 167 2 0 7 142 7
         185 2 0 142 142 142 146 2 0 67 0 0 189 3 0 0 0 132 0 133 2 0
         0 0 132 136 1 0 0 0 137 2 0 0 0 12 139 2 0 10 0 12 16 2 0 19
         0 14 20 2 0 0 0 12 124 2 0 119 0 0 160 2 0 155 142 0 156 1 0
         0 12 50))))))
        (QUOTE |lookupComplete|)))
```

21.69 URAGG.lsp BOOTSTRAP

URAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*URAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnaryRecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |UnaryRecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |UnaryRecursiveAggregate| (#1=#:G84596)
 (LET (#2=#:G84597)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnaryRecursiveAggregate;AL|))
     (CDR #2#))
   (T
    (SETQ |UnaryRecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|UnaryRecursiveAggregate;| #1#)))
      |UnaryRecursiveAggregate;AL|))
    #2#))))

(DEFUN |UnaryRecursiveAggregate;| (|t#1|)
 (PROG (#1=#:G84595)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|UnaryRecursiveAggregate;CAT|)
       ((QUOTE T)
        (LETT |UnaryRecursiveAggregate;CAT|
         (|Join|
          (|RecursiveAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|concat| (|$| |$| |$|)) T)
          ((|concat| (|$| |t#1| |$|)) T)
          ((|first| (|t#1| |$|)) T)
          ((|elt| (|t#1| |$| "first")) T)
          ((|first| (|$| |$| (|NonNegativeInteger|))) T)
          ((|rest| (|$| |$|)) T)
          ((|elt| (|$| |$| "rest")) T)
          ((|rest| (|$| |$| (|NonNegativeInteger|))) T)
          ((|last| (|t#1| |$|)) T)
          ((|elt| (|t#1| |$| "last")) T)
          ((|last| (|$| |$| (|NonNegativeInteger|))) T)
          ((|tail| (|$| |$|)) T)
          ((|second| (|t#1| |$|)) T)
          ((|third| (|t#1| |$|)) T)
          ((|cycleEntry| (|$| |$|)) T)
          ((|cycleLength| ((|NonNegativeInteger|) |$|)) T)
          ((|cycleTail| (|$| |$|)) T)
          ((|concat!| (|$| |$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|concat!| (|$| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|cycleSplit!| (|$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setfirst!| (|t#1| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|t#1| |$| "first" |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setrest!| (|$| |$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|$| |$| "rest" |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setlast!| (|t#1| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|t#1| |$| "last" |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|split!| (|$| |$| (|Integer|)))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
         NIL
         (QUOTE ((|Integer|) (|NonNegativeInteger|)))
         NIL))
       . #2=(|UnaryRecursiveAggregate|)))))
   . #2#)
  (SETELT #1# 0
   (LIST (QUOTE |UnaryRecursiveAggregate|) (|devaluate| |t#1|)))))))
```

21.70 URAGG-.lsp BOOTSTRAP

URAGG- depends on **URAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*URAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |URAGG-;elt;AfirstS;1| (|x| G84610 |$|)
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |URAGG-;elt;AlastS;2| (|x| G84612 |$|)
 (SPADCALL |x| (QREFELT |$| 11)))

(DEFUN |URAGG-;elt;ArestA;3| (|x| G84614 |$|)
 (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |URAGG-;second;AS;4| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 8)))

(DEFUN |URAGG-;third;AS;5| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 14))
  (QREFELT |$| 8)))

(DEFUN |URAGG-;cyclic?;AB;6| (|x| |$|)
 (COND
  ((OR
    (SPADCALL |x| (QREFELT |$| 20))
    (SPADCALL (|URAGG-;findCycle| |x| |$|) (QREFELT |$| 20)))
     (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |URAGG-;last;AS;7| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) (QREFELT |$| 8)))

(DEFUN |URAGG-;nodes;AL;8| (|x| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| NIL |URAGG-;nodes;AL;8|)
```

```
     (SEQ
      G190
      (COND
       ((NULL
         (COND
          ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
          ((QUOTE T) (QUOTE T))))
         (GO G191)))
      (SEQ
       (LETT |l| (CONS |x| |l|) |URAGG-;nodes;AL;8|)
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;nodes;AL;8|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (NREVERSE |l|))))))

(DEFUN |URAGG-;children;AL;9| (|x| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| NIL |URAGG-;children;AL;9|)
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 20)) |l|)
      ((QUOTE T) (CONS (SPADCALL |x| (QREFELT |$| 14)) |l|))))))))

(DEFUN |URAGG-;leaf?;AB;10| (|x| |$|)
 (SPADCALL |x| (QREFELT |$| 20)))

(DEFUN |URAGG-;value;AS;11| (|x| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 20)) (|error| "value of empty object"))
  ((QUOTE T) (SPADCALL |x| (QREFELT |$| 8)))))

(DEFUN |URAGG-;less?;ANniB;12| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;less?;ANniB;12|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((|<| 0 |i|)
          (COND
           ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;less?;ANniB;12|)
      (EXIT (LETT |i| (|-| |i| 1) |URAGG-;less?;ANniB;12|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (|<| 0 |i|))))))

(DEFUN |URAGG-;more?;ANniB;13| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;more?;ANniB;13|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((|<| 0 |i|)
          (COND
           ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;more?;ANniB;13|)
      (EXIT (LETT |i| (|-| |i| 1) |URAGG-;more?;ANniB;13|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((ZEROP |i|)
       (COND
        ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;size?;ANniB;14| (|l| |n| |$|)
  (PROG (|i|)
   (RETURN
    (SEQ
     (LETT |i| |n| |URAGG-;size?;ANniB;14|)
     (SEQ
      G190
      (COND
       ((NULL
        (COND
         ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (|<| 0 |i|))))
        (GO G191)))
      (SEQ
       (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;size?;ANniB;14|)
       (EXIT (LETT |i| (|-| |i| 1) |URAGG-;size?;ANniB;14|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT
      (COND
       ((SPADCALL |l| (QREFELT |$| 20)) (ZEROP |i|))
       ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;#;ANni;15| (|x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| 0 |URAGG-;#;ANni;15|)
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((EQL |k| 1000)
         (COND
          ((SPADCALL |x| (QREFELT |$| 33)) (EXIT (|error| "cyclic list"))))))
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;#;ANni;15|)))
     (LETT |k| (QSADD1 |k|) |URAGG-;#;ANni;15|)
     (GO G190)
     G191
```

```
      (EXIT NIL))
     (EXIT |k|)))))

(DEFUN |URAGG-;tail;2A;16| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) (|error| "empty list"))
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;tail;2A;16|)
       (SEQ
        (LETT |k| 0 |URAGG-;tail;2A;16|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (COND
          ((EQL |k| 1000)
            (COND
             ((SPADCALL |x| (QREFELT |$| 33))
               (EXIT (|error| "cyclic list"))))))
         (EXIT
          (LETT |y|
           (SPADCALL (LETT |x| |y| |URAGG-;tail;2A;16|) (QREFELT |$| 14))
           |URAGG-;tail;2A;16|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;tail;2A;16|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |URAGG-;findCycle| (|x| |$|)
 (PROG (#1=#:G84667 |y|)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
      (SEQ
       G190
```

```
         (COND
          ((NULL
            (COND
             ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (COND
           ((SPADCALL |x| |y| (QREFELT |$| 36))
             (PROGN (LETT #1# |x| |URAGG-;findCycle|) (GO #1#))))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
          (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)
          (COND
           ((SPADCALL |y| (QREFELT |$| 20))
             (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
          (COND
           ((SPADCALL |x| |y| (QREFELT |$| 36))
             (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
          (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |y|)))
      #1#
      (EXIT #1#)))))

  (DEFUN |URAGG-;cycleTail;2A;18| (|x| |$|)
   (PROG (|y| |z|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL
         (LETT |y|
          (LETT |x| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleTail;2A;18|)
          |URAGG-;cycleTail;2A;18|)
         (QREFELT |$| 20))
        |x|)
       ((QUOTE T)
         (SEQ
          (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)
          (SEQ
           G190
           (COND
            ((NULL
              (COND
             ((SPADCALL |x| |z| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |y| |z| |URAGG-;cycleTail;2A;18|)
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |y|))))))))

(DEFUN |URAGG-;cycleEntry;2A;19| (|x| |$|)
 (PROG (|l| |z| |k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) |x|)
     ((SPADCALL
       (LETT |y| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleEntry;2A;19|)
       (QREFELT |$| 20))
      |y|)
     ((QUOTE T)
      (SEQ
       (LETT |z| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
       (SEQ
        (LETT |l| 1 |URAGG-;cycleEntry;2A;19|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| |z| (QREFELT |$| 36)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT |z|
           (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
        (LETT |l|
         (QSADD1 |l|) |URAGG-;cycleEntry;2A;19|) (GO G190) G191 (EXIT NIL))
       (LETT |y| |x| |URAGG-;cycleEntry;2A;19|)
       (SEQ
        (LETT |k| 1 |URAGG-;cycleEntry;2A;19|)
        G190
        (COND ((QSGREATERP |k| |l|) (GO G191)))
        (SEQ
         (EXIT
          (LETT |y|
           (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;cycleEntry;2A;19|)
        (GO G190)
        G191
        (EXIT NIL))
       (SEQ
        G190
        (COND
          ((NULL
            (COND
             ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ
         (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
         (EXIT
          (LETT |y|
           (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |URAGG-;cycleLength;ANni;20| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL |x| (QREFELT |$| 20))
       (SPADCALL
        (LETT |x| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleLength;ANni;20|)
        (QREFELT |$| 20)))
      0)
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)
       (SEQ
        (LETT |k| 1 |URAGG-;cycleLength;ANni;20|)
        G190
        (COND
          ((NULL
            (COND
             ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |y|
            (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)))
         (LETT |k| (QSADD1 |k|) |URAGG-;cycleLength;ANni;20|)
         (GO G190)
          G191
          (EXIT NIL))
        (EXIT |k|))))))))

(DEFUN |URAGG-;rest;ANniA;21| (|x| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |i| 1 |URAGG-;rest;ANniA;21|)
     G190
     (COND ((QSGREATERP |i| |n|) (GO G191)))
     (SEQ
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 20)) (|error| "Index out of range"))
        ((QUOTE T)
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;rest;ANniA;21|)))))
     (LETT |i| (QSADD1 |i|) |URAGG-;rest;ANniA;21|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |URAGG-;last;ANniA;22| (|x| |n| |$|)
 (PROG (|m| #1=#:G84694)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 42)) |URAGG-;last;ANniA;22|)
    (EXIT
     (COND
      ((|<| |m| |n|) (|error| "index out of range"))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| |m| |n|) |URAGG-;last;ANniA;22|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 43))
         (QREFELT |$| 44))))))))

(DEFUN |URAGG-;=;2AB;23| (|x| |y| |$|)
 (PROG (|k| #1=#:G84705)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |k| 0 |URAGG-;=;2AB;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 20))
               (SPADCALL |y| (QREFELT |$| 20)))
               (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (COND
           ((EQL |k| 1000)
            (COND
             ((SPADCALL |x| (QREFELT |$| 33))
               (EXIT (|error| "cyclic list"))))))
          (COND
           ((NULL
             (SPADCALL
              (SPADCALL |x| (QREFELT |$| 8))
              (SPADCALL |y| (QREFELT |$| 8))
              (QREFELT |$| 46)))
            (EXIT (PROGN (LETT #1# (QUOTE NIL) |URAGG-;=;2AB;23|) (GO #1#)))))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)
          (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)))
         (LETT |k| (QSADD1 |k|) |URAGG-;=;2AB;23|)
         (GO G190)
         G191
         (EXIT NIL))
```

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          (EXIT
           (COND
            ((SPADCALL |x| (QREFELT |$| 20)) (SPADCALL |y| (QREFELT |$| 20)))
            ((QUOTE T) (QUOTE NIL))))))))
      #1#
      (EXIT #1#)))))

  (DEFUN |URAGG-;node?;2AB;24| (|u| |v| |$|)
   (PROG (|k| #1=#:G84711)
    (RETURN
     (SEQ
      (EXIT
       (SEQ
        (SEQ
         (LETT |k| 0 |URAGG-;node?;2AB;24|)
         G190
         (COND
           ((NULL
            (COND
             ((SPADCALL |v| (QREFELT |$| 20)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
          (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((SPADCALL |u| |v| (QREFELT |$| 48))
              (PROGN (LETT #1# (QUOTE T) |URAGG-;node?;2AB;24|) (GO #1#)))
            ((QUOTE T)
             (SEQ
              (COND
               ((EQL |k| 1000)
                (COND
                 ((SPADCALL |v| (QREFELT |$| 33))
                   (EXIT (|error| "cyclic list"))))))
              (EXIT
               (LETT |v|
                (SPADCALL |v| (QREFELT |$| 14))
                |URAGG-;node?;2AB;24|)))))))
         (LETT |k| (QSADD1 |k|) |URAGG-;node?;2AB;24|) (GO G190) G191 (EXIT NIL))
        (EXIT (SPADCALL |u| |v| (QREFELT |$| 48)))))
      #1# (EXIT #1#)))))

  (DEFUN |URAGG-;setelt;Afirst2S;25| (|x| G84713 |a| |$|)
   (SPADCALL |x| |a| (QREFELT |$| 50)))

  (DEFUN |URAGG-;setelt;Alast2S;26| (|x| G84715 |a| |$|)
 (SPADCALL |x| |a| (QREFELT |$| 52)))

(DEFUN |URAGG-;setelt;Arest2A;27| (|x| G84717 |a| |$|)
 (SPADCALL |x| |a| (QREFELT |$| 54)))

(DEFUN |URAGG-;concat;3A;28| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 56)))

(DEFUN |URAGG-;setlast!;A2S;29| (|x| |s| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 20)) (|error| "setlast: empty list"))
   ((QUOTE T)
    (SEQ
     (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) |s| (QREFELT |$| 50))
     (EXIT |s|))))))

(DEFUN |URAGG-;setchildren!;ALA;30| (|u| |lv| |$|)
 (COND
  ((EQL (LENGTH |lv|) 1) (SPADCALL |u| (|SPADfirst| |lv|) (QREFELT |$| 54)))
  ((QUOTE T) (|error| "wrong number of children specified"))))

(DEFUN |URAGG-;setvalue!;A2S;31| (|u| |s| |$|)
 (SPADCALL |u| |s| (QREFELT |$| 50)))

(DEFUN |URAGG-;split!;AIA;32| (|p| |n| |$|)
 (PROG (#1=#:G84725 |q|)
  (RETURN
   (SEQ
    (COND
     ((|<| |n| 1) (|error| "index out of range"))
     ((QUOTE T)
      (SEQ
       (LETT |p|
        (SPADCALL |p|
         (PROG1
          (LETT #1# (|-| |n| 1) |URAGG-;split!;AIA;32|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         (QREFELT |$| 43))
        |URAGG-;split!;AIA;32|)
       (LETT |q| (SPADCALL |p| (QREFELT |$| 14)) |URAGG-;split!;AIA;32|)
       (SPADCALL |p| (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54))
       (EXIT |q|))))))))

(DEFUN |URAGG-;cycleSplit!;2A;33| (|x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL
        (LETT |y| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleSplit!;2A;33|)
        (QREFELT |$| 20))
       (SPADCALL |x| |y| (QREFELT |$| 36))) |y|)
     ((QUOTE T)
      (SEQ
       (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)
       (SEQ G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |z| |y| (QREFELT |$| 36)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |x| |z| |URAGG-;cycleSplit!;2A;33|)
         (EXIT
          (LETT |z|
           (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (SPADCALL |x|
        (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54)) (EXIT |y|))))))))

(DEFUN |UnaryRecursiveAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|UnaryRecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |UnaryRecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
```

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      (QSETREFV |$| 7 |#2|)
      (COND
        ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
          (QSETREFV |$| 45
            (CONS (|dispatchFunction| |URAGG-;last;ANniA;22|) |$|))))
      (COND
       ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
        (PROGN
         (QSETREFV |$| 47 (CONS (|dispatchFunction| |URAGG-;=;2AB;23|) |$|))
         (QSETREFV |$| 49
          (CONS (|dispatchFunction| |URAGG-;node?;2AB;24|) |$|)))))
      (COND
       ((|testBitVector| |pv$| 1)
        (PROGN
         (QSETREFV |$| 51
          (CONS (|dispatchFunction| |URAGG-;setelt;Afirst2S;25|) |$|))
         (QSETREFV |$| 53
          (CONS (|dispatchFunction| |URAGG-;setelt;Alast2S;26|) |$|))
         (QSETREFV |$| 55
          (CONS (|dispatchFunction| |URAGG-;setelt;Arest2A;27|) |$|))
         (QSETREFV |$| 57
          (CONS (|dispatchFunction| |URAGG-;concat;3A;28|) |$|))
         (QSETREFV |$| 58
          (CONS (|dispatchFunction| |URAGG-;setlast!;A2S;29|) |$|))
         (QSETREFV |$| 59
          (CONS (|dispatchFunction| |URAGG-;setchildren!;ALA;30|) |$|))
         (QSETREFV |$| 60
          (CONS (|dispatchFunction| |URAGG-;setvalue!;A2S;31|) |$|))
         (QSETREFV |$| 63
          (CONS (|dispatchFunction| |URAGG-;split!;AIA;32|) |$|))
         (QSETREFV |$| 64
          (CONS (|dispatchFunction| |URAGG-;cycleSplit!;2A;33|) |$|)))))
      |$|))))

  (MAKEPROP
   (QUOTE |UnaryRecursiveAggregate&|)
   (QUOTE |infovec|)
   (LIST
    (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
     (0 . |first|) (QUOTE "first") |URAGG-;elt;AfirstS;1| (5 . |last|)
     (QUOTE "last") |URAGG-;elt;AlastS;2| (10 . |rest|) (QUOTE "rest")
     |URAGG-;elt;ArestA;3| |URAGG-;second;AS;4| |URAGG-;third;AS;5|
     (|Boolean|) (15 . |empty?|) |URAGG-;cyclic?;AB;6| (20 . |tail|)
     |URAGG-;last;AS;7| (|List| |$|) |URAGG-;nodes;AL;8|
     |URAGG-;children;AL;9| |URAGG-;leaf?;AB;10| |URAGG-;value;AS;11|
     (|NonNegativeInteger|) |URAGG-;less?;ANniB;12| |URAGG-;more?;ANniB;13|
 |URAGG-;size?;ANniB;14| (25 . |cyclic?|) |URAGG-;#;ANni;15|
 |URAGG-;tail;2A;16| (30 . |eq?|) (36 . |cycleEntry|)
 |URAGG-;cycleTail;2A;18| |URAGG-;cycleEntry;2A;19|
 |URAGG-;cycleLength;ANni;20| |URAGG-;rest;ANniA;21| (41 . |#|)
 (46 . |rest|) (52 . |copy|) (57 . |last|) (63 . |=|) (69 . |=|)
 (75 . |=|) (81 . |node?|) (87 . |setfirst!|) (93 . |setelt|)
 (100 . |setlast!|) (106 . |setelt|) (113 . |setrest!|)
 (119 . |setelt|) (126 . |concat!|) (132 . |concat|) (138 . |setlast!|)
 (144 . |setchildren!|) (150 . |setvalue!|) (156 . |empty|) (|Integer|)
 (160 . |split!|) (166 . |cycleSplit!|) (QUOTE "value")))
(QUOTE #(|value| 171 |third| 176 |tail| 181 |split!| 186 |size?| 192
 |setvalue!| 198 |setlast!| 204 |setelt| 210 |setchildren!| 231 |second|
 237 |rest| 242 |nodes| 248 |node?| 253 |more?| 259 |less?| 265 |leaf?|
 271 |last| 276 |elt| 287 |cyclic?| 305 |cycleTail| 310 |cycleSplit!|
 315 |cycleLength| 320 |cycleEntry| 325 |concat| 330 |children| 336 |=|
 341 |#| 347))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 64 (QUOTE (1 6 7 0 8 1 6 7 0 11 1 6 0 0 14 1 6
     19 0 20 1 6 0 0 22 1 6 19 0 33 2 6 19 0 0 36 1 6 0 0 37 1 6 29 0
     42 2 6 0 0 29 43 1 6 0 0 44 2 0 0 0 29 45 2 7 19 0 0 46 2 0 19 0
     0 47 2 6 19 0 0 48 2 0 19 0 0 49 2 6 7 0 7 50 3 0 7 0 9 7 51 2 6
     7 0 7 52 3 0 7 0 12 7 53 2 6 0 0 0 54 3 0 0 0 15 0 55 2 6 0 0 0 56
     2 0 0 0 0 57 2 0 7 0 7 58 2 0 0 0 24 59 2 0 7 0 7 60 0 6 0 61 2 0
     0 0 62 63 1 0 0 0 64 1 0 7 0 28 1 0 7 0 18 1 0 0 0 35 2 0 0 0 62 63
     2 0 19 0 29 32 2 0 7 0 7 60 2 0 7 0 7 58 3 0 7 0 12 7 53 3 0 0 0 15
     0 55 3 0 7 0 9 7 51 2 0 0 0 24 59 1 0 7 0 17 2 0 0 0 29 41 1 0 24 0
     25 2 0 19 0 0 49 2 0 19 0 29 31 2 0 19 0 29 30 1 0 19 0 27 2 0 0 0
     29 45 1 0 7 0 23 2 0 7 0 12 13 2 0 0 0 15 16 2 0 7 0 9 10 1 0 19 0
     21 1 0 0 0 38 1 0 0 0 64 1 0 29 0 40 1 0 0 0 39 2 0 0 0 0 57 1 0 24
     0 26 2 0 19 0 0 47 1 0 29 0 34))))))
(QUOTE |lookupComplete|)))
```

Chapter 22

Chunk collections

```
⟨algebra⟩≡
  ⟨category ABELGRP AbelianGroup⟩
  ⟨category ABELMON AbelianMonoid⟩
  ⟨category AMR AbelianMonoidRing⟩
  ⟨category ABELSG AbelianSemiGroup⟩
  ⟨category AGG Aggregate⟩
  ⟨category ALGEBRA Algebra⟩
  ⟨category ACF AlgebraicallyClosedField⟩
  ⟨category ACFS AlgebraicallyClosedFunctionSpace⟩
  ⟨category AHYP ArcHyperbolicFunctionCategory⟩
  ⟨category ATRIG ArcTrigonometricFunctionCategory⟩
  ⟨category ALAGG AssociationListAggregate⟩
  ⟨category ATTREG AttributeRegistry⟩

  ⟨category BGAGG BagAggregate⟩
  ⟨category BASTYPE BasicType⟩
  ⟨category BLMETCT BlowUpMethodCategory⟩
  ⟨category BMODULE BiModule⟩
  ⟨category BRAGG BinaryRecursiveAggregate⟩
  ⟨category BTCAT BinaryTreeCategory⟩
  ⟨category BTAGG BitAggregate⟩

  ⟨category CACHSET CachableSet⟩
  ⟨category CABMON CancellationAbelianMonoid⟩
  ⟨category CHARNZ CharacteristicNonZero⟩
  ⟨category CHARZ CharacteristicZero⟩
  ⟨category KOERCE CoercibleTo⟩
  ⟨category CLAGG Collection⟩
  ⟨category CFCAT CombinatorialFunctionCategory⟩
  ⟨category COMBOPC CombinatorialOpsCategory⟩
```

⟨*category COMRING CommutativeRing*⟩
⟨*category COMPCAT ComplexCategory*⟩
⟨*category KONVERT ConvertibleTo*⟩

⟨*category DQAGG DequeueAggregate*⟩
⟨*category DSTRCAT DesingTreeCategory*⟩
⟨*category DIAGG Dictionary*⟩
⟨*category DIOPS DictionaryOperations*⟩
⟨*category DIFEXT DifferentialExtension*⟩
⟨*category DPOLCAT DifferentialPolynomialCategory*⟩
⟨*category DIFRING DifferentialRing*⟩
⟨*category DIVCAT DivisorCategory*⟩
⟨*category DVARCAT DifferentialVariableCategory*⟩
⟨*category DIRPCAT DirectProductCategory*⟩
⟨*category DIVRING DivisionRing*⟩
⟨*category DLAGG DoublyLinkedAggregate*⟩

⟨*category ELEMFUN ElementaryFunctionCategory*⟩
⟨*category ELTAB Eltable*⟩
⟨*category ELTAGG EltableAggregate*⟩
⟨*category ENTIRER EntireRing*⟩
⟨*category EUCDOM EuclideanDomain*⟩
⟨*category EVALAB Evalable*⟩
⟨*category ES ExpressionSpace*⟩
⟨*category ELAGG ExtensibleLinearAggregate*⟩
⟨*category XF ExtensionField*⟩

⟨*category FIELD Field*⟩
⟨*category FPC FieldOfPrimeCharacteristic*⟩
⟨*category FILECAT FileCategory*⟩
⟨*category FNCAT FileNameCategory*⟩
⟨*category FINITE Finite*⟩
⟨*category FAMR FiniteAbelianMonoidRing*⟩
⟨*category FAXF FiniteAlgebraicExtensionField*⟩
⟨*category FDIVCAT FiniteDivisorCategory*⟩
⟨*category FFIELDC FiniteFieldCategory*⟩
⟨*category FLAGG FiniteLinearAggregate*⟩
⟨*category FINRALG FiniteRankAlgebra*⟩
⟨*category FINAALG FiniteRankNonAssociativeAlgebra*⟩
⟨*category FSAGG FiniteSetAggregate*⟩
⟨*category FPS FloatingPointSystem*⟩
⟨*category FORTFN FortranFunctionCategory*⟩
⟨*category FMTC FortranMachineTypeCategory*⟩
⟨*category FMC FortranMatrixCategory*⟩
⟨*category FMFUN FortranMatrixFunctionCategory*⟩
⟨*category FORTCAT FortranProgramCategory*⟩

⟨*category FVC FortranVectorCategory*⟩
⟨*category FVFUN FortranVectorFunctionCategory*⟩
⟨*category FRAMALG FramedAlgebra*⟩
⟨*category FRNAALG FramedNonAssociativeAlgebra*⟩
⟨*category FAMONC FreeAbelianMonoidCategory*⟩
⟨*category FLALG FreeLieAlgebra*⟩
⟨*category FMCAT FreeModuleCat*⟩
⟨*category FEVALAB FullyEvalableOver*⟩
⟨*category FLINEXP FullyLinearlyExplicitRingOver*⟩
⟨*category FPATMAB FullyPatternMatchable*⟩
⟨*category FRETRCT FullyRetractableTo*⟩
⟨*category FFCAT FunctionFieldCategory*⟩
⟨*category FS FunctionSpace*⟩

⟨*category GCDDOM GcdDomain*⟩
⟨*category GRALG GradedAlgebra*⟩
⟨*category GRMOD GradedModule*⟩
⟨*category GROUP Group*⟩

⟨*category HOAGG HomogeneousAggregate*⟩
⟨*category HYPCAT HyperbolicFunctionCategory*⟩

⟨*category IXAGG IndexedAggregate*⟩
⟨*category IDPC IndexedDirectProductCategory*⟩
⟨*category INFCLCT InfinitlyClosePointCategory*⟩
⟨*category IEVALAB InnerEvalable*⟩
⟨*category INS IntegerNumberSystem*⟩
⟨*category INTDOM IntegralDomain*⟩
⟨*category INTCAT IntervalCategory*⟩

⟨*category KDAGG KeyedDictionary*⟩

⟨*category LZSTAGG LazyStreamAggregate*⟩
⟨*category LALG LeftAlgebra*⟩
⟨*category LMODULE LeftModule*⟩
⟨*category LIECAT LieAlgebra*⟩
⟨*category LNAGG LinearAggregate*⟩
⟨*category LINEXP LinearlyExplicitRingOver*⟩
⟨*category LODOCAT LinearOrdinaryDifferentialOperatorCategory*⟩
⟨*category LFCAT LiouvillianFunctionCategory*⟩
⟨*category LSAGG ListAggregate*⟩
⟨*category LOCPOWC LocalPowerSeriesCategory*⟩
⟨*category LOGIC Logic*⟩

⟨*category MATCAT MatrixCategory*⟩
⟨*category MODULE Module*⟩

⟨*category MONAD Monad*⟩
⟨*category MONADWU MonadWithUnit*⟩
⟨*category MONOGEN MonogenicAlgebra*⟩
⟨*category MLO MonogenicLinearOperator*⟩
⟨*category MONOID Monoid*⟩
⟨*category MDAGG MultiDictionary*⟩
⟨*category MSETAGG MultisetAggregate*⟩
⟨*category MTSCAT MultivariateTaylorSeriesCategory*⟩

⟨*category NAALG NonAssociativeAlgebra*⟩
⟨*category NASRING NonAssociativeRing*⟩
⟨*category NARNG NonAssociativeRng*⟩
⟨*category NTSCAT NormalizedTriangularSetCategory*⟩
⟨*category NUMINT NumericalIntegrationCategory*⟩
⟨*category OPTCAT NumericalOptimizationCategory*⟩

⟨*category OC OctonionCategory*⟩
⟨*category A1AGG OneDimensionalArrayAggregate*⟩
⟨*category OM OpenMath*⟩
⟨*category OAGROUP OrderedAbelianGroup*⟩
⟨*category OAMON OrderedAbelianMonoid*⟩
⟨*category OAMONS OrderedAbelianMonoidSup*⟩
⟨*category OASGP OrderedAbelianSemiGroup*⟩
⟨*category OCAMON OrderedCancellationAbelianMonoid*⟩
⟨*category ORDFIN OrderedFinite*⟩
⟨*category OINTDOM OrderedIntegralDomain*⟩
⟨*category ORDMON OrderedMonoid*⟩
⟨*category OMSAGG OrderedMultisetAggregate*⟩
⟨*category ORDRING OrderedRing*⟩
⟨*category ORDSET OrderedSet*⟩
⟨*category ODECAT OrdinaryDifferentialEquationsSolverCategory*⟩

⟨*category PADICCT PAdicIntegerCategory*⟩
⟨*category PDECAT PartialDifferentialEquationsSolverCategory*⟩
⟨*category PDRING PartialDifferentialRing*⟩
⟨*category PTRANFN PartialTranscendentalFunctions*⟩
⟨*category PATAB Patternable*⟩
⟨*category PATMAB PatternMatchable*⟩
⟨*category PERMCAT PermutationCategory*⟩
⟨*category PLACESC PlacesCategory*⟩
⟨*category PPCURVE PlottablePlaneCurveCategory*⟩
⟨*category PSCURVE PlottableSpaceCurveCategory*⟩
⟨*category PTCAT PointCategory*⟩
⟨*category POLYCAT PolynomialCategory*⟩
⟨*category PFECAT PolynomialFactorizationExplicit*⟩
⟨*category PSETCAT PolynomialSetCategory*⟩

⟨*category PSCAT PowerSeriesCategory*⟩
⟨*category PRIMCAT PrimitiveFunctionCategory*⟩
⟨*category PID PrincipalIdealDomain*⟩
⟨*category PRQAGG PriorityQueueAggregate*⟩
⟨*category PRSPCAT ProjectiveSpaceCategory*⟩
⟨*category PACEXTC PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory*⟩
⟨*category PACFFC PseudoAlgebraicClosureOfFiniteFieldCategory*⟩
⟨*category PACPERC PseudoAlgebraicClosureOfPerfectFieldCategory*⟩
⟨*category PACRATC PseudoAlgebraicClosureOfRationalNumberCategory*⟩

⟨*category QUATCAT QuaternionCategory*⟩
⟨*category QUAGG QueueAggregate*⟩
⟨*category QFCAT QuotientFieldCategory*⟩

⟨*category RADCAT RadicalCategory*⟩
⟨*category RCFIELD RealClosedField*⟩
⟨*category REAL RealConstant*⟩
⟨*category RNS RealNumberSystem*⟩
⟨*category RRCC RealRootCharacterizationCategory*⟩
⟨*category RMATCAT RectangularMatrixCategory*⟩
⟨*category RCAGG RecursiveAggregate*⟩
⟨*category RPOLCAT RecursivePolynomialCategory*⟩
⟨*category RSETCAT RegularTriangularSetCategory*⟩
⟨*category RETRACT RetractableTo*⟩
⟨*category RMODULE RightModule*⟩
⟨*category RING Ring*⟩
⟨*category RNG Rng*⟩

⟨*category SEGCAT SegmentCategory*⟩
⟨*category SEGXCAT SegmentExpansionCategory*⟩
⟨*category SGROUP SemiGroup*⟩
⟨*category SETAGG SetAggregate*⟩
⟨*category SETCAT SetCategory*⟩
⟨*category SETCATD SetCategoryWithDegree*⟩
⟨*category SEXCAT SExpressionCategory*⟩
⟨*category SPFCAT SpecialFunctionCategory*⟩
⟨*category SNTSCAT SquareFreeNormalizedTriangularSetCategory*⟩
⟨*category SFRTCAT SquareFreeRegularTriangularSetCategory*⟩
⟨*category SMATCAT SquareMatrixCategory*⟩
⟨*category SKAGG StackAggregate*⟩
⟨*category STEP StepThrough*⟩
⟨*category STAGG StreamAggregate*⟩
⟨*category SRAGG StringAggregate*⟩
⟨*category STRICAT StringCategory*⟩

⟨*category TBAGG TableAggregate*⟩

⟨*category SPACEC ThreeSpaceCategory*⟩
⟨*category TSETCAT TriangularSetCategory*⟩
⟨*category TRIGCAT TrigonometricFunctionCategory*⟩
⟨*category TRANFUN TranscendentalFunctionCategory*⟩
⟨*category ARR2CAT TwoDimensionalArrayCategory*⟩
⟨*category TYPE Type*⟩

⟨*category URAGG UnaryRecursiveAggregate*⟩
⟨*category UFD UniqueFactorizationDomain*⟩
⟨*category ULSCAT UnivariateLaurentSeriesCategory*⟩
⟨*category ULSCCAT UnivariateLaurentSeriesConstructorCategory*⟩
⟨*category UPOLYC UnivariatePolynomialCategory*⟩
⟨*category UPSCAT UnivariatePowerSeriesCategory*⟩
⟨*category UPXSCAT UnivariatePuiseuxSeriesCategory*⟩
⟨*category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory*⟩
⟨*category OREPCAT UnivariateSkewPolynomialCategory*⟩
⟨*category UTSCAT UnivariateTaylorSeriesCategory*⟩

⟨*category VECTCAT VectorCategory*⟩
⟨*category VSPACE VectorSpace*⟩

⟨*category XALG XAlgebra*⟩
⟨*category XFALG XFreeAlgebra*⟩
⟨*category XPOLYC XPolynomialsCat*⟩

⟨*dotabb*⟩≡

```
digraph dotabb {
 ranksep=1.25;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

⟨CATEGORY.dotabb⟩
⟨A1AGG.dotabb⟩
⟨ABELGRP.dotabb⟩
⟨ABELMON.dotabb⟩
⟨ABELSG.dotabb⟩
⟨ACF.dotabb⟩
⟨ACFS.dotabb⟩
⟨AGG.dotabb⟩
⟨AHYP.dotabb⟩
⟨ALAGG.dotabb⟩
⟨ALGEBRA.dotabb⟩
⟨AMR.dotabb⟩
⟨ARR2CAT.dotabb⟩
⟨ATRIG.dotabb⟩
⟨ATTREG.dotabb⟩
⟨BASTYPE.dotabb⟩
⟨BGAGG.dotabb⟩
⟨BLMETCT.dotabb⟩
⟨BMODULE.dotabb⟩
⟨BRAGG.dotabb⟩
⟨BTAGG.dotabb⟩
⟨BTCAT.dotabb⟩
⟨CABMON.dotabb⟩
⟨CACHSET.dotabb⟩
⟨CFCAT.dotabb⟩
⟨CHARNZ.dotabb⟩
⟨CHARZ.dotabb⟩
⟨CLAGG.dotabb⟩
⟨COMBOPC.dotabb⟩
⟨COMPCAT.dotabb⟩
⟨COMRING.dotabb⟩
⟨DIAGG.dotabb⟩
⟨DIFEXT.dotabb⟩
⟨DIFRING.dotabb⟩
⟨DIOPS.dotabb⟩
⟨DIRPCAT.dotabb⟩
⟨DIVCAT.dotabb⟩
⟨DIVRING.dotabb⟩
⟨DLAGG.dotabb⟩
⟨DPOLCAT.dotabb⟩
```

⟨*DQAGG.dotabb*⟩
⟨*DSTRCAT.dotabb*⟩
⟨*DVARCAT.dotabb*⟩
⟨*ELAGG.dotabb*⟩
⟨*ELEMFUN.dotabb*⟩
⟨*ELTAB.dotabb*⟩
⟨*ELTAGG.dotabb*⟩
⟨*ENTIRER.dotabb*⟩
⟨*ES.dotabb*⟩
⟨*EUCDOM.dotabb*⟩
⟨*EVALAB.dotabb*⟩
⟨*FAMONC.dotabb*⟩
⟨*FAMR.dotabb*⟩
⟨*FAXF.dotabb*⟩
⟨*FDIVCAT.dotabb*⟩
⟨*FEVALAB.dotabb*⟩
⟨*FFCAT.dotabb*⟩
⟨*FFIELDC.dotabb*⟩
⟨*FIELD.dotabb*⟩
⟨*FILECAT.dotabb*⟩
⟨*FINAALG.dotabb*⟩
⟨*FINITE.dotabb*⟩
⟨*FINRALG.dotabb*⟩
⟨*FLAGG.dotabb*⟩
⟨*FLALG.dotabb*⟩
⟨*FLINEXP.dotabb*⟩
⟨*FMC.dotabb*⟩
⟨*FMCAT.dotabb*⟩
⟨*FMFUN.dotabb*⟩
⟨*FMTC.dotabb*⟩
⟨*FNCAT.dotabb*⟩
⟨*FORTCAT.dotabb*⟩
⟨*FORTFN.dotabb*⟩
⟨*FPATMAB.dotabb*⟩
⟨*FPC.dotabb*⟩
⟨*FPS.dotabb*⟩
⟨*FRAMALG.dotabb*⟩
⟨*FRETRCT.dotabb*⟩
⟨*FRNAALG.dotabb*⟩
⟨*FS.dotabb*⟩
⟨*FSAGG.dotabb*⟩
⟨*FVC.dotabb*⟩
⟨*FVFUN.dotabb*⟩
⟨*GCDDOM.dotabb*⟩
⟨*GRALG.dotabb*⟩
⟨*GRMOD.dotabb*⟩

$\langle GROUP.dotabb \rangle$
$\langle HOAGG.dotabb \rangle$
$\langle HYPCAT.dotabb \rangle$
$\langle IDPC.dotabb \rangle$
$\langle INFCLCT.dotabb \rangle$
$\langle IEVALAB.dotabb \rangle$
$\langle INS.dotabb \rangle$
$\langle INTCAT.dotabb \rangle$
$\langle INTDOM.dotabb \rangle$
$\langle IXAGG.dotabb \rangle$
$\langle KDAGG.dotabb \rangle$
$\langle KOERCE.dotabb \rangle$
$\langle KONVERT.dotabb \rangle$
$\langle LALG.dotabb \rangle$
$\langle LFCAT.dotabb \rangle$
$\langle LIECAT.dotabb \rangle$
$\langle LINEXP.dotabb \rangle$
$\langle LMODULE.dotabb \rangle$
$\langle LNAGG.dotabb \rangle$
$\langle LODOCAT.dotabb \rangle$
$\langle LOGIC.dotabb \rangle$
$\langle LSAGG.dotabb \rangle$
$\langle LZSTAGG.dotabb \rangle$
$\langle MATCAT.dotabb \rangle$
$\langle MDAGG.dotabb \rangle$
$\langle MLO.dotabb \rangle$
$\langle MODULE.dotabb \rangle$
$\langle MONAD.dotabb \rangle$
$\langle MONADWU.dotabb \rangle$
$\langle MONOGEN.dotabb \rangle$
$\langle MONOID.dotabb \rangle$
$\langle MSETAGG.dotabb \rangle$
$\langle MTSCAT.dotabb \rangle$
$\langle NAALG.dotabb \rangle$
$\langle NARNG.dotabb \rangle$
$\langle NASRING.dotabb \rangle$
$\langle NTSCAT.dotabb \rangle$
$\langle NUMINT.dotabb \rangle$
$\langle OAGROUP.dotabb \rangle$
$\langle OAMON.dotabb \rangle$
$\langle OAMONS.dotabb \rangle$
$\langle OASGP.dotabb \rangle$
$\langle OC.dotabb \rangle$
$\langle OCAMON.dotabb \rangle$
$\langle ODECAT.dotabb \rangle$
$\langle OINTDOM.dotabb \rangle$

⟨*OM.dotabb*⟩
⟨*OMSAGG.dotabb*⟩
⟨*OPTCAT.dotabb*⟩
⟨*ORDFIN.dotabb*⟩
⟨*ORDMON.dotabb*⟩
⟨*ORDRING.dotabb*⟩
⟨*ORDSET.dotabb*⟩
⟨*OREPCAT.dotabb*⟩
⟨*PACFFC.dotabb*⟩
⟨*PACPERC.dotabb*⟩
⟨*PACRATC.dotabb*⟩
⟨*PADICCT.dotabb*⟩
⟨*PATAB.dotabb*⟩
⟨*PATMAB.dotabb*⟩
⟨*PDECAT.dotabb*⟩
⟨*PDRING.dotabb*⟩
⟨*PERMCAT.dotabb*⟩
⟨*PFECAT.dotabb*⟩
⟨*PID.dotabb*⟩
⟨*PLACESC.dotabb*⟩
⟨*POLYCAT.dotabb*⟩
⟨*PPCURVE.dotabb*⟩
⟨*PRIMCAT.dotabb*⟩
⟨*PRQAGG.dotabb*⟩
⟨*PRSPCAT.dotabb*⟩
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⟨*PSCURVE.dotabb*⟩
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⟨*PTCAT.dotabb*⟩
⟨*PTRANFN.dotabb*⟩
⟨*QFCAT.dotabb*⟩
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⟨*RADCAT.dotabb*⟩
⟨*RCAGG.dotabb*⟩
⟨*RCFIELD.dotabb*⟩
⟨*REAL.dotabb*⟩
⟨*RETRACT.dotabb*⟩
⟨*RING.dotabb*⟩
⟨*RMATCAT.dotabb*⟩
⟨*RMODULE.dotabb*⟩
⟨*RNG.dotabb*⟩
⟨*RNS.dotabb*⟩
⟨*RPOLCAT.dotabb*⟩
⟨*RRCC.dotabb*⟩
⟨*RSETCAT.dotabb*⟩

```
⟨SEGCAT.dotabb⟩
⟨SEGXCAT.dotabb⟩
⟨SETAGG.dotabb⟩
⟨SETCAT.dotabb⟩
⟨SETCATD.dotabb⟩
⟨SEXCAT.dotabb⟩
⟨SFRTCAT.dotabb⟩
⟨SGROUP.dotabb⟩
⟨SKAGG.dotabb⟩
⟨SMATCAT.dotabb⟩
⟨SNTSCAT.dotabb⟩
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⟨SPFCAT.dotabb⟩
⟨SRAGG.dotabb⟩
⟨STAGG.dotabb⟩
⟨STEP.dotabb⟩
⟨STRICAT.dotabb⟩
⟨TBAGG.dotabb⟩
⟨TRANFUN.dotabb⟩
⟨TRIGCAT.dotabb⟩
⟨TSETCAT.dotabb⟩
⟨TYPE.dotabb⟩
⟨UFD.dotabb⟩
⟨ULSCAT.dotabb⟩
⟨ULSCCAT.dotabb⟩
⟨UPOLYC.dotabb⟩
⟨UPSCAT.dotabb⟩
⟨UPXSCAT.dotabb⟩
⟨UPXSCCA.dotabb⟩
⟨URAGG.dotabb⟩
⟨UTSCAT.dotabb⟩
⟨VECTCAT.dotabb⟩
⟨VSPACE.dotabb⟩
⟨XALG.dotabb⟩
⟨XF.dotabb⟩
⟨XFALG.dotabb⟩
⟨XPOLYC.dotabb⟩
}
```

⟨dotfull⟩≡

```
digraph dotfull {
 ranksep=1.25;
 nodesep=1.5;
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

⟨CATEGORY.dotfull⟩
⟨A1AGG.dotfull⟩
⟨ABELGRP.dotfull⟩
⟨ABELMON.dotfull⟩
⟨ABELSG.dotfull⟩
⟨ACF.dotfull⟩
⟨ACFS.dotfull⟩
⟨AGG.dotfull⟩
⟨AHYP.dotfull⟩
⟨ALAGG.dotfull⟩
⟨ALGEBRA.dotfull⟩
⟨AMR.dotfull⟩
⟨ARR2CAT.dotfull⟩
⟨ATRIG.dotfull⟩
⟨ATTREG.dotfull⟩
⟨BASTYPE.dotfull⟩
⟨BGAGG.dotfull⟩
⟨BLMETCT.dotfull⟩
⟨BMODULE.dotfull⟩
⟨BRAGG.dotfull⟩
⟨BTAGG.dotfull⟩
⟨BTCAT.dotfull⟩
⟨CABMON.dotfull⟩
⟨CACHSET.dotfull⟩
⟨CFCAT.dotfull⟩
⟨CHARNZ.dotfull⟩
⟨CHARZ.dotfull⟩
⟨CLAGG.dotfull⟩
⟨COMBOPC.dotfull⟩
⟨COMPCAT.dotfull⟩
⟨COMRING.dotfull⟩
⟨DIAGG.dotfull⟩
⟨DIFEXT.dotfull⟩
⟨DIFRING.dotfull⟩
⟨DIOPS.dotfull⟩
⟨DIRPCAT.dotfull⟩
⟨DIVCAT.dotfull⟩
⟨DIVRING.dotfull⟩
```

⟨*DLAGG.dotfull*⟩
⟨*DPOLCAT.dotfull*⟩
⟨*DQAGG.dotfull*⟩
⟨*DSTRCAT.dotfull*⟩
⟨*DVARCAT.dotfull*⟩
⟨*ELAGG.dotfull*⟩
⟨*ELEMFUN.dotfull*⟩
⟨*ELTAB.dotfull*⟩
⟨*ELTAGG.dotfull*⟩
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⟨*ES.dotfull*⟩
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⟨*FAMONC.dotfull*⟩
⟨*FAMR.dotfull*⟩
⟨*FAXF.dotfull*⟩
⟨*FDIVCAT.dotfull*⟩
⟨*FEVALAB.dotfull*⟩
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⟨*FILECAT.dotfull*⟩
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⟨*FINRALG.dotfull*⟩
⟨*FLAGG.dotfull*⟩
⟨*FLALG.dotfull*⟩
⟨*FLINEXP.dotfull*⟩
⟨*FMC.dotfull*⟩
⟨*FMCAT.dotfull*⟩
⟨*FMFUN.dotfull*⟩
⟨*FMTC.dotfull*⟩
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⟨*FS.dotfull*⟩
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⟨*FVFUN.dotfull*⟩
⟨*GCDDOM.dotfull*⟩

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⟨*GROUP.dotfull*⟩
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⟨*IDPC.dotfull*⟩
⟨*INFCLCT.dotfull*⟩
⟨*IEVALAB.dotfull*⟩
⟨*INS.dotfull*⟩
⟨*INTCAT.dotfull*⟩
⟨*INTDOM.dotfull*⟩
⟨*IXAGG.dotfull*⟩
⟨*KDAGG.dotfull*⟩
⟨*KOERCE.dotfull*⟩
⟨*KONVERT.dotfull*⟩
⟨*LALG.dotfull*⟩
⟨*LFCAT.dotfull*⟩
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⟨*LINEXP.dotfull*⟩
⟨*LMODULE.dotfull*⟩
⟨*LNAGG.dotfull*⟩
⟨*LODOCAT.dotfull*⟩
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⟨*LZSTAGG.dotfull*⟩
⟨*MATCAT.dotfull*⟩
⟨*MDAGG.dotfull*⟩
⟨*MLO.dotfull*⟩
⟨*MODULE.dotfull*⟩
⟨*MONAD.dotfull*⟩
⟨*MONADWU.dotfull*⟩
⟨*MONOGEN.dotfull*⟩
⟨*MONOID.dotfull*⟩
⟨*MSETAGG.dotfull*⟩
⟨*MTSCAT.dotfull*⟩
⟨*NAALG.dotfull*⟩
⟨*NARNG.dotfull*⟩
⟨*NASRING.dotfull*⟩
⟨*NTSCAT.dotfull*⟩
⟨*NUMINT.dotfull*⟩
⟨*OAGROUP.dotfull*⟩
⟨*OAMON.dotfull*⟩
⟨*OAMONS.dotfull*⟩
⟨*OASGP.dotfull*⟩
⟨*OC.dotfull*⟩
⟨*OCAMON.dotfull*⟩

⟨*ODECAT.dotfull*⟩
⟨*OINTDOM.dotfull*⟩
⟨*OM.dotfull*⟩
⟨*OMSAGG.dotfull*⟩
⟨*OPTCAT.dotfull*⟩
⟨*ORDFIN.dotfull*⟩
⟨*ORDMON.dotfull*⟩
⟨*ORDRING.dotfull*⟩
⟨*ORDSET.dotfull*⟩
⟨*OREPCAT.dotfull*⟩
⟨*PACFFC.dotfull*⟩
⟨*PACPERC.dotfull*⟩
⟨*PACRATC.dotfull*⟩
⟨*PADICCT.dotfull*⟩
⟨*PATAB.dotfull*⟩
⟨*PATMAB.dotfull*⟩
⟨*PDECAT.dotfull*⟩
⟨*PDRING.dotfull*⟩
⟨*PERMCAT.dotfull*⟩
⟨*PFECAT.dotfull*⟩
⟨*PID.dotfull*⟩
⟨*PLACESC.dotfull*⟩
⟨*POLYCAT.dotfull*⟩
⟨*PPCURVE.dotfull*⟩
⟨*PRIMCAT.dotfull*⟩
⟨*PRQAGG.dotfull*⟩
⟨*PRSPCAT.dotfull*⟩
⟨*PSCAT.dotfull*⟩
⟨*PSCURVE.dotfull*⟩
⟨*PSETCAT.dotfull*⟩
⟨*PTCAT.dotfull*⟩
⟨*PTRANFN.dotfull*⟩
⟨*QFCAT.dotfull*⟩
⟨*QUAGG.dotfull*⟩
⟨*QUATCAT.dotfull*⟩
⟨*RADCAT.dotfull*⟩
⟨*RCAGG.dotfull*⟩
⟨*RCFIELD.dotfull*⟩
⟨*REAL.dotfull*⟩
⟨*RETRACT.dotfull*⟩
⟨*RING.dotfull*⟩
⟨*RMATCAT.dotfull*⟩
⟨*RMODULE.dotfull*⟩
⟨*RNG.dotfull*⟩
⟨*RNS.dotfull*⟩
⟨*RPOLCAT.dotfull*⟩

```
⟨RRCC.dotfull⟩
⟨RSETCAT.dotfull⟩
⟨SEGCAT.dotfull⟩
⟨SEGXCAT.dotfull⟩
⟨SETAGG.dotfull⟩
⟨SETCAT.dotfull⟩
⟨SETCATD.dotfull⟩
⟨SEXCAT.dotfull⟩
⟨SFRTCAT.dotfull⟩
⟨SGROUP.dotfull⟩
⟨SKAGG.dotfull⟩
⟨SMATCAT.dotfull⟩
⟨SNTSCAT.dotfull⟩
⟨SPACEC.dotfull⟩
⟨SPFCAT.dotfull⟩
⟨SRAGG.dotfull⟩
⟨STAGG.dotfull⟩
⟨STEP.dotfull⟩
⟨STRICAT.dotfull⟩
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⟨TRANFUN.dotfull⟩
⟨TRIGCAT.dotfull⟩
⟨TSETCAT.dotfull⟩
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⟨ULSCCAT.dotfull⟩
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⟨UPSCAT.dotfull⟩
⟨UPXSCAT.dotfull⟩
⟨UPXSCCA.dotfull⟩
⟨URAGG.dotfull⟩
⟨UTSCAT.dotfull⟩
⟨VECTCAT.dotfull⟩
⟨VSPACE.dotfull⟩
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⟨XPOLYC.dotfull⟩
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